Adaptive Carrier Interferometry MC-CDMA

S. Sureshkumar, Ha H. Nguyen, *Senior Member, IEEE*, and Ed Shwedyk

***Abstract*—An adaptive carrier interferometry (CI) scheme is proposed for multicarrier code-division multiple-access (MC-CDMA) systems where it is assumed that there is a feedback channel between the receiver and the transmitter. By exploiting the additional degree of freedom in selecting the amplitudes of the subcarriers in accordance with the channel condition, the proposed scheme attains a significant performance gain over the conventional CI-MC-CDMA systems in which a constant amplitude is set for all the carriers. Two adaptation strategies, namely 1) local adaptation and 2) global adaptation, are considered for estimating the appropriate subcarrier amplitudes at the receiver in the proposed systems. Both single-user adaptation, where the other users do not adapt, and multiuser adaptation, in which all users adapt, are investigated. A further advantage of the proposed scheme is that it eliminates the peak-to-average power ratio problem present in the conventional CI-MC-CDMA systems.**

***Index Terms*—Adaptive algorithms, interferometry codes, multicarrier code-division multiple access (MC-CDMA), multiple access interference, peak-to-average power ratio (PAPR).**

I. Introduction

FOR FUTURE high-data-rate wireless applications, multicarrier code-division multiple access (MC-CDMA) has emerged as the most promising multiple access scheme due to its numerous advantages over conventional direct-sequence CDMA (DS-CDMA) in both uplink and downlink communications. Several types of MC-CDMA have been proposed in the literature [1]–[5], which can be categorized into two main groups, namely 1) MC-CDMA, where spreading is done in the frequency domain [1], [5] and 2) MC-DS-CDMA, where each parallel transmission constitutes a time-domain spreading [2]–[4]. Although both groups have similar advantages, MC-DS-CDMA employs a smaller number of carriers compared to MC-CDMA. On the other hand, MC-CDMA exploits frequency diversity in frequency-selective fading channels [6].

Although promising, the performance of MC-CDMA systems is limited by the multiple access interference (MAI), which is similar to conventional DS-CDMA systems. For a given total bandwidth and number of users, the level of MAI in an MC-CDMA system is influenced by the following factors: 1) the number of carriers, 2) the carrier spacing, and 3) the users' signature waveforms [7]. It then follows that when the number of carriers and the carrier spacing are also fixed, the MAI is only determined by the set of users' signature waveforms.

It is relevant to point out that the signature waveforms in both MC-CDMA and traditional DS-CDMA systems are constructed as linear combinations of a given set of orthonormal basis functions. Any specific weighting coefficient vector is generally referred to as the signature sequence. The orthonormal basis functions are commonly chosen to be the delayed versions of a Nyquist pulse (called the chip pulse or chip waveform), whereas the signature sequences are binary sequences (i.e., the weighting coefficients are $+1$ or -1). The use of other basis functions and/or signature sequences to minimize the MAI in traditional DS-CDMA systems is studied in [8] and [9].

To date, most MC-CDMA systems adopt signature sequences that were previously devised for DS-CDMA systems. In [10], a thorough analysis and comparison of existing spreading codes, including the Hadamard-Walsh, Gold, orthogonal Gold, and Zadoff-Chu sequences, is presented for MC-CDMA systems. More recently, a new family of spreading sequences, known as carrier interferometry (CI) codes, has also been introduced specifically for frequency-spreading MC-CDMA systems [11]. The CI codes, which are of length N, have a unique feature that allows an MC-CDMA system to support N users orthogonally and, as the system demand increases, to accommodate up to an additional $N-1$ users pseudoorthogonally. Moreover, there is no restriction on the length N of the CI codes (i.e., the length N can be any integer), making it more robust to the diverse requirements of the wireless environment.

Essentially, the CI codes are designed by appropriately varying the phases of the orthogonal carriers while assuming a constant amplitude for all carriers. Allowing both the amplitudes and phases of the carriers to be changed provides an additional freedom to design the signature waveforms for MC-CDMA systems. While such a design still preserves all the properties of CI codes, it can provide performance enhancement by adapting the carrier amplitudes according to the condition of the fading channel. Signature sequence adaptation in fading channels has been extensively studied for traditional DS-CDMA systems via transmitter/receiver optimization under different performance criteria [12]–[17]. A similar adaptation strategy is studied for MC-DS-CDMA systems in [18] by varying the carrier powers of each user.

The system of interest in this paper is an MC-CDMA over a multipath-fading channel. Given the system's total bandwidth and transmission power, the objective is to adapt the amplitudes of the orthogonal carriers according to the channel condition to

Manuscript received November 24, 2004; revised May 16, 2005 and October 19, 2005. The review of this paper was coordinated by Prof. T. Lok.

S. Sureshkumar was with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB R3T 5V6, Canada. He is now with Nokia, Espoo, Finland (e-mail: Sivapathasundaram.Sureshkumar@nokia.com).

H. H. Nguyen is with the Department of Electrical Engineering, University of Saskatchewan, Saskatoon, SK S7N 5A9, Canada (e-mail: ha.nguyen@usask.ca).

E. Shwedyk is with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB R3T 5V6, Canada (e-mail: shwedyk@ee.umanitoba.ca).

Digital Object Identifier 10.1109/TVT.2006.874583

0018-9545/$20.00 © 2006 IEEE

optimize the system performance. Because the phases of the sinusoidal carriers are also defined in [11], the CI property still applies, and hence, it is appropriate to refer to the proposed MC-CDMA scheme as adaptive CI-MC-CDMA (ACI-MC-CDMA). For simplicity, it is assumed that there is an ideal feedback channel to transmit the updated carrier amplitudes from the receiver back to the transmitter. A minimum mean-square-error (MMSE) receiver is also assumed. The signal-to-interference-plus-noise ratio (SINR) will be used as the performance index at the output of the MMSE receiver.

The carrier amplitude adaptation can be performed in either carrier-based or user-based mode. In carrier-based adaptation, the amplitude of each carrier is adapted by maximizing the SINR corresponding to that carrier. On the other hand, user-based adaptation maximizes the overall SINR, which is the sum of SINR of all carriers. In this paper, only user-based adaptation is considered. Two algorithms, namely 1) local adaptation and 2) global adaptation, are examined for user-based adaptation. These algorithms are implemented based on the technique presented in [17].

In local adaptation, an individual user is allowed to adapt his/her own carrier amplitudes to optimize his/her own performance, without considering the performance of other users in the system. Because the signals from other users are treated as noise, the performance criterion for this adaptation strategy is the SINR of that particular user. This type of strategy is well suited for situations where each user needs to achieve a different quality of service, as typical in multimedia wireless communications as well as multirate communications. Although this adaptation is applicable for both uplink and downlink communications, it is more appropriate for the downlink, where other users' information is not generally available at a particular user's receiver.

In contrast, global adaptation updates each user's carrier amplitudes in order to optimize the overall system performance. Hence, the total mean square error (or equivalently, the average SINR of all users) is a suitable performance criterion. Because overall system performance is the objective in this adaptation, global adaptation is more appropriate for uplink communications, where all the users' information is generally available at the receiver. This scheme is applicable in downlink communications as well, where different groups of users' operate at different data rates.

The paper is organized as follows: Section II describes the system model under consideration. Section III establishes the optimization problems and also provides the solutions. Section IV illustrates the performance of the proposed ACI-MC-CDMA systems and compares it with that of the conventional CI-MC-CDMA systems studied in [11]. Finally, conclusions are drawn in Section V.

II. System Model

This section describes the model of the MC-CDMA systems considered in this paper. Assume that there are K users. Each user employs N subcarriers to transmit information over a channel of bandwidth B_T. The N subcarriers are overlapped with carrier spacing $\Delta f = 1/T_b$, where T_b is the bit duration.

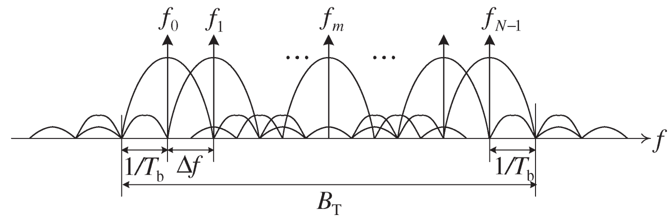

Fig. 1. PSD of an MC-CDMA system.

It follows that the number of subcarriers N is related to B_T and T_b as

$$N = B_T T_b - 1. \tag{1}$$

Observe that for a fixed bit duration (i.e., fixed bit rate), the number of carriers is limited by the total system bandwidth B_T. Fig. 1 illustrates the power spectral density (PSD) of the MC-CDMA system under consideration.

The complex baseband transmitted signal of the kth user can be expressed as

$$X_k(t) = \sum_{n=-\infty}^{\infty} b_k(n) s_k(t - nT_b) \tag{2}$$

where $\{b_k(n)\}$ is the binary data sequence of the kth user, which is modeled as a sequence of independent and identically distributed (i.i.d.) random variables taking values in $\{+1, -1\}$ with equal probability. For the MC-CDMA systems considered in this paper, the signature waveform $s_k(t)$ of the kth user is constructed as follows:

$$\begin{aligned} s_k(t) &= \left[\sum_{m=0}^{N-1} c_k^{(m)} e^{jm(2\pi \Delta f t + \Delta\Theta_k)} \right] \phi(t) \\ &= \left[\sum_{m=0}^{N-1} s_k^{(m)}(t) \right] \phi(t) \end{aligned} \tag{3}$$

where $\phi(t)$ is a unit-energy chip pulse limited to $[0, T_b]$ and $s_k^{(m)}(t) = c_k^{(m)} e^{jm(2\pi\Delta f t + \Delta\Theta_k)}$. Note that the set of waveforms $\{s_k^{(m)}(t)\}_{m=0}^{N-1}$ determines how the total power of the transmitted signal is spread (or distributed) over the total bandwidth. In addition, observe from (3) that the signature waveform of a specific user is described by the following parameters: 1) the number of carriers N (which is related to the total bandwidth), 2) the shape of the chip waveform $\phi(t)$, 3) the carrier spacing Δf, 4) the amplitude $c_k^{(m)}$, and 5) the phase $\Delta\Theta_k$. This construction of the signature waveforms unifies several variants of spreading waveforms proposed in the literature. By setting $\Delta\Theta_k = 0$, the current spreading system becomes the MC-CDMA described in [1]. On the other hand, the CI-MC-CDMA scheme proposed in [11] is realized by setting $c_k^{(m)} = 1$.

In this paper, the phases of the signature waveforms are defined in the same way as in [11]. That is

$$\Delta\Theta_k = \begin{cases} \frac{2\pi}{N} \cdot k, & 0 \le k \le N-1 \\ \frac{2\pi}{N} \cdot k + \frac{\pi}{N}, & N \le k \le 2N-1 \end{cases}. \tag{4}$$

However, different from [11], the amplitudes of the carriers $c_k^{(m)}$ are allowed to be varied by the transmitter in accordance with the feedback information from the receiver. Therefore, the proposed MC-CDMA systems are referred to as ACI-MC-CDMA systems.

The channel under consideration is a frequency-selective Rayleigh-fading channel. The number of carriers is chosen such that each carrier undergoes frequency nonselective slow Rayleigh fading. With this assumption, one can model the channel gains $h_k^{(m)}$, $k = 1, 2, \ldots, K$, and $m = 0, 1, \ldots, N-1$ as zero-mean complex Gaussian random variables. The magnitude of each channel gain is therefore Rayleigh distributed. Furthermore, although the channel gains $h_k^{(m)}$s are generally correlated [11], here, for simplicity, it is assumed that $h_k^{(m)}$s are i.i.d. for different k and m. Note that such a simplified channel model is also considered in [19]–[21].

As in [11], to make the analysis simple, the system is assumed to be synchronized. The complex baseband received signal during the nth bit duration is given by

$$y(t) = \sum_{k=1}^{K} b_k(n) \sum_{m=0}^{N-1} h_k^{(m)} c_k^{(m)} e^{jm(2\pi\Delta ft + \Delta\Theta_k)} \times \phi(t - nT_b) + n(t) \quad (5)$$

where $n(t)$ is additive white Gaussian noise (AWGN) with two-sided power spectral density of σ^2. For slow-fading channels, the channel gains can be assumed to be invariant over the time interval of transmitter adaptation.

At the receiver of every user, the received signal is first projected onto N orthogonal carriers to obtain the vector $\mathbf{r} = [r^{(0)}, \ldots, r^{(m)}, \ldots, r^{(N-1)}]^T$. With exact phase and carrier synchronization and the use of a rectangular pulse for $\phi(t)$, the mth component of $\mathbf{r}$ is given by

$$r^{(m)} = \sum_{k=1}^{K} b_k(n) h_k^{(m)} c_k^{(m)} e^{jm\Delta\Theta_k} + n^{(m)} \quad (6)$$

where $n^{(m)}$ is a Gaussian random variable with zero mean and variance σ^2. Furthermore, the vector $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \sum_{k=1}^{K} b_k(n) \mathbf{P}_k \mathbf{H}_k \mathbf{c}_k + \mathbf{n} \quad (7)$$

where $\mathbf{c}_k = [c_k^{(0)}, \ldots, c_k^{(m)}, \ldots, c_k^{(N-1)}]^T$ is the carrier amplitude vector chosen by the kth user, $\mathbf{H}_k$ is an $N \times N$ diagonal matrix whose mth diagonal element is $h_k^{(m)}$, and $\mathbf{P}_k$ is an $N \times N$ diagonal matrix whose mth diagonal element is $e^{jm\Delta\Theta_k}$.

The vector $\mathbf{r}$ is then fed into a receive filter in order to combine the signal components from all the N carriers to give the decision statistic. The decision statistic for the jth user is

$$z_j = \mathbf{w}_j^H \mathbf{r} \quad (8)$$

where $\mathbf{w}_j = [w_j^{(0)}, w_j^{(1)}, \ldots, w_j^{(N-1)}]^T$ is an N-dimensional weight vector of the receive filter of the jth user. The superscripts H and T denote the Hermitian and transpose operations, respectively. Here, a minimum mean-square-error (MMSE) receiver is employed, and to achieve the best performance, the weight vector is designed jointly to minimize the composite mean square error [22]

$$\text{MSE} = E\left\{\|b_j(n) - z_j\|^2\right\} = E\left\{\left\|b_j(n) - \mathbf{w}_j^H \mathbf{r}\right\|^2\right\}. \quad (9)$$

The optimum weight vector can be shown to be [23]

$$\mathbf{w}_j = \mathbf{R}^{-1} \mathbf{H}_j \mathbf{P}_j \mathbf{c}_j. \quad (10)$$

In (10), $\mathbf{R}$ is the received correlation matrix, which is defined as

$$\begin{aligned}\mathbf{R} &= \sum_{k=1}^{K} \mathbf{H}_k \mathbf{P}_k \mathbf{c}_k \mathbf{c}_k^H \mathbf{P}_k^H \mathbf{H}_k^H + \sigma^2 \mathbf{I} \\ &= \mathbf{R}_j + \mathbf{H}_j \mathbf{P}_j \mathbf{c}_j \mathbf{c}_j^H \mathbf{P}_j^H \mathbf{H}_j^H \end{aligned} \quad (11)$$

where $\mathbf{I}$ denotes the identity matrix and $\mathbf{R}_j$ is the interference-plus-noise correlation matrix corresponding to the jth user. This matrix is defined as

$$\mathbf{R}_j = \sum_{\substack{k=1 \\ k \neq j}}^{K} \mathbf{H}_k \mathbf{P}_k \mathbf{c}_k \mathbf{c}_k^H \mathbf{P}_k^H \mathbf{H}_k^H + \sigma^2 \mathbf{I}. \quad (12)$$

It can also be shown that the optimal weight vector aforementioned also maximizes the SINR at the output of the MMSE receiver [23], which is given by

$$\text{SINR}_j = \mathbf{c}_j^H \mathbf{P}_j^H \mathbf{H}_j^H \mathbf{R}_j^{-1} \mathbf{H}_j \mathbf{P}_j \mathbf{c}_j. \quad (13)$$

It is obvious from (13) that the SINR_j achieved by the MMSE receiver filter still depends on the carrier amplitudes of the jth user. Thus, it is possible to further improve the system performance by suitably choosing the users' carrier amplitudes.

III. Signature Waveform Adaptation

Two different adaptation strategies, namely 1) local adaptation and 2) global adaptation, are first presented in this section. In local adaptation, an individual user adapts his/her carrier amplitudes without taking any other users into account. Thus, one is interested in only the desired user's performance. On the other hand, in global adaptation, the users adapt their carrier amplitudes simultaneously by considering the whole system's performance. In both methods, it is assumed that the channel information is available at the receiver. More specifically, in local adaptation, the receiver needs to know only the desired user's channel information. In contrast, global adaptation requires that each receiver needs to know all other users' channel information as well as their receive filters. Moreover, a centralized receiver may be used to perform a joint detection (i.e., multiuser detection) for multiple users.

Both adaptation strategies can be implemented using either the forward–backward or the joint method, which are similar to the methods presented in [17] for single-carrier CDMA systems. The forward–backward method switches between optimizing the receiver with a fixed transmitter and optimizing the transmitter with a fixed receiver. On the other hand, in the joint method, one jointly optimizes the carrier amplitudes of a user and the receive filters. Although both methods should give the same performance, their convergence and complexity properties are different.

Finally, the last part of this section presents two procedures for adapting the carrier amplitude vectors of a group of users (i.e., multiuser adaptation). Specifically, one procedure is performed iteratively with local or global adaptation on the assumption that each user has his/her own receivers. The other procedure is noniterative and can be performed using a multiuser detector.

A. Local Adaptation

The problem at hand can be formulated as follows: Assume that user j is the user of interest for performance optimization. The goal is to obtain the optimal carrier amplitude vector $\mathbf{c}_j$ that minimizes the MSE (or maximizes the SINR) at the output of the receive filter, subject to a constraint on the transmitted power of the jth user. Because $\mathbf{P}_j\mathbf{P}_j^H = \mathbf{I}$, the transmitted power can be computed simply as $\|\mathbf{P}_j\mathbf{c}_j\|^2 = \|\mathbf{c}_j\|^2$. Let κ_j be the power constraint of the jth user, then the aforementioned goal can be achieved by solving the following optimization problem:

$$\begin{aligned} \min_{\mathbf{c}_j} \quad & \text{MSE}_j = E\left\{\left\|b_j(n) - \mathbf{w}_j^H\mathbf{r}\right\|^2\right\} \\ \text{subject to} \quad & \|\mathbf{c}_j\|^2 \le \kappa_j. \end{aligned} \tag{14}$$

The solution to the preceding problem can be obtained by two methods, namely 1) the forward–backward method and 2) the joint method.

In the forward–backward method, one first optimizes the receiver, which is given by (10) in the forward step and then optimizes the transmitter in the backward step. Using the Lagrange multiplier μ_j, the objective function in (14) is written as

$$L_j = E\left\{\left\|b_j(n) - \mathbf{w}_j^H\mathbf{r}\right\|^2\right\} + \mu_j\left(\|\mathbf{c}_j\|^2 - \kappa_j\right). \tag{15}$$

By assuming $\mathbf{w}_j$ was optimized (and hence, fixed) and setting the gradient of (15) with respect to the carrier amplitudes to zero, one obtains the following condition for the optimal carrier amplitude vector:

$$\mathbf{c}_j = \left(\mathbf{H}_j^H\mathbf{P}_j^H\mathbf{w}_j\mathbf{w}_j^H\mathbf{H}_j\mathbf{P}_j + \mu_j\mathbf{I}\right)^{-1}\mathbf{H}_j^H\mathbf{P}_j^H\mathbf{w}_j. \tag{16}$$

Using the matrix inversion lemma [24] and invoking the equality in the constraint, the optimal solution for $\mathbf{c}_j$ can be further simplified to

$$\mathbf{c}_j = \nu\mathbf{H}_j^H\mathbf{P}_j^H\mathbf{w}_j \tag{17}$$

where $\nu = \sqrt{\kappa_j/(\mathbf{w}_j^H\mathbf{H}_j^H\mathbf{H}_j\mathbf{w}_j)}$ is a scalar. The iterative procedure to implement this forward–backward method is illustrated in the following:

- Step 1) The jth user's transmitter sends the 1st data symbol using the initial carrier amplitudes $\mathbf{c}_j$. The initial carrier amplitudes can be set as $c_j^{(m)} = 1/\sqrt{N}$ for $m = 0, 1, \ldots, N-1$.
- Step 2) The weight vector $\mathbf{w}_j$ of the jth user's receive filter is then determined using (10) based on the current carrier amplitudes $\mathbf{c}_j$.
- Step 3) With the weight vector $\mathbf{w}_j$ computed in Step 2), the receiver estimates the new carrier amplitudes $\mathbf{c}_j$ using (17).
- Step 4) The receiver repeats Step 2) and Step 3) until the convergence of the MMSE is achieved.
- Step 5) Once the MMSE is achieved, the receiver transmits the updated carrier amplitudes $\mathbf{c}_j$ back to transmitter via the feedback channel. The transmitter uses the updated carrier amplitudes to send the subsequent data symbols.

Unlike the forward–backward method, which switches between the transmitter and the receiver, the joint method provides a closed-form solution by jointly optimizing the carrier amplitudes and the receive filter for the jth user. This optimization problem can be formulated as

$$\begin{aligned} \max_{\mathbf{c}_j} \quad & \text{SINR}_j = \mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j \\ \text{subject to} \quad & \|\mathbf{c}_j\|^2 \le \kappa_j. \end{aligned} \tag{18}$$

Using the method of Lagrange multiplier, the solution to the preceding optimization problem yields the following necessary condition:

$$\left\{\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j\mathbf{P}_j\right\}\mathbf{c}_j = \mu_j\mathbf{c}_j \tag{19}$$

where the Lagrange multiplier μ_j is chosen to satisfy the constraint in (18). It can be shown that the optimal $\mathbf{c}_j$ is the eigenvector of $\{\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j\mathbf{P}_j\}$ that maximizes the SINR_j. In other words, $\mathbf{c}_j$ should be chosen to align with the channel having the strongest signal component and the least interference.

Note that, if there is no multipath interference (i.e., $\mathbf{H}_j = \mathbf{I}$), then $\mathbf{c}_j$ is simply chosen as an eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_j^{-1}$. This implies that the optimum $\mathbf{c}_j$ lies in the subspace containing the least interference and noise. A similar observation was also made for optimal spreading sequence selection in DS-CDMA in [12] and [17]. It should be pointed out that in this optimization, SINR_j is used instead of MSE_j. If MSE_j is considered, then a similar necessary condition to (19) can be obtained, where the only difference is that $\mathbf{R}_j^{-1}$ is replaced by $\mathbf{R}^{-1}$. Because $\mathbf{R}$ depends on $\mathbf{c}_j$, the condition obtained using SINR_j is more convenient to solve.

B. Global Adaptation

In this adaptation strategy, one wishes to find the carrier amplitudes from the perspective of optimizing the performance of the system as a whole. A single user or a group of users is allowed to adapt the carrier amplitudes in order to improve the global system's performance for given transmitted powers. Because the overall system's performance is of particular interest, instead of minimizing MSE_j individually, the minimization of their sum (i.e., the total mean square error, or TMSE) shall be considered as the performance criterion. Note that, compared to local adaptation, global adaptation penalizes any additional interference to other users as a result of the change in an individual user's carrier amplitudes.

In this adaptation, one wishes to find the optimum carrier amplitude vector of the jth user by minimizing the TMSE subject to the constraints on the transmitter powers and on the assumption that each user implements his/her own MMSE receiver. Hence, the optimization problem can be formulated as follows:

$$\min_{\mathbf{c}_j} \quad \mathrm{TMSE} = \sum_{j=1}^{K} \mathrm{MSE}_j = E\left\{\left\|\mathbf{b}(n) - \mathbf{W}^H\mathbf{r}\right\|^2\right\}$$
$$\text{subject to} \quad \|\mathbf{c}_j\|^2 \le \kappa_j, \qquad j = 1, 2, \ldots, K \tag{20}$$

where $\mathbf{b}(n) = [b_1(n), b_2(n), \ldots, b_K(n)]^T$, $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_j, \ldots, \mathbf{w}_K]$ is the $N \times K$ matrix of the receive filters, and $\mathbf{r}$ and $\mathbf{w}_j$ are given in (7) and (10), respectively. Using the Lagrange multiplier method again, the objective function in the preceding optimization problem can be written as

$$L = E\left\{\left\|\mathbf{b}(n) - \mathbf{W}^H\mathbf{r}\right\|^2\right\} + \sum_{j=1}^{K} \mu_j \left(\|\mathbf{c}_j\|^2 - \kappa_j\right). \tag{21}$$

As in local adaptation, the solution for (21) can be obtained using either the forward–backward or the joint method.

In the forward–backward method, if the receive filters are assumed optimized (i.e., $\mathbf{w}_j$, $j = 1, 2, \ldots, K$, is fixed), one obtains the optimal $\mathbf{c}_j$ to minimize (21) as

$$\mathbf{c}_j = \left(\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{W}\mathbf{W}^H\mathbf{H}_j\mathbf{P}_j + \mu_j\mathbf{I}\right)^{-1}\mathbf{H}_j^H\mathbf{P}_j^H\mathbf{w}_j. \tag{22}$$

As with local adaptation, one iterates updating the transmitter in (22) and the receiver in (10) until the TMSE is minimized. It should be noted, however, that each update of $\mathbf{c}_j$ should satisfy the power constraint. This can be achieved by properly adjusting the Lagrange multipliers via a numerical search algorithm.

It should be noted that, to obtain the optimal carrier amplitude vector as in (22), it requires the knowledge of the receive filters as well as the channel information of all the users. This is in contrast to the optimal carrier amplitude vector given in (17) for local adaptation, where it requires only the knowledge of the desired users' channel information and the receive filter. Furthermore, the convergence of this method is slower than that of the forward–backward method in local adaptation. This is because this method needs to process the information of all other users, as shown in (22).

As with local adaptation, the optimal carrier amplitude vector can also be obtained using the joint method. In this case, instead of minimizing TMSE, maximizing the total SINR is considered to be the performance criterion because the related derivation is somewhat simpler. The optimization problem can be formulated as follows:

$$\max_{\mathbf{c}_j} \quad \mathrm{TSNIR} = \sum_{j=1}^{K} \mathrm{SINR}_j$$
$$= \sum_{j=1}^{K} \mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j \tag{23}$$
$$\text{subject to} \quad \|\mathbf{c}_j\|^2 \le \kappa_j, \qquad j = 1, 2, \ldots, K. \tag{24}$$

The Lagrange function L is then given by

$$L = \sum_{j=1}^{K} \mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_j^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j + \sum_{j=1}^{K} \mu_j \left(\|\mathbf{c}_j\|^2 - \kappa_j\right) \tag{25}$$

where μ_j is the Lagrange multiplier. The derivative of L with respect to $\mathbf{c}_j$ can be obtained as follows: Observe that $\mathbf{R}_j^{-1}$, where $\mathbf{R}_j$ is defined in (12), is not a function of $\mathbf{c}_j$, whereas $\mathbf{R}_k^{-1}$ (for $k \neq j$) can be expressed explicitly as a function of $\mathbf{c}_j$ using the matrix inversion lemma[1] [24] as follows:

$$\mathbf{R}_k^{-1} = \left(\mathbf{R}_{k,j} + \mathbf{H}_j\mathbf{P}_j\mathbf{c}_j\mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\right)^{-1}$$
$$= \mathbf{R}_{k,j}^{-1} - \frac{1}{1 + \mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j}$$
$$\times \mathbf{R}_{k,j}^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j\mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_{k,j}^{-1} \tag{26}$$

where

$$\mathbf{R}_{k,j} = \sum_{\substack{i=1\\ i\neq j,k}}^{K} \mathbf{H}_i\mathbf{P}_i\mathbf{c}_i\mathbf{c}_i^H\mathbf{P}_i^H\mathbf{H}_i^H + \sigma^2\mathbf{I}. \tag{27}$$

Therefore, the derivative of L with respect to $\mathbf{c}_j$ is given by

$$\frac{\partial L}{\partial \mathbf{c}_j} = 2\left\{1 - \sum_{\substack{k=1\\ k\neq j}}^{K} \frac{\eta_{k,k}}{(1+\eta_{j,k})^2}\right\}\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j + 2\mu_j\mathbf{c}_j \tag{28}$$

where $\eta_{j,k} = \mathbf{c}_j^H\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{H}_j\mathbf{P}_j\mathbf{c}_j$. Setting $\partial L/\partial \mathbf{c}_j = 0$ gives the following condition for the optimal carrier amplitude vector:

$$\mathbf{M}_j\mathbf{c}_j = \nu\mathbf{c}_j \tag{29}$$

where

$$\mathbf{M}_j = \left\{1 - \sum_{\substack{k=1\\ k\neq j}}^{K} \frac{\eta_{k,k}}{(1+\eta_{j,k})^2}\right\}\mathbf{P}_j^H\mathbf{H}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{H}_j\mathbf{P}_j \tag{30}$$

[1] If $\mathbf{B} = (\mathbf{A} + \mathbf{XRY})$, then the inverse of $\mathbf{B}$ is given by $\mathbf{B}^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{X}(\mathbf{R}^{-1} + \mathbf{Y}\mathbf{A}^{-1}\mathbf{X})^{-1}\mathbf{Y}\mathbf{A}^{-1}$.

and $\nu = -\mu_j$. The optimal $\mathbf{c}_j$ is therefore the eigenvector of matrix $\mathbf{M}_j$, which corresponds to the maximum eigenvalue of $\mathbf{M}_j$. Although solving (29) might look straightforward, it is complicated by the fact that $\mathbf{M}_j$ depends on $\mathbf{c}_j$ through $\eta_{j,k}$.

Instead of solving (23) directly, an iterative approach suggested in [18] can be used to seek a stationary point of the Lagrange function (25). Specifically, at each step, update $\mathbf{c}_j$ and μ_j according to the following relationships:

$$\mathbf{c}_j \quad \leftarrow \quad \mathbf{c}_j - \epsilon \frac{\partial L}{\partial \mathbf{c}_j} \tag{31}$$

$$\mu_j \quad \leftarrow \quad \mu_j + \epsilon \left(\|\mathbf{c}_j\|^2 - \kappa_j \right) \tag{32}$$

where ϵ is a parameter that can be numerically chosen so that $\mathbf{c}_j$ satisfies the power constraint at each update. A gradient descent algorithm is used to update $\mathbf{c}_j$, whereas a gradient ascent algorithm can be used to update μ_j. Equations (31) and (32) are repeated until the TSINR is maximized. It should be pointed out that the computational complexity of the joint method in global optimization is considerably higher than the forward–backward method in global optimization. Consequently, results are only obtained with the forward–backward method for performance comparison in the next section.

C. Multiuser Adaptation

In multiuser adaptation, a group of users is allowed to adapt their carrier amplitudes simultaneously. This can be performed iteratively by assuming that each user implements its own receiver or noniteratively using a multiuser detector.

The iterative update can be performed with either global or local adaptation. In both adaptation strategies, the forward–backward and the joint methods lead to two different iterative algorithms to seek for the optimum points for multiuser adaptation [17]. Assume that the users' carrier amplitude vectors are assigned initially as $\mathbf{c}_1 = \mathbf{c}_2 \cdots = \mathbf{c}_j = \cdots = \mathbf{c}_K$ with $c_j^{(m)} = 1/\sqrt{N}$, $j = 1, 2 \ldots, K$. In global adaptation, the forward–backward method implies that all the receive filters are optimized according to (10) in the forward step, followed by the optimization of the carrier amplitude vectors of all the users as in (22) in the backward step, and so on. This updating procedure is referred to as horizontal optimization because all the receivers in the forward step or all the transmitters in the backward step are lined up horizontally. On the other hand, in the joint method, (29) can be applied successively across all users, which can be referred to as vertical or user-by-user optimization. This is because the transmitter and the receiver for a particular user are lined up vertically. In either case, the TMSE must converge. The horizontal and vertical optimizations can also be applied by iterating the optimal conditions obtained in local adaptation across all the users. In this case, it is shown in [13] and [17] that the convergence to a fixed point is more difficult to establish. Due to this difficulty, this algorithm shall not be considered for multiuser adaptation in this paper.

The preceding discussion shows that the iterative algorithm takes time to converge. In addition, its computational complexity is quite high. For example, consider horizontal optimization. In each backward step, one needs to compute the carrier amplitude vector for every user. Computing the carrier amplitude vector for a specific user is time consuming because one needs to obtain the Lagrange multiplier numerically in order to satisfy the power constraint of that particular user. Furthermore, this procedure needs to be repeated for all users.

Rather than using the horizontal or vertical iterative algorithm across all the users, one can obtain the carrier amplitude vectors of all the users in one step by performing multiuser detection and jointly optimizing the carrier amplitude vectors of all the users. This noniterative method is described in the following. The optimization problem now can be formulated as

$$\begin{aligned} \min_{\mathbf{C}} \quad & \text{TMSE} = E\left\{ \left\| \mathbf{b}(n) - \mathbf{W}^H \mathbf{r} \right\|^2 \right\} \\ \text{subject to} \quad & \|\mathbf{c}_j\|^2 \le \kappa_j, \quad j = 1, 2, \ldots, K \end{aligned} \tag{33}$$

where $\mathbf{W} = \mathbf{R}^{-1}\mathbf{HPC}$ is the optimal weight matrix of the MMSE multiuser receiver, $\mathbf{H} = [\mathbf{H}_1, \ldots, \mathbf{H}_k, \ldots, \mathbf{H}_K]$ is an $N \times NK$ channel matrix of all users in the group, $\mathbf{P} = \text{diag}(\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_K)$ is an $NK \times NK$ block-diagonal matrix formed by grouping all the K users' carrier phases together, and $\mathbf{C} = \text{diag}(\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_K)$ is the $NK \times K$ block-diagonal matrix containing the carrier amplitudes of all the users in the group. Note that the formulation of this optimization problem is different from (20).

Let $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$ be the singular-value decomposition of matrix $\mathbf{H}$, where $\mathbf{V}$ and $\mathbf{U}$ are $NK \times NK$ and $N \times N$ unitary matrices, respectively. It is assumed that the singular values α_n are arranged in descending order in the diagonal matrix Σ, which takes the form

$$\Sigma = \begin{bmatrix} \alpha_1 & & & & \\ & \ddots & & \mathbf{0} & \\ & & \alpha_n & & \\ & \mathbf{0} & & \ddots & \\ & & & & \alpha_N \end{bmatrix}. \tag{34}$$

It is assumed that $\mathbf{H}$ is of full rank, i.e., $\alpha_n > 0$ for $n = 1, 2, \ldots, N$. For simplicity, assume also that the system is fully loaded, i.e., $K = N$. Let $\beta_1, \beta_2, \ldots, \beta_n, \ldots, \beta_N$ ($\beta_n > 0, \quad n = 1, 2, \ldots, N$) be the eigenvalues of $\mathbf{F}^H\mathbf{F}$ arranged in descending order, where $\mathbf{F} = (\mathbf{H}^H\mathbf{H})^{1/2}\mathbf{A}/\sigma$ and $\mathbf{A} = \mathbf{PC}$. Then, (33) can be simplified to

$$\begin{aligned} \min_{\beta_n} \quad & \text{TMSE} = \sum_{n=1}^{N} \frac{1}{\beta_n + 1} \\ \text{subject to} \quad & \sum_{n=1}^{N} \frac{\beta_n}{\alpha_n^2} \le \frac{\kappa}{\sigma^2}. \end{aligned} \tag{35}$$

The proof that (33) is equivalent to (35) is provided in Appendix I. Solving (35) and combining the solution with (33), the optimal carrier amplitude matrix that achieves the minimum TMSE is given by the following proposition:

Proposition 1: Given K, N, σ, and $\mathbf{H}$, let $\mathbf{W}$ be a $K \times K$ unitary matrix such that

$$\operatorname{diag}\left(\mathbf{W}\operatorname{diag}\left(\frac{\beta_1}{\alpha_1^2}, \frac{\beta_2}{\alpha_2^2}, \ldots, \frac{\beta_N}{\alpha_N^2}\right)\mathbf{W}^H\right) = \operatorname{diag}(\kappa_1, \kappa_2, \ldots, \kappa_K). \quad (36)$$

Then, the optimal carrier amplitude matrix is

$$\mathbf{C} = \sigma\mathbf{P}^H\mathbf{V}\begin{bmatrix} \frac{\sqrt{\beta_1}}{\alpha_1} & & & & \\ & \ddots & & \mathbf{0} & \\ & & \frac{\sqrt{\beta_n}}{\alpha_n} & & \\ & \mathbf{0} & & \ddots & \\ & & & & \frac{\sqrt{\beta_N}}{\alpha_N} \end{bmatrix}\mathbf{W}^H \quad (37)$$

where

$$\beta_n = \alpha_n \left[\frac{\frac{\kappa}{\sigma^2} + \sum_{l=1}^{M}\frac{1}{\alpha_l^2}}{\sum_{l=1}^{N}\frac{1}{\alpha_l}}\right] - 1. \quad (38)$$

The proof of the above proposition is given in Appendix II.

The results provided by Proposition 1 implies the following: If one thinks about the eigenvectors of the matrix $\mathbf{H}^H\mathbf{H}$ as the vectors that define a space spanned by the transmitted signals of the MC-CDMA systems having the channel matrix $\mathbf{H}$, the selection of the preceding optimal carrier amplitude matrix $\mathbf{C}$ chooses good channels (corresponding to large eigenvalues) while discarding the bad channels (with small eigenvalues). In the absence of fading (such as AWGN channels), the channel matrix is an identity matrix, i.e., $\mathbf{H} = \mathbf{I}$, which implies that the channel is equally good. In this case, carrier amplitudes are evenly occupied by the whole channel space (see Appendix III), which is consistent with the results obtained in [18].

IV. Performance Comparisons

A. SINR Performance

To investigate the performance of the proposed ACI-MC-CDMA systems with various adaptation algorithms presented in the previous section, consider a system with $N = 16$ carriers. The channel gains $h_k^{(m)}$, $k = 1, 2, \ldots, K$ and $m = 0, 1, \ldots, N-1$ are normalized so that the average power is unity, i.e., $E\{\|\mathbf{H}_k\|^2\} = 1$. A power constraint $\|\mathbf{c}_k\| = 1$ is set for all users $k = 1, 2, \ldots, K$.

First, the performance of single-user adaptation is considered. Fig. 2 illustrates the convergence of SINR at the output of the receive filter for the first user and for different adaptation algorithms. Here, the number of users in the system is $K = 8$, and the signal-to-noise ratio (SNR) is set at 16 dB. The first user is assumed to be the desired user, whereas other users are treated as interference users. Observe that, as the number of iterations increases, the performance of the forward–backward method in local adaptation converges to that of the joint method. Furthermore, it is shown that the forward–backward method in global adaptation needs more iterations to converge than local adaptation. The results of the joint method in global adaptation is not presented here because, as explained in Section III, this method is much more complicated.

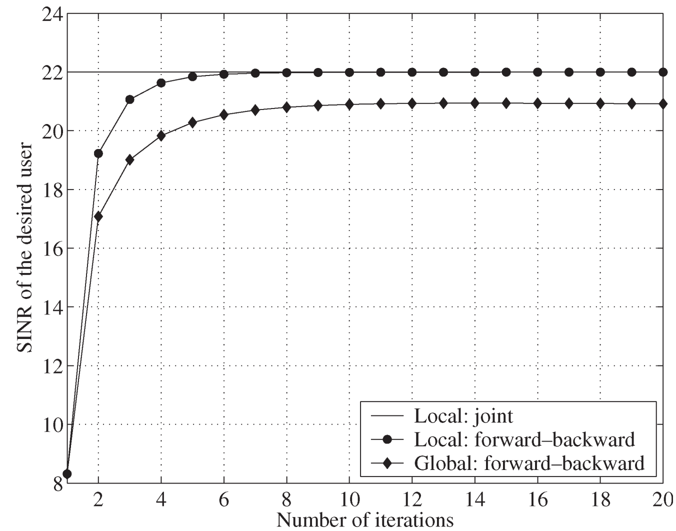

Fig. 2. Convergence of the SINR with single-user adaption ($N = 16$, $K = 8$, and SNR $= 16$ dB).

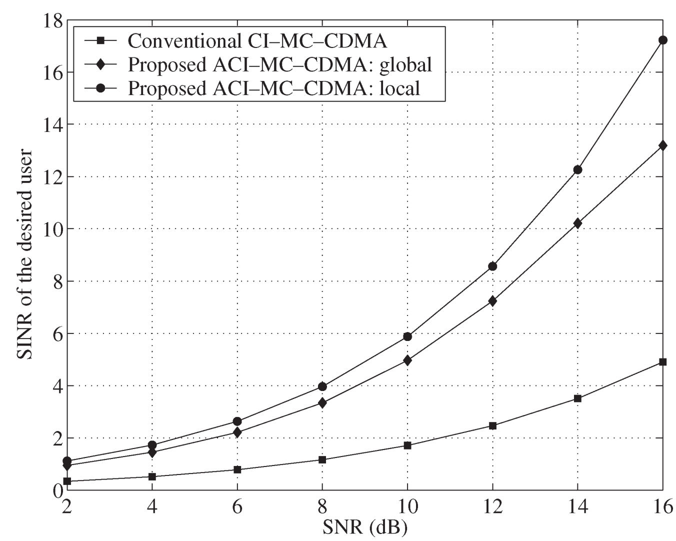

Fig. 3. Performance for single-user adaptation ($N = 16$ and $K = 16$).

Fig. 3 shows the SINR performance of the first user versus the channel SNR for the case $K = 16$. As expected, local adaptation performs better than global adaptation. This is because global adaptation takes the performance of other users into account, which compromises the performance of the desired user. In addition, it was observed before that global adaptation needs more computation time to converge. Performance comparison of the proposed ACI-MC-CDMA and the conventional CI-MC-CDMA in [11] clearly shows that ACI-MC-CDMA outperforms CI-MC-CDMA. For example, at SNR $= 10$ dB,

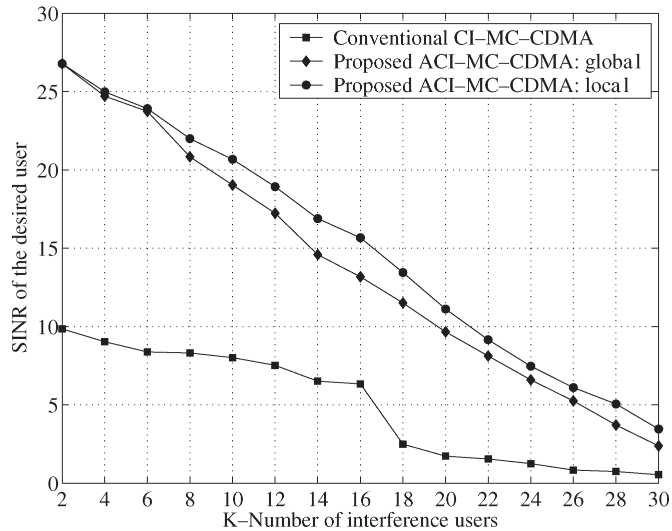

Fig. 4. Influence of the number of interference users in single adaptation ($N = 16$ and SNR $= 16$ dB).

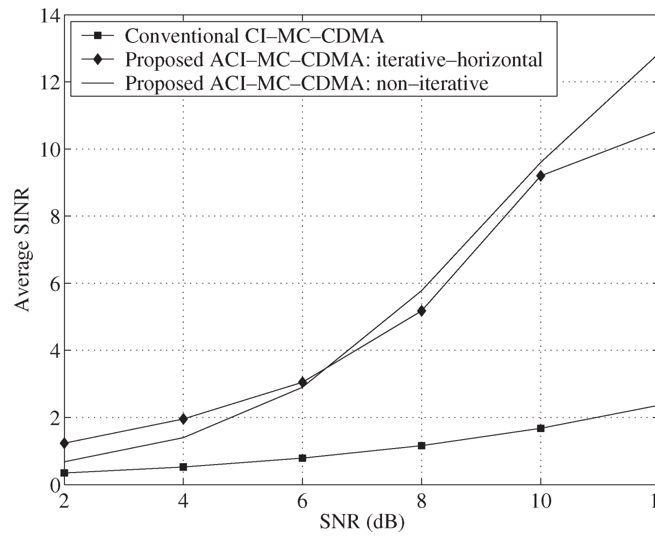

Fig. 5. Performance for multiuser adaptation ($N = 16$ and $K = 16$).

local adaptation in ACI-MC-CDMA provides a gain of about 5.0 dB in SINR over the conventional CI-MC-CDMA. Although global adaptation is unable to achieve the same performance, it still offers a 4.0-dB gain over the conventional CI-MC-CDMA.

Fig. 4 also demonstrates the performance of various systems as a function of the number of users K. As the number of users increases, there is a performance degradation for all systems due to the increase in multiuser interference. Observe from this figure that global adaptation performs very close to local adaptation for small number of users. When K becomes larger, there is a clear performance gap between the two adaptation strategies. This again is due to the fact that global adaptation takes the performance of other users into account, which compromises the performance of the desired user.

In addition, observe that the performance of the conventional CI-MC-CDMA decreases slowly when K increases up to 16. However, there is an abrupt performance degradation once K increases beyond 16. This observation is consistent with the fact that the CI codes are orthogonal when $K \leq N$, whereas the CI codes are pseudoorthogonal for $K > N$. This observation is also in contrast to the proposed ACI-MC-CDMA where, for both the local and global adaptations, the performance degradation is more graceful when K increases.

Finally, the performance of multiuser adaptation is presented in Fig. 5. Because multiuser adaptation is considered, the average SINR is the performance measure used in this figure. Similar to single-user adaptation, a substantial performance gain is also obtained by the proposed ACI-MC-CDMA using multiuser adaptation over the conventional CI-MC-CDMA. Moreover, it is shown in this figure that the performance of multiuser adaptation using the noniterative method leads to a noticeable performance enhancement compared to the iterative method using horizontal optimization, especially when SNR $\geq$ 10 dB. For example, at SNR $= 12$ dB, the performance gap between the noniterative method and the iterative method using horizontal optimization is about 0.86 dB, whereas there is a 7.4-dB gain that is achieved by the multiuser adaptation with the noniterative method over the conventional CI-MC-CDMA. It should be mentioned here again that the computational complexity of iterative algorithm is higher than that of the proposed noniterative method.

In this section, one has demonstrated that the proposed ACI-MC-CDMA systems outperform the conventional CI-MC-CDMA. However, it is important to keep in mind that such performance improvement comes at the expense of a higher system complexity, which depends on the particular adaptation strategy chosen.

B. Peak-to-Average Power Ratio (PAPR) Performance

One common problem regarding the use of MC-CDMA with CI codes is the high PAPR [25]. High peaks in the power result from highly fluctuating envelopes, which are a consequence of using independently modulated carriers. This, in turn, leads to an inefficient operation of the transmit power amplifier because an increased signal dynamic range requires power amplifiers with a greater linear region of operation.

The signal envelope compactness can be measured using the so-called crest factor (CF) [26], [27], which relates to the PAPR as

$$\text{CF} = \sqrt{\text{PAPR}} = \frac{\|u(t)\|_\infty}{\|u(t)\|_2} \tag{39}$$

where $u(t)$ is the multicarrier signal, $\|u(t)\|_\infty$ corresponds to the maximum absolute value of $u(t)$, and $\|u(t)\|_2$ is the root-mean-square (rms) value of $u(t)$. For the uplink, $u(t) = b_k(n)s_k(t) = s_k(t)$, and for the downlink, $u(t) = \sum_{k=0}^{K-1} b_k(n)s_k(t)$, where $s_k(t)$ is the signature waveform of the kth user defined in (3). It then follows that the PAPR of an uplink system directly depends only on the amplitudes of the carriers. On the other hand, the PAPR of a downlink system depends on the amplitudes of the carriers as well as the users' transmitted bits.

In CI-MC-CDMA systems, the amplitudes of all the carriers are the same (i.e., $c_k^{(m)} = 1, \forall m$). Hence, the amplitude of the signature waveform of each user becomes N. This is the reason why a CI-MC-CDMA system produces a higher CF for the uplink. Although the signature waveform of an individual user has a poor CF, it is shown in [28] that the combined signal in the downlink improves the CF tremendously. Such a reduction in the CF is due to the averaging effect of the users' random transmitted information bits. Furthermore, in [29], the CF in the downlink of a fully loaded CI-MC-CDMA system was theoretically analyzed. It is proved in [29] that the peak behavior of a fully loaded CI-MC-CDMA system is better than the traditional MC-CDMA.

More recently, Natarajan and Nassar [25] show that when $K < N$, the downlink CF degrades gradually and approaches the uplink CF as K tends to 1. Furthermore, it is demonstrated that the CFs of a fully loaded CI-MC-CDMA uplink system as well as a partially loaded CI-MC-CDMA downlink system can be brought to very low values by applying Schroeder's simple CF reduction technique [30].

Specifically, the CF reduction technique used in [25] introduces a phase correction into each carrier at the transmitter side. Consequently, the amplitudes of carriers are allowed to be different and can be positive or negative. This modification of the carrier amplitudes helps to reduce the amplitudes of the signature waveforms compared to that of the traditional CI-MC-CDMA and as a direct consequence, produces a low CF.

Although the ACI-MC-CDMA system is proposed to primarily achieve a better system performance in terms of the SINR, it shall be demonstrated in this subsection that the proposed ACI-MC-CDMA possesses a second desirable property, namely, a low CF. In ACI-MC-CDMA, as explained in the previous sections, the carrier amplitudes $c_k^{(m)}$ are allowed to vary. Therefore, unlike CI-MC-CDMA but similar to the phase-corrected CI-MC-CDMA, the carriers have different amplitudes and different signs. Note, however, that rather than varying the carrier amplitudes based on a fixed (and optimal) set of Schroeder codes (usually found by a computer search) as in the phased-corrected CI-MC-CDMA, the carrier amplitudes are varied adaptively in ACI-MC-CDMA according to the channel conditions.

From the preceding discussion, it is reasonable to predict that the CF of the proposed ACI-MC-CDMA is lower than that of the conventional CI-MC-CDMA, but it might be higher than the CF of the phase-corrected CI-MC-CDMA. Such a prediction is confirmed by the numerical results in the following paragraphs.

Fig. 6 shows the CF levels of the uplink ACI-MC-CDMA with local adaptation over 100 000 transmissions. The results were obtained with $N = 16$ carriers and for SNR $= 4$ [Fig. 6(a)] and 16 dB [Fig. 6(b)]. Notice in Fig. 6(a) that most of the CF values stay closer to the mean CF level of 1.62, and some values exceed 2.2 (a typical acceptable CF value [25]). However, no CF values are higher than 3.47. Similarly, for SNR $= 16$ dB, Fig. 6(b) displays no CF values above 4.35, and most of the CF values stay closer to the mean CF level of 1.89. Compared to the case when SNR $= 4$ dB, the percentage of the CF values exceeding 2.2 is higher for SNR $= 16$ dB. Although there is a small variation in CF levels between SNRs of 4 and 16 dB, the proposed ACI-MC-CDMA generally has much lower CF values compared to the conventional CI-MC-CDMA system, where the CF level is 4.00 for $N = 16$ carriers.

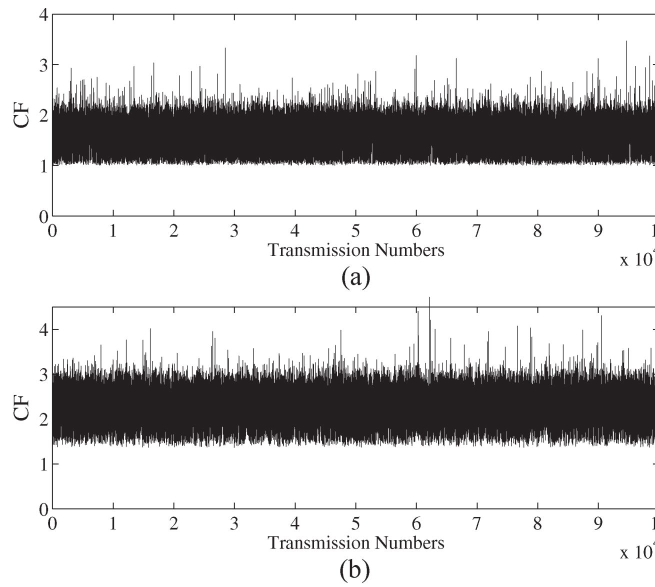

Fig. 6. CF per transmission with local adaptation for (a) SNR $= 4$ dB and (b) SNR $= 16$ dB ($N = 16$).

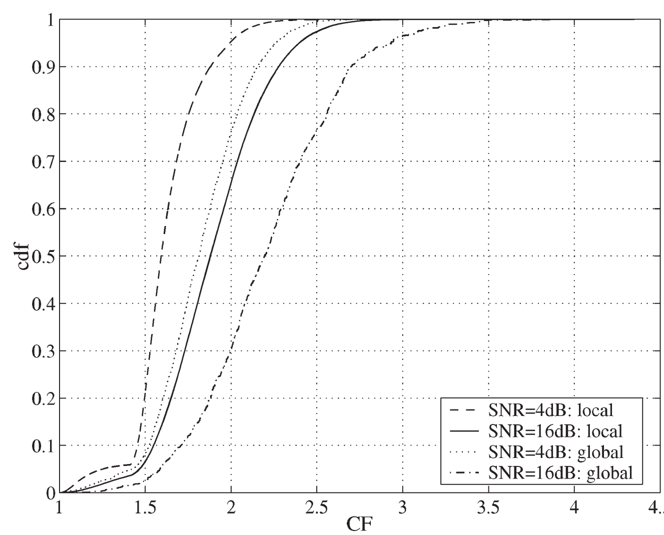

Fig. 7. CDFs of CF with different SNRs and different adaptation strategies ($N = 16$).

The cumulative density functions (CDFs) of CF with different adaptation strategies are illustrated in Fig. 7 for $N = 16$ carriers. Similar to the SINR performance, it is observed that the CF performance with local adaptation is better than that that with global adaptation. More specifically, as shown in Fig. 7, with local adaptation, the probability of CF ≥ 2.2 is less than 3% and 15% for SNR $= 4$ dB and SNR $= 16$ dB, respectively. On the other hand, with global adaptation, the probability of CF ≥ 2.2 is less than 10% for SNR $= 4$ dB and 40% for SNR $= 16$ dB, respectively.

TABLE I
UPLINK $\sqrt{\text{PAPR}}$ VALUES FOR DIFFERENT NUMBERS OF CARRIERS N

N	$CF = \sqrt{PAPR}$					
	CI-MC-CDMA		ACI-MC-CDMA			
	Conventional	Phase-Corrected	Local		Global	
			SNR=4dB	SNR=16dB	SNR=4dB	SNR=16dB
8	2.82	1.36	1.68	1.88	1.88	2.12
16	4.00	1.37	1.62	1.89	1.82	2.21
32	5.65	1.37	1.58	1.85	1.71	2.30

Table I summarizes and compares the (mean) CF values of different MC-CDMA schemes in the uplink and for various number of carriers. The CF values of the conventional CI-MC-CDMA system can be computed analytically, whereas the CF values of the phase-corrected CI-MC-CDMA system are taken from [25]. Observe from Table I that the CF values of the conventional CI-MC-CDMA increases with increasing N and are the highest compared to the CF values of the other two schemes. Although the CF values of the ACI-MC-CDMA are higher than that of the phase-corrected CI-MC-CDMA, they are well within the tolerable levels of power amplifiers. Similar to the phase-corrected CI-MC-CDMA, the CF values of the ACI-MC-CDMA remain almost constant regardless of the number of carriers. However, different from both the conventional and phase-corrected CI-MC-CDMA, the CF values of the ACI-MC-CDMA slightly increase with the increasing channel SNR. This is, of course, the direct consequence of adapting the carrier amplitudes in accordance with the channel condition in ACI-MC-CDMA.

V. CONCLUSION

ACI-MC-CDMA systems have been proposed and studied in this paper. The novelty in the proposed systems is that the amplitudes of the subcarriers are allowed to change according to the channel conditions. Two adaptive strategies, namely 1) local adaptation and 2) global adaptation, were presented to update the carrier amplitudes. In multiuser adaptations, the proposed noniterative algorithm performs slightly worse than the iterative algorithm at low channel SNR, but it gives a considerable performance gain at high channel SNR. In single-user adaptation, the local adaptive algorithm performs much better than the global adaptive algorithm. Other than having different performances, each of the considered algorithms has its own advantages and disadvantages in terms of implementation. Which adaptation strategy to use therefore depends on the particular application. More importantly, numerical results show that there is a considerable performance gain provided by the proposed ACI-MC-CDMA over the conventional CI-MC-CDMA considered in [11]. In addition to this, it has also been shown that the proposed ACI-MC-CDMA is also attractive in terms of PAPR reduction.

APPENDIX I
PROOF THAT (33) IS EQUIVALENT TO (35)

With the optimal weight matrix $\mathbf{W} = \mathbf{R}^{-1}\mathbf{HPC}$, the TMSE can be simplified to

$$\text{TMSE} = \text{tr}\{\mathbf{I} - \tilde{\mathbf{C}}^H\mathbf{R}^{-1}\tilde{\mathbf{C}}\} \tag{40}$$

where $\mathbf{R} = \tilde{\mathbf{C}}\tilde{\mathbf{C}}^H + \sigma^2\mathbf{I}$ is the receiver correlation matrix and $\tilde{\mathbf{C}} = \mathbf{HPC}$. Note that

$$\begin{aligned}\tilde{\mathbf{C}}^H\mathbf{R}^{-1}\tilde{\mathbf{C}} &= \tilde{\mathbf{C}}^H[\tilde{\mathbf{C}}\tilde{\mathbf{C}}^H + \sigma^2\mathbf{I}]^{-1}\tilde{\mathbf{C}} \\ &= \mathbf{A}^H\mathbf{H}^H[\mathbf{HAA}^H\mathbf{H}^H + \sigma^2\mathbf{I}]\mathbf{HA} \\ &= \frac{\mathbf{A}^H\mathbf{H}^H}{\sigma}\left[\frac{\mathbf{HA}}{\sigma}\frac{\mathbf{A}^H\mathbf{H}^H}{\sigma} + \mathbf{I}\right]^{-1}\frac{\mathbf{HA}}{\sigma} \\ &= \mathbf{I} - \left[\mathbf{I} + \frac{\mathbf{A}^H\mathbf{H}^H\mathbf{HA}}{\sigma^2}\right]^{-1} \\ &= \mathbf{I} - [\mathbf{I} + \mathbf{F}^H\mathbf{F}]^{-1}\end{aligned} \tag{41}$$

where the relationship $\mathbf{X}^H[\mathbf{XX}^H + \mathbf{I}]^{-1}\mathbf{X} = \mathbf{I} - [\mathbf{I} + \mathbf{X}^H\mathbf{X}]^{-1}$ is invoked in (41) and recall that $\mathbf{A} = \mathbf{PC}$. Substituting (41) into (40) yields

$$\text{TMSE} = \text{tr}\left\{[\mathbf{I} + \mathbf{F}^H\mathbf{F}]^{-1}\right\}. \tag{42}$$

Furthermore, the constraint stated in (20) can also be written in terms of matrix $\mathbf{F}$ as follows:

$$\begin{aligned}\text{diag}[\mathbf{C}^H\mathbf{C}] &= \text{diag}\left[\sigma^2\mathbf{F}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\right] \\ &\le [\kappa_1, \kappa_2, \ldots, \kappa_j, \ldots, \kappa_K]\end{aligned} \tag{43}$$

where the operator diag$[\cdot]$ takes the diagonal elements of a matrix to form a row vector. Furthermore, the inequality is interpreted as element-by-element comparisons. Hence, one can reformulate (20) in terms of matrix $\mathbf{F}$ as

$$\begin{aligned}\min_{\mathbf{C}} \quad & \text{TMSE} = \text{tr}\left\{[\mathbf{I} + \mathbf{F}^H\mathbf{F}]^{-1}\right\} \\ \text{subject to} \quad & \text{diag}\left[\sigma^2\mathbf{F}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\right] \\ & \le [\kappa_1, \kappa_2, \ldots, \kappa_j, \ldots, \kappa_K].\end{aligned} \tag{44}$$

Next, consider a related optimization problem with a weaker constraint

$$\begin{aligned}\min_{\mathbf{C}} \quad & \text{TMSE} = \text{tr}\left\{[\mathbf{I} + \mathbf{F}^H\mathbf{F}]^{-1}\right\} \\ \text{subject to} \quad & \text{tr}\left\{\sigma^2\mathbf{F}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\right\} \le \kappa\end{aligned} \tag{45}$$

where $\kappa = \sum_{j=1}^{K}\kappa_j$. With the singular value decomposition $\mathbf{H} = \mathbf{U\Sigma V}^H$ in (34), one has the following spectral decomposition of matrix $[\mathbf{H}^H\mathbf{H}]^{-1}$:

$$[\mathbf{H}^H\mathbf{H}]^{-1} = \mathbf{V}\text{diag}\left[\frac{1}{\alpha_1^2}, \frac{1}{\alpha_2^2}, \ldots, \frac{1}{\alpha_N^2}, 0, \ldots, 0\right]\mathbf{V}^H \tag{46}$$

where the eigenvalues $1/\alpha_1^2, 1/\alpha_2^2, \ldots, 1/\alpha_N^2$ are arranged in ascending order. Furthermore, let $\beta_1, \beta_2, \ldots, \ldots, \beta_N$ ($\beta_n > 0,\ n = 1, 2, \ldots, N$) be the eigenvalues of $\mathbf{F}^H\mathbf{F}$, which are arranged in descending order. By applying a lemma in [31, p. 249] to $\mathrm{tr}\{\mathbf{F}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\}$, one has

$$\mathrm{tr}\left\{\mathbf{F}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\right\} = \mathrm{tr}\left\{[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{F}\mathbf{F}^H\right\} \geq \sum_{n=1}^{N}\frac{\beta_n}{\alpha_n^2}. \quad (47)$$

Performing a similar spectral factorization gives

$$\mathrm{TMSE} = \mathrm{tr}\left\{\left[\mathbf{I}+\mathbf{F}^H\mathbf{F}\right]^{-1}\right\} = \sum_{n=1}^{N}\frac{1}{\beta_n+1}. \quad (48)$$

Therefore, the optimization problem reduces to

$$\min_{\beta_n} \quad \mathrm{TMSE} = \sum_{n=1}^{N}\frac{1}{\beta_n+1}$$
$$\text{subject to} \quad \sum_{n=1}^{N}\frac{\beta_n}{\alpha_n^2} \leq \frac{\kappa}{\sigma^2} \quad (49)$$

which is the same as that of (35).

Appendix II
Proof of Proposition 1

To solve (49), form the Lagrange function

$$L = \sum_{n=1}^{N}\frac{1}{\beta_n+1} + \mu\left(\sum_{n=1}^{N}\frac{\beta_n}{\alpha_n^2} - \frac{\kappa}{\sigma^2}\right) \quad (50)$$

where μ is the Lagrange multiplier. Taking the derivative with respect to β_n and setting it to zero yield

$$\beta_n = \pm\frac{\alpha_n}{\sqrt{\mu}} - 1. \quad (51)$$

Because $\beta_n > 0$, one selects

$$\beta_n = \frac{\alpha_n}{\sqrt{\mu}} - 1. \quad (52)$$

Selecting the Lagrange multiplier to satisfy the power constraint, the optimal eigenvalues can be found from (52) as

$$\beta_n = \alpha_n\left[\frac{\frac{\kappa}{\sigma^2} + \sum_{l=1}^{N}\frac{1}{\alpha_l^2}}{\sum_{l=1}^{N}\frac{1}{\alpha_l}}\right] - 1. \quad (53)$$

Then, it is not hard to see that the following choice of $\mathbf{F}$ attains the minimum TMSE:

$$\mathbf{F} = \mathbf{V}\begin{bmatrix} \sqrt{\beta_1} & & & & \\ & \ddots & & \mathbf{0} & \\ & & \sqrt{\beta_n} & & \\ & \mathbf{0} & & \ddots & \\ & & & & \sqrt{\beta_N} \end{bmatrix}\mathbf{W}^H. \quad (54)$$

In (54), $\mathbf{V}$ is an $NK \times NK$ unitary matrix that can be obtained from (46), and $\mathbf{W}$ is a $K \times K$ unitary matrix that can be constructed to satisfy the constraint

$$\mathrm{diag}\left[\mathbf{W}\mathrm{diag}\left(\frac{\beta_1}{\alpha_1^2}, \frac{\beta_2}{\alpha_2^2}, \ldots, \frac{\beta_N}{\alpha_N^2}\right)\mathbf{W}^H\right] = \mathrm{diag}[\kappa_1, \kappa_2, \ldots, \kappa_K]. \quad (55)$$

Therefore, with $\mathbf{C} = \sigma\mathbf{P}^H[\mathbf{H}^H\mathbf{H}]^{-1/2}\mathbf{F}$, (33) can be solved as indicated in (37).

Appendix III
Solution to the Optimization Problem in (35) for the Case of AWGN Channels

In the absence of multipath, the channel matrix is an identity matrix, i.e., $\mathbf{H} = \mathbf{I}$. In this case, (45) reduces to

$$\min_{\mathbf{C}} \quad \mathrm{TMSE} = \mathrm{tr}\left\{\left[\mathbf{I} + \frac{\mathbf{C}^H\mathbf{P}^H}{\sigma}\frac{\mathbf{PC}}{\sigma}\right]^{-1}\right\}$$
$$\text{subject to} \quad \mathrm{tr}[\mathbf{C}^H\mathbf{C}] \leq \kappa. \quad (56)$$

As shown previously, the constraint is equivalent to

$$\mathrm{tr}[\mathbf{C}^H\mathbf{C}] = \mathrm{tr}[\mathbf{C}\mathbf{C}^H] = \sum_{n=1}^{N}\beta_n \leq \kappa \quad (57)$$

where β_n, $n = 1, 2, \ldots, M$ are the eigenvalues of $\mathbf{C}^H\mathbf{C}$. Performing a similar spectral factorization, the TMSE can be written as

$$\mathrm{TMSE} = \mathrm{tr}\left\{\left[\mathbf{I} + \frac{\mathbf{C}^H\mathbf{P}^H}{\sigma}\frac{\mathbf{PC}}{\sigma}\right]^{-1}\right\} = \sum_{n=1}^{N}\frac{1}{(1/\sigma^2)\beta_n+1} = \sum_{n=1}^{N}\frac{1}{\gamma\beta_n+1} \quad (58)$$

where $\gamma = 1/\sigma^2$. Now, the problem is to find N positive eigenvalues $\{\beta_1, \beta_2, \ldots, \beta_N\}$ that minimize $\sum_{n=1}^{N}(\gamma\beta_n+1)^{-1}$ subject to $\sum_{n=1}^{N}\beta_n = \kappa$. The Lagrange method can be used to show that the optimal eigenvalues are simply equal to κ/N, i.e., $\beta_1 = \beta_2 = \cdots = \beta_N = \kappa/N$.

Thus, the minimum TMSE is obtained when $\mathbf{C}^H\mathbf{C}$ is a scalar times the identity matrix or, equivalently

$$\mathbf{C}^H\mathbf{C} = \mathbf{S}\Lambda\mathbf{S}^H = \mathbf{S}\left[(\kappa/N)\mathbf{I}_{N\times N}\right]\mathbf{S}^H \quad (59)$$

where $\mathbf{S}$ can be constructed to satisfy the following constraint:

$$\mathrm{diag}\left[\mathbf{S}\left[(\kappa/N)\mathbf{I}_{N\times N}\right]\mathbf{S}^H\right] = \mathrm{diag}[\kappa_1, \kappa_2, \ldots, \kappa_K]. \quad (60)$$

Given $\mathbf{S}$, one can construct the carrier amplitude matrix $\mathbf{C}$ as follows:

$$\mathbf{C} = \sigma\mathbf{P}^H\mathbf{V}\begin{bmatrix}(\sqrt{\kappa/N})\mathbf{I}_{N\times N} & \\ & \mathbf{0}_{NK-N\times NK-N}\end{bmatrix}\mathbf{S}^H \quad (61)$$

where $\mathbf{V}$ is an $NK \times NK$ unitary matrix constructed such that the columns of matrix $\mathbf{C}$ are orthogonal and have norm squared equal to $\kappa_1, \kappa_2, \ldots, \kappa_K$.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their helpful comments and suggestions, which improved the presentation of this paper. They would also like to thank one of the reviewers for pointing out reference [25]. This work was performed while S. Sureshkumar was visiting the Department of Electrical Engineering, University of Saskatchewan.

REFERENCES

[1] N. Yee, J. P. Linnartz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," in *Proc. IEEE Int. Symp. Pers., Indoor, Mobile Radio Commun.*, Yokohama, Japan, Sep. 1993, pp. 109–113.
[2] S. Kondo and L. B. Milstein, "Performance of multicarrier DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, no. 2, pp. 238–246, Feb. 1996.
[3] L. Vandendorpe, "Multitone spread spectrum multiple access communications system in a multipath Rician fading channel," *IEEE Trans. Veh. Technol.*, vol. 44, no. 2, pp. 327–337, May 1995.
[4] E. A. Sourour and M. Nakagawa, "Performance of orthogonal multicarrier CDMA in a multipath fading channel," *IEEE Trans. Commun.*, vol. 44, no. 3, pp. 356–367, Mar. 1996.
[5] A. Chouly, A. Brajal, and S. Jourdan, "Orthogonal multicarrier techniques applied to direct sequence spread spectrum CDMA systems," in *Proc. IEEE GLOBECOM*, Houston, TX, Nov. 1993, pp. 1723–1728.
[6] S. Hara and R. Prasad, "DC-CDMA, MC-CDMA and MT-CDMA for mobile multi-media communications," in *Proc. IEEE VTC*, Atlanta, GA, Apr. 1996, pp. 1106–1110.
[7] H. H. Nguyen, "Effect of chip shaping on the performance of band-limited multicarrier CDMA systems," *IEEE Trans. Veh. Technol.*, vol. 54, no. 3, pp. 1022–1029, May 2005.
[8] H. H. Nguyen and E. Shwedyk, "Bandwidth constrained signature waveforms for maximizing the network capacity of synchronous CDMA systems," *IEEE Trans. Commun.*, vol. 49, no. 6, pp. 961–965, Jun. 2001.
[9] ——, "Bandwidth constrained signature waveforms and the Walsh signal space receivers for synchronous CDMA systems," *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1137–1149, Jul. 2002.
[10] B. M. Popović, "Spreading sequences for multicarrier CDMA systems," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 918–926, Jun. 1999.
[11] B. Natarajan, C. R. Nassar, M. Michelini, and Z. Wu, "High-performance MC-CDMA via carrier interferometry codes," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1344–1353, Nov. 2001.
[12] T. M. Lok and T. F. Wong, "Transmitter adaptation in multicode DS-CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 1, pp. 69–82, Jan. 2001.
[13] S. Ulukus and R. D. Yates, "Iterative construction of optimum signature sequence sets in synchronous CDMA systems," *IEEE Trans. Inf. Theory*, vol. 47, no. 5, pp. 1989–1998, Jul. 2001.
[14] O. Kaya and S. Ulukus, "Jointly optimal power and signature sequence allocation for fading CDMA," in *Proc. IEEE GLOBECOM*, San Francisco, CA, Dec. 2003, pp. 1872–1876.
[15] L. Gao and T. F. Wong, "Power control and spreading sequence allocation in a CDMA forward link," *IEEE Trans. Inf. Theory*, vol. 50, no. 1, pp. 105–124, Jan. 2004.
[16] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. Inf. Theory*, vol. 45, no. 6, pp. 1984–1991, Sep. 1999.
[17] G. S. Rajappan and M. L. Honig, "Signature sequence adaptation for DS-CDMA with multipath," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 2, pp. 384–395, Feb. 2002.
[18] T. M. Lok and T. F. Wong, "Transmitter and receiver optimization in multicarrier CDMA systems," *IEEE Trans. Commun.*, vol. 48, no. 7, pp. 1197–1207, Jul. 2000.
[19] J. G. Andrews and T. H. Y. Meng, "Performance of multicarrier CDMA with successive interference cancellation in a multipath fading channel," *IEEE Trans. Commun.*, vol. 52, no. 5, pp. 811–822, May 2004.
[20] J. Jang and K. B. Lee, "Effect of frequency offset on MC/CDMA system performance," *IEEE Commun. Lett.*, vol. 3, no. 7, pp. 196–198, Jul. 1999.
[21] M. Abdel-Hafez, Z. Li, and M. Latva-aho, "On frequency domain equalization for MC-CDMA in Nakagami fading channels," *Eur. Trans. Telecommun.*, vol. 15, no. 1, pp. 7–13, Jan. 2004.
[22] S. L. Miller and B. J. Rainbolt, "MMSE detection of multicarrier CDMA," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 11, pp. 2356–2362, Nov. 2000.
[23] S. Verdu, *Multiuser Detection*, 1st ed. Cambridge, U.K.: Cambridge Univ. Press, 1998.
[24] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1999.
[25] B. Natarajan and C. R. Nassar, "Crest factor reduction in MC-CDMA employing carrier interferometry codes," *EURASIP J. Wireless Commun. Netw.*, vol. 2, no. 2, pp. 374–379, Dec. 2004.
[26] B. M. Popovic, "Synthesis of power efficient multitone signals with flat amplitude spectrum," *IEEE Trans. Commun.*, vol. 39, no. 7, pp. 1031–1033, Jul. 1991.
[27] E. V. D. Ouderaa, J. Schoukens, and J. Renneboog, "Peak factor minimization of input and output signals of linear systems," *IEEE Trans. Instrum. Meas.*, vol. 37, no. 2, pp. 207–212, Jun. 1988.
[28] B. Natarajan, "Carrier interferometry for next generation CDMA and TDMA wireless systems: A multicarrier framework," Ph.D. dissertation, Dept. of Elect. Comput. Eng., Colorado State Univ., Fort Collins, 2002.
[29] G. Wunder and K. G. Paterson, "Crest-factor analysis of carrier interferometry MC-CDMA and OFDM systems," in *Proc. Int. Symp. Inf. Theory*, Chicago, IL, Jul. 2004, p. 426.
[30] M. R. Schroeder, "Synthesis of low-peak-factor signals and binary sequences with low autocorrelation," *IEEE Trans. Inf. Theory*, vol. IT-16, no. 1, pp. 85–89, Jan. 1970.
[31] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, 1st ed. New York: Academic, 1979.

S. Sureshkumar received the B.Sc. degree from the University of Peradeniya, Peradeniya, Sri Lanka, in 1992, the M.Eng. degree from Asian Institute of Technology, Bangkok, Thailand, in 1994, and the Ph.D. degree in electrical engineering from the University of Manitoba, Winnipeg, MB, Canada, in 2005.

He joined Nokia, Espoo, Finland, in 2005. His main research interests are spread spectrum techniques and multicarrier modulations.

Ha H. Nguyen (M'01–SM'05) received the B.Eng. degree from Hanoi University of Technology, Hanoi, Vietnam, in 1995, the M.Eng. degree from Asian Institute of Technology, Bangkok, Thailand, in 1997, and the Ph.D. degree in electrical engineering from the University of Manitoba, Winnipeg, MB, Canada, in 2001.

He joined the Department of Electrical Engineering, University of Saskatchewan, Saskatoon, SK, Canada, in 2001 as an Assistant Professor and was promoted to Associate Professor in 2005. His research interests include digital communications, spread spectrum systems, and error control coding.

Dr. Nguyen is a registered Member of the Association of Professional Engineers and Geoscientists of Saskatchewan.

Ed Shwedyk received the B.Sc. and M.Sc. degrees in electrical engineering from the University of Manitoba, Winnipeg, MB, Canada, in 1965 and 1968, respectively, and the Ph.D. degree in electrical engineering from the University of New Brunswick, Fredericton, NB, Canada, in 1974.

Since 1974, he has been with the Department of Electrical and Computer Engineering, University of Manitoba, where he is currently a Full Professor. His research interests are digital communications, principally estimation and detection, and biosignal processing.