Game Theory in Cooperative Communications

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Abstract

Cooperative communication has great potential to improve the wireless channel capacity by exploiting the antennas on wireless devices for spatial diversity. However, applications of cooperative communication are barely seen in reality. A main obstacle blocking its wide applications is the lack of incentives for wireless nodes to participate in cooperative communication. We first survey the existing game theoretic solutions for providing cooperation incentives in cooperative communications. We then discuss the challenges in applying game theory to cooperative communications.

Keywords:

I. Introduction

Cooperative communication has been proposed to improve the channel capacity in wireless networks. It takes advantage of the broadcast nature of wireless transmission and utilizes the antennas on wireless nodes to achieve spacial diversity. Depending on how the relay node processes the overheard signal, two primary cooperative communication modes are widely used: *Amplify-and-Forward* (AF) and *Decode-and-Forward* (DF). In the AF mode, the relay node amplifies the signal before forwarding it to the destination node. In the DF mode, the relay node decodes and encodes the signal before forwarding it to the destination node. Cooperative communication has potential applications in many different networks, including cellular networks, ad-hoc networks, and cognitive networks, as shown in Fig. 1.

To achieve cooperative communication, cooperation from other nodes is required. In network applications for military actions and disaster relief, cooperation among nodes can be assumed since the nodes belong to a single authority and thus voluntarily cooperate to achieve a common goal. However, in commercial applications, where nodes usually belong to different independent entities, there is no good reason to assume that the nodes will cooperate. In fact, nodes are selfish and consume their resources only when doing so can maximize their own benefits. Game theory

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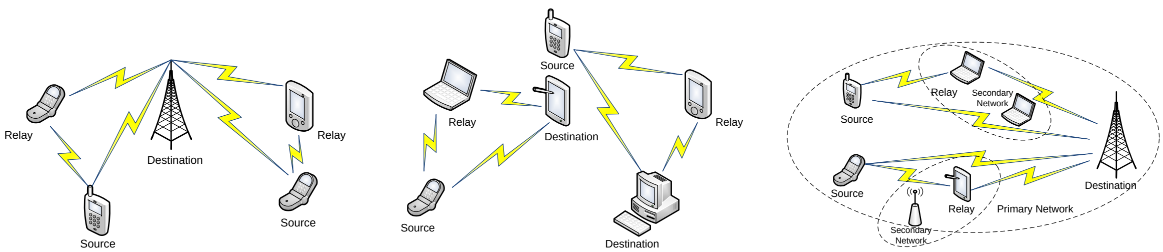
is naturally the most appropriate tool to model, analyze and solve the problems in cooperative communications.



(a) Cellular network (b) Ad-hoc Network (c) Cognitive Radio Network

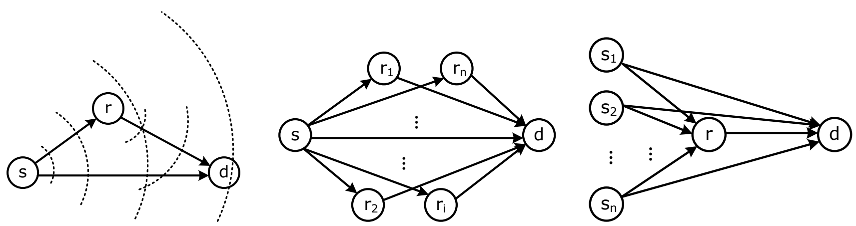
Fig. 1. Networks where cooperation communication can be applied

Game theory is the study that analyzes the strategic interactions among autonomous decision makers, whose actions have mutual, probably conflicting, consequences. Originally developed to model problems in the field of economics, game theory has recently been applied to network problems, in most cases to solve the resource allocation problems in a competitive environment. The reason that game theory is an appropriate choice for studying cooperative communications is multifold. First, nodes in the network are autonomous agents, making decisions only for their own interests. Game theory provides us sufficient theoretical tools to analyze the network users' behaviors and actions. Second, game theory primarily deals with distributed optimization, which often requires local information only. Thus it enables us to design distributed algorithms. Finally, auction, a market game of incomplete information, allows us to design mechanisms where relay nodes can sell their resources to the source nodes for cooperative communications. Such approach is desirable when neither the source node nor the relay node knows each other's private valuation on the resources to trade.

In the following sections, we first introduce the basic concepts of cooperative communication and game theory. We then survey the existing game theoretic solutions to cooperative communication problems, with the focus on cooperation incentive provisioning. Finally, we discuss the research challenges in applying game theory to the problems in cooperative communications.

II. Cooperative Communication

Depending on the availability of nodes and the cooperative communication protocols, there are three different communication topologies: one-to-one, one-to-many, and many-to-one, as shown in Fig. 2. Take the simplest topology, one-to-one, as an example to illustrate the basic idea of cooperative communication. In this example, s is the source node that transmits information, d is the destination node that receives information, and r is the relay node that relays information to enhance the communication between the source and the destination. Let P_s and P_r denote the transmission power of s and r, respectively. Let W denote the bandwidth of the transmission channel. Assume that transmission proceeds in a frame-by-frame fashion, as shown in Fig. 3. Each frame is divided into two phases. In the first phase, the source node s transmits its data to the destination node d. Due to the broadcast nature of wireless transmission, the relay node r can overhear the data. In the second phase, the relay node r forwards the data to the destination d after processing, depending on the underlying cooperative communication mode.



(a) One-to-one (b) One-to-many (c) Many-to-one

Fig. 2. Three cooperative communication topologies

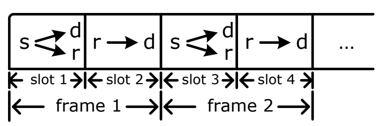


Fig. 3. Illustration of cooperative communication

We next compute the achievable capacity under cooperative communications [1]. When node u transmits a signal to node v with power P_u, the signal-to-noise ratio (SNR) at node v, denoted by SNR_{uv}, is

$$SNR_{uv} = \frac{P_u}{N_0 \cdot ||u,v||^{\alpha}},$$

where N_0 is the abient noise, $||u,v||$ is the Euclidean distance between nodes u and v, and α is the path loss exponent which is between 2 and 4 in general, depending on the characteristics of the communication medium.

Amplify-and-Forward (AF): In the amplify-and-forward mode, the relay node amplifies the signal transmitted by the source node in the first time slot and then transmits the amplified signal to the destination node in the second time slot. The achievable capacity from s to d is

$$C_{AF}(s,r,P_r,d) = \frac{W}{2}\log_2\left(1 + SNR_{sd} + \frac{SNR_{sr}\cdot SNR_{rd}}{SNR_{sr}+SNR_{rd}+1}\right).$$

Decode-and-Forward (DF): In the decode-and-forward mode, the relay node decodes and estimates the signal transmitted by the source node in the first time slot and then transmits the data to the destination node in the second time slot. The achievable capacity from s to d is

$$C_{DF}(s,r,P_r,d) = \frac{W}{2}\min\left\{\log_2(1+SNR_{sr}), \log_2(1+SNR_{sd}+SNR_{rd})\right\}.$$

III. Game Theory Basics in a Nutshell

Game theory [2] is a discipline aimed at modeling scenarios where individual decision-makers have to choose specific actions that have mutual or possibly conflict consequences. A game consists of three major components:

- **players**: The decision makers are called players, denoted by a finite set $\mathcal{N} = \{1, 2, \ldots, n\}$.
- **strategy**: Each player $i \in \mathcal{N}$ has a non-empty strategy set S_i. Let s_i denote the selected strategy by player i. A strategy profile s consists of all players' strategies, i.e., $s = (s_1, s_2, \ldots, s_n)$. Obviously, we have $s \in S = \times_{i\in\mathcal{N}} S_i$, where $\times$ is the Cartesian product.
- **utility/payoff**: The utility of player i is a measurement function, denoted by $u_i : S \mapsto \mathbb{R}$, on the possible outcome determined by the strategies of all players, where $\mathbb{R}$ is the set of real numbers.

The mapping from the components in a game to the elements in cooperative communications is shown in Table I.

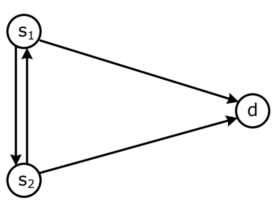
The players of the game are assumed to be rational and selfish, which means each player is only interested in maximizing its own utility without respecting others' and the system's performance. Let s_{-i} denote the strategy profile excluding s_i. As a notational convention, we have $s = (s_i, s_{-i})$. We say that player i prefers s_i to s'_i if $u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$. When other players' strategies are fixed, player i can select a strategy, denoted by $b_i(s_{-i})$, which maximizes its utility function. Such a strategy is called a *best response* of player i. A strategy is called a *dominant strategy* of player i if, regardless of what other players do, the strategy earns player i a larger utility

TABLE I

COMPONENTS OF GAMES IN COOPERATIVE COMMUNICATIONS

Components in the Game	Elements in Cooperative Communications
Players	Source nodes and/or relay nodes
Strategy	Power control [3–7]
	Spectrum allocation [8]
	Relay node(s) or source node(s) selection [9]
	To cooperate or not [10]
	Price [3, 9, 11, 12]
Utility/Payoff	Data rate [6, 10]
	Profit, e.g., revenue minus cost [3–5, 7–9, 11]

than any other strategy. In order to study the interactions among players, the concept of *Nash Equilibrium* (NE) is introduced. A strategy profile constitutes an NE if none of the players can improve its utility by unilaterally deviating from its current strategy. To characterize and quantify the inefficiency of the system performance due to the lack of cooperation among the players, we use the concept of *price of anarchy* (POA). The POA of the game is the ratio of the system performance in the worst NE to the system performance in the social optimal solution.



Player 1 \ Player 2	Cooperate	Not Cooperate
Cooperate	4, 4	2, 5
Not Cooperate	5, 2	3, 3

Fig. 4. Two-player cooperative communication

To illustrate the basic concepts of game theory, we use a simple two-player example. This game is essentially equivalent to the well-studied game, *Prisoners' Dilemma*. There are two players in this game, player 1 and player 2. Each player can choose from two strategies, Cooperate (C) and Not Cooperate (NC). If player 1 takes strategy C, it will act as a relay for player 2 for cooperative communication. Otherwise, player 1 only transmits its own data to the destination. Therefore there are totally four different strategy profiles. The utilities of different profiles are shown in the table in Fig. 4. It is straightforward to see that (NC, NC) is an NE with social performance 6. However, the social optima is the strategy profile (C, C), which gives the social performance 8. Thus the POA of this game is $\frac{3}{4}$.

Games can be classified into two categories, *strategic form game (or static game)* and *extensive form game (or dynamic game)*. The strategic form game is a one-shot game. In this game, the players make their decisions simultaneously without knowing what others will do. On the contrary, the extensive form game represents the structure of interactions between players and defines possible orders of moves. The *repeat game* is a class of the extensive form game, in which each stage is a repetition of the same strategic game. At the beginning of each stage, players observe the past history of strategies before making decisions. The number of stages may be finite or infinite. The utility of each player is the accumulated utility through all the stages. Therefore, players care not only the current utility but also the future utilities.

The *Stackelberg game* is an extensive form game, which is used to model the competition between one player, called the leader, and a set of players, called the followers. In this game, the leader takes action first and then the followers take actions. The leader knows ex ante that the followers observe its action and take actions accordingly. The NE in the Stackelberg game is called *Stackelberg Equilibrium*.

As game theory studies interactions between rational and intelligent players, it can be applied to the economic world where people interact with each other in the market. The marriage of game theory and economic models yields interesting games and fruitful theoretical results in microeconomics and auction theory. Auction is a decentralized market mechanism for allocating resources. The essence of auction is a game of incomplete information, where the players are the bidders, the strategies are the bids, and both allocations and payments are functions of the bids. In an auction mechanism, each bidder i has some private information t_i, called its *type*, and its strategy is the bid b_i. A mechanism then computes an *output* $o = o(b_1, b_2, \ldots, b_n)$ and a *payment* vector $p = (p_1, p_2, \ldots, p_n)$, where $p_i = p_i(b_1, b_2, \ldots, b_n)$ is the money given to the participating agent i. For each possible output o, bidder i's valuation is $v_i(t_i, o)$. The utility of bidder i is $u_i(t_i, o) = v_i(t_i, o) + p_i$.

Based on the number of objects auctioned on the market, auctions can be categorized into *single-object auction* and *multi-object auction*. Two basic single-object auction schemes are the *first-price auction* and the *second-price auction*. In the first-price auction, the auctioneer grants the item to the highest bidder and charges the highest bid. In the second-price auction, also known as Vickrey auction, the auctioneer grants the item to the highest bidder, but charges the second highest bid. Multi-object auction can be *homogeneous auction* or *heterogeneous auction*, depending on whether the objects are identical.

There are three desirable economic properties while designing an auction scheme:

- **Truthfulness**: An auction is truthful if revealing true private valuation is the dominant strategy for each bidder. In other words, no bidder can improve its utility by submitting a bid different from its true valuation, no matter how others submit.
- **Individual Rationality**: each agent participating in the auction can expect a non-negative profit.
- **System Efficiency**: An auction is system-efficient if the sum of valuations of all bidders is maximized.

IV. Cooperation Incentives

Cooperative communication has been proposed for years [1] and is gaining popularity since it has great potential to increase the capacity of wireless networks. Nevertheless, its applications are rarely seen in reality. A main obstacle blocking its wide applications is the lack of incentives for the wireless nodes to serve as relay nodes.

There are three primary mechanisms designed to provide such incentives: *reputation-based mechanism*, *resource-exchange-based mechanism*, and *pricing-based mechanism*. We will investigate these mechanisms in the following subsections.

A. Reputation-based Mechanism

In this mechanism, a centralized authority, e.g. base station, keeps records of the cooperative behavior and punish non-cooperating nodes. Consider a simple scenario, as shown in Fig. 4, where two nodes wish to transmit data to a common destination. Each node is a player and its strategy is whether to cooperate with the other node. Based on the utility table in Fig. 4, if player 1 chooses to cooperate, then player 2 will choose not to cooperate since player 2's utility is improved from 4 to 5. If player 1 chooses not to cooperate, player 2 will also choose not to cooperate since player 2's utility is improved from 2 to 3. Thus NC is player 2's dominant strategy. Similarly, NC is also player 1's dominant strategy. Therefore (NC, NC) is an NE of the static game. However, the (NC, NC) strategy profile is undesirable from the system perspective, as it does not efficiently utilize the system resource. Intuitively, even if a player is willing to cooperate, the other user's utility drives it to not to cooperate but rather to free-ride. In fact, a selfish player will always take advantage of a cooperating player and free-ride to maximize its own utility. In addition, such free-riding behavior has no consequence, e.g. punishment, in the static game.

For these reasons, a repeated game was modeled in [10]. In this game, free-riders from the previous stage will be punished and forced to reduce their transmission power, while players taking advantage of cooperative communications will be awarded to transmit with higher power. Since players need to care about their future utility, an NE in which players mutually cooperate can be achieved.

B. Resource-exchange-based Mechanism

In this mechanism, the source node takes other nodes as relays for cooperative communication. In return, the source node provides its own resource to help the relay nodes achieve certain objectives.

For the same network in Fig. 4, a different game was modeled in [4]. The strategy of each player is to determine the power for transmitting its own data and the power for relaying the other player's data. The utility of each player is defined as the difference between the achieved data rate and the energy cost. This game was modeled as a Stackelberg game by taking one of the two players as the leader and the other as the follower. It was shown that there are more benefits when cooperation is done between node pairs who are closer to each other.

In cognitive radio networks as shown in Fig. 1, the primary user (PU) can involve secondary users (SUs) as the cooperative relays. In return, the SUs obtain the opportunity to access the wireless channel for their own transmissions. In [6], the authors formulated the problem as a Stackelberg game, where the PU is the leader and the SUs are the followers. The strategy of the PU is to decide the portion of time it allocates to SUs and select a set of SUs as relays, based on the cooperative transmission power from the SUs. The utility of the PU is the achieved data rate with SUs' help. The strategy of the SU is the cooperative transmission power dedicated to the PU, since the channel access time of each SU is proportional to the contribution it makes in the cooperative communication. The utility of each SU is defined as the difference between its achieved data rate and the energy cost for helping with PU's transmission. The authors proved the existence of a Stackelberg Equilibrium and obtained the corresponding strategies.

C. *Pricing-based Mechanism*

In this mechanism, virtual currency or tokens are assumed in the network. Relay nodes sell their resources, e.g., power, bandwidth and time, for a certain price. Source nodes make payment to relay nodes for using their resources. Depending on the relationship between demand and supply, the game can be formulated as a buyer's market or a seller's market.

When there is one source node and multiple relay nodes, the game is formulated as a buyer's market [7]. The source node selects a subset of relay nodes for relaying based on the channel condition between itself and each relay node and the price asked by the relay node. Each relay node determines its price according to the conditions of the channel between itself and source node, and the channel between itself and the destination node, as well as the other relay nodes' prices. Since the game is a buyer's market, it is essentially a Stackelberg game with the source node as the leader and the relay nodes as the followers.

When there is one relay node and multiple source nodes, the market becomes a seller's market [8]. The authors assumed that only the source nodes are players and the relay node has a fixed cost function, which is known to all the source nodes. The strategy of each source node is the bandwidth it wants to buy. A distributed algorithm was developed to search the NE. In [3], the source nodes are bidders and submit bids to the relay node. The relay node allocates its transmission power proportional to the source nodes' bids. Two different payments were defined, of which one is a function of the extra SNR due to the cooperative communication and the other is a function of the power allocated to the source node. It was proved that the NE exists and is unique. The distributed best response bid updates converge globally to the unique NE in a completely asynchronous manner.

The above works only consider the selfish behavior of players, but not the *cheating* behavior. It has been shown both theoretically and practically that a market could be vulnerable to market manipulation and produce very poor outcomes if players are dishonest on their prices. Therefore truthfulness is the most critical property of the mechanism design. In [11], the authors designed an auction scheme for cooperative communications, which satisfies not only truthfulness, but also individual rationality and budget balanced properties. In this auction, source nodes are buyers, relay nodes are sellers, and the base station is the auctioneer. Buyers bid for relaying service for cooperative communications, while sellers offer cooperative service at the cost of their resources, e.g. energy, and receive monetary payment in return. Each buyer has different valuations of the relay nodes as it can achieve different capacities by cooperating with different relay nodes.

V. Challenges in Applying Game Theory

While game theory has been extensively applied to model the problems in cooperative communications, there are still many challenging research issues unsolved. We list some of them below to inspire interested readers on future research directions.

- **Selection of the utility function**: utility function is undoubtedly a very important component in the game. It should precisely reflect the true valuation of the player on the outcome of the game. In the games modeling cooperative communications, data rate and profit are widely used as utility functions. In the design of pricing-based mechanism, most works choose the transmission power as the cost of cooperation, which appears in the utility function as a linear term. However, in reality, the cost of transmission power may depend on the specific device and the remaining power level, and thus is probably not linear in the transmission power.
- **Existence and uniqueness of NE**: The existence of NEs in a game is always one of the properties investigated by the researchers. The reason is that an NE is a solution concept that describes a steady state condition of the game. If the existence of NE is not guaranteed, it is possible that players oscillate their strategies to improve their utilities, generating a significant amount of communication overhead and wasting computing resources. Besides the existence, the uniqueness of NE is another desirable property, which has been largely neglected by the existing works. If there is only one NE, players will not be confused while selecting their NE strategies. In addition, we can predict the NE of the game and the resulting performance.
- **Computation of NE**: Once the existence of NE is proved, the next question would be how to compute an NE. Computationally heavy algorithms for computing an NE are not desirable in networks, like cellular networks and mobile networks, where devices are powered by batteries. In such networks, computing power is a valuable resource. Therefore designing efficient algorithms for NE computation is necessary.

- **Efficiency of NE**: It is known that NE is usually an inefficient solution from the system's perspective. This inefficiency is captured by the concept of price of anarchy. Pricing has been adopted for steering players to converge to an equilibrium with better system performance. For example, in [9], the authors designed a payment scheme which induces the source nodes to select the relay nodes resulting in an optimal relay node assignment. However, the designed payment scheme is based on the condition that the optimal solution can be obtained. This condition does not hold in general. Therefore designing a pricing-based mechanism to influence players to converge to an efficient NE is still a challenging task for cooperative communications.
- **Mechanism design**: Most of the mechanism designs in the literature only consider single-side auction, where source nodes are buyers. The only work that studies the double auction for cooperative communications is presented in [11]. The authors showed that it is desirable to design an auction satisfying truthfulness, individual rationality, budget balance, and system efficiency properties. Unfortunately, the *impossibility theorem* [13] shows that no double auction can simultaneously achieve all four economic properties. Thus they designed an auction scheme, which satisfies the first three properties while ignoring the last. It is still open and very challenging to design a double auction scheme, which satisfies the first three properties while approximately maximizing the system efficiency.

VI. Conclusion

In this article, we have briefly surveyed the game theoretic solutions to the problems in cooperative communications, with the focus on designing cooperation incentive mechanisms. While game theory has been extensively applied to cooperative communications, there are still many challenges that demand extra effort from researchers.

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