Unit-rate and Low-complexity Space-time Block Coding for Multiple Antennas

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Abstract: Space-time block coding (STBC) has been an effective transmit diversity technique for combating fading due to its orthogonal design, simple decoding and high diversity gains. In this paper, a unit-rate and low-complexity STBC scheme for multiple antennas is proposed, and turbo coding is employed as channel coding to improve the proposed code scheme performance further. Compared with full-diversity multiple antennas STBC schemes, the proposed scheme can implement unit rate, complex orthogonality and partial diversity; and under the same system throughput, it has much smaller computational complexity. Moreover, using the proposed scheme can form efficient spatial interleaving, thus the performance loss due to partial diversity is effectively compensated by the concatenation of turbo coding. Simulation results show that on the condition of the same system throughput and concatenation of turbo coding, the proposed scheme has lower BER than those low-rate and full-diversity multiple antennas STBC schemes.

1. Introduction

Recently, transmit diversity has been studied extensively as a method of combating detrimental effects in wireless fading channels due to its relative simplicity of implement and feasibility of having multiple antennas at the BS [1]. A simple transmit diversity scheme using 2 antennas is proposed by Alamouti in [2]. The Alamouti transmit diversity approach was then generalized to an arbitrary number of antennas in [3], where it is shown that the Alamouti scheme is a special case of space-time block (STB) code. The STB code scheme can achieve full transmit diversity and has a simple maximum likelihood (ML) decoding algorithm when used at the decoder. For this reason, STB code is very attractive approach for practical purposes. But in [3], it is proved that for STB code, a complex orthogonal design which provide full diversity and unit rate is not possible for more than two antennas, and the 1/2-rate or 3/4-rate STB code for three and four transmit antennas (4Tx) are also given in [3] with the code-rate <1. And 2/3-rate STB code for five transmit antennas is proposed in [4] recently. Considering that the full data rate is the important means to implement high data rate service and very important for low signal to noise ratios [1]; moreover, low complexity decoding algorithm of STB code is necessary due to the restriction of receiver size and power; we present a complex orthogonal STB code scheme for 3Tx or 4Tx in terms of Alamouti code, it can provide unit rate and partial diversity, and it has low computational complexity; thus it does not affect the data transmit rate and implementation complexity. Compared with those full-diversity and low-rate STB codes, the proposed scheme has small performance loss due to partial diversity. But when concatenated with channel coding (especially turbo coding [5]), its performance will be improved significantly due to efficient spatial interleaving. As a result, it is superior to corresponding full-diversity STB code schemes. Moreover, under the same throughput, it can adopt low order modulation scheme due to unit rate. Besides, its design is based on simple Alamouti code. So its implementation complexity is much lower.

2. Unit-rate and low-complexity STB code scheme

2.1 Full-diversity space-time block codes review

In this subsection, we briefly review the STB code scheme in multiple antennas system. Let L, M and T be positive integers, a complex orthogonal STB code is defined by a $T\times M$ transmission matrix G, every entry of which is complex linear combination of the L input symbols $d_1, d_2,\ldots,d_L$ and their conjugates $d_1^*, d_2^*,\ldots,d_L^*$, and the columns of G are required to be mutually complex orthogonal, i.e., they satisfy that $\mathbf{d}_k^H\cdot\mathbf{d}_i = 0$ for $k\neq i$; where $\mathbf{d}_k$ and $\mathbf{d}_i$ are column vectors of G, superscripts '^{H}' and '*'denote Hermitian transpose and complex conjugation, respectively. M and T are the numbers of transmit antennas and time slots used to transmit L input symbols, respectively. Considering that L symbols are transmitted over T time slots, the rate of the code G is defined as $R_{STBC} = L/T$. In Ref.[2], the first full-diversity complex orthogonal STB code for two transmit antennas (i.e. G_2 code in [6]) is presented by Alamouti as $G_2=[d_1, d_2 ; -d_2^*, d_1^*]$. So corresponding code rate R_{STBC} =2/2=1, thus the G_2 code achieve unit rate. And in [3], the full-diversity complex orthogonal STB codes for three transmit antennas are proposed by Tarokh *et al.* with G_3 code and H_3 code (i.e. (39) in [3]), respectively. And for the G_3 code, R_{STBC} =4/8=1/2, while for the H_3 code, R_{STBC} =3/4.

2.2 Unit-rate space-time block coding scheme

In this subsection, consider a wireless communication system comprising 3 transmit antennas (3Tx) and 1 receive antenna (1Rx) that operates in a Rayleigh flat-fading environment for simplicity of analysis. At the transmitter, the data source bits are firstly encoded by the channel encoder, then are mapped into corresponding constellation symbols; the symbols are space-time block encoded, the resulting encoded symbols are modulated onto a pulse waveform and then transmitted from three transmit antennas respectively. At the receiver, a demodulator firstly samples the output of a filter matched to the pulse waveform synchronously, then the sampling symbols are space-time block decoded, symbol demapped and channel decoded. Finally, the detected source bits are achieved. The channel is assumed to be a quasi-static fading so that the path gains keep constant over a frame and vary from one frame to another. The path gain from transmit antenna i (i=1,2,3) to receive antenna is defined as h_i, and $\{h_i\}$ are modeled as samples of independent complex Gaussian random variables with variance 0.5 per real dimension. For a fair comparison, we also define the system throughput as follows:

$$\mathit{Throughput} = R_c \times R_{stbc} \times \eta \qquad (1)$$

where R_{STBC} and R_c are the code rate of the STB code and the channel code, respectively. η denotes the bandwidth efficiency of modulation scheme.

Based on the encoding principle of Alamouti code, we develop a unit-rate and low-complexity STBC scheme with complex orthogonal design for 3Tx. That is: the input information bits are firstly mapped into the constellation symbols; then every four consecutive symbols (e.g. d_1, d_2, d_3, d_4) are space-time block coded in terms of the following designed code matrix D:

$$D = \begin{bmatrix} d_1 & d_2 & 0 \\ d_2^* & -d_1^* & 0 \\ 0 & d_3 & d_4 \\ 0 & -d_4^* & d_3^* \end{bmatrix} \qquad (2)$$

At time slot 1, d_1, d_2 are transmitted via the first two transmit antennas (i.e. antenna 1 and antenna 2), respectively; while at time slot 2, d_2*, -d_1* are also transmitted via these two transmit antennas, respectively. At time slot 3, d_3, d_4 are transmitted via the last two transmit antennas (i.e. antenna 2 and antenna 3), respectively; while at time slot 4, -d_4*, d_3* are also transmitted via corresponding two transmit antennas, respectively. Considering that 4 symbols are transmitted over 4 time slots, the rate of the proposed code matrix D is 4/4=1, i.e, the D code has unity rate, thus it can implement full data rate transmission. Moreover, the columns of D keep complex orthogonal mutually, i.e., for $i\neq j$, $\mathbf{d}_i^H \cdot \mathbf{d}_j = 0$; where $\mathbf{d}_i$ and $\mathbf{d}_j$ are column vectors of code matrix. Thus our scheme can realize unit rate and complex orthogonal design.

Based on the proposed coding scheme above, at the receiver, the received signal matrix at four consecutive slots can be expressed by

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \end{bmatrix}^T = \sqrt{\lambda E} D \mathbf{h} + \mathbf{z} = \sqrt{\frac{E}{2}} \begin{bmatrix} d_1 & d_2 & 0 \\ d_2^* & -d_1^* & 0 \\ 0 & d_3 & d_4 \\ 0 & -d_4^* & d_3^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \qquad (3)$$

The normalized constant λ is to ensure that the total transmitted energy is the same as E, E is the transmitted energy, here λ=1/2. $\mathbf{z}$ is the 4×1 Gaussian noise matrix whose elements are independent identically distributed (i.i.d.) Gaussian random variables with mean zero and variance N_0. The signal to noise ratio is defined as E/N_0.

The (3) can be further changed as follows:

$$\tilde{\mathbf{y}} = \begin{bmatrix} y_1 \\ y_2^* \\ y_3 \\ y_4^* \end{bmatrix} = \sqrt{\frac{E}{2}} \begin{bmatrix} h_1 & h_2 & 0 & 0 \\ -h_2^* & h_1^* & 0 & 0 \\ 0 & 0 & h_2 & h_3 \\ 0 & 0 & h_3^* & -h_2^* \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2^* \\ z_3 \\ z_4^* \end{bmatrix} = \sqrt{E/2}\mathbf{H}\mathbf{c} + \tilde{\mathbf{z}} \qquad (4)$$

By left-multiplying by $\mathbf{H}^H$, we have following linear combining:

$$\begin{cases} h_1^* y_1 - h_2 y_2^* = \sqrt{E/2}(|h_1|^2 + |h_2|^2)c_1 + h_1^* z_1 - h_2 z_2^* \\ h_2^* y_1 + h_1 y_2^* = \sqrt{E/2}(|h_1|^2 + |h_2|^2)c_2 + h_2^* z_1 + h_1 z_2^* \\ h_2^* y_3 + h_3 y_4^* = \sqrt{E/2}(|h_2|^2 + |h_3|^2)c_3 + h_2^* z_3 + h_3 z_4^* \\ h_3^* y_3 - h_2 y_4^* = \sqrt{E/2}(|h_2|^2 + |h_3|^2)c_4 + h_3^* z_3 - h_2 z_4^* \end{cases} \qquad (5)$$

Then the decoding is performed via linear combining and maximum likelihood (ML) decision as follows:

$$\begin{cases} \hat{c}_1 = \arg\min_{\hat{c}_1 \in \Psi} \| h_1^* y_1 - h_2 y_2^* - \sqrt{E/2}(|h_1|^2 + |h_2|^2)\hat{c}_1 \|^2 \\ \hat{c}_2 = \arg\min_{\hat{c}_2 \in \Psi} \| h_2^* y_1 + h_1 y_2^* - \sqrt{E/2}(|h_1|^2 + |h_2|^2)\hat{c}_2 \|^2 \\ \hat{c}_3 = \arg\min_{\hat{c}_3 \in \Psi} \| h_2^* y_3 + h_3 y_4^* - \sqrt{E/2}(|h_2|^2 + |h_3|^2)\hat{c}_3 \|^2 \\ \hat{c}_4 = \arg\min_{\hat{c}_4 \in \Psi} \| h_3^* y_3 - h_2 y_4^* - \sqrt{E/2}(|h_2|^2 + |h_3|^2)\hat{c}_4 \|^2 \end{cases} \qquad (6)$$

where Ψ represents energy-normalized signal constellation, and the ML decision metric is equivalent to the search for minimum metric in constellation Ψ. In fact, we often employ maximum ratio combing (MRC) method to simply the ML decoding in terms of (5), thus the computational complexity can be reduced further. According to (5) or (6), the signal to noise ratio (SNR) for $\hat{c}_1, \hat{c}_2$ can be obtained from the decoder output, i.e.

$$SNR_{\hat{c}_1,\hat{c}_2} = (|h_1|^2 + |h_2|^2)E/(2N_0) \qquad (7)$$

and the signal to noise ratio for $\hat{c}_3, \hat{c}_4$ is obtained from the decoder output by

$$SNR_{\hat{c}_3,\hat{c}_4} = (|h_2|^2 + |h_3|^2)E/(2N_0) \qquad (8)$$

According to the above analysis, our scheme achieves partial diversity, i.e. 2 diversity degrees.

Similarly, we can calculate the SNR of decoding output for G_2 code by $SNR_{G2}=(|h_1|^2+|h_2|^2)E/(2N_0)$, and the SNR for G_3 by $SNR_{G3}=2(|h_1|^2+|h_2|^2+|h_3|^2)E/(3N_0)$, as well as the SNR for H_3 code by $SNR_{H3}=(4/3)(|h_1|^2+|h_2|^2+|h_3|^2)E/(3N_0)$. Thus these code schemes (i.e. G_2, G_3 and H_3 code) all achieve full diversity. From the above results, we can see that the SNR for the G_2, G_3 and H_3 code keep constant in quasi-static fading channel, where the path gains are invariable over a frame. For this reason, the conventional time interleaver will be ineffective, and efficiency of channel coding will also be lowered. Whereas the SNR for our scheme no longer keeps constant in quasi-static fading channel, it is changed. Thus the spatial interleaving is formed by using our scheme. So when our scheme is concatenated with channel coding, the spatial interleaving effect will become very obvious, and high performance gain can also be obtained by the application of the inherent advantage of channel coding after making full use of the effective spatial interleaving. In addition, considering that turbo coding has good ability to combat the burst error of fading channel, we employ turbo coding as channel coding to improve the system performance and make up for performance loss due to partial diversity. Following simulation results also show that our new code scheme is indeed superior to the other compared STB codes when they are concatenated with turbo coding.

Table 1. Arithmetic complexity comparison for linear combining of one symbol in ML decoding

Code schemes	Complex addition	Complex multiplication
G_3-STBC	5	6
H_3-STBC	5	6
Proposed STBC	1	2

Based on the linear combining of one symbol in ML decoding, the needed complex addition and multiplication times of different STBC schemes are shown in Table 1. From the Table, we can observe that the proposed scheme has lower computational complexity than other full-diversity STBC schemes with 3Tx. Moreover, the full-diversity multiple antennas STBC schemes can not implement unit rate, so they must employ higher order modulation to maintain the same throughput, which requires higher accuracy of channel estimation, and will bring about the significant increase of implementation complexity. Hence, our STBC scheme has considerably lower implementation complexity.

In addition, based on the above-mentioned encoding scheme, our scheme is easily extended to four transmit antennas system. Thus we can apply the above method to analysis corresponding system performance, and a similar conclusion can also be reached. Here we no longer give specific details due to space limitations.

3. Simulation results

In this section, we provide simulation results for the performance of various STBC schemes with turbo coding or without turbo coding, and the G_2, G_3, H_3 code and proposed STBC scheme are investigated and compared. In simulation, the channel is assumed to be quasi-static Rayleigh fading; the receiver has perfect channel knowledge and system synchronization. Every data frame includes 960 information bits, and Gray mapping of the bits to symbol is adopted. The turbo code consists of two identical recursive systematic convolutional codes $(1, g_n/g_d)$ connected in parallel via a pseudorandom interleaver, where g_n and g_d are the forward and feedback generator polynomials, and g_n and g_d are chosen to be 15_{octal} and 13_{octal}, respectively. The code rate of turbo code is 1/3, and the Log-maximum a posteriori probability is used for turbo decoding, the number of iterations is 5. We obtain the simulation results from the 10^6 Monte-Carlo simulation runs, and the results are illustrated in Fig.1-Fig.4, respectively.

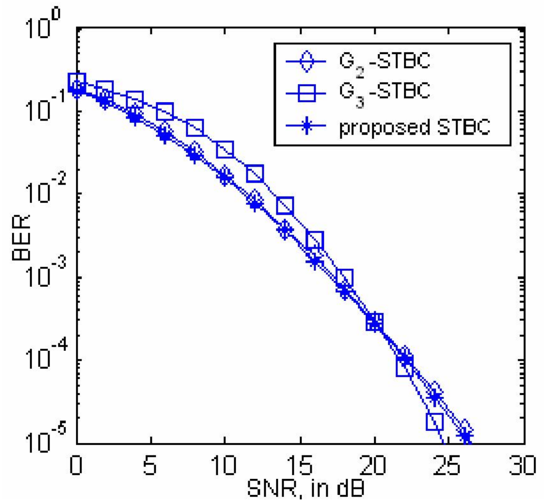

Figure 1. BER versus SNR for different STBC schemes

In Fig.1 and Fig.2, the QPSK modulation scheme is employed in conjunction with the G_2 code and the proposed scheme respectively, and the 16QAM modulation is applied to the G_3 code. Thus the system throughput without turbo coding equals 2 bit/s/Hz, while the turbo-coded system throughput is 2/3bit/s/Hz. It is shown in Fig.1 that the proposed scheme almost achieves the same performance as the G_2 code, and outperforms the G_3 code at low SNR, but performs worse than the G_3 code at high SNR due to partial diversity. When these STBC schemes are concatenated with turbo coding, the proposed scheme has superior performance over other STBC schemes. Namely, on the condition of same throughput, the proposed scheme gives about 3 dB gains over the G_3 code and 1 dB gains over the G_2 code at a BER of 10^{-4}. This is because our scheme can implement effective spatial interleaving, and has relative more spatial redundancy information. Besides, the more vulnerable 16QAM scheme is used for the low-rate G_3 code to maintain the same throughput, whereas the 16QAM signal constellation points are more densely packed when compared with QPSK scheme, thus they are more prone to errors in fading channel. As a result, the performance of the G_3 code will be affected greatly. Moreover, the proposed scheme is also superior to the G_2 code due to effective spatial interleaving.

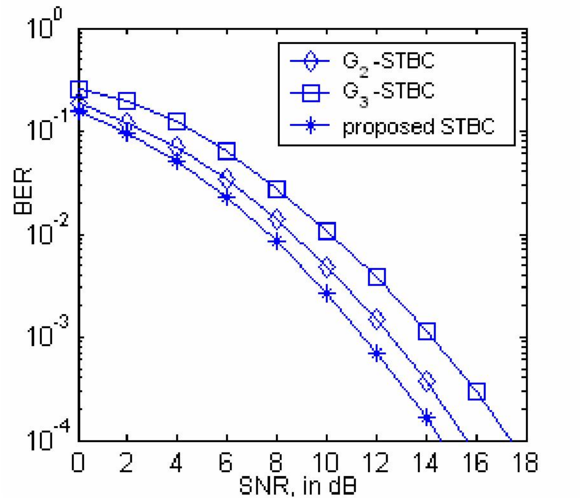

Figure 2. BER versus SNR for different STBC schemes with turbo coding

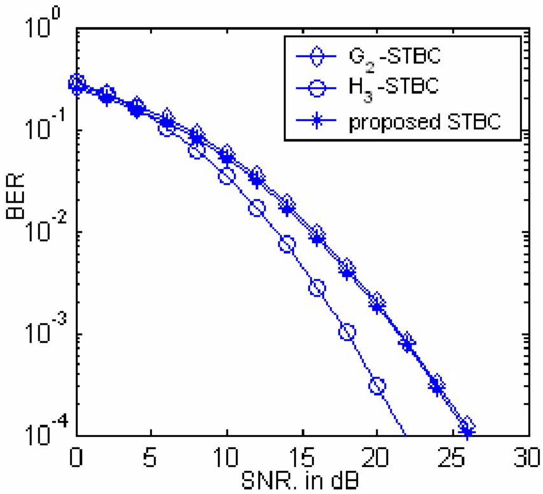

Figure 3. BER versus SNR for different STBC schemes

In Fig.3 and Fig.4, we compare the performance of the space-time block codes G_2, H_3 and the proposed STBC scheme. For the G_2 code and the proposed scheme, 8PSK modulation is employed, while for the H_3 code, the 16QAM modulation is used. Thus the system throughput without turbo coding equals 3 bit/s/Hz, while the turbo coded system throughput is 1bit/s/Hz. We can see in Fig.3 that proposed scheme still achieves nearly the same performance as the G_2 code, but is much worse than the H_3 code due to partial diversity. It is also because the performance of 8PSK scheme is only slightly better than that of high order modulation scheme 16QAM when compared with QPSK scheme. In Fig.4, however, when these STBC schemes are concatenated with turbo coding, the proposed scheme has better performance than other schemes. On the condition of same throughput, the proposed scheme gives about 2 dB gains over the H_3 code and less than 1 dB gain over the G_2 code at a BER of 10^{-4}.

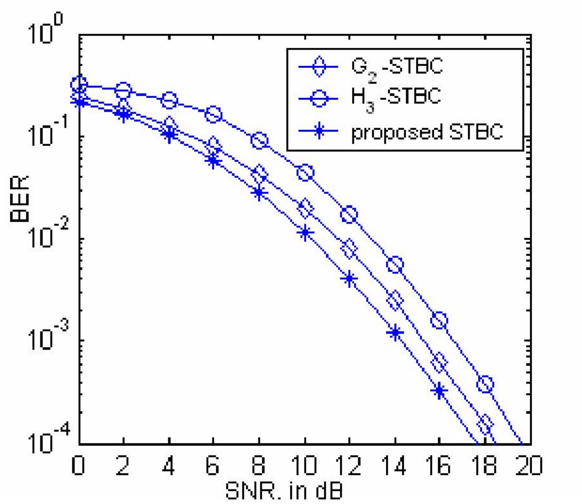

Figure 4. BER versus SNR for different STBC schemes with turbo coding

4. Summary

We have presented a unit-rate and low-complexity space-time block coding scheme with complex orthogonal design for 3 transmit antennas, and compare its performance with other full-diversity space-time block codes, such as the G_2, G_3 and H_3 code. Although the scheme does not achieve full diversity, its performance loss can be compensated by concatenating channel coding. It is shown that on the condition of same system throughput, the proposed scheme has better performance when these multiple antennas space-time block codes are concatenated with turbo coding, namely, it has lower bit error rate at the same SNR. Compared with those full-diversity space-time block code schemes, the new scheme can implement full data rate and partial diversity, and has a smaller computational complexity.

Acknowledgments

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