Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

Estimation over Wireless Sensor Networks

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Abstract— **Remote estimation problems are critical to many novel applications enabled by large-scale dense wireless sensor network. Individual sensors simultaneously sense, process and transmit measured information over a lossy wireless network to a central base station, which processes the data and produces an optimal estimate of the state. In this paper, we investigate the tradeoff between the estimation performance and the number of communicating nodes with respect to the major MAC protocols used in wireless sensor networks. We first construct a Markov model of the node behavior to study the correlation between packet reception probability and the number of communicating nodes. We then develop a multi-sensor measurement fusion model. This is used to feed a multi-sensor Kalman filtering algorithm to assess the impact of MAC protocols on estimation performance. We offer a target tracking example to illustrate our approach.**

I. Introduction

Wireless sensor networks (WSN) are composed of low power devices that integrate general-purpose computing with heterogeneous sensing and wireless communication. Their emergence enables observation of the physical world at an unprecedented level of granularity [4].

Some of the most promising applications have been successfully implemented in in the fields of industrial and home automation, consumer electronics, military security and health monitoring [2],[3].

In the monitoring and control of moving machinery, for example, wireless sensor networks have compelling economic and engineering advantages over their wired counterparts. They may also deliver crucial information in real-time from environments and processes where data collection is impossible or impractical with wired sensors.

Installed at a fraction of the labor and the cost of wired devices [12], the deployment of wireless sensor network in heating ventilation air conditioning (HVAC) systems improves system performance and increases energy efficiency by providing the control systems with more complete information about the building environment.

In home automation applications, SK Telecom, the largest cellular provider in South Korea, has rolled out a new digital smart home service in October 2005 for customers to monitor and control their homes remotely using cell phones and/or the Internet. A key enabler of this service is 802.15.4/ZigBee-compliant wireless networking technology.

In all of the above WSN application examples, remote estimation is a central problem. Multiple *sensor nodes* in a common neighborhood sense an event and subsequently transmit sensed information to a remote processing unit or *base station*. Base stations are responsible for collecting and processing data. The Medium Access Control (MAC) sublayer sits directly on top of the physical layer and controls data communication.

In an attempt to compute an optimal estimate of the system state upon available observations, the use of more sensors can potentially improve the performance of the estimation algorithms (essentially by averaging). However, using too many sensors can generate bottlenecks in the communication infrastructure when they all compete for bandwidth. As a result, having too many sensors can actually degrade the estimation performance. It is this tradeoff that we wish to explore in this paper.

A. Related Work

In recent years, state estimation over lossy networks has received considerable attention. The work by Sinopoli et al. [14] shows the existence of a critical value of the packet loss rate for bounded estimation error covariance.

For multi-sensor scenario, Liu and Goldsmith [8] extend the analytical work [14] to a two-sensor case. Matveev and Savkin [9] consider one out of N sensors sending its measurement to the estimator with delay and study the stability conditions of the system. Gupta et al. [6] address the multi-sensor joint state-estimation of a plant, allowing only one sensor to take measurement and access communication channel at each time step.

The tradeoff between communication and estimation performance is explored as *controlled communication* in the works of Yook et al. [20], Xu and Hespanha [18] [19], to actively reduce network traffic. They construct within the (only) smart sensor a stationary Kalman filter besides a copy of the remote estimator. By comparing the two copies, the system decides whether or not the local estimate should be sent to the network.

To our best knowledge, our paper is the first attempt to explicitly analyze this tradeoff in terms of the optimal number of communicating sensor nodes among a large wireless sensor network.

The remainder of this paper is organized as follows. A summary of major MAC protocols used in wireless sensor network, along with characterization of packet reception probability based on a Markov model of node transmission

This research is supported in part by NEST, TRUST and NSF under Grant ECS0302554.

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is described in Section 2. Section 3 presents our multi-sensor measurement model and lossy multi-sensor Kalman filtering algorithm. An illustrative target tracking example is offered in Section 4. Finally, we summarize our results and draw conclusions.

II. Modeling of Media Access Control

The Media Access Control (MAC) sublayer arbitrates which sensor node is allowed to access the radio channel at any given time. In general, MAC protocols can be categorized into Time Division Multiple Access (TDMA) and contention based Carrier Sense Multiple Access (CSMA). TDMA requires global synchronization and each node to maintain a list of its neighbors' schedules, which may be hard to achieve in resource constrained wireless sensor networks. In the case of power deprivation, sensor nodes may stop functioning, thus this type of schemes is not scalable and hard to implement in large scale sensor networks.

The channel access schemes for wireless sensor networks are generally contention based due to their simplicity and flexibility. Among them the two most widely used MAC protocols are B-MAC [11] and IEEE 802.15.4 MAC [7]. B-MAC is still the de-facto standard networking stack while IEEE 802.15.4 is emerging as the new global standard in the wireless sensor network stack. Zigbee, a global wireless industry open standard that is built on top of IEEE 802.15.4, has become the preferred technology for home automation and control, among other applications.

Under these consideration we focus on CSMA-type MAC schemes to study the correlation between the packet reception probability [1] and the number of reporting sensors.

Before going into the details of node transmission modeling, let us state our main assumptions.

A. Network Model and Assumption

There are N identical sensors observing a dynamical system and reporting to a central location over the wireless sensor network with one radio channel. For simplicity, we assume our sensor network to be a single hop network with star topology where every node has always a packet ready for transmission.

In our estimation application, we consider a report is lost when it is not correctly received after one sample period [2].

Given a single radio channel and multiple reporting sensors, the network needs to arbitrate media access by using the protocol specified for this end.

B. Node Model

We illustrate the basic idea through the *Contention Access Period* (CAP) of unslotted IEEE 802.15.4 MAC, the standard for the low rate wireless personal area network. Fig 1 provides its flow chart.

[1]For tractability, we omit other network impact such as interference and coding errors.

[2] Depending on the network conditions such as the channel gain and the network traffic, there is a possibility that a report is corrupted or substantially delayed during channel access and transmission.

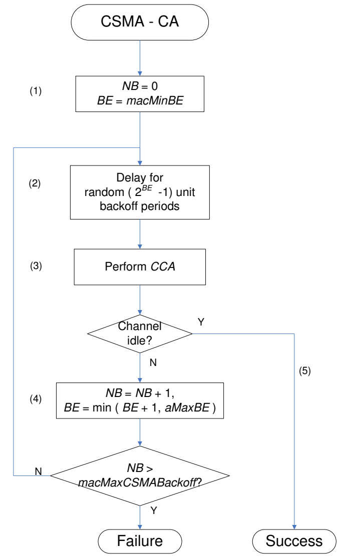


Fig. 1. Contention Access Period of unslotted IEEE 802.15.4, where *NB* is the number of backoff stage

During the CAP, a node with a packet ready for transmission first backs off for a random unit period, chosen uniformly between 0 and $2^{BE}-1$, or the *backoffWindow* before sensing the channel, where the parameter BE is the *backoff exponent* which is initially set to *macMinBE*. This random backoff serves to reduce the probability of collision among contending nodes. Once the backoff time counter decrements to zero, a node starts to sense whether the channel is busy. The channel sensing mechanism then ensures that the channel is clear of activity for a *contention window* duration, expressed in terms of units of backoff periods, before the node can attempt transmission. If the channel is found to be busy, the backoff exponent is incremented by one and a new number of units is drawn for the node to wait, until the channel can be sensed again. This process is repeated until either BE equals the parameter *aMaxBE* (which has a default value of 5), at which point it is frozen at aMaxBE, or until a certain maximum number of permitted random backoff stages is reached, at which point an access failure is declared to the upper layer. The maximum number of permitted random backoff stages is determined by the parameter *macMaxCSMABackoffs*, which has a default value of 5 [3].

We formulate the behavior of a single node with a Markov model, a similar idea used for modeling IEEE 802.11 in [1]. The transmission success of a particular sensor is identified with the absence of collisions, i.e. none of the $N-1$ remaining sensors accesses the channel when it does. Our assumption is that at each channel access attempt, the channel is busy with constant and independent probability c regardless of the number of access attempts. This assumption

[3]Similarly, B-MAC has an initial backoff and activates congestion backoff after collision takes place. The upper layer protocol, Mint, typically, used with BMAC specifies the number of retransmission to be 5 too.

is reasonable and accurate for a large scale network with a large N and a fixed packet size.

The Markov chain is depicted in Fig 2

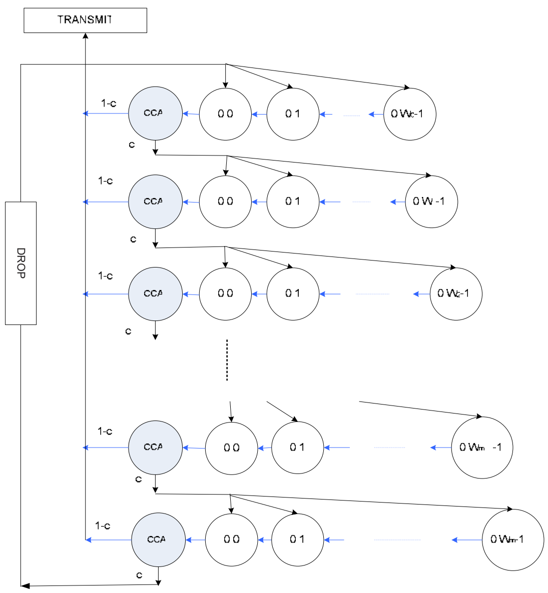


Fig. 2. Markov chain model of the node state including backoff stages and channel sensing. c refers to the probability that the channel is busy. The backoff timer decrements till zero and commences the *Clear Channel Assessment*(CCA).

Let $s(t)$ be the stochastic process representing the backoff stage $(0,...,m)$ of a node at time t with m being determined by $macMaxCSMABackoff$. Let $b(t)$ be the stochastic process representing the time counter at each backoff stage including backoff and clear channel assessment. Denote the backoff window size $W_i = 2^i W$, where $W = 2^{macMinBE} - 1$. The transition probabilities of the Markov chain for the bidimensional stochastic process $\{s(t), b(t)\}$ are

$$\begin{cases} P\{s(t+1)=i, b(t+1)=k | s(t)=i, b(t)=k+1\} = 1 \\ i \in [0,m] \qquad k \in [CCA, W_i - 2] \\ P\{s(t+1)=0, b(t+1)=k | s(t)=i, b(t)=0\} = (1-c)/W_0 \\ k \in [0, W_i - 1] \\ P\{s(t+1)=0, b(t+1)=k | s(t)=m, b(t)=0\} = 1/W_0 \\ k \in [0, W_0 - 1] \\ P\{s(t+1), b(t+1)=k | s(t)=i-1, b(t)\} = c/W_i \\ i \in [1,m] \qquad k \in [0, W_i - 1] \end{cases} \tag{1}$$

The first equation in (1) describes the fact that the time counter always decrements. The second equation says that following either a successful access attempt and transmission a node starts to transmit a new packet and takes an initial random backoff, uniformly chosen in the interval $(0, W_0 - 1)$. The third one reflects the fact that after the maximum number of backoff stages, regardless of the status of the access attempt, a node will transmit a new packet. The last equation explains that any unsuccessful access attempt before the maximum backoff stage leads to a new round of backoff with the new backoff value uniformly chosen from a longer backoff window, i.e. $(0, W_i)$.

Let $b_{i,k} = \lim_{t->\infty} P\{s(t)=i, b(t)=k\}, i \in (0,m), k \in (CCA, W_i - 1)$ be the stationary distribution of the chain. We have

$$\begin{aligned} b_{i-1,0}c &= b_{i,0} \qquad \Rightarrow b_{i,0} = c^i b_{0,0} \qquad i \in [1,m] \\ b_{0,0} &= (1-c)\sum_{i=0}^{m} b_{i,0} + c b_{m,0}. \end{aligned}$$

By the chain regularity, we have $b_{i,k} = \frac{W_i - k}{W_i} b_{i,0}$.

We can express all the values of $b_{i,k}$ as functions of the value $b_{0,0}$ and the conditional channel busy probability c. By balancing the Markov Chain, we have

$$1 = \sum_{i=0}^{m}\sum_{k=0}^{W_i - 1} b_{i,k} = \frac{b_{0,0}}{2}\left[\frac{1-c^{m+1}2^{m+1}}{1-2c}W + \frac{1-c^{m+1}}{1-c}\right]. \tag{2}$$

Thus

$$b_{0,0} = \frac{2(1-2c)(1-c)}{(1-c)(1-c^{m+1}2^{m+1})W + (1-2c)(1-c^{m+1})}. \tag{3}$$

We are now in the position to evaluate the probability p_{tr} that a node transmits in a randomly chosen time period. Any transmission occurs when a node finds the channel to be idle after its backoff time counter decrements to zero, regardless of the backoff stage, namely

$$\begin{aligned} p_{tr} &= (1-c)\sum_{i=0}^{m} b_{i,0} = (1-c^{m+1}) b_{0,0} \\ &= \frac{2(1-2c)(1-c)(1-c^{m+1})}{(1-c)(1-c^{m+1}2^{m+1})W + (1-2c)(1-c^{m+1})} \end{aligned} \tag{4}$$

The fundamental independence assumption implies that each access attempt "sees" the system in the same state, i.e. in steady state. At steady state, each node transmits with probability p_{tr}. Thus for a given node, at each channel access attempt, the conditional channel busy probability c is

$$c = 1 - (1 - p_{tr})^{N-1} \tag{5}$$

We can derive the value of p_{tr} by solving the set of non-linear fixed point equations. (4, 5). It is straightforward to show there is only one fixed point for each N.

C. Successful Packet Reception Probability

The ideal channel condition assumes that a node has a successful transmission if it is the only one that transmits. Without loss of generality, we consider that successful packet reception probability for an individual node is equivalent to successful transmission probability,

$$\lambda_t(N) = p_{tr}(1-p_{tr})^{N-1} \tag{6}$$

III. Remote Estimation Problem

The discrete time linear dynamical system and measurement models are the following, where i is the sensor index.

$$x_{t+1} = Ax_t + w_t \tag{7}$$
$$y_{i,t} = Cx_t + v_{i,t} \tag{8}$$

where $x_t \in \Re^n$ is the state vector, $y_t \in \Re^m$ is the output vector, $w_t \in \Re^n$ is white Gaussian noise with zero mean and covariance $Q > 0$ and v_i's $\in \Re^m$ are white Gaussian noises with covariances $R_i > 0$. w_t and v_i's are independent. The initial system state x_0 is Gaussian with zero mean and covariance $\sum_0$. We assume x_0 is independent of w_t and v_i's.

We use Kalman filter to estimate the state of the dynamical system.

A. Measurement Fusion

The accuracy of measurement improves as more sensors collaborate.

The two most commonly used methods for Kalman filter based data fusion are state-vector fusion and measurement fusion [5]. State-vector fusion involves fusing a joint state estimate through individual estimates produced by each sensor from its individual Kalman filter whereas measurement fusion method directly fuses the sensor measurements to obtain a weighted measurement and feeds it into a single Kalman filter to derive a final state estimate.

The measurement fusion method provides a better overall estimation performance and demands a relative lower computation load on each sensor node. State-vector fusion method is only effective when the Kalman filters are consistent [5], whereas modeling errors introduced by linearization in many realistic applications often violate this condition. Thus for our sensor network, measurement fusion is preferable.

Among several possible methods for measurement fusion, we choose to fuse observations from different sensors with the inverse of sensor's variance as the weighting factor.

$$y_t = [\sum_{i=1}^{N} R_i^{-1}(t)]^{-1} \sum_{i=1}^{N} R_i^{-1}(t) y_{i,t} \tag{9}$$

This method is optimal in the sense of minimum-mean-square-error (MMSE) with a consistent observation vector dimension to have a lower computational load. Note that the noise covariance of fused measurement takes the form $R_t = [\sum_{i=1}^{N} R_i^{-1}(t)]^{-1}$.

B. Multi-sensor lossy Estimation

We define the arrival of an observation y_i at time t as a binary random variable $\gamma_{i,t}$, i.i.d Bernoulli with probability distribution $p_{\gamma_{i,t}}(1) = \lambda_t(N)$, and with $\gamma_{i,t}$ independent of $\gamma_{i,s}$ if $t \neq s$.

Under this lossy network condition, the measurement y_t 9 of the dynamical system is fused as

$$y_t = Cx_t + [\sum_{i=1}^{N} \gamma_{i,t} R_i^{-1}(t)]^{-1} \sum_{i=1}^{N} \gamma_{i,t} R_i^{-1}(t) v_{i,t} \tag{10}$$

The measurement noise of sensor i is defined in the following fashion:

$$p(v_{i,t}|\gamma_{i,t}) = \begin{cases} \mathcal{N}(0, R_i) & : \gamma_{i,t} = 1 \\ \mathcal{N}(0, \infty I) & : \gamma_{i,t} = 0 \end{cases}$$

When the observation from every sensor $y_{i,t}$ is lost, i.e. $\gamma_{i,t} = 0$ for $i = 1...n$, then the measurement of y_t is fully lost and it's equivalent to receiving the measurement with an infinite noise variance. Otherwise, the variance matrix of the measurement noise is $R_t^{(N)} = R_t = [\sum_{i=1}^{N} \gamma_{i,t} R_i^{-1}(t)]^{-1}$. That is to say the more packets arrive the higher is the measurement accuracy.

Let us define the following vectors $\boldsymbol{\gamma_t} = \{\gamma_0, ..., \gamma_t\}$ and $\mathbf{y_t} = \{y_0, ..., y_t\}$. Then

$$\begin{aligned} \hat{x}_{t|t} &= \mathbb{E}[x_t|\mathbf{y_t}, \boldsymbol{\gamma_t}] \\ P_{t|t} &= \mathbb{E}[(x_t - \hat{x}_{t|t})(x_t - \hat{x}_{t|t})'|\mathbf{y_t}, \boldsymbol{\gamma_t}] \\ \hat{x}_{t+1|t} &= \mathbb{E}[x_{t+1}|\mathbf{y_t}, \boldsymbol{\gamma_{t+1}}] \\ P_{t+1|t} &= \mathbb{E}[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\mathbf{y_t}, \boldsymbol{\gamma_{t+1}}] \\ \hat{y}_{t+1|t} &= \mathbb{E}[y_{t+1}|\mathbf{y_t}, \boldsymbol{\gamma_{t+1}}]. \end{aligned}$$

The prediction phase for $\hat{x}_{t+1|t}$ and $P_{t+1|t}$ of the Kalman filter is independent of the observation process with:

$$\hat{x}_{t+1|t} = A\hat{x}_{t|t} \tag{11}$$

$$P_{t+1|t} = AP_{t|t}A' + Q \tag{12}$$

The measurement update is stochastic as the received measurements are functions of $\boldsymbol{\gamma_t}$, which are random.

If only one sensor observes the dynamical system, Sinopoli et al [14] have detailed exposition.

Liu et al [8] discuss the related case of 2 reporting nodes, in which one measurement is partitioned between measurements from two sensor nodes with different loss rate.

For illustration purposes, we will derive our equations for 2 sensors before generalizing to N. In our setting, we have the following measurement updates corresponding to the four possible outcomes.

For $\gamma_{1,t}, \gamma_{2,t} = 0$, i.e., both measurements are lost, then the final measurement update runs one step open loop.

$$\begin{aligned} \hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} \\ P_{t+1|t+1} &= P_{t+1|t} \end{aligned} \tag{13}$$

For $\gamma_{1,t} = 1$ and $\gamma_{2,t} = 0$, or $\gamma_{1,t} = 0$ and $\gamma_{2,t} = 1$, then we are back to the case with one sensing sensor.

$$\begin{aligned} \hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} \\ &\quad + P_{t+1|t}C'(CP_{t+1|t}C' + R^{(1)})^{-1}(y_{1,t+1} - C\hat{x}_{t+1|t}) \end{aligned} \tag{14}$$

$$P_{t+1|t+1} = P_{t+1|t} - P_{t+1|t}C'(CP_{t+1|t}C' + R^{(1)})^{-1}CP_{t+1|t} \tag{15}$$

For $\gamma_{1,t}, \gamma_{2,t} = 1$, i.e., both measurements successfully provide a fused final measurement to the Kalman filter.

$$\begin{aligned} \hat{x}_{t+1|t+1} &= \hat{x}_{t+1|t} + P_{t+1|t}C'(CP_{t+1|t}C' + R^{(2)})^{-1} \\ &\quad (R^{(2)}(R_1^{-1}y_{1,t+1} + R_2^{-1}y_{2,t+1}) - C\hat{x}_{t+1|t}) \end{aligned} \tag{16}$$

Combine (13), (14), and (16), the modified Kalman filter formulation with two reporting sensors can be rewritten as follows:

$$\begin{aligned} P_{t+1}^{(2)} &= AP_tA' + Q - [\gamma_{1,t}(1-\gamma_{2,t}) + (1-\gamma_{1,t})\gamma_{2,t}]M^{(1)}(P_t) \\ &- \gamma_{1,t}\gamma_{2,t}M^{(2)}(P_t) \end{aligned} \tag{17}$$

where we use the simplified notation $P_t = P_{t|t-1}$, $M^{(i)}(P_t) = AP_tC'(CP_tC' + R^{(i)})^{-1}CP_tA'$ and $R^{(i)} = [\sum_{j=1}^{i} R_j^{-1}]^{-1}$.

Without loss of generality, by omitting the correspondence of the indexes and the sensors, the covariance update may have following compact expression,

$$P_{t+1}^{(N)} = AP_tA' + Q - \sum_{n=1}^{N}\prod_{i=1}^{n}\gamma_i(N)\prod_{j=1}^{N-n}(1-\gamma_j(N))M^{(n)}(P_t) \quad (18)$$

where $M^{(n)}(P_t) = AP_tC'(CP_tC' + R^{(n)})^{-1}CP_tA'$ and $R^{(n)} = [\sum_{j=1}^{n} R_j^{-1}]^{-1}$. Note that $R^{(n)}$ is the variance of n random sensors out of the total N .

Given the initial condition P_0, the sequence $\{P_t\}_{t=0}^{\infty}$ is a random process dependent on both $\{\gamma_{i,t}\}_{t=0}^{\infty}$ and the number of reporting sensors n. Therefore, we only focus on the statistical properties of the error covariance matrix iteration. Since P_t is bounded with probability 1 if and only if $\overline{P_t} = \mathbb{E}[P_t]$ is bounded, we will study $\overline{P_t} = \mathbb{E}[P_{t+1}]$ as the metric of the estimation performance.

We define the modified algebraic Riccati equation (MARE) for the Kalman filter with multiple reporting sensors as follows,

$$g_{\boldsymbol{\lambda}(N)}(X) = AXA' + Q - \sum_{n=1}^{N}\lambda(N)^n(1-\lambda(N))^{N-n}M^{(n)}(X) \quad (19)$$

where $M^{(n)}(X) = AXC'(CXC' + R^{(n)})^{-1}CXA'$ and $R^{(n)} = [\sum_{i=1}^{n} R_j^{-1}]^{-1}$.

It follows that

$$g_{\boldsymbol{\lambda}}(P_t) = \mathbb{E}[P_{t+1}|P_t], \text{ and} \quad (20)$$

$$\overline{P_{t+1}} = \mathbb{E}[P_{t+1}] = \mathbb{E}[g_{\boldsymbol{\lambda}}(P_t)] \quad (21)$$

C. Special Case

To show the tradeoff between the number of reporting nodes and the estimation performance, in this section we will first discuss a simpler scenario where the number of reporting sensors doesn't contribute to packet loss rate and the measurement noise of a particular sensor is a constant regardless of the distance between the sensor and the sensed object.

Denote the successful packet arrival rate for each sensor, λ_t. This is a special case of the (10), where measurements for fusion are equally weighted, namely

$$y_t = \frac{\sum_{i=1}^{N}\lambda_t y_{i,t}}{\sum_{i=1}^{N}\lambda_t} = Cx_t + \frac{\sum_{i=1}^{N}\lambda_t v_{i,t}}{\sum_{i=1}^{N}\lambda_t}$$

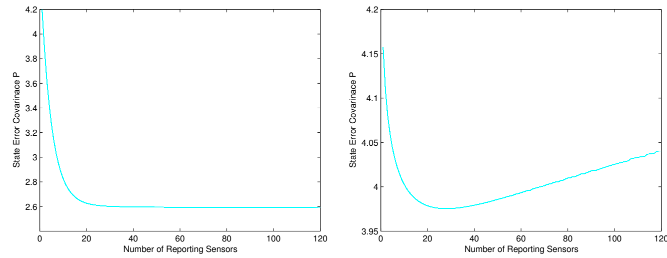
The covariance update simplifies into the following form,

$$\overline{P_{t+1}} = A\overline{P_t}A' + Q - \sum_{n=1}^{N}\binom{N}{n}\lambda(N)^n(1-\lambda(N))^{N-n}M^{(n)}(\overline{P_t}) \quad (22)$$

where $M^{(n)}(\overline{P_t}) = A\overline{P_t}C'(C\overline{P_t}C' + \frac{R}{n})^{-1}C\overline{P_t}A'$.

Lemma 1: For a fixed $0 \le \lambda \le 1$ and a given X, $g_{\boldsymbol{\lambda}}(X)$ is a non-increasing function in N, i.e. if $0 \le N_1 < N_2$, then $g_{\boldsymbol{\lambda}(N_1)}(X) \ge g_{\boldsymbol{\lambda}(N_2)}(X)$. Thus $\mathbb{E}[P_{t+1}|P_t]$ is non-increasing in N.

Remark: Statistically speaking, the more samples, the smaller the variance of "sample mean" and the better estimation as well. By the same token, the more measurements of sensor nodes for multi-sensor Kalman filter, the smaller the error covariance should be, provided constant packet reception rate, as shown in Fig. 3(a)



(a)Constant Packet Reception Rate (b)Varying Packet Reception Rate

Fig. 3. Estimation Performance v.s. Number of Reporting Sensors with (a)Constant Packet Reception Rate (b)Varying Packet Reception Rate that is dependent of the number of reporting sensors

However, as shown in Fig. 3(b), if the packet reception probability varies with the number of reporting sensors, the estimation performance is no longer a monotonic function in N, i.e. the number of reporting sensors.

IV. Illustrative Example: Tracking

A specific estimation example – tracking over wireless sensor network is presented in this section to further illustrate the impact of the number of communicating sensors on the estimation performance.

A. Simple Example with Same Measurement Noise

There are 150 identical sensors uniformly distributed over the surveillance region. We model the discrete dynamics and measurement of the agent as

$$\begin{aligned} x_{t+1} &= A^e x_t + w_t \\ y_{i,t} &= C_i x_t + v_{i,t} \end{aligned} \quad (23)$$

where w and v are white Gaussian noises with zero mean and covariance Q^e = diag $(0.15^2, 0.15^2, 0.15^2, 0.15^2)$ and $R_i = R$ = diag$(0.15^2, 0.15^2)$, and $\delta = 0.5$ is the sampling period.

$$A^e = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad C_i = C\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T \quad (24)$$

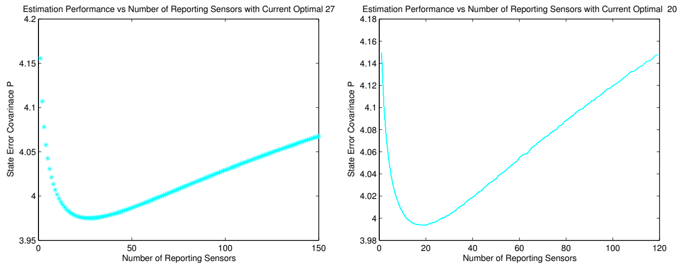
Using (19), we can can compute the optimal number of sensor, which is 27 for this case as illustrated in Fig 4(a).

The tracking performance comparison of wireless sensor networks with different number of communicating sensors, 100 and 27 respectively, is shown in Fig 5,

Fig. 6 gives the performance comparison of the two sensor networks in terms of tracking error. It shows that the sensor network comprised of 27 communicating sensors generates better results than that of 100 case.

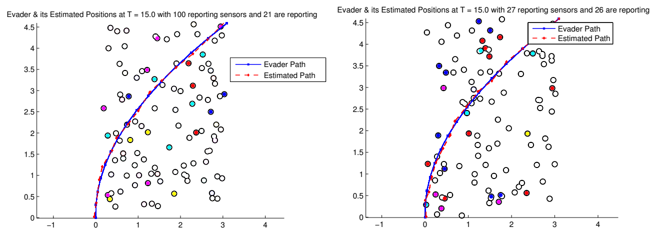
B. Example of Sensors with Distance Determined Variance

In this example, the sensor measurement noise variance is determined by the distance between the sensing sensor and sensed target.



(a)Distance-independent Variance (b)Distance Determined Variance

Fig. 4. Estimation Performance vs number of Reporting Sensors



100 reporting sensors 27 reporting sensors

Fig. 5. Tracking Performance with Different Number of Reporting Sensors, best view with color as the color indicates at which attempt the sensor gets to transmit. The color indicates at which transmission stage the sensor successfully transmits if any.

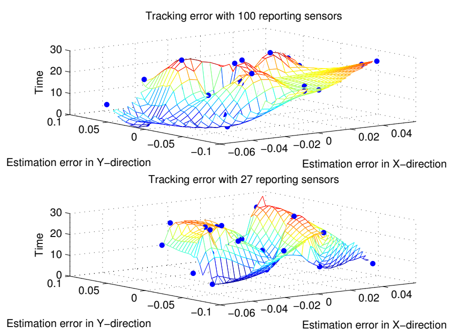


Fig. 6. Tracking Error Comparison

As shown in Fig 4(b), the optimal number of sensors decreases, comparing with the special case of distance independent variance. In fact, sensors further away from the target have larger measurement noise variance.

V. CONCLUSION

The tradeoff between estimation performance and the number of communicating sensors in lossy wireless sensor network is analyzed using a Markov model of the MAC protocol and a lossy multi-sensor Kalman filtering algorithm based on fused measurements. This cross-layer analysis shows that " the more, the merrier " is not applicable when it comes to multiple sensors communicating their observations to the center location. A target tracking example in wireless sensor network is provided to illustrate the analysis. In terms of how to engage the optimal number of sensors into communicating, sensor scheduling problem is a very attractive question to be addressed next, especially in the setting of multi-hop wireless network.

VI. ACKNOWLEDGEMENT

Authors would like thank professor Jean Warland for sharing his expertise on wireless communication protocols.

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