Change detection for multi-polarization, multi-pass SAR

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ABSTRACT

This paper develops a unified mathematical theory of change detection for the multi-polarization, multi-pass synthetic aperture radar (SAR) change detection problem. A generalized likelihood ratio test (glrt) for optimum processing of the measured data is developed; an evaluation of the performance of the optimum glrt is also given in the form of ROC curves (PD versus PFA), quantifying the benefits of multi-polarization, multi-pass SAR change detection.

Keywords: synthetic aperture radar, polarimetric SAR, change detection, GLRT, PWF

1. INTRODUCTION

This paper presents a unified mathematical theory of change detection (CD) for multi-polarization, multi-pass Synthetic Aperture Radar (SAR). The fundamental problem addressed is that of detecting targets in a multi-polarization SAR image by comparing a recently gathered image of a region of interest with a prior image (or multiple prior images) of the same region of interest. Specifically, for the two-pass change detection problem we assume that a single reference image of the region (denoted as the "day-1" image) has previously been gathered by the SAR and has been stored in memory. At some later time, a test image (the "mission image") of the region of interest (denoted as the "day-2" image) is gathered by the SAR and a comparison of the day-1 and day-2 images is performed in order to locate targets that have either (1) entered the region (denoted "arrivals") or (2) departed the region or possibly moved to another location within the region of interest (denoted "departures"). For the more general multi-pass change detection problem we assume there are two or more reference images that have been gathered (day-1, day-2,…., day-(n-1) images) and it is desired to compare the test image gathered on "day-n" with a reference image formed from all these previously gathered reference images. Finally, for the most general problem considered we assume that several independent test images of the region of interest have been gathered and it is desired to use all the available reference and test images to perform SAR change detection.

2. MATHEMATICAL MODEL OF THE SAR MEASUREMENT DATA

A fully-polarimetric Synthetic Aperture Radar (SAR) measures for each pixel in the SAR image three complex polarization returns: HH, HV, and VV; these returns are represented by a three dimensional complex vector, $\underline{X}$. We will assume the measurement data vector, $\underline{X} = \begin{pmatrix} HH \\ HV \\ VV \end{pmatrix}$, is a complex Gaussian vector of dimension "p=3" characterized by the PDF:

$$f(\underline{X})=\frac{1}{\pi^{p}\left|C\right|}\exp\{-\underline{X}^{\dagger}C^{-1}\underline{X}\}=\frac{1}{\pi^{p}\left|C\right|}\exp\{-tr\left[C^{-1}\underline{X}\underline{X}^{\dagger}\right]\} \qquad (1)$$

where $\left|C\right|$ is the determinant of the positive-definite covariance matrix $C=E\left(\underline{X}\underline{X}^{\dagger}\right)$ and $\underline{X}^{\dagger}$ is the complex conjugate transpose of the complex vector $\underline{X}$.

We also assume there are "N" independent data vectors in the measurement set, thus the joint PDF of the set $\underline{X}_1$, $\underline{X}_2$, …..,$\underline{X}_N$ can be written:

$$f(\underline{X}_1,\underline{X}_2,...,\underline{X}_N)=\frac{\exp\{-tr\left[C^{-1}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}}{\pi^{Np}\left|C\right|^{N}} \qquad (2)$$

where C = C_1 for the "day-1" data set, and C = C_2 for the "day-2" data set.

There are "N" independent data vectors in each of the day-1 and day-2 measurement sets; under the Null Hypothesis, "H_0", C = C_1 = C_2 and all 2N data vectors are characterized by the joint PDF:

$$f(\underline{X}_1,...,\underline{X}_N,\underline{X}_{N+1},...,\underline{X}_{2N})=\frac{\exp\{-tr\left[C_1^{-1}\sum_{i=1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}}{\pi^{2Np}\left|C_1\right|^{2N}} \qquad (3)$$

Under the alternative Hypothesis "H_1", $C_2 \neq C_1$ and the joint PDF of the 2N data vectors is:

$$f(\underline{X}_1,...,\underline{X}_N)f(\underline{X}_{N+1},...,\underline{X}_{2N})=\frac{\exp\{-tr\left[C_1^{-1}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}}{\pi^{Np}\left|C_1\right|^{N}}\frac{\exp\{-tr\left[C_2^{-1}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}}{\pi^{Np}\left|C_2\right|^{N}} \qquad (4)$$

3. DERIVATION OF THE GENERALIZED LIKELIHOOD RATIO TEST (GLRT)

In order to derive the "generalized" likelihood ratio test (glrt), we start with the following expression for the likelihood ratio test:

$$lrt=\frac{f(\underline{X}_1,...,\underline{X}_N,\underline{X}_{N+1},...,\underline{X}_{2N}\left|H_0\right.)}{f(\underline{X}_1,...,\underline{X}_N\left|H_0\right.)f(\underline{X}_{N+1},...,\underline{X}_{2N}\left|H_1\right.)} \qquad (5)$$

Substituting the appropriate density functions from Equations 3 and 4 into Equation 5 gives following expression:

$$lrt = \frac{\left(\pi^{-2Np} \left| C_1 \right|^{-2N} \exp\{-tr\left[C_1^{-1} \sum_{i=1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right]\} \right)}{\left(\pi^{-Np} \left| C_1 \right|^{-N} \exp\{-tr\left[C_1^{-1} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} \right]\} \right)\left(\pi^{-Np} \left| C_2 \right|^{-N} \exp\{-tr\left[C_2^{-1} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right]\} \right)} \quad (6)$$

To obtain the glrt we substitute maximum likelihood estimates of the unknown covariance matrices directly into the above expression, and in the above expression the numerator and denominator are treated separately. The MLE estimates we substitute into the above lrt are given below.

Under Hypothesis "H_0", the MLE estimate of the unknown covariance C_1 is:

$$\hat{C}_1 = \frac{1}{2N} \sum_{i=1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} = \frac{1}{2}\left[\frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} + \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right] \quad (7)$$

We substitute this estimated covariance matrix into the numerator of Equation 6.

Under Hypothesis "H_1", the MLE estimates of the unknown covariance matrices C_1 and C_2 are:

$$\hat{C}_1 = \frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} \quad (8)$$

$$\hat{C}_2 = \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \quad (9)$$

We substitute these estimated covariance matrices into the denominator of Equation 6.

After performing these substitutions we obtain the generalized likelihood ratio test:

$$glrt = \frac{\left(\left| \frac{1}{2N} \sum_{i=1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right|^{-2N} e^{-2Np} \right)}{\left(\left| \frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} \right|^{-N} e^{-Np} \right)\left(\left| \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right|^{-N} e^{-Np} \right)} \quad (10)$$

After canceling numerator and denominator scale factors, Equation 10 becomes:

$$glrt = \frac{\left| \frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} \right|^{N} \left| \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right|^{N}}{\left| \frac{1}{2}\left(\frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} + \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right) \right|^{2N}} \quad (11)$$

The above glrt provides a useful detection test for determining whether a change has occurred (or has not occurred) between the day-1 and day-2 SAR images. Note that when the sample covariance matrices of the day-1 and day-2 data are equal, i.e., when $\frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} = \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger}$, the glrt = 1; and when these sample covariance matrices are not equal, implying that a change has occurred, then $0 \le glrt \le 1$.

In implementing our polarimetric change detection algorithm we would like to have the algorithm to provide an output of "zero" for "no change"; and we would like the algorithm to provide a positive output when a target "arrival" has occurred in the day-2 image; and we also would like to have the algorithm provide a negative output when a target "departure" has occurred in the day-2 image. To decide whether a detected change is due to an arrival or a departure, we compare the relative sizes of the day-1 and day-2 image data; we do this by comparing the numerator determinants according to the rule:

If $\left|\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right| > \left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|$ the change is an "arrival", and

if $\left|\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right| < \left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|$ the change is a "departure".

Before proceeding with an evaluation of the performance characteristics (ROC's) for this change detection algorithm we will show how to modify the algorithm to provide (1) an output near "zero" when no change has occurred and (2) an output near "one" when a change has occurred. To accomplish this we first take the Nth root of the glrt:

$$glrt^{\frac{1}{N}} = \frac{\left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|\left|\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right|}{\left|\frac{1}{2}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^2} \tag{12}$$

This operation preserves the monotonic property of the algorithm, i.e., provides an output in the range (0,1) where for no change the output is near "one". (See Appendix A for a proof).

We take as the final form of the change detection metric the following algorithm:

$$y = 1 - glrt^{\frac{1}{N}} = 1 - \frac{\left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|\left|\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right|}{\left|\frac{1}{2}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^2} \tag{13}$$

4. MULTI-PASS CHANGE DETECTION: TWO (OR MORE) REFERENCE IMAGES

In a typical SAR surveillance scenario there may exist several (two or more) reference images of a region of interest which were gathered a priori from previous SAR passes over the region. We assume these SAR images have been gathered at the same (or nearly the same) aspect and depression angles. We also conjecture that these reference images may be combined to form a more accurate reference for detecting changes in a recently gathered mission image. We consider the simplest case of two available reference images and a single mission image; the algorithm for combining several available reference images and a single mission image to achieve optimum change detection performance is obtained as an obvious extension of this simpler case.

There are three sets of SAR image data, two reference images and a mission image; these data sets each consist of "N" data vectors which we denote as follows:

(i) day-1 data vectors $\underline{X}_1, \underline{X}_2, ..., \underline{X}_N \sim N(\underline{0}, C_1)$

(ii) day-2 data vectors $\underline{X}_{N+1}, \underline{X}_{N+2}, ..., \underline{X}_{2N} \sim N(\underline{0}, C_1)$

(iii) day-3 data vectors $\underline{X}_{2N+1}, \underline{X}_{2N+2}, ..., \underline{X}_{3N} \sim N(\underline{0}, C_3)$

Repeating the approach used in the previous lrt derivation given in Equation 6, we obtain the following result:

$$lrt = \frac{\left(\pi^{-3Np}\left|C_1\right|^{-3N}\exp\{-tr\left[C_1^{-1}\sum_{i=1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}\right)}{\left(\pi^{-2Np}\left|C_1\right|^{-2N}\exp\{-tr\left[C_1^{-1}\sum_{i=1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}\right)\left(\pi^{-Np}\left|C_3\right|^{-N}\exp\{-tr\left[C_3^{-1}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right]\}\right)} \quad (14)$$

Under Hypothesis "H_0", the MLE estimate of the unknown covariance C_1 is:

$$\hat{C}_1 = \frac{1}{3N}\sum_{i=1}^{3N}\underline{X}_i\underline{X}_i^{\dagger} = \frac{1}{3}\left[\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger} + \frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger} + \frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right] \quad (15)$$

We substitute this MLE estimate of the covariance matrix C_1 into the numerator of Equation 14.

Under Hypothesis "H_1", the MLE estimates of the unknown covariance matrices C_1 and C_3 are:

$$\hat{C}_1 = \frac{1}{2N}\sum_{i=1}^{2N}\underline{X}_i\underline{X}_i^{\dagger} \quad (16)$$

$$\hat{C}_3 = \frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger} \quad (17)$$

We substitute these MLE estimates of covariance matrices C_1 and C_3 into the denominator of Equation 14 and evaluate the $glrt^{\frac{1}{N}}$ as follows:

$$glrt^{\frac{1}{N}} = \frac{\left|\frac{1}{2}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger} + \frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^2\left|\frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right|}{\left|\frac{1}{3}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger} + \frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger} + \frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^3} \quad (18)$$

The above derivation is easily extended to the general case of M-passes (a single mission image and M-1 reference images).

5. MULTI-PASS CHANGE DETECTION: TWO (OR MORE) MISSION IMAGES

In this SAR surveillance scenario we assume there exists a single reference image of a region of interest gathered during a previous SAR imaging pass of the region; and we have recently gathered two or more mission images. We conjecture that the multiple mission images (i.e., multiple looks of the target area) may be combined to achieve improved change detection performance.

Consider the case of a single (day-1) reference and two mission images (denoted day-2 and day-3 images). As in the previous derivation, we have three sets of SAR image data, one reference image and two independent mission images; and each data set consists of "N" data vectors. There are two possible likelihood ratio tests possible for this scenario, depending on the assumptions made about the polarization covariance matrices for the mission images.

(i) Under Hypothesis "H_1" the mission covariance matrices are equal ($C_2 = C_3$), which implies very nearly identical SAR imaging geometries for the day-2 and day-3 passes.
(ii) Under Hypothesis "H_1" the mission covariance matrices are not equal ($C_2 \neq C_3$), which implies non-identical SAR imaging geometries for the day-2 and day-3 passes.

For case (i) above, the appropriate $glrt^{\frac{1}{N}}$ expression is evaluated to be:

$$glrt^{\frac{1}{N}} = \frac{\left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|\left|\frac{1}{2}\left(\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^2}{\left|\frac{1}{3}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^3} \tag{19}$$

For case (ii) above, the appropriate $glrt^{\frac{1}{N}}$ expression is evaluated to be:

$$glrt^{\frac{1}{N}} = \frac{\left|\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}\right|\left|\left(\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|\left|\frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right|}{\left|\frac{1}{3}\left(\frac{1}{N}\sum_{i=1}^{N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=N+1}^{2N}\underline{X}_i\underline{X}_i^{\dagger}+\frac{1}{N}\sum_{i=2N+1}^{3N}\underline{X}_i\underline{X}_i^{\dagger}\right)\right|^3} \tag{20}$$

The above derivation is easily extended to the general case of multiple mission images and multiple reference images.

6. CHANGE DETECTION FOR SINGLE-POLARIZATION SAR

In this section of the paper we consider the application of the previously derived multi-polarization, multi-channel change detection algorithm to a single-polarization SAR[1]. We show that the glrt algorithm reduces to a normalized non-coherent difference of the mission and reference images. This normalized difference algorithm is then shown to be equivalent to a simple non-coherent ratio CD test. The derivation proceeds as follows.

For a single-polarization SAR the change detection algorithm given in Equation 13 reduces to the following scalar version:

$$y = 1 - \frac{\left(\frac{1}{N}\sum_{i=1}^{N}|x_i|^2\right)\left(\frac{1}{N}\sum_{i=N+1}^{2N}|x_i|^2\right)}{\left(\frac{1}{2}\left(\frac{1}{N}\sum_{i=1}^{N}|x_i|^2+\frac{1}{N}\sum_{i=N+1}^{2N}|x_i|^2\right)\right)^2} \tag{21}$$

This expression is easily shown to simplify to the following:

$$y = \left(\frac{\sum_{i=N+1}^{2N} |x_i|^2 - \sum_{i=1}^{N} |x_i|^2}{\sum_{i=N+1}^{2N} |x_i|^2 + \sum_{i=1}^{N} |x_i|^2} \right)^2 \geq T_y^2 \qquad (22)$$

In Equation 22 we have specified a target detection threshold given by the parameter T_y^2. The assumption is that T_y^2 has been selected to achieve a desired CD false alarm probability, PFA.

Since we are interested in detecting the departures as well as the arrivals we have the additional arrival/departure test:

If $\sum_{i=N+1}^{2N} |x_i|^2 > \sum_{i=1}^{N} |x_i|^2$ the detected change is declared an "arrival";

if $\sum_{i=N+1}^{2N} |x_i|^2 < \sum_{i=1}^{N} |x_i|^2$ the detected change is declared a "departure".

For detecting arrivals in the day-2 SAR image we use the normalized difference CD test with a detection threshold, T_y:

$$\frac{\sum_{i=N+1}^{2N} |x_i|^2 - \sum_{i=1}^{N} |x_i|^2}{\sum_{i=N+1}^{2N} |x_i|^2 + \sum_{i=1}^{N} |x_i|^2} \geq T_y \qquad (23)$$

But the change detection test in Equation 23 is equivalent to the ratio test:

If $\frac{\sum_{i=N+1}^{2N} |x_i|^2}{\sum_{i=1}^{N} |x_i|^2} > T_z$, the detected change is declared an "arrival", where $T_z = \frac{1+T_y}{1-T_y}$.

For detecting departures in the day-2 SAR image we use the normalized difference CD test with threshold, T_y:

$$\frac{\sum_{i=1}^{N} |x_i|^2 - \sum_{i=N+1}^{2N} |x_i|^2}{\sum_{i=1}^{N} |x_i|^2 + \sum_{i=N+1}^{2N} |x_i|^2} > T_y \qquad (24)$$

And the change detection test in Equation 24 is equivalent to the ratio test:

If $\frac{\sum_{i=N+1}^{2N}|x_i|^2}{\sum_{i=1}^{N}|x_i|^2} < \frac{1}{T_z}$ then the detected change is declared a "departure".

These results are summarized as follows. To detect both arriving and departing targets, we use the simple ratio change detection algorithm:

$$\text{"Arrival threshold"}, T_z < \frac{\sum_{i=N+1}^{2N}|x_i|^2}{\sum_{i=1}^{N}|x_i|^2} < \frac{1}{T_z}, \text{"Departure threshold"} \quad (25)$$

7. CHANGE DETECTION ALGORITHM PERFORMANCE RESULTS

This section of the paper presents simulated change detection performance results. These performance results were obtained using a Monte Carlo simulation of the algorithms developed in the previous sections of this paper. In the simulation we used a 5 by 5 "box" of pixels (N=25) and we simulated sets of 25 independent, 3-dimensional complex Gaussian vectors representing the day-1, day-2, and day-3 data vectors. To evaluate PD versus detection threshold "T" we used the following covariance matrices:

$$C_1 = \begin{pmatrix} 1 & 0 & 0.5 \\ 0 & 0.2 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}, \quad C_2 = 2\begin{pmatrix} 1 & 0 & 0.5 \\ 0 & 0.2 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} \quad (26)$$

To evaluate PFA versus detection threshold "T" we used $C_2 = C_1$.

The detection performance curves (ROC curves) obtained from our Monte Carlo simulations are shown in Figure 1; the curves indicate the detection probability (PD) versus the corresponding false alarm probability (PFA) as the detection threshold "T" is varied. There are three curves shown in the figure and these correspond to the three algorithms described below.

(1) The curve denoted CD-2 in Figure 1 shows the simulated performance for the two-pass change detection algorithm given in Equation 13.

(2) The curve denoted CD-3 in Figure 1 shows the simulated performance for the multi-pass (three-pass) change detection algorithm given in Equation 18 (two reference images and a single mission image).

(3) The curve denoted PWF^2 in Figure 1 shows the corresponding single-pass detection performance obtained by PWF processing the fully-polarimetric data vectors into a SAR PWF intensity image and picking the maximum of the 5 by 5 "box" of PWF pixels; this amplitude statistic was compared to a detection threshold and the PD and PFA were calculated.

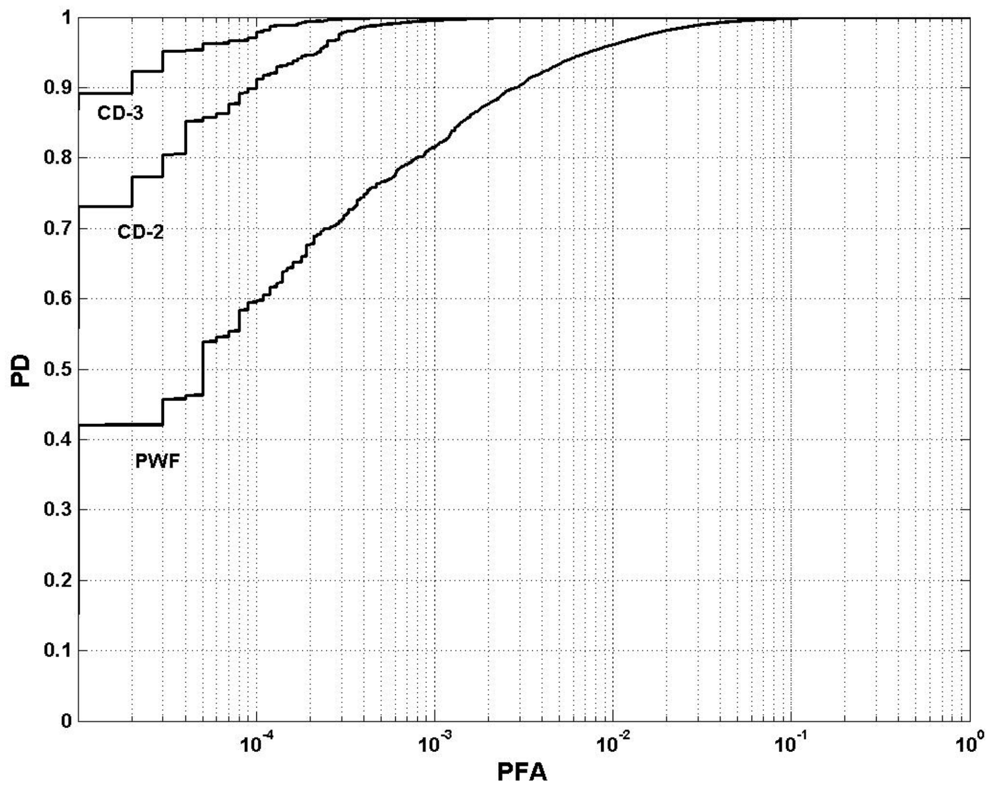

Figure 1: Change Detection Performance Comparison (Monte Carlo Simulation using computer generated Gaussian random vectors)

8. SUMMARY AND CONCLUSIONS

The simulated detection performance for several target detection approaches is shown in Figure 1. A single-pass detection result for a fully-polarimetric SAR that combines the three polarizations into a single PWF image is presented; this single-pass result (denoted PWF in the figure) provides a baseline performance against which we compare the two-pass (denoted CD-2) and multi-pass (denoted CD-3) change detection results.

The improved detection performance achieved using a change detection approach is clearly demonstrated in the curves of Figure 1. For example, at a specific detection probability of 0.9, the probability of false alarm achieved by the single-pass PWF detector is estimated from the figure to be approximately 30×10^{-4}. At the same detection probability (PD = 0.9) the two-pass change detection algorithm (Equation 13) achieved PFA $\approx 1 \times 10^{-4}$ – a reduction in the number of false alarms of approximately 30. The multi-pass result shown in the figure as CD-3 demonstrates additional performance improvement can be achieved by combining two reference images with a single mission image; at the same Pd = 0.9, the multi-pass change detection algorithm (Equation 18) achieved PFA $\approx 0.2 \times 10^{-4}$. Compared to the two-pass change detection performance, CD-3 reduced the number of false alarms by an additional factor of 5.

9. REFERENCES

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APPENDIX A: PROPERTIES OF THE GLRT

This appendix presents a mathematical proof that the generalize likelihood ratio test given in Equation 12 satisfies the property: $0 \le glrt^{\frac{1}{N}} \le 1$.

We consider the ratio of determinants $\dfrac{|C_1||C_2|}{\left|\frac{1}{2}(C_1+C_2)\right|^2}$ where C_1 and C_2 are the polarization covariance matrices of the day-1 and day-2 data vectors, respectively.
Assume these covariance matrices are of the form $C_1 = C_C$ and $C_2 = C_C + C_T$ where "C_C" corresponds to the day-1 "clutter" covariance, and "$C_C + C_T$" corresponds to the day-2 "clutter plus target" covariance. Thus, the model assumes that a target has entered the clutter area imaged previously on day-1, and so the target's presence has resulted in an additive target covariance matrix, C_T. The proof proceeds as follows:

Let the ratio be denoted "R" given below.

$$R = \frac{|C_C||C_C+C_T|}{\left|\frac{1}{2}(C_C+(C_C+C_T))\right|^2} = \frac{|C_C|\left|C_C\left(I+C_C^{-1}C_T\right)\right|}{\left|\frac{1}{2}(2C_C+C_T)\right|^2} = \frac{|C_C|^2\left|I+C_C^{-1}C_T\right|}{\left|C_C\left(I+\frac{1}{2}C_C^{-1}C_T\right)\right|^2} \qquad \text{(A-1)}$$

We rewrite the above ratio "R" as follows:

$$R = \frac{\left|I+C_C^{-1}C_T\right|}{\left|I+\frac{1}{2}C_C^{-1}C_T\right|^2}$$

, and when $C_T \to 0, \quad R \to 1$. We need to show that if $C_T \ne 0, \quad R > 0$.

To do this we make use of the well-known result from matrix theory. Since the covariance matrices C_C and C_T are assumed to be positive definite, the eigenvalues of the following equation are all positive (i.e., greater than zero).

$\left(C_C^{-1}C_T\right)Z_i = \lambda_i Z_i, \quad i = 1,2,3$ so we may diagonalize; assume an invertible transformation "M" exists so that $M^{-1}\left(C_C^{-1}C_T\right)M = \Lambda$, where $\Lambda = diag(\lambda_1, \lambda_2, \lambda_3)$. The ratio "R" is then written as follows:

$$R = \frac{\left|I+M\Lambda M^{-1}\right|}{\left|I+\frac{1}{2}M\Lambda M^{-1}\right|^2} = \frac{|I+\Lambda|}{\left|I+\frac{1}{2}\Lambda\right|^2} = \frac{(1+\lambda_1)}{\left(1+\frac{\lambda_1}{2}\right)^2}\frac{(1+\lambda_2)}{\left(1+\frac{\lambda_2}{2}\right)^2}\frac{(1+\lambda_3)}{\left(1+\frac{\lambda_3}{2}\right)^2} \qquad \text{(A-2)}$$

Since each of the factors in the above expression satisfy the inequality:

$\frac{(1+\lambda_i)}{\left(1+\frac{\lambda_i}{2}\right)^2} < 1, \quad i = 1,2,3$ the ratio $R < 1 \; for \; C_T \neq 0$. We can easily demonstrate that

the ratio R satisfies the inequality $0 \leq R < 1$ as the Figure A1 shows.

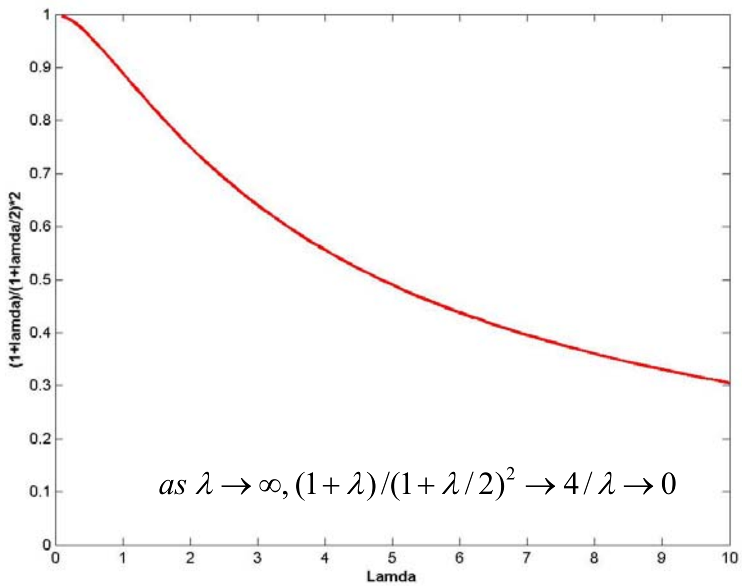

Figure A1: Plot of the factor $(1+\lambda)/(1+\lambda/2)^2$ versus λ

APPENDIX B: AN ALTERNATIVE CHANGE DETECTION ALGORITHM

This appendix presents a derivation of the likelihood ratio test (lrt) under the assumption that the covariance matrices C_1 and C_2 are known a priori. This derivation leads to the design of an alternative change detection algorithm to the glrt derived previously. We form the likelihood ratio test (lrt) from the known probability densities as follows:

$$lrt = \frac{f(\underline{X}_1,\dots,\underline{X}_N,\underline{X}_{N+1},\dots,\underline{X}_{2N} \,|H_0)}{f(\underline{X}_1,\dots,\underline{X}_N \,|H_0) f(\underline{X}_{N+1},\dots,\underline{X}_{2N} \,|H_1)} \qquad \text{(B-1)}$$

Omitting the details of the derivation, one may easily evaluate (using Equations 3 and 4 given above), the following expression for the lrt :

$$lrt = \frac{|C_2|^N}{|C_1|^N} \exp\{tr\left[(C_2^{-1} - C_1^{-1}) \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right]\} \qquad \text{(B-2)}$$

Note under the assumption that C_1 and C_2 are exactly known, Equation B-2 indicates that the likelihood ratio test uses only the measured data vectors from day-2 to detect the change. Next we write the lrt in terms of a log likelihood test, $\ln(lrt)$; this gives the expression:

$$\frac{1}{N}\ln(lrt) = tr\left[(C_2^{-1} - C_1^{-1}) \left(\frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \right) \right] + \ln\left|C_1^{-1} C_2\right| \qquad \text{(B-3)}$$

The above $\ln(lrt)$ test requires we know the covariance matrices C_1 and C_2, and since these covariance matrices in reality will not be known a priori, the test is not a useful one. In our change detection application we estimate the unknown covariance matrices C_1 and C_2 from the measured day-1 and day-2 data vectors. Substituting estimated covariance matrices for the unknown covariance matrices we obtain a simple, useful change detection algorithm. The estimated covariance matrices we use are the sample covariance matrices given by the following expressions:

$$\hat{C}_1 = \frac{1}{N} \sum_{i=1}^{N} \underline{X}_i \underline{X}_i^{\dagger} \text{ (day-1 data) and } \hat{C}_2 = \frac{1}{N} \sum_{i=N+1}^{2N} \underline{X}_i \underline{X}_i^{\dagger} \text{ (day-2 data).} \qquad \text{(B-4)}$$

This yields a simple test for the equality of two covariance matrices (i.e., for detecting a change in the covariance between the day-2 data and the day-1 data):

$$l = tr(\hat{C}_1^{-1}\hat{C}_2) - \ln\left|\hat{C}_1^{-1}\hat{C}_2\right| - p \qquad \text{(B-5)}$$

" p " is the dimension of the data vectors, $\underline{X}_i$. The above metric (referred to as the "trace metric") has two desirable properties which make the test a useful change detection metric:

$$(i) \quad l = 0 \; when \; \hat{C}_2 = \hat{C}_1$$
$$(ii) \quad l > 0 \; when \; \hat{C}_2 \neq \hat{C}_1$$

These change metric properties are easily verified. Property (i) is trivially verified by substituting $\hat{C}_2 = \hat{C}_1$ in Equation B-5. Property (ii) is verified using well-known eigenvalue relationships for positive-definite covariance matrices:

$$tr(\hat{C}_1^{-1}\hat{C}_2) = \sum_{i=1}^{p} \lambda_i \quad and \quad \ln\left|\hat{C}_1^{-1}\hat{C}_2\right| = \sum_{i=1}^{p} \ln \lambda_i \qquad \text{(B-6)}$$
$$where \quad \left(\hat{C}_1^{-1}\hat{C}_2\right)\underline{Z}_i = \lambda_i \underline{Z}_i, \quad i = 1,2,\ldots,p \qquad \text{(B-7)}$$

For the fully-polarimetric change detection problem, p =3, and the change metric becomes:

$$l = \sum_{i=1}^{3} [\lambda_i - \ln(\lambda_i)] - 3 \qquad \text{(B-8)}$$

The proof of property (ii) is presented graphically as follows. A plot of the factor "$\lambda - \ln(\lambda)$" versus "λ" shown in Figure B1 demonstrates that each term in the summation given in Equation B-8 is greater than or equal to the value +1, thus the metric $l > 0 \quad \forall\, \hat{C}_2 \neq \hat{C}_1$ and $l = 0 \quad iff\ \hat{C}_2 = \hat{C}_1$

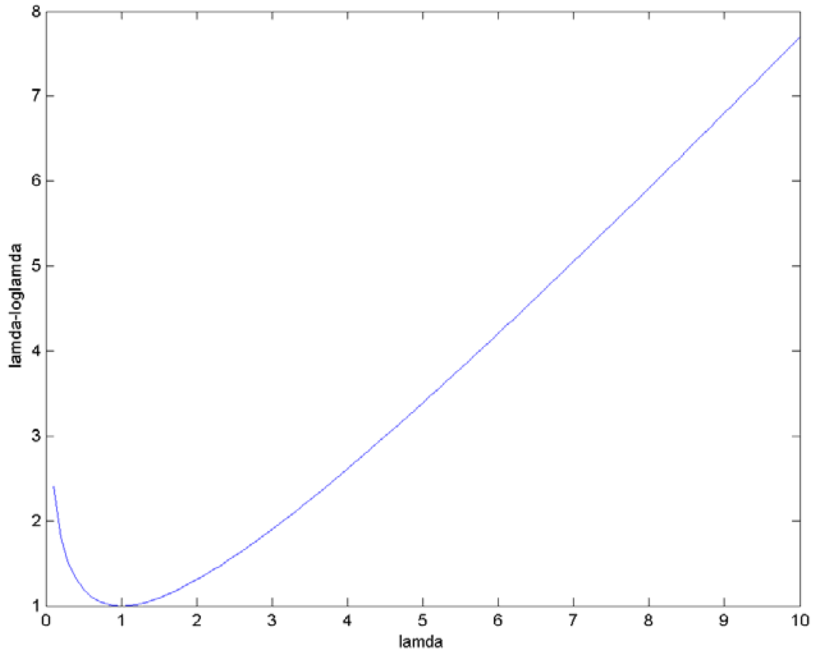

Figure B1: Plot of factor "$\lambda - \ln(\lambda)$" versus "λ"