A BRIEF HISTORY OF THE "SMARANDACHE FUNCTION"

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This function is originated from the exiled Romanian professor Florentin Smarandache. It is defined as follows:

For any non-null integers n, S(n) is the smallest integer such that (S(n))! is divisible by n.

The importance of the notion is that it characterizes a prime number, i.e.:

Let $p > 4$, then: p is prime if and only if $S(p) = p$.

Another properties:

If $(a,b) = 1$, then $S(ab) = \max\{ S(a), S(b) \}$;

and

For any non-null integers, $S(ab) \leq S(a) + S(b)$.

{All three found and proved by the author in 1979 (see [3], 15, 12-13, 65).}

If $n > 1$, then $S(n)$ and n have a proper common divisor.

{Found and proved by student Prodănescu in 1993: as a lemma needed to solve the conjecture formulated by the author in 1979 that:

the equation $S(n) = S(n + 1)$ has no solutions

(see [3], 37, and [30]).}

Etc.

Also, an infinity of open/unsolved problems, involving this function, provoked mathematicians around the world to study it and its applications (computational mathematics, simulation, quantum theory, etc.).

Thus, the unsolved question:

$$\text{Calculate } \lim_{n\to\infty} \left[1 + \sum_{k=2}^{n} \frac{1}{S(k)} - \log S(n) \right], \quad \text{(see [3], 29)}$$

made by the author in 1979, has been separately proved by J. Thompson from USA in 1992 (see [18], 1), by Nigel Backhouse from United Kingdom in 1993 (see [25]), and by Pål Grønås from Norway in 1993 (see [51]) that this limit is equal to ∞.

The author wonderred if it's posible to approach the function (see [3], 1979, 25-6), but Ian Parberry expressed that one can immediately find an algorithm that computes S(n) in $O(n\log n/\log\log n)$ time (see [38], 1993).

Some unsolved (by now!) other problems stated by the author in 1979 (see [3], 27-30):

a) To find a general form of the continued fraction expansion of $S(n)/n$, for all $n\geq 2$.

b) What is the smallest k such that for any integer n at least one of the numbers S(n), S(n+1), ..., S(n+k-1) is a perfect square?

c) To build the largest arithmetical progression $a_1, a_2, \ldots, a_r$ for which their images by the function are also an arithmetical progression.

Etc.

In 1975 Smarandache was a student at the University of Craiova, and he was attracted by the Number Theory. He created and published a lot of proposed problems of mathematics in various scientific journals. He liked to play with the numbers ...
Thus, in 1980 his research paper "A Function in the Number Theory", based on a special representation of integers, was published (for the first time) in <Analele Universităţii Timişoara>, Seria Ştiinţe Matematice, Vol. 18, pp. 79-88,
and was reviewed in <Zentralblatt fur Mathematik>, 471.10004, 1982, by P. Kiss, and in the <Mathematical Reviews>, 83c:10008, 1983, by R. Meyer.

In 1988 he escaped from the Ceauşescu's dictatorship, spent almost two years in a political refugee camp in Turkey (Istambul and Ankara), and finally emigrated to the United States.

Articles, notes, quickies, comments, proposals related to the Smarandache Function were presented to international conferences within the Mathematical Association of America or the American Mathematical Society at the New Mexico State University (Las Cruces), New Mexico Tech. (Socorro), University of Arizona (Tucson), University of San Antonio, University of Victoria (Canada) etc. or published in <Octogon> (Sacele), <Gazeta Matematică> (Bucharest), <The Mathematical Spectrum> (UK), <Elemente der Mathematik> (Switzerland), <The Fibonacci Quarterly> (USA) etc.

In 1992 Dr. J. R. Sutton from United Kingdom designed a BASIC PROCedure to calculate S(n) for all powers of a prime number up to a maximum. (see [26])

Jim Duncan from United Kingdom computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. (see [17])

Also, John McCarthy from United Kingdom estimated that his machine would take several years to just calculate and store S(n) to disk for the entire range of n it can handle (0<n<2^32), and using the compression detailed in ncld9207.c at least 12 Gigabytes of disk space would be needed. It took about 3 hours for his program to work out that 3,303,302 pages (!) would be needed to list the full range of n and S(n). (see [15])

In 1993 Henry Ibstedt from Sweden used a dtk-computer with 486/33MHz processor in Borland's Turbo Basic and calculated S(n) for n upto 10^6 which took 2 hours and 50 minutes! (see [52])

A group of professors (V. Seleacu, C. Dumitrescu, L. Tuţescu, I. Pătrascu, M. Mocanu) and scientific students from the University of Craiova, having a weekly meeting, are doing research on the function and its applicability.

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The Smarandache Function, together with a sample of The Infinity of Unsolved Problems associated with it, presented by Mike Mudge.[1]

The Smarandache Function, $S(n)$ (originated by Florentin Smarandache — *Smarandache Function Journal*, vol 1, no 1, December 1990. ISSN 1053-4792) is defined for all non-null integers, n, to be the smallest integer such that $(S(n))!$ is divisible by n.

Note N! denotes the factorial function, $N!=1\times2\times3\times\ldots\times N$: for all positive integer N. In addition $0! = 1$ by definition.

$S(n)$ is an even function. That is, $S(n) = S(-n)$ since if $(S(n))!$ is divisible by n it is also divisible by -n.

$S(p) = p$ when p is a prime number, since no factorial less than p! has a factor p in this case where p is prime.

The values of $S(n)$ in *Fig 1* are easily verified. For example, $S(14) = 7$ because 7 is the smallest number such that 7! is divisible by 14.

Problem (i) Design and implement an algorithm to generate and store/tabulate $S(n)$ as a function of n.

Hint It may be advantageous to consider the STANDARD FORM of n, *viz* $n = e p_1^{a_1} p_2^{a_2} \ldots\ldots p_r^{a_r}$ where $e = \pm 1$, $p_1, p_2, \ldots p_r$ denote the distinct prime factors of n and $a_1, a_2, \ldots a_r$ are their respective multiplicities.

Problem (ii) Investigate those sets of consecutive integers $(i, i+1, i+2, \ldots i+x)$ for which S generates a monotonic increasing (or indeed monotonic decreasing) sequence.

Note For (1,2,3,4,5) S generates the monotonic increasing sequence 0,2,3,4,5; here $i = 1$ & $x = 4$.

If possible estimate the largest value of x.

Problem (iii) Investigate the existence of integers m,n,p,q & k with $n \neq m$ and $p \neq q$ for which:

either **(A)**: $S(m) + S(m+1) + \ldots S(m+p) = S(n) + S(m+1) + \ldots S(n+q)$ or **(B)**:

$$\frac{S(m)^2 + S(m+1)^2 + \ldots S(m+p)^2}{S(n)^2 + S(n+1)^2 + \ldots S(n+p)^2} = k$$

Problem (iv) Find the smallest integer k for which it is true that <u>for all n less than some given n_0</u> at least one of:

$S(n), S(n+1), \ldots S(n+k-1)$ is:

A) a perfect square

B) a divisor of k^n

C) a factorial of a positive integer.

Conjecture what happens to k as n_0 tends to infinity: i.e. becomes larger and larger.

Problem (v) <u>Construct prime numbers of the form</u> $\overline{S(n)\,S(n+1)\ldots S(n+k)}$: where $\overline{abcdefg}$ denotes the integer formed by the concatenation of a,b,c,d,e,f & g. For example, trivially $\overline{S(2)\,S(3)} = 23$ which is prime, but no so trivially $\overline{S(14)S(15)S(16)S(17)} = 75617$, also prime!

Definition An A-SEQUENCE is an integer sequence $a_1, a_2, \ldots$ with $1 \le a_1 < a_2 < \ldots$ such that no a_i is the sum of distinct members of the sequence (other than a_i).

Problem (vi) Investigate the construction of A-SEQUENCES $a_1, a_2, \ldots$ such that the associated sequences $S(a_1), S(a_2), \ldots$ are also A-SEQUENCES.

Definition The k^{th} order forward finite differences of the Smarandache function are defined thus:

$D_s(x) = {}^{*}\text{modulus}(S(x+1) - S(x))$,

$D_s^{(k)}(x) = D(D(\ldots k\text{-times}\, D_s(x)\ldots))$

Problem (vii) Investigate the conjecture that $D_s^{(k)}(1) = 1$ or 0 for all k greater than or equal to 2.

c.f. Gilbreath's conjecture on prime numbers, discussed in 'Numbers Count' *PCW* Dec 1983. * Here modulus is taken to mean the absolute value of (ABS.), modulus $(y) = y$ if y is positive and modulus $(y) = -y$ if y is negative.

The following selection of Diophantine Equations (i.e. solutions are sought in integer values of x) are taken from the Smarandache Journal and make up:

Problem (viii) If m & n are given integers, solve each of:

a) $S(x) = S(x+1)$, conjectured to have no solution

b) $S(mx+n) = x$

c) $S(mx+n) = m+nx$

d) $S(mx+n) = x!$

e) $S(x^m) = x^n$

f) $S(x)^m = S(x^n)$

g) $S(x) + y = x + S(y)$, x&y not prime

h) $S(x) + S(y) = S(x+y)$

i) $S(x+y) = S(x)S(y)$

j) $S(xy) = S(x)S(y)$

Review, July 1992
The Smarandache Function: a first visit? [2]

This topic is certain to be revisited in the near future, and the lack of space available here will certainly be remedied on that occasion. Suffice it to report that Jim Duncan computed up to S(1499999), the first million taking 50 hours in Lattice C on an Atari 1040ST. In Problem (ii), no evidence for a largest value of x was found, while in Problem (vii) the conjecture was verified for the first 32,000 values of $S(n)$. The very worthy prizewinner is John McCarthy of 17 Mount Street, Mansfield, Notts NG19 7AT, who has extensively *investigated* the computation of $S(n)$ up to 2^{32}; arriving at conclusions such as: 'several years of computing', 'at least 12Gb of disk space' and '3,303,302 pages of output'. John's concluding comment, 'Am I mad?', is clearly answered NO! by examining his specimen pages of output including those relating to 10-digit values of n. Listings supplied. Details from John directly upon request.

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1 *Republished from <Personal Computer World>, No.112, 420, July 1992 (with the author permission), because some of the following research papers are referring to these open problems.*

2 *Republished from <Personal Computer World>, No.117, 412, December 1992 (with the author permission).*