Fuzzy-Model-Based Robust Fault Detection with Stochastic Mixed Time-Delays and Successive Packet Dropouts

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Abstract— **This paper is concerned with the network-based robust fault detection problem for a class of uncertain discrete-time T-S fuzzy systems with stochastic mixed time-delays and successive packet dropouts. The mixed time-delays comprise both the multiple discrete time-delays and the infinite distributed delays. A sequence of stochastic variables is introduced to govern the random occurrences of the discrete time-delays, distributed time-delays and successive packet dropouts, where all the stochastic variables are mutually independent but obey the Bernoulli distribution. The main purpose of this paper is to design a fuzzy fault detection filter such that the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions are first established via intensive stochastic analysis for the existence of the desired fuzzy fault detection filters, and then the corresponding solvability conditions for the desired filter gains are established. Also, the optimal performance index for the addressed robust fuzzy fault detection problem is obtained by solving an auxiliary convex optimization problem. An illustrative example is provided to show the usefulness and effectiveness of the proposed design method.**

Keywords— **Discrete-time fuzzy systems; fault detection; networked control systems; packet dropouts; randomly occurring mixed time-delays.**

I. INTRODUCTION

Over the past decades, the fault detection problem has been an active field of research because of the ever increasing demand for higher performance, higher safety and reliability standards [3, 24, 28, 33]. Generally speaking, a fault detection process consists of constructing a residual signal and computing a residual evaluation function which can then be compared with a pre-defined threshold. When the residual exceeds the threshold, the fault is detected and an alarm of fault is generated. As is well known, for a fault detection problem, the residual signal should be made sensitive to faults (in order to detect faults in a timely way) and robust to modeling errors or disturbances (in order to avoid false alarms). Recently, the model-based approaches to fault detection problems have been widely adopted for dynamic systems. The main idea of these approaches is to introduce a performance index and then convert the fault detection problem into an associated optimization problem. Accordingly, a variety of important results have been reported in the literature. For example, the fault detection problems have been addressed in [39] for linear time-varying systems, in [41] for singular systems, in [16, 34] for sampled-data systems, in [28] for stochastic systems, in [40] for Markovian jump linear systems and in [12, 23] for networked control systems. It is worth noting that most of the above mentioned results are concerned with *linear* models.

Nonlinearities are recognized to exist universally in practical systems. Takagi-Sugeno (T-S) fuzzy models have proven to be capable of approximating any smooth nonlinear systems to any specified accuracy within any compact set, which is achieved by smoothly blending a family of local linear models through fuzzy membership functions. Based on this local linearity, many complex nonlinear systems can be represented by using T-S fuzzy models. As a result, the last decade has witnessed a rapidly growing interest in T-S fuzzy systems, and many important results have been reported in the literature. To mention a few, the stability and stabilization problems of the fuzzy systems have been addressed in [4, 9, 11, 14, 32, 37]. The filtering problems have been investigated in [8, 18] and the control problems have been studied in [2, 10, 17, 21, 29]. Since nonlinear systems can be approximated by T-S fuzzy models, an applicable design scheme of the fault detectors for nonlinear systems can be transformed into the fault detection problem for T-S fuzzy systems [28].

On another active research front, networked control systems (NCSs) have received a great deal of attention in recent years owing to their successful applications in a wide range of areas such as industrial automation, aerospace, nuclear power station and internet-based control [5, 6, 12, 13, 20, 25, 31, 35, 36]. The research into NCSs has mainly focused on the networked-induced challenging problems, such as the communication delays and data packet dropouts that could degrade the system performances or

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even cause fault. Over the past few years, compared with the rich literature on filtering and control problems for NCSs, only a limited number of results have been available on the fault detection problems of NCSs with time-delays. For example, the fault detection problem has been investigated for a class of discrete-time networked linear systems with constant time-delays [12]. In [38], the fault detection problem for NCSs has been studied where the communication delays are described as a random Markov jump process. It is worth mentioning that most of the reported results have been concerned with the discrete time-delays. Another kind of time-delays, namely, distributed time-delays, has been largely overlooked due mainly to the complexity and difficulty. The application insight of the distributed delays has been widely recognized and a number of results involving continuously distributed delays have been published, see e.g. [15, 26, 30]. The distributed delays in the discrete-time setting, on the other hand, have received little attention despite their application significance in digital control systems [27]. Furthermore, it is fairly unrealistic to assume that the discrete-time distributed delays is deterministic in a networked environment. Instead, due to the unpredictable changes of the network conditions, the discrete-time distributed delays may occur in a probabilistic way. As such, it makes practical sense to investigate how the randomly occurring phenomena (e.g. discrete time-delay, distributed time-delay and packet dropout) affect the dynamic behavior of the NCSs as well as the fault detection process.

A thorough literature review on the fault detection problems for NCSs and fuzzy control systems has revealed that, up to now, little attention has been paid to the study of fault detection for nonlinear NCSs with both communication delays and packet dropouts, especially when the randomly occurred phenomena are taken into consideration. Summarizing the above discussion, in this paper, we are motivated to study the robust fault detection problem for a class of uncertain discrete-time T-S fuzzy systems involving stochastic mixed time-delays and successive packet dropouts. By augmenting the state of the original system and the fault detection filter, the addressed fault detection problem is converted into an auxiliary $\mathcal{H}_\infty$ filtering problem. Sufficient conditions are established for the existence of the desired fuzzy fault detection filter, and then the corresponding solvability conditions for the desired filter gains are established. A practical simulation example is provided to show the usefulness and effectiveness of the proposed design method. *The main contributions of this paper, which lie primarily on the novelty of the research problem and system models, are summarized as follows. 1) A model is proposed to describe the randomly occurring multiple time-varying communication delays, randomly occurring infinite distributed delays and successive packet dropouts, all of which occur according to a specified Bernoulli distribution. 2) The investigation on the T-S fuzzy model is carried out for a class of complex systems that account for the modeling errors, disturbance rejection attenuation, probabilistic delay and packet dropouts within*
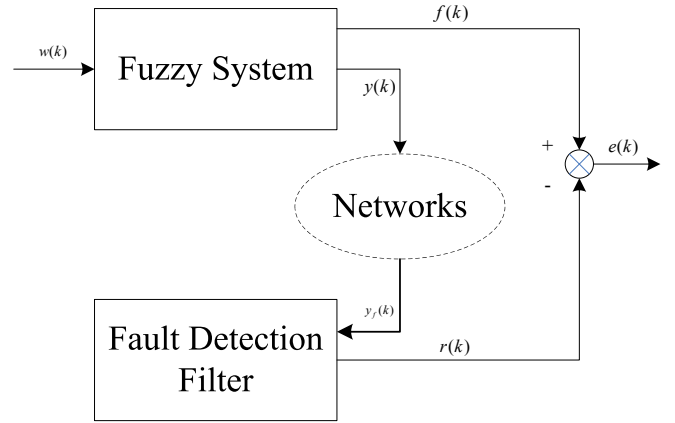

Fig. 1. The framework of the fuzzy fault detection filter design over networks environments

the same framework. 3) Stochastic analysis is conducted to enforce multiple requirements including the $\mathcal{H}_\infty$-norm constraints, the fault detection specification and the usual mean-square convergence of the detection errors.

Notation The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m \times n}$ is the set of all real matrices of dimension $m \times n$; The set of all non-positive integers is denoted by $\mathbb{Z}^-$. 0 represents zero matrix of compatible dimensions. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. The notation $P > 0$ means that P is real symmetric and positive definite; $l_2[0, \infty)$ is the space of square summable sequences; tr(M) refers to the trace of the matrix M; the notation $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\operatorname{tr}(A^T A)}$ and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions, we use an asterisk "$*$" to represent a term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x \,|\, y\}$ will, respectively, denote expectation of the stochastic variable x and expectation of x conditional on y. Prob$\{\cdot\}$ means the occurrence probability of the event "·". The symbol $\otimes$ denotes the Kronecker product; Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. PROBLEM FORMULATION

In this paper, we consider the fault detection problem for a class of uncertain discrete-time fuzzy system with stochastic mixed time-delays and successive packet dropouts in NCSs, where the framework is shown in Figure 1. The sensors are connected to the fault detection filter via a network which is subject to possible successive packet dropouts.

A. The Physical Plant

Consider a discrete-time nonlinear system with stochastic mixed time-delays which can be represented by the fol-

lowing T-S fuzzy dynamic model:

△ **Plant Rule i:** IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and ... and $\theta_p(k)$ is M_{ip} THEN

$$x(k+1) = A_i(k)x(k) + A_{d1i} \sum_{m=1}^{h} \alpha_m(k)x(k - \tau_m(k))$$

$$+ \beta(k)A_{d2i} \sum_{d=1}^{\infty} \mu_d x(k - d)$$

$$+ D_{1i}w(k) + G_i f(k) \tag{1}$$

$$y(k) = C_i x(k) + D_{2i}w(k)$$

$$x(k) = \psi(k), \ \forall k \in \mathbb{Z}^-, \ i = 1, \ldots, r$$

where M_{ij} is the fuzzy set, r is the number of IF-THEN rules and $\theta(k) = \begin{bmatrix} \theta_1(k), \ \theta_2(k), \ldots, \theta_p(k) \end{bmatrix}$ is the premise variable vector. $x(k) \in \mathbb{R}^n$ represents the state vector; $y(k) \in \mathbb{R}^m$ is the process output; $w(k) \in \mathbb{R}^q$ is the unknown disturbance input; $f(k) \in \mathbb{R}^l$ is the fault to be detected; $w(k)$ and $f(k)$ belong to $l_2[0, \infty)$; $\tau_m(k)$ denotes the discrete communication delay that occurs according to the stochastic variable $\alpha(k)$; d describes the distributed time delay; $A_i(k) = A_i + \Delta A_i(k)$ and (A_i, A_{d1i}, A_{d2i}, D_{1i}, G_i, C_i and D_{2i}) are known constant matrices of compatible dimensions; $\psi(k)$, $k \in \mathbb{Z}^-$ are given random initial conditions satisfying $\sup_{k \in \mathbb{Z}^-} \mathbb{E}\left\{\|\psi(k)\|^2\right\} < \infty$.

The real-valued matrix $\Delta A_i(k)$ represents the norm-bounded parameter uncertainty of the following structure:

$$\Delta A_i(k) = H_{ai}F(k)E_a, \ i = 1, \ldots, r \tag{2}$$

where H_{ai}, E_a are known constant matrices of appropriate dimensions, and $F(k)$ is an unknown matrix function satisfying

$$F^T(k)F(k) \le I. \tag{3}$$

The parameter uncertainty $\Delta A_i(k)$ is said to be admissible if both (2) and (3) hold.

The variable $\tau_m(k)$ denotes the time-varying delay satisfying

$$d_{\min} \le \tau_m(k) \le d_{\max}$$

where $d_{\min}$ and $d_{\max}$ are constant positive integers representing the lower and upper bounds on the communication delay, respectively. The constants $\mu_d \ge 0$ ($d = 1, 2, \ldots, \infty$) satisfy the following convergence conditions:

$$\bar{\mu} := \sum_{d=1}^{\infty} \mu_d < +\infty \tag{4}$$

To account for the phenomena of randomly occurring discrete time-delays and distributed time-delays, we introduce the following stochastic variables $\alpha_m(k) \in \mathbb{R}$ ($m = 1, 2, \ldots, h$) and $\beta(k) \in \mathbb{R}$, which are mutually independent Bernoulli distributed white sequences and also independent with the premise variables $\theta(k)$. A natural assumption on $\alpha_m(k)$ and $\beta(k)$ are as follows:

$$\text{Prob}\{\alpha_m(k) = 1\} = \mathbb{E}\{\alpha_m(k)\} = \bar{\alpha}_m,$$
$$\text{Prob}\{\alpha_m(k) = 0\} = 1 - \bar{\alpha}_m,$$
$$\text{Prob}\{\beta(k) = 1\} = \mathbb{E}\{\beta(k)\} = \bar{\beta},$$
$$\text{Prob}\{\beta(k) = 0\} = 1 - \bar{\beta}.$$

Remark 1: In model (1), both the discrete time-delay term $x(k - \tau_m(k))$ and the distributed time-delay term $\sum_{d=1}^{\infty} \mu_d x(k - d)$ behave probabilistically due to the introduction of the stochastic variables $\alpha_m(k)$ and $\beta(k)$. As such, they are called randomly occurring mixed time-delays. Note that the term $\sum_{d=1}^{\infty} \mu_d x(k-d)$ may be considered as the discretization of the semi-infinite integral form $\int_{-\infty}^{t} k(t - s)x(s)ds$ in the continuous-time system. It is noted that distributed delays occur very often in practical systems and have attracted much attention in the literature (see, e.g., [15, 30]). Most results have been concerned with the continuous deterministic time-delays, and there have been very few results for randomly occurring mixed time-delays especially when the fault detection problem becomes a research focus.

By using a center average defuzzifier, product interference and a singleton fuzzifier, the global dynamics of the T-S fuzzy systems (1) can be inferred as follows:

$$x(k+1) = \sum_{i=1}^{r} h_i(\theta(k)) \Bigg[A_i(k)x(k)$$

$$+ A_{d1i} \sum_{m=1}^{h} \alpha_m(k)x(k - \tau_m(k))$$

$$+ \beta(k)A_{d2i} \sum_{d=1}^{\infty} \mu_d x(k - d) \tag{5}$$

$$+ D_{1i}w(k) + G_i f(k) \Bigg]$$

$$y(k) = \sum_{i=1}^{r} h_i(\theta(k))[C_i x(k) + D_{2i}w(k)]$$

$$x(k) = \psi(k), \ \forall k \in \mathbb{Z}^-$$

where the fuzzy basis functions are given by

$$h_i(\theta(k)) = \frac{\vartheta_i(\theta(k))}{\sum_{i=1}^{r} \vartheta_i(\theta(k))}$$

with $\vartheta_i(\theta(k)) = \prod_{j=1}^{p} M_{ij}(\theta_j(k))$. Where $\vartheta_i(\theta(k)) \ge 0$, $i = 1, 2, \ldots, r$, $\sum_{i=1}^{r} \vartheta_i(\theta(k)) > 0$, and $M_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in M_{ij}. Hence, we have

$$h_i(\theta(k)) \ge 0, \ i = 1, 2, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta(k)) = 1.$$

In what follows, we write $h_i := h_i(\theta(k))$ for brevity.

B. Communication Channel with Packet Dropouts

In this paper, we assume that an unreliable network medium is present between the physical plant and the fault detection filter, and the packet dropout phenomenon constitutes another focus of our present research. The signal received by the fault detection filter can be described by

$$y_f(k) = \hat{\gamma}(k)y(k) + (1 - \hat{\gamma}(k))y_f(k-1) \tag{6}$$

where $y_f(k) \in \mathbb{R}^m$ is the *actual* measurement signal of $y(k)$ and $\hat{\gamma}(k) \in \mathbb{R}$ is a binary distributed random variable with $\text{Prob}\{\hat{\gamma}(k) = 1\} = \mathbb{E}\{\hat{\gamma}(k)\} = \bar{\gamma}$ and $\text{Prob}\{\hat{\gamma}(k) = 0\} = 1 - \bar{\gamma}$. In this paper, we assume that the premise variables $\theta(k)$ don't depend on the stochastic variables $\alpha(k)$, $\beta(k)$ and $\hat{\gamma}(k)$. Also, all the stochastic variables are assumed to be mutually independent Bernoulli distributed white sequences.

Remark 2: The dropout model (6) has been introduced in [19] to describe the successive packet dropouts. For example, if $\hat{\gamma}(k) = 1$, we have $y(k) = y_f(k)$ which means that there is no packet dropout; if $\hat{\gamma}(k) = 0$ but $\hat{\gamma}(k-1) = 1$, we have $y(k) = y_f(k-1)$ which means that the measured output at time point k is missing but one at time point $k-1$ has been received. As shown in [22], it is easy to further confirm that (6) can be a model for multiple consecutive packet dropouts where the latest measurement received in the buffers will be utilized if the current measurement is lost during packet transmissions. Such a scheme is certainly more realistic than the one setting the measurement signals to zero when the current measurements are lost [6, 8, 25].

C. Fuzzy Fault Detection Filter

As discussed previously, the key step of fault detection schemes is the construction of a dynamic system called a fault detection observer/filter, in which the residual signal is generated in order to decide whether a fault has occurred or not [28].

In this paper, for the physical plant represented by (1) and (5), we adopt a fuzzy fault detection filter whose model is described as follows:

△ **Filter Rule i:** IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\cdots$ and $\theta_p(k)$ is M_{ip} THEN

$$
\begin{aligned}
\hat{x}(k+1) &= A_{fi}\hat{x}(k) + B_{fi}y_f(k) \\
r(k) &= C_{fi}\hat{x}(k) + D_{fi}y_f(k)
\end{aligned} \tag{7}
$$

where $\hat{x}(k) \in \mathbb{R}^n$ represents the filter state vector, $r(k) \in \mathbb{R}^l$ is the so-called residual that is compatible with the fault vector $f(k)$, and $A_{fi}, B_{fi}, C_{fi}, D_{fi}$ are appropriately dimensioned filter matrices to be determined. Then, the overall fuzzy fault detection filter can be represented in the following form:

$$
\begin{aligned}
\hat{x}(k+1) &= \sum_{i=1}^{r} h_i \Big[A_{fi}\hat{x}(k) + B_{fi}y_f(k) \Big] \\
r(k) &= \sum_{i=1}^{r} h_i \Big[C_{fi}\hat{x}(k) + D_{fi}y_f(k) \Big]
\end{aligned} \tag{8}
$$

Our aim in this paper is to design a fault detection filter of the form in (7) that makes the error between residual signal $r(k)$ and fault signal $f(k)$ as small as possible. From (5), (6) and (8), we have the overall fault detection dynam-

ics governed by the following system:

$$
\begin{aligned}
\eta(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Bigg[\big(\bar{A}_{ij}(k) + \tilde{\gamma}(k)\hat{A}_{ij}\big)\eta(k) \\
&\quad + \sum_{m=1}^{h} \big(\bar{A}_{d1mi} + \tilde{\alpha}_m(k)\hat{A}_{d1i}\big)\eta(k - \tau_m(k)) \\
&\quad + \big(\bar{A}_{d2i} + \tilde{\beta}(k)\hat{A}_{d2i}\big)\sum_{d=1}^{\infty}\mu_d\eta(k-d) \tag{9} \\
&\quad + \big(\bar{B}_{ij} + \tilde{\gamma}(k)\hat{B}_{ij}\big)v(k) \Bigg] \\
e(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \Bigg[\big(\bar{C}_{ij} + \tilde{\gamma}(k)\hat{C}_{ij}\big)\eta(k) \\
&\quad + \big(\bar{D}_{ij} + \tilde{\gamma}(k)\hat{D}_{ij}\big)v(k) \Bigg] \tag{10}
\end{aligned}
$$

where

$$
\begin{aligned}
\eta(k) &= \begin{bmatrix} x^T(k) & \hat{x}^T(k) & y_f^T(k-1) \end{bmatrix}^T, \\
e(k) &= r(k) - f(k), \quad v(k) = \begin{bmatrix} w^T(k) & f^T(k) \end{bmatrix}^T, \\
\bar{A}_{d1mi} &= \text{diag}\{\bar{\alpha}_m A_{d1i}, 0, 0\}, \quad \hat{A}_{d1i} = \text{diag}\{A_{d1i}, 0, 0\}, \\
\bar{D}_{ij} &= \begin{bmatrix} \bar{\gamma}D_{fj}D_{2i} & -I \end{bmatrix}, \quad \bar{A}_{d2i} = \text{diag}\{\bar{\beta}A_{d2i}, 0, 0\}, \\
\hat{A}_{d2i} &= \text{diag}\{A_{d2i}, 0, 0\}, \quad \hat{D}_{ij} = \begin{bmatrix} D_{fj}D_{2i} & 0 \end{bmatrix}, \\
\bar{C}_{ij} &= \begin{bmatrix} \bar{\gamma}D_{fj}C_i & C_{fj} & (1-\bar{\gamma})D_{fj} \end{bmatrix}, \\
\hat{C}_{ij} &= \begin{bmatrix} D_{fj}C_i & 0 & -D_{fj} \end{bmatrix}, \\
\bar{A}_{ij}(k) &= \begin{bmatrix} A_i(k) & 0 & 0 \\ \bar{\gamma}B_{fj}C_i & A_{fj} & (1-\bar{\gamma})B_{fj} \\ \bar{\gamma}C_i & 0 & (1-\bar{\gamma})I \end{bmatrix}, \\
\hat{A}_{ij} &= \begin{bmatrix} 0 & 0 & 0 \\ B_{fj}C_i & 0 & -B_{fj} \\ C_i & 0 & -I \end{bmatrix}, \\
\bar{B}_{ij} &= \begin{bmatrix} D_{1i} & G_i \\ \bar{\gamma}B_{fj}D_{2i} & 0 \\ \bar{\gamma}D_{2i} & 0 \end{bmatrix}, \quad \hat{B}_{ij} = \begin{bmatrix} 0 & 0 \\ B_{fj}D_{2i} & 0 \\ D_{2i} & 0 \end{bmatrix} \tag{11}
\end{aligned}
$$

with $\tilde{\alpha}_m(k) = \alpha_m(k) - \bar{\alpha}_m$, $\tilde{\beta}(k) = \beta(k) - \bar{\beta}$, $\tilde{\gamma}(k) = \hat{\gamma}(k) - \bar{\gamma}$. It is clear that $\mathbb{E}\{\tilde{\alpha}_m(k)\} = 0$, $\mathbb{E}\{\tilde{\beta}(k)\} = 0$, $\mathbb{E}\{\tilde{\gamma}(k)\} = 0$ and $\mathbb{E}\{\tilde{\alpha}_m^2(k)\} = \bar{\alpha}_m(1 - \bar{\alpha}_m)$, $\mathbb{E}\{\tilde{\beta}^2(k)\} = \bar{\beta}(1 - \bar{\beta})$, $\mathbb{E}\{\tilde{\gamma}^2(k)\} = \bar{\gamma}(1 - \bar{\gamma})$.

Before formulating the problem to be investigated, we first introduce the following definition which plays an important role in deriving our main results in the sequel.

Definition 1: With system (10) and every initial conditions ψ, the fault detection dynamics in (10) is said to be exponentially mean square stable if, in case of $v(k) = 0$, there exist constants $\delta > 0$ and $0 < \kappa < 1$ such that

$$
\mathbb{E}\left\{\|\eta(k)\|^2\right\} \leq \delta\kappa^k \sup_{i \in \mathbb{Z}^-} \mathbb{E}\left\{\|\psi(i)\|^2\right\}, \quad \forall k \geq 0
$$

To this end, the fault detection problem to be addressed in this paper can be described by the following two steps:
Step 1: Generate a residual signal. For system (1), design a fuzzy fault detection filter in the form of (7) to generate a

residual signal $r(k)$. Furthermore, the filter is designed so that the overall fault detection system (10) is exponentially mean square stable with the following $\mathcal{H}_\infty$ performance constraint under zero initial condition:

$$\sum_{k=0}^{\infty} \mathbb{E}\{\|e(k)\|^2\} \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2 \qquad (12)$$

where $v(k) \neq 0$, and $\gamma > 0$ is made as small as possible in the feasibility of (12).

Step 2: Set up a fault detection measure. In this paper, we adopt a residual evaluation stage including an evaluation function $J(k)$ and a threshold J_{th} of the following form:

$$J(k) = \left\{ \sum_{k=s-\mathcal{L}}^{k=s} r^T(k) r(k) \right\}^{\frac{1}{2}}, \quad J_{th} = \sup_{w \in l_2, f=0} \mathbb{E}\{J(k)\} \qquad (13)$$

where $\mathcal{L}$ denotes the length of the finite evaluating time-horizon. Based on (13), the occurrence of faults can be detected by comparing $J(k)$ with J_{th} according to the following rule:

$$J(k) > J_{th} \Longrightarrow \text{with faults} \Longrightarrow \text{alarm},$$
$$J(k) \leq J_{th} \Longrightarrow \text{no faults}$$

III. Main Results

In this section, the analysis and synthesis problem of the desired fault detection filter for the T-S fuzzy stochastic systems presented in the previous section is considered. Before proceeding further, we give the following lemmas which will be used in establishing our main results.

Lemma 1: [15] Let $\mathcal{M} \in \mathbb{R}^{n \times n}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^n$, and constant $a_i > 0$ $(i = 1, 2, \ldots, \infty)$. If the series concerned are convergent, then we have

$$\left(\sum_{i=1}^{\infty} a_i x_i \right)^T \mathcal{M} \left(\sum_{i=1}^{\infty} a_i x_i \right) \leq \left(\sum_{i=1}^{\infty} a_i \right) \sum_{i=1}^{\infty} a_i x_i \mathcal{M} x_i \qquad (14)$$

Lemma 2: (S-procedure) Let $L = L^{\mathrm{T}}$ and H and E be real matrices of appropriate dimensions with F satisfying $FF^T \leq I$, then $L + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$, if and only if there exists a positive scalar $\varepsilon > 0$ such that $L + \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E < 0$, or equivalently,

$$\begin{bmatrix} L & H & \varepsilon E^{\mathrm{T}} \\ H^{\mathrm{T}} & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0 \qquad (15)$$

Lemma 3: [10] For any real matrices X_{ij} for $i, j = 1, 2, \ldots, r$, and $\Lambda > 0$ with appropriate dimensions, we have

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} \sum_{l=1}^{r} h_i h_j h_k h_l X_{ij}^T \Lambda X_{kl} \leq \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j X_{ij}^T \Lambda X_{ij} \qquad (16)$$

For convenience of presentation, we first discuss the nominal system of (10) (that is, without parameter uncertainty ΔA_i) and will eventually extend our main results to the general case. We have the following analysis result that serves as a theoretical basis for the subsequent design problem.

Theorem 1: Consider the nominal fuzzy system of (1) with given filter parameters and a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$. The nominal fuzzy fault detection system in (10) is exponentially mean square stable with a disturbance attenuation level γ if there exist matrices $P > 0$, $Q_k > 0$ $(k = 1, 2, \ldots, h)$ and $R > 0$ satisfying

$$\Psi_{ii}^T \check{P} \Psi_{ii} + \hat{\Psi}_{ii}^T \check{P} \hat{\Psi}_{ii} + \bar{P}_{ii} < 0, \qquad (17)$$
$$(\Psi_{ij} + \Psi_{ji})^T \check{P} (\Psi_{ij} + \Psi_{ji}) + (\hat{\Psi}_{ij} + \hat{\Psi}_{ji})^T \check{P}$$
$$\times (\hat{\Psi}_{ij} + \hat{\Psi}_{ji}) + 2(\bar{P}_{ij} + \bar{P}_{ji}) < 0 \qquad (18)$$

where

$$\check{\mathcal{A}}_{ij} = \begin{bmatrix} g\hat{A}_{ij} & 0 & 0 & g\hat{B}_{ij} \end{bmatrix}^T, \quad \hat{\Psi}_{ij} = \begin{bmatrix} \check{\mathcal{A}}_{ij}^T & \check{\mathcal{C}}_{ij}^T \end{bmatrix}^T,$$
$$\bar{\mathcal{A}}_{ij} = \begin{bmatrix} \bar{A}_{ij} & \hat{Z}_{1mi} & \bar{A}_{d2ij} & \bar{B}_{ij} \end{bmatrix}, \quad g^2 = \bar{\gamma}(1-\bar{\gamma}),$$
$$\mathcal{C}_{ij} = \begin{bmatrix} \bar{C}_{ij} & 0 & 0 & \bar{D}_{ij} \end{bmatrix}, \quad \Psi_{ij} = \begin{bmatrix} \bar{\mathcal{A}}_{ij}^T & \mathcal{C}_{ij}^T \end{bmatrix}^T,$$
$$\check{\mathcal{C}}_{ij} = \begin{bmatrix} g\hat{C}_{ij} & 0 & 0 & g\hat{D}_{ij} \end{bmatrix}^T, \quad \check{P} = \text{diag}\{P, I\},$$
$$\bar{P}_{ij} = \text{diag}\{\hat{P}_{ij}, -\gamma^2 I\}, \quad \hat{P}_{ij} = \text{diag}\{\bar{Q}_k, \mathcal{F}_{ij}, \check{A}_{d2ij}\},$$
$$\tilde{P} = I_h \otimes P, \quad \hat{Z}_{1mi} = \begin{bmatrix} \bar{A}_{d11i} & \cdots & \bar{A}_{d1hi} \end{bmatrix},$$
$$\bar{Q}_k = \sum_{k=1}^{h} (d_{\max} - d_{\min} + 1)Q_k + \bar{\mu}R - P,$$
$$\mathcal{F}_{ij} = \check{A}_{d1i}^T \tilde{P} \check{A}_{d1j} - \hat{Q}, \quad \check{A}_{d2ij} = g_\beta^2 \hat{A}_{d2i}^T P \hat{A}_{d2j} - \frac{1}{\bar{\mu}}R,$$
$$\hat{Q} = \text{diag}\{Q_1, \ldots, Q_h\}, \quad \check{A}_{d1i} = \hat{g}_m \otimes \hat{A}_{d1i},$$
$$\hat{g}_m^2 = \text{diag}\{\bar{\alpha}_1(1-\bar{\alpha}_1), \ldots, \bar{\alpha}_h(1-\bar{\alpha}_h)\},$$
$$g_\beta^2 = \bar{\beta}(1-\bar{\beta}). \qquad (19)$$

Proof: Choose a Lyapunov functional candidate as

$$V(k) = \sum_{i=1}^{3} V_i(k) \qquad (20)$$

where

$$V_1(k) = \eta^T(k) P \eta(k),$$
$$V_2(k) = \sum_{d=1}^{\infty} \mu_d \sum_{\tau=k-d}^{k-1} \eta^T(\tau) R \eta(\tau),$$
$$V_3(k) = \sum_{j=1}^{h} \sum_{i=k-\tau_j(k)}^{k-1} \eta^T(i) Q_j \eta(i)$$
$$+ \sum_{j=1}^{h} \sum_{m=-d_{\max}+1}^{-d_{\min}} \sum_{i=k+m}^{k-1} \eta^T(i) Q_j \eta(i)$$

with $P > 0$, $R > 0$, $Q_j > 0$ $(j = 1, 2, \ldots, h)$ being matrices to be determined. Then, along the trajectory of system (10), we have

$$\mathbb{E}\{\Delta V_1(k)\}$$
$$= \mathbb{E}\left\{ \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{s=1}^{r} \sum_{t=1}^{r} h_i h_j h_s h_t \left[(\bar{A}_{ij} + \tilde{\gamma}(k)\hat{A}_{ij}) \eta(k) \right. \right.$$
$$\left. + \sum_{m=1}^{h} (\bar{A}_{d1mi} + \tilde{\alpha}_m(k)\hat{A}_{d1i}) \eta(k - \tau_m(k)) \right.$$

$$+\big(\bar{A}_{d2i}+\tilde{\beta}(k)\hat{A}_{d2i}\big)\sum_{d=1}^{\infty}\mu_d\eta(k-d)$$

$$+\big(\bar{B}_{ij}+\tilde{\gamma}(k)\hat{B}_{ij}\big)v(k)\Big]^T P\Big[\big(\bar{A}_{st}+\tilde{\gamma}(k)\hat{A}_{st}\big)\eta(k)$$

$$+\sum_{m=1}^{h}\big(\bar{A}_{d1ms}+\tilde{\alpha}_m(k)\hat{A}_{d1s}\big)\eta(k-\tau_m(k))+\big(\bar{A}_{d2s}$$

$$+\tilde{\beta}(k)\hat{A}_{d2s}\big)\sum_{d=1}^{\infty}\mu_d\eta(k-d)+\big(\bar{B}_{st}+\tilde{\gamma}(k)\hat{B}_{st}\big)v(k)\Big]$$

$$-\eta^T(k)P\eta(k)\bigg\} \tag{21}$$

Next, by applying Lemma 1, it can be derived that

$$\mathbb{E}\{\Delta V_2(k)\}$$

$$\leq \mathbb{E}\bigg\{\bar{\mu}\eta^T(k)R\eta(k)-\frac{1}{\bar{\mu}}\bigg(\sum_{d=1}^{\infty}\mu_d\eta(k-d)\bigg)^T R$$

$$\times\bigg(\sum_{d=1}^{\infty}\mu_d\eta(k-d)\bigg)\bigg\} \tag{22}$$

$$\mathbb{E}\{\Delta V_3(k)\}$$

$$\leq \mathbb{E}\bigg\{\sum_{j=1}^{h}\bigg(\eta^T(k)Q_j\eta(k)-\eta^T(k-\tau_j(k))Q_j$$

$$\times\eta(k-\tau_j(k))\bigg)+\sum_{j=1}^{h}(d_{\max}-d_{\min})\eta^T(k)Q_j\eta(k)\bigg\} \tag{23}$$

For notational convenience, we denote

$$\hat{\xi}(k)=\begin{bmatrix} \eta^T(k) & \eta^T(k-\tau) & \sum_{d=1}^{\infty}\mu_d\eta^T(k-d) \end{bmatrix}^T,$$
$$\xi(k)=\begin{bmatrix} \hat{\xi}^T(k) & v^T(k) \end{bmatrix}^T, \quad \mathcal{P}_{ij}=\mathrm{diag}\{\hat{P}_{ij},0\},$$
$$\eta(k-\tau)=\begin{bmatrix} \eta^T(k-\tau_1(k)) & \cdots & \eta^T(k-\tau_h(k)) \end{bmatrix}^T,$$
$$\mathcal{A}_{ij}=\begin{bmatrix} \bar{A}_{ij} & \hat{Z}_{1mi} & \bar{A}_{d2i} \end{bmatrix}, \quad \hat{\mathcal{A}}_{ij}=\begin{bmatrix} g\hat{A}_{ij} & 0 & 0 \end{bmatrix}. \tag{24}$$

In the following, we first prove the exponential stability of the fault detection dynamics system (10) with $v(k)=0$. Considering (20)–(24) and by using Lemma 3, we have

$$\mathbb{E}\{\Delta V(k)\}$$

$$\leq \mathbb{E}\bigg\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\hat{\xi}^T(k)\Big(\mathcal{A}_{ij}^T P$$

$$\times\mathcal{A}_{st}+\hat{\mathcal{A}}_{ij}^T P\hat{\mathcal{A}}_{st}+\hat{P}_{is}\Big)\hat{\xi}(k)\bigg\}$$

$$\leq \mathbb{E}\bigg\{\sum_{i=1}^{r}h_i^2\hat{\xi}^T(k)\Big(\mathcal{A}_{ii}^T P\mathcal{A}_{ii}+\hat{\mathcal{A}}_{ii}^T P\hat{\mathcal{A}}_{ii}+\hat{P}_{ii}\Big)\hat{\xi}(k)$$

$$+\frac{1}{2}\sum_{i,j=1,i<j}^{r}h_ih_j\hat{\xi}^T(k)\Big[\Big(\mathcal{A}_{ij}+\mathcal{A}_{ji}\Big)^T P\Big(\mathcal{A}_{ij}+\mathcal{A}_{ji}\Big)$$

$$+\Big(\hat{\mathcal{A}}_{ij}+\hat{\mathcal{A}}_{ji}\Big)^T P\Big(\hat{\mathcal{A}}_{ij}+\hat{\mathcal{A}}_{ji}\Big)+2(\hat{P}_{ij}+\hat{P}_{ji})\Big]\hat{\xi}(k)\bigg\}. \tag{25}$$

By utilizing the Schur Complement Lemma [1], we know that $\mathbb{E}\{\Delta V(k)\}<0$ if (17) and (18) are true. Furthermore, along the same line of the proof for Theorem 1 in [25], it can be concluded that the discrete-time nominal fuzzy system of (10) with $v(k)=0$ is exponentially mean square stable.

Now, let us establish the $\mathcal{H}_\infty$ performance of the nominal fuzzy system of (10). Assume zero initial condition and introduce the following index:

$$J(n) = \mathbb{E}\sum_{k=0}^{n}\big[e^T(k)e(k)-\gamma^2 v^T(k)v(k)\big]$$

$$\leq \mathbb{E}\sum_{k=0}^{n}\big[e^T(k)e(k)-\gamma^2 v^T(k)v(k)+\Delta V(k)\big]$$

Along the trajectory of the nominal system of (10) and taking (11) and (24)–(25) into consideration, we have

$$J(n) \leq \mathbb{E}\bigg\{\sum_{k=0}^{n}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\xi^T(k)\big(\Psi_{ij}^T\check{P}\Psi_{st}$$

$$+\hat{\Psi}_{ij}^T\check{P}\hat{\Psi}_{st}+\bar{P}_{is}\big)\xi(k)\bigg\}$$

$$\leq \mathbb{E}\bigg\{\sum_{k=0}^{n}\bigg\{\sum_{i=1}^{r}h_i^2\xi^T(k)\big(\Psi_{ii}^T\check{P}\Psi_{ii}+\hat{\Psi}_{ii}^T\check{P}\hat{\Psi}_{ii}$$

$$+\bar{P}_{ii}\big)\xi(k)+\frac{1}{2}\sum_{i,j=1,\ i<j}^{r}h_ih_j\xi^T(k)\big[\big(\Psi_{ij}$$

$$+\Psi_{ji}\big)^T\check{P}\big(\Psi_{ij}+\Psi_{ji}\big)+\big(\hat{\Psi}_{ij}+\hat{\Psi}_{ji}\big)^T\check{P}$$

$$\times\big(\hat{\Psi}_{ij}+\hat{\Psi}_{ji}\big)+2(\bar{P}_{ij}+\bar{P}_{ji})\big]\xi(k)\bigg\}\bigg\} \tag{26}$$

which leads to $J(n)<0$ by considering Theorem 1. Letting $n\to\infty$, we obtain

$$\sum_{k=0}^{\infty}\mathbb{E}\Big\{\|e(k)\|^2\Big\}\leq\gamma^2\sum_{k=0}^{\infty}\|v(k)\|^2$$

which is equivalent to the inequality in (12), and therefore the proof of Theorem 1 is complete. ∎

Having established the analysis results, we are now in a position to deal with the fuzzy fault detection filter design problem.

Theorem 2: Consider the nominal fuzzy system of (10) and let $\gamma>0$ be a given scalar. A desired full-order fault detection filter of the form (7) exists if there exist matrices $P>0$, $R>0$, $Q_k>0$ ($k=1,2,\ldots,h$), matrices X_i and K_i satisfying

$$\Omega_1=\begin{bmatrix} \bar{P}_{ii} & * \\ \Gamma_{ii} & -\check{\mathcal{P}} \end{bmatrix}<0, \quad (i=1,2,\ldots,r) \tag{27}$$

$$\Omega_2=\begin{bmatrix} 2(\bar{P}_{ij}+\bar{P}_{ij}) & * \\ \Gamma_{ij}+\Gamma_{ji} & -\check{\mathcal{P}} \end{bmatrix}<0,$$
$$(1\leq i<j\leq r) \tag{28}$$

where

$$\Gamma_{11ij} = \begin{bmatrix} P\hat{A}_{0i} + X_i\hat{R}_{1j} & P\hat{Z}_{1mi} \\ K_i\hat{R}_{1j} & 0 \end{bmatrix},$$

$$\Gamma_{12ij} = \begin{bmatrix} P\bar{A}_{d2i} & P\bar{B}_{0i} + \bar{\gamma}X_i\hat{R}_{2j} \\ 0 & \hat{E}_0 + \bar{\gamma}K_i\hat{R}_{2j} \end{bmatrix},$$

$$\Gamma_{21ij} = \begin{bmatrix} gP\hat{A}_{1i} + gX_i\hat{R}_{4j} & 0 \\ gK_i\hat{R}_{4j} & 0 \end{bmatrix},$$

$$\Gamma_{22ij} = \begin{bmatrix} 0 & gP\hat{D}_{0i} + gX_i\hat{R}_{2j} \\ 0 & gK_i\hat{R}_{2j} \end{bmatrix},$$

$$\hat{R}_{1j} = \begin{bmatrix} 0 & I & 0 \\ \bar{\gamma}C_j & 0 & (1-\bar{\gamma})I \end{bmatrix}, \quad \Gamma_{ij} = \begin{bmatrix} \Gamma_{11ij} & \Gamma_{12ij} \\ \Gamma_{21ij} & \Gamma_{22ij} \end{bmatrix},$$

$$\hat{A}_{0i} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & 0 & 0 \\ \bar{\gamma}C_i & 0 & (1-\bar{\gamma})I \end{bmatrix}, \quad \hat{A}_{1i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_i & 0 & -I \end{bmatrix},$$

$$\bar{B}_{0i} = \begin{bmatrix} D_{1i} & G_i \\ 0 & 0 \\ \bar{\gamma}D_{2i} & 0 \end{bmatrix}, \quad \hat{D}_{0i} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ D_{2i} & 0 \end{bmatrix},$$

$$\hat{R}_{2j} = \begin{bmatrix} 0 & 0 \\ D_{2j} & 0 \end{bmatrix}, \quad \hat{R}_{4j} = \begin{bmatrix} 0 & 0 & 0 \\ C_j & 0 & -I \end{bmatrix},$$

$$\hat{E}_0 = \begin{bmatrix} 0 & -I \end{bmatrix}, \quad \hat{E} = \begin{bmatrix} 0 & I & 0 \end{bmatrix}^T, \quad \check{\mathcal{P}} = I_2 \otimes \check{P}$$

and $\bar{P}_{ii}$, $\check{P}$ are defined in Theorem 1. Furthermore, if (P, R, Q_k, X_i, K_i) is a feasible solution of (27)–(28), then the fault detection filter parameters in the form of (7) are given as follows:

$$\begin{bmatrix} A_{fi} & B_{fi} \end{bmatrix} = (\hat{E}^T P \hat{E})^{-1} \hat{E}^T X_i, \quad \begin{bmatrix} C_{fi} & D_{fi} \end{bmatrix} = K_i \quad (29)$$

Proof: In order to avoid partitioning the positive definite matrices P, Q_k and R, we rewrite the parameters in Theorem 1 in the following form:

$$\begin{aligned}
\bar{A}_{ij} &= \hat{A}_{0i} + \hat{E}L_i\hat{R}_{1j}, & \hat{A}_{ij} &= \hat{A}_{1i} + \hat{E}L_i\hat{R}_{4j}, \\
\bar{B}_{ij} &= \bar{B}_{0i} + \bar{\gamma}\hat{E}L_i\hat{R}_{2j}, & \hat{B}_{ij} &= \hat{D}_{0i} + \hat{E}L_i\hat{R}_{2j}, \\
\bar{C}_{ij} &= K_i\hat{R}_{1j}, & \hat{C}_{ij} &= K_i\hat{R}_{4j}, \\
\bar{D}_{ij} &= \hat{E}_0 + \bar{\gamma}K_i\hat{R}_{2j}, & \hat{D}_{ij} &= K_i\hat{R}_{2j}
\end{aligned} \quad (30)$$

where $L_i = [A_{fi}\ B_{fi}]$. Noticing (30), we can rewrite (17) and (18) as follows:

$$\begin{bmatrix} \bar{P}_{ii} & * \\ \hat{\Gamma}_{ii} & -\check{\mathcal{P}}^{-1} \end{bmatrix} < 0, \quad (i = 1, 2, \ldots, r) \quad (31)$$

$$\begin{bmatrix} 2(\bar{P}_{ij} + \bar{P}_{ij}) & * \\ \hat{\Gamma}_{ij} + \hat{\Gamma}_{ji} & -\check{\mathcal{P}}^{-1} \end{bmatrix} < 0, \quad (1 \le i < j \le r) \quad (32)$$

where

$$\hat{\Gamma}_{ij} = \begin{bmatrix} \hat{\Gamma}_{11ij} & \hat{\Gamma}_{12ij} \\ \hat{\Gamma}_{21ij} & \hat{\Gamma}_{22ij} \end{bmatrix},$$

$$\hat{\Gamma}_{11ij} = \begin{bmatrix} \hat{A}_{0i} + \hat{E}L_i\hat{R}_{1j} & \hat{Z}_{1mi} \\ K_i\hat{R}_{1j} & 0 \end{bmatrix},$$

$$\hat{\Gamma}_{12ij} = \begin{bmatrix} \bar{A}_{d2i} & \bar{B}_{0i} + \bar{\gamma}\hat{E}L_i\hat{R}_{2j} \\ 0 & \hat{E}_0 + \bar{\gamma}K_i\hat{R}_{2j} \end{bmatrix},$$

$$\hat{\Gamma}_{21ij} = \begin{bmatrix} g\hat{A}_{1i} + g\hat{E}L_i\hat{R}_{4j} & 0 \\ gK_i\hat{R}_{4j} & 0 \end{bmatrix},$$

$$\text{(33)}$$

$$\hat{\Gamma}_{22ij} = \begin{bmatrix} 0 & g\hat{D}_{0i} + g\hat{E}L_i\hat{R}_{2j} \\ 0 & gK_i\hat{R}_{2j} \end{bmatrix} \quad (34)$$

Pre- and post multiplying the inequalities (31) and (32) by $\text{diag}\{I, \check{\mathcal{P}}\}$ and letting $X_i = P\hat{E}L_i$, we can obtain (27) and (28) readily, and the proof is then complete. ∎

In the following, the results obtained for nominal systems will be extended to fuzzy system with uncertainty described in (1).

Theorem 3: Consider the uncertain fuzzy fault detection system (10) and let $\gamma > 0$ be a given scalar. A desired full-order fault detection filter of the form (7) exists if there exist matrices $P > 0$, $R > 0$, $Q_k > 0$ ($k = 1, 2, \ldots, h$), matrices X_i, K_i and positive constant scalars $\varepsilon_{ij} > 0$ satisfying

$$\begin{bmatrix} \Omega_1 & * & * \\ \bar{H}_{ai}^T & -\varepsilon_{ii}I & * \\ \varepsilon_{ii}\overline{E}_a & 0 & -\varepsilon_{ii}I \end{bmatrix} < 0, \quad (i = 1, 2, \ldots, r) \quad (35)$$

$$\begin{bmatrix} \Omega_2 & * & * \\ \bar{H}_{ai}^T + \bar{H}_{aj}^T & -\varepsilon_{ij}I & * \\ \varepsilon_{ij}\overline{E}_a & 0 & -\varepsilon_{ij}I \end{bmatrix} < 0,$$

$$(1 \le i < j \le r) \quad (36)$$

where

$$\begin{aligned}
\bar{H}_{ai} &= \begin{bmatrix} 0 & 0 & 0| & 0 & |\hat{H}_{ai}^T & 0| & 0 & 0 \end{bmatrix}^T, \\
\overline{E}_a &= \begin{bmatrix} \hat{E}_a & 0 & 0| & 0 & |0 & 0| & 0 & 0 \end{bmatrix}, \\
\hat{H}_{ai} &= \begin{bmatrix} H_{ai}^T & 0 & 0 \end{bmatrix}^T, \quad \hat{E}_a = \begin{bmatrix} E_a & 0 & 0 \end{bmatrix} \quad (37)
\end{aligned}$$

Ω_1 and Ω_2 are defined in Theorem 2. Moreover, if $(P, R, Q_k, X_i, K_i, \varepsilon_{ij})$ is a feasible solution of (35)–(36), then the fault detection filter parameters in the form of (7) are given as follows:

$$\begin{bmatrix} A_{fi} & B_{fi} \end{bmatrix} = (\hat{E}^T P \hat{E})^{-1} \hat{E}^T X_i, \quad \begin{bmatrix} C_{fi} & D_{fi} \end{bmatrix} = K_i \quad (38)$$

Proof: Replace A_i in (27)–(28) with $A_i + H_{ai}F(k)E_a$ and rewrite (27)–(28) in the following form

$$\Omega_1 + \bar{H}_{ai}F(k)\overline{E}_a + \overline{E}_a^T F^T(k)\bar{H}_{ai}^T < 0,$$

$$\Omega_2 + (\bar{H}_{ai} + \bar{H}_{aj})F(k)\overline{E}_a + \overline{E}_a^T F^T(k)(\bar{H}_{ai} + \bar{H}_{aj})^T < 0$$

where the corresponding parameters have been defined in (37). According to Lemma 2, we can easily obtain (35) and (36), and the proof is then complete. ∎

Remark 3: In Theorem 3, the fuzzy fault detection filter is designed such that the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions are first established for the existence of the desired fuzzy fault detection filters, and then the corresponding solvability conditions for the desired filter gains are established. Also, the optimal performance index for the addressed robust fuzzy fault detection problem can be obtained by solving an auxiliary convex optimization problem. Note that

the sufficient conditions involve the occurrence probabilities of the discrete time-delays, distributed time-delays and packet dropouts, thereby reflecting the nature of the randomly occurring phenomena.

IV. An Illustrative Example

In this section, we use a nonlinear pendulum to demonstrate the effectiveness and applicability of the proposed method. The pendulum system is modified from [7] by adding one damping term for stability of the system. It is assumed that two components of the system (that is, angle and angular velocity) are randomly perturbed by time delays. The equations of motion of the pendulum are described as follows:

$$
\begin{aligned}
\dot{\theta}(t) &= \lambda\bar{\theta}(t) + \alpha_1(t)(1-\lambda)\bar{\theta}(t-d(t)) + \alpha_2(t)(1-\lambda) \\
&\quad \times \theta(t-d(t)) + \beta(t)\int_{-\infty}^{t}\bar{\lambda}(t-s)\theta(s)ds \\
\dot{\bar{\theta}}(t) &= -\frac{g\sin(\theta(t)) + (b/lm)[\lambda\bar{\theta}(t) + (1-\lambda)\bar{\theta}(t-d(t))]}{\frac{2}{3}l - \frac{a}{2}ml\cos^2(\theta(t))} \\
&\quad -\frac{(aml/4)[\lambda\bar{\theta}(t) + (1-\lambda)\bar{\theta}(t-d(t))]^2\sin(2\theta(t))}{\frac{2}{3}l - \frac{a}{2}ml\cos^2(\theta(t))} \\
&\quad +w_1(t) \\
y(t) &= \sin(\theta(t)) + \lambda\bar{\theta}(t) + w_2(t) \qquad (39)
\end{aligned}
$$

where θ denotes the angle of the pendulum from the vertical, $\bar{\theta}$ is the angular velocity, $g = 9.8$ m/s^2 is the gravity constant, m is the mass of the pendulum, $a = 1/(m+M)$, M is the mass of the cart, l is the length of the pendulum, b is the damping coefficient of the pendulum around the pivot, and w_1 and w_2 are the disturbance applied to the cart and measurement noise, respectively. In this simulation, the pendulum parameters are chosen as $m = 2$ kg, $M = 8$ kg, $l = 0.5$ m and $b = 0.7$ Nm/s, and the retarded coefficient $\lambda = 0.6$.

Letting $x_1(t) = \theta(t)$, $x_2(t) = \bar{\theta}(t)$, we linearize the plant around the origin $x = (\pm\pi/2)$ and $x = (\pm\pi/3)$, and consider the differences between the linearized local model and the original nonlinear model as the uncertainties. By discretizing the plant with a sampling period 0.05 s, we obtain the following discrete-time T-S fuzzy model:

$$
\begin{aligned}
x(k+1) &= \sum_{i=1}^{3} h_i(\theta(k))\Bigg[(A_i + \Delta A_i(k))x(k) + A_{d1i} \\
&\quad \times \sum_{m=1}^{h} \alpha_m(k)x(k-\tau_m(k)) + \beta(k)A_{d2i} \\
&\quad \times \sum_{d=1}^{\infty} \mu_d x(k-d) + D_{1i}w(k)\Bigg] \\
y(k) &= \sum_{i=1}^{3} h_i(\theta(k))[C_i x(k) + D_{2i}w(k)]
\end{aligned}
$$

The model parameters are given as follows:

$$
A_1 = \begin{bmatrix} 1.000 & 0.0450 \\ 0.8558 & 0.7894 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1.000 & 0.0450 \\ 0.6315 & 0.8018 \end{bmatrix},
$$

$$
A_3 = \begin{bmatrix} 1.000 & 0.0450 \\ -0.4679 & 0.8055 \end{bmatrix},
$$

$$
A_{d11} = A_{d12} = A_{d13} = \begin{bmatrix} 0.14 & 0.02 \\ 0 & 0.094 \end{bmatrix},
$$

$$
A_{d21} = A_{d22} = A_{d23} = \begin{bmatrix} 0 & 0.12 \\ 0.1 & 0.02 \end{bmatrix},
$$

$$
D_{11} = D_{12} = D_{13} = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, \quad C_1 = \begin{bmatrix} 0.9949 & 0.9 \end{bmatrix},
$$

$$
C_2 = \begin{bmatrix} 0.8270 & 0.9 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0.6366 & 0.9 \end{bmatrix},
$$

$$
H_{a1} = H_{a2} = H_{a3} = \begin{bmatrix} 0.2 & 0.01 \end{bmatrix}^T, \quad E_a = \begin{bmatrix} 0 & 2 \end{bmatrix},
$$

$$
F(k) = \sin(k), \quad D_{21} = D_{22} = D_{23} = 1
$$

and the membership functions are shown in Figure 2. Assume that the time-varying communication delays $\tau_1(k)$ and $\tau_2(k)$ are random variables whose elements are uniformly distributed in the interval $[2,6]$, and

$$
\bar{\alpha}_1 = \mathbb{E}\{\alpha_1(k)\} = 0.8, \quad \bar{\alpha}_2 = \mathbb{E}\{\alpha_2(k)\} = 0.6,
$$
$$
\bar{\beta} = \mathbb{E}\{\beta(k)\} = 0.9.
$$

Choosing the constants $\mu_d = 2^{-3-d}$, we can easily find that $\bar{\mu} = \sum_{d=1}^{\infty} \mu_d = 2^{-3} < \sum_{d=1}^{\infty} d\mu_d = 2^{-2} < \infty$, which satisfies the convergence condition (4).

Assume that there are faults on the angular velocity of the pendulum, with fault matrices given by

$$
\begin{aligned}
G_1 &= \begin{bmatrix} 0.9887 & -0.0180 \end{bmatrix}^T, G_2 = \begin{bmatrix} 0.9033 & -0.0172 \end{bmatrix}^T, \\
G_3 &= \begin{bmatrix} 0.6237 & 0.0180 \end{bmatrix}^T
\end{aligned}
$$

Let the probability of $\hat{\gamma}(k)$ be given by $\bar{\gamma} = 0.7$. Applying Theorem 3, we can obtain the desired $\mathcal{H}_\infty$ filter parameters as follows:

$$
\begin{aligned}
A_{f1} &= \begin{bmatrix} -0.3879 & -0.4043 \\ -0.3840 & 0.4032 \end{bmatrix}, \\
A_{f2} &= \begin{bmatrix} 0.4279 & -0.4243 \\ -0.4840 & -0.5132 \end{bmatrix}, \\
A_{f3} &= \begin{bmatrix} 0.3868 & 0.4093 \\ 0.5420 & 0.5132 \end{bmatrix}, \\
B_{f1} &= \begin{bmatrix} -0.4690 & -0.4690 \end{bmatrix}^T, \\
B_{f2} &= \begin{bmatrix} 0.5679 & 0.4420 \end{bmatrix}^T, \\
B_{f3} &= \begin{bmatrix} -0.3868 & -0.3420 \end{bmatrix}^T, \\
C_{f1} &= \begin{bmatrix} -0.7846 & -0.6585 \end{bmatrix}, \\
C_{f2} &= \begin{bmatrix} -0.7579 & -0.5664 \end{bmatrix}, \\
C_{f3} &= \begin{bmatrix} -0.5052 & 0.4335 \end{bmatrix}, \\
D_{f1} &= -3.5656, \quad D_{f2} = -1.3585, \quad D_{f3} = -0.1792
\end{aligned}
$$

with the optimized performance index $\gamma^* = 1.1598$.

Now, let us show how the probabilities in the measurement equation (6) affect the $\mathcal{H}_\infty$ performance of the fault detection filtering process. In figure 3, after 100 Monte Carlo simulations, the plot of the average optimal disturbance attenuation level γ^* versus the probability of packet dropouts is provided. It can be seen clearly that a better
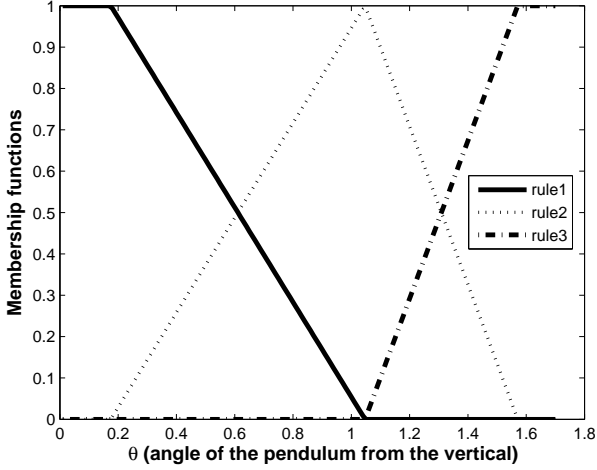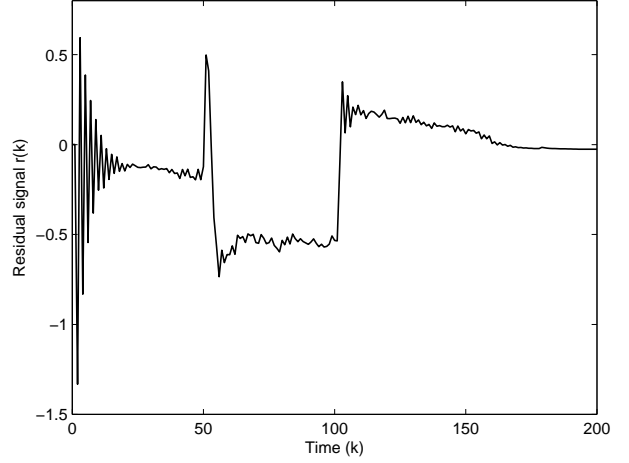
Fig. 2.   Membership functions
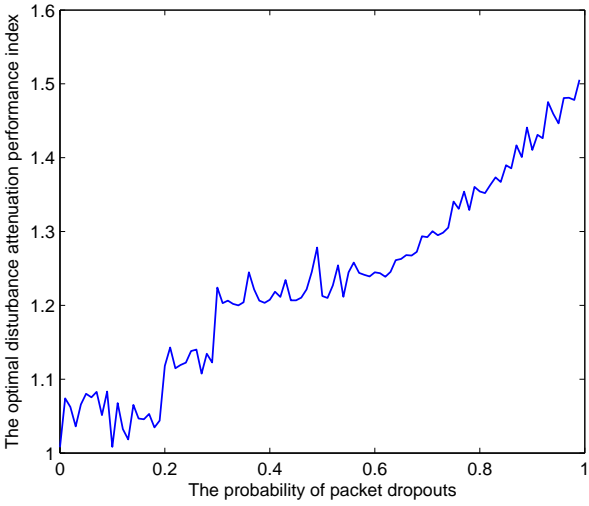

Fig. 4.   Residual signal without $w(k)$

Fig. 3.   The probability of packet dropouts versus the optimal performance γ^*

Fig. 5.   Evolution of residual evaluation function $J(k)$ without $w(k)$

performance can be achieved with less missing measurements.

To further illustrate the effectiveness of the designed fault detection filter, for $k = 0, 1, \ldots, 150$, let the fault signal $f(k)$ be given as

$$f(k) = \begin{cases} 1, & 50 \leq k \leq 100 \\ 0, & \text{else} \end{cases} \tag{40}$$

First, in the case that the initial conditions $\psi(k)$, $\forall k \in \mathbb{Z}^-, \psi \in \mathbb{R}^2$ are 200 random state vectors whose elements are uniformly distributed in the interval $[0, 0.1]$, $\tau_1(0) = 3$, $\tau_2(0) = 4$, $x(0) = [\pi/8 \ 0]^T$, $\hat{x}(0) = [0 \ 0]^T$, $y_f(-1) = 0$, $T = 20$ and the external disturbance is $w(k) = 0$. The residual signal $r(k)$ and evolution of residual evaluation function $J(k)$ are shown in Figure 4 and Figure 5, respectively, which indicate that the designed filter can detect the fault effectively when it occurs.
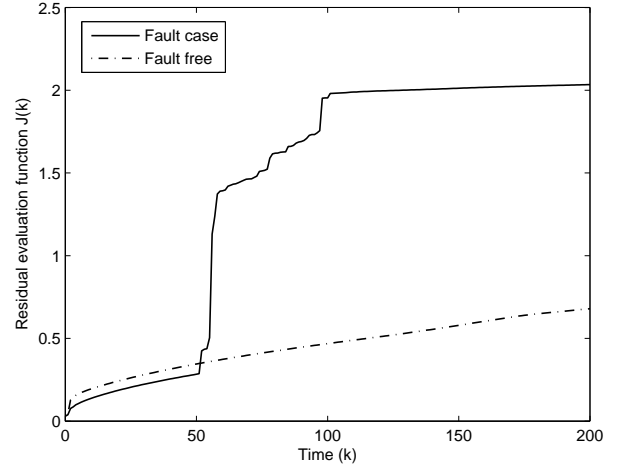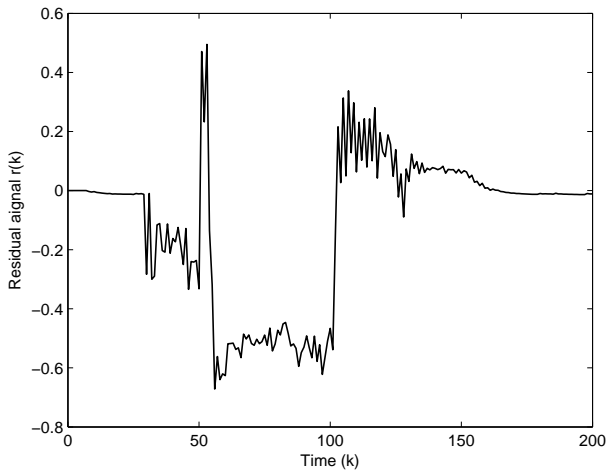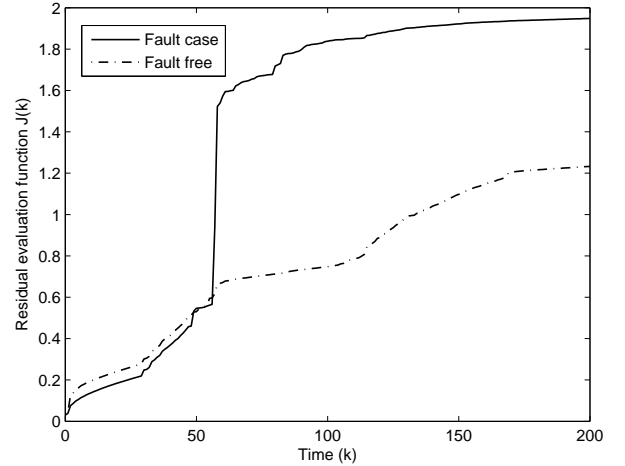
Next, assume that the disturbance is given by

$$w(k) = \begin{cases} 0.5 \times \text{rand}[0, 1], & 30 \leq k \leq 130 \\ 0, & \text{else} \end{cases} \tag{41}$$

where the rand function generates arrays of random numbers whose elements are uniformly distributed in the interval $[0, 1]$.

The **rand** distribution of successive packet dropout numbers is shown in Figure 6 from which we can see that if the number on the Y-axis is 0, it means that the current measurement output of the physical plant is transmitted to the fault detection filter successfully. Furthermore, when the number is i $(i = 1, 2, \ldots)$, it means that we have experienced i successive packet dropouts and the received measurement at the time $k - i$ will be used for current estimation. The residual signal $r(k)$ and evolution of residual evaluation function $J(k)$ are shown in Figure 7 and Figure 8, respectively. It can be seen that the residual not only can reflect the fault in time, but also detect the fault without confusing it with the disturbance $w(k)$.
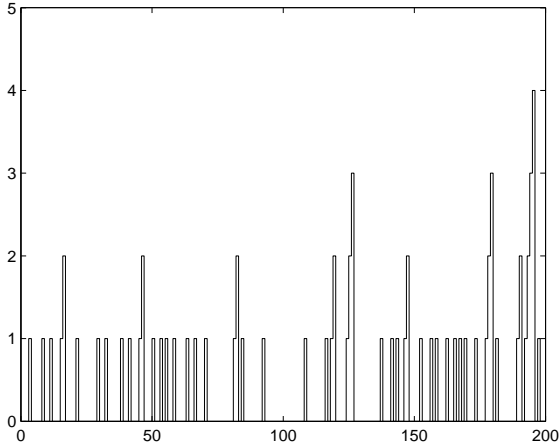
Fig. 6. The distribution of the packet dropout number under $\bar{\gamma} = 0.7$

Fig. 8. Evolution of residual evaluation function $J(k)$ with $w(k)$ in equation (41)

Fig. 7. Residual signal with $w(k)$
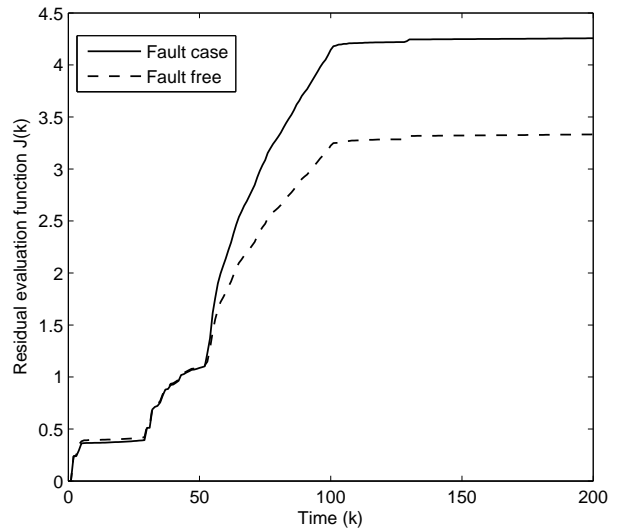
Fig. 9. Evolution of residual evaluation function $J(k)$ with $w(k) = 1 \times \mathrm{rand}[0, 1], 30 \leq k \leq 130$

Remark 4: In the simulation, we increase the magnitude of $w(k)$ in equation (41) with hope to see how a larger disturbance would influence the performance of the fault detection filter. For example, we take $w(k)$ as $1 \times \mathrm{rand}[0, 1]$ and $2 \times \mathrm{rand}[0, 1]$, and then show the corresponding evolutions of residual evaluation function $J(k)$ in Figure 9 and Figure 10, respectively. For simulation purpose, the threshold is selected as $J_{th} = \sup_{f=0} \mathbb{E}\{\sum_{k=0}^{200} r^T(k)r(k)\}^{1/2}$ and, accordingly, it can be obtained that $J_{th} = 1.2643$ in Figure 9 after 200 Monte Carlo simulations with no faults. From Figure 9, it can be seen that $1.1036 = J(111) < Jth < J(112) = 1.3657$, which means that the fault can be detected in 12 time steps after its occurrence. Similarly, we can conclude from Figure 10 that the fault can be detected in 21 time steps after its occurrence. From simulation results, it can be clearly observed that, the smaller $w(k)$ we have, the smaller the threshold we obtain and the faster the fault detection will take.

V. CONCLUSIONS

In this paper, we have addressed the robust fault detection problem for a class of uncertain discrete-time T-S fuzzy systems comprising randomly occurred mixed time-delays and successive packet dropouts. The mixed time-delays involve both the multiple time-varying discrete delays and the infinite distributed delays. The successive packet dropouts are modeled by a stochastic variable satisfying the Bernoulli random binary distribution. A fuzzy fault detection filter has been designed such that the fault detection dynamics is exponentially stable while preserving a guaranteed performance. A practical simulation example has been used to demonstrate the effectiveness of the fault detection techniques presented in this paper.
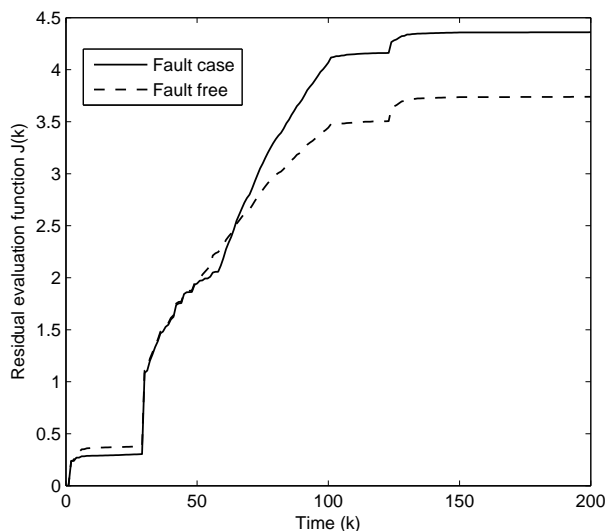
Fig. 10. Evolution of residual evaluation function $J(k)$ with $w(k) = 2 \times \text{rand}[0,1], 30 \le k \le 130$

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