

# Frame synchronization, channel estimation scheme and signal compensation using regression method in OFDM systems

Jyh-Horng Wen<sup>a</sup>, Gwo-Ruey Lee<sup>b,\*</sup>, Jia-Wei Liu<sup>c</sup>, Te-Lung Kung<sup>b</sup>

<sup>a</sup> Department of Electrical Engineering, Tunghai University No. 181, Section 3, Taichung Harbor Road, Taichung, Taiwan, ROC

<sup>b</sup> Department of Electrical Engineering, National Chung Cheng University No. 168, University Road, Min-Hsiung ChiaYi, Taiwan, ROC

<sup>c</sup> Department of Communication Engineering, National Chung Cheng University No. 168, University Road, Min-Hsiung ChiaYi, Taiwan, ROC

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## Abstract

In this paper, a joint frame synchronization and channel estimation scheme using the regression method is proposed. Based on an OFDM frame format with adding the pseudo-noise code, the frame timing and the channel impulse response could be obtained by the proposed scheme. With the estimated channel information, a frequency domain one-tap equalizer is presented to achieve the signal compensation. The simulation results show the missing probability of frame start with the proposed algorithm performs better than that with the moving average scheme and Beek's scheme under the Rayleigh channel. Also, the frame timing could be exactly obtained with the proposed schemes at SNR is large than  $-10$  dB. Besides, the estimator mean square error with the proposed channel estimation scheme is better than that with the least squared scheme and minimum mean square error scheme. With the estimated channel response, the signal could be compensated by the one-tap equalizer and the BER performance is shown in this paper.

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**Keywords:** OFDM systems; Frame synchronization; Channel estimation; Regression method

## 1. Introduction

The significant technology, orthogonal frequency division multiplexing (OFDM), with the capacity for a high data-rate transmission has been applied into many digital transmission systems such as digital audio broadcasting (DAB) system, digital video broadcasting terrestrial TV (DVB-T) system, asymmetric digital subscriber line (ADSL), wireless local area network (WLAN), broadband wireless access (BWA) network, worldwide interoperability for microwave access (WiMAX) systems and ultra-wide-band (UWB) systems [1–4]. With a high data transmission capacity, the advantages of OFDM are robust to against the selective fading effects, to combat inter-symbol interference (ISI) and so on [1–4].

However, the knowledge of symbol timing and channel estimation is required for signal demodulation in the OFDM systems. At the receiver, the symbol boundaries and the optimal timing instants are required to minimize the effects of ISI [1,5]. In the OFDM systems, both data-aided and non-data-aided synchronization algorithms have been proposed previously [6–9]. Within the non-data-aided algorithms, the cyclic property of the guard interval could be employed for the symbol synchronization without any training symbol in the OFDM systems. Among those non-data-aided algorithms, the estimator exploits the second-order cyclostationarity of the received signals and, then, it obtains the information of symbol-timing offset by the cyclic correlation [6]. On the other hand, the maximum likelihood (ML) estimator proposed by J.J. Van de Beek applies the correlation between the cyclic prefix and the OFDM symbol to find the symbol timing [7]. It uses the redundant information contained within the cyclic prefix. The results show that the Beek's estimator could have a lower error variance when the number of cyclic prefix sam-

\* Corresponding author. Tel.: +886 5 2720411 23240; fax: +886 5 2720862.

E-mail address: [raye@wireless.eccu.edu.tw](mailto:raye@wireless.eccu.edu.tw) (G.-R. Lee).

ples is larger. Within the data-aided algorithms, at the receiver, the symbol synchronization could be implemented with the aid of the dedicated training symbols or pilot symbols [8,9]. The frame synchronization using the designed training symbol has been proposed. Reliable frame synchronization could be obtained under a low signal-to-noise (SNR) environment. Based on the pseudo-noise (PN) sequence preambles, OFDM synchronization gives a better detection in terms of the low false error and low missing error [8]. Based on the constant envelope preamble, the synchronization algorithm exploits the correlation property of the PN sequence and the two identical parts of the preamble to estimate the timing offset [9].

Channel estimation is necessary before the demodulation of OFDM received signal since the radio channel is frequency selective and time-varying for wideband mobile communication systems. With the help of channel estimation, the system performance could be improved. Both blind channel estimation scheme and data-aided channel estimation scheme are proposed. Blind channel estimation techniques try to estimate the channel information without any knowledge of the transmitted signal [10]. With the data-aided schemes, pilots or training sequences are inserted in the transmitted signal to estimate the channel response [11–15]. In pilot-based schemes, the channel estimation can be performed by either inserting pilots into all of subcarriers in OFDM symbols with a specific periodicity or inserting pilots into reserved subcarriers in each OFDM symbol [12,13]. The first method is called as block-type pilot channel estimation [11,12]. The method is developed under a slow fading channel, i.e., the channel transfer function is assumed to be not changing rapidly. Although block-type pilot channel estimation could estimate the overall channel response and provide a better performance, it suffers the bandwidth efficiency. The other is called as com-type pilot channel estimation. In the scheme, the pilots are inserted into the reserved subcarriers to obtain the partial channel response based on least square (LS), minimum mean square error (MMSE), or least mean square (LMS) algorithms [13]. The overall channel response in frequency domain can be got with the interpolation techniques such as linear interpolation, second order interpolation, low-pass interpolation and the cubic spline interpolation [11,14,15]. Training sequence-based channel estimation is widely used in packet-based communications. For example, long training symbol is used in IEEE 802.11a wireless Local area network (WLAN) standard to estimate the channel information [15,16]. In this paper, PN code is selected as the training sequence within the OFDM symbol. With applying the correlated properties of pseudo-noise code, a scheme is proposed to jointly implement the timing estimation and channel estimation. The regression method based on the minimum mean square error criterion could provide a solution to predict the symbol timing. Based on the correlation of the PN code, the scheme applying the regression method is simple to realize the frame synchronization. In addition, based on the maximum correlation of

the PN code, the channel response could be obtained with applying the proposed scheme. Hence, the scheme could jointly estimate the frame timing and channel response in the OFDM system. It is also provide a simple way to implement the frame timing estimation and channel estimation in the OFDM system. According to the estimated channel response, an equalizer could be designed to compensate the channel effect and, then, to provide a better performance. In the following section, an OFDM frame format with adding the time domain PN code and the system model are presented. In Section 3, a joint frame synchronization and channel estimation scheme is proposed to estimate the frame timing and channel response in the OFDM system. The simulation results are shown in Section 4. Finally, a conclusion is given in Section 5.

## 2. OFDM system description

Basically, in an OFDM system, it contains a series-to-parallel converter, a signal mapping scheme, a fast Fourier transform (FFT)/inverse FFT (IFFT) module [1–4]. The series-to-parallel converter is considered to realize the concept of parallel data transmission to provide a high data rate transmission.  $M$ -ary phase shift keying (PSK) or quadrature amplitude modulation (QAM) mapping scheme is modulated in each parallel subcarrier to provide a different data rate service in the system. Besides, the IFFT and FFT modules are employed to replace the banks of sinusoidal generator for the modulation and demodulation with different carrier frequencies to reduce the complexity of OFDM modem implementation [1,4]. The baseband OFDM system model is shown in Fig. 1. The serial bit stream is transformed to a parallel form. Each parallel data is mapped with the PSK or the QAM scheme and, then, those data are modulated by means of an IFFT on  $N$ -parallel subcarriers. The resulting OFDM symbols extended with a cyclic prefix and the PN code are serially transmitted over a discrete-time channel. At the receiver, based on the known PN code, the proposed algorithm using the regression method is used to achieve the frame synchronization and channel estimation. Besides, the receiver performs the inverse process of the transmitter, the received data are demodulated by a FFT. Then, based on the estimated channel response, the signal could be compensated through one-tap frequency domain equalizer. And, the parallel data are demapped with corresponding PSK or QAM scheme to obtain the estimated bit stream.

Without timing and frequency offset, the baseband discrete-time data symbol  $x(n)$  is as

$$x(n) = \sum_{k=0}^{N-1} X(k)e^{j\left(\frac{2\pi nk}{N}\right)}, \quad (1)$$

where  $N$  denotes the number of subcarriers,  $T$  denotes the symbol duration, and  $X(k)$  is the complex modulated data of  $k$ -th subcarrier. The data symbol extended with a cyclic prefix forms an OFDM symbol  $x_t(n)$ .

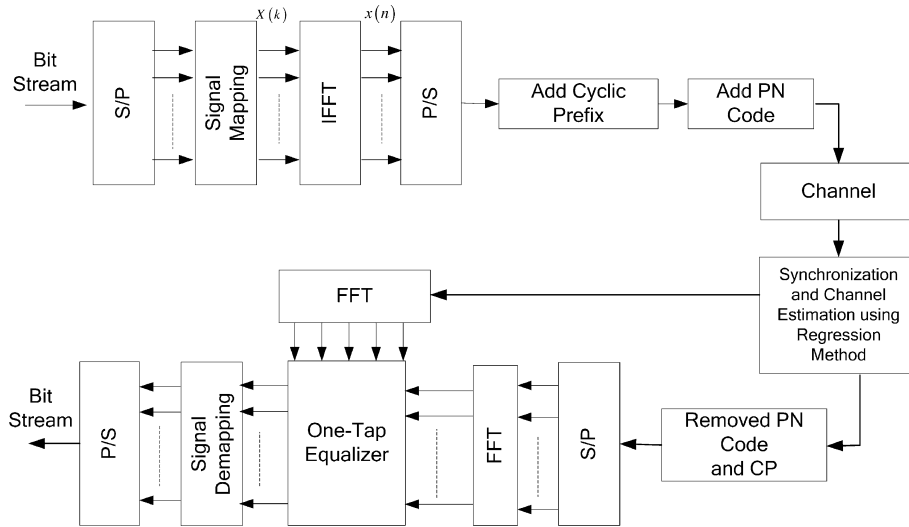


Fig. 1. The baseband OFDM system.

$$x_i(n) = \begin{cases} x(n - N_g + N), & 0 \leq n \leq N_g - 1 \\ x(n - N_g), & N_g \leq n \leq N + N_g - 1 \end{cases}, \quad (2)$$

where  $N_g$  denotes the sampling number of guard interval. The proposed frame format is shown in Fig. 2. The PN code is added in the OFDM symbol. The transmitted OFDM symbol is as

$$s(n) = \begin{cases} c(n), & 0 \leq n \leq N_c - 1 \\ x_i(n - N_c), & N_c \leq n \leq N_g + N + N_c - 1 \end{cases}, \quad (3)$$

where  $c(n)$  denotes the PN code with the chip rate  $CR$ ,  $N_c$  is the sampling number of the PN code before spreading.

Multipath is the main effect that makes the distortion of received signal in wireless communications. In this paper, the multipath channel is referred to the OFDM multipath channel model in the 802.11 wireless LAN [16]. The multipath channel is expressed as

$$h(n) = \sum_{l=1}^L h_l \delta(n - \tau_l), \quad (4)$$

where  $L$  denotes total path number,  $\tau_l$  is the delay time of  $l$ th path and  $\tau_l \neq 0$ . The channel impulse response is composed of complex samples with random uniformly distributed phase and Rayleigh distributed magnitude with average power decaying exponentially. Hence, the  $l$ th channel impulse response is given by

$$h_l = N \left( 0, \frac{1}{2} \sigma_l^2 \right) + jN \left( 0, \frac{1}{2} \sigma_l^2 \right), \quad (5)$$

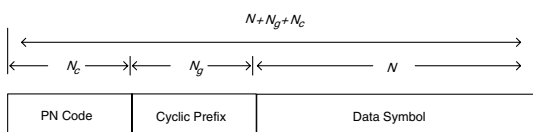


Fig. 2. Proposed frame format.

where  $N(0, \sigma_l^2/2)$  is a Gaussian random variable with zero mean and variance  $\sigma_l^2/2$ , where  $\sigma_l^2 = \sigma_0^2 \cdot e^{-l \cdot T_s/T_{RMS}}$  and  $\sigma_0^2 = 1 - e^{-T_s/T_{RMS}}$ , where  $T_s$  is the sampling period and  $T_{RMS}$  is the delay spread of the channel. Besides, the maximum delay spread value is assumed to be smaller than the cyclic prefix in this paper. Hence, the received signal  $y(n)$  could be expressed as

$$y(n) = \sum_{l=0}^{L-1} h_l s(n - \tau_l) + w(n), \quad (6)$$

where  $w(n)$  is an Gaussian noise.

Before demodulating the received signal in OFDM system, the receiver has to work on the frame synchronization, frequency synchronization and channel estimation. On the OFDM link, the orthogonality is required within the sub-carriers at the receiver. Actually, the carrier frequency synchronization algorithm in [17], for instance, could be used to compensate the effect of frequency offset. In this study, the carrier frequency synchronization is not considered. However, the frame synchronization and channel estimation have to be considered. The receiver should remove the cyclic prefix and PN code, the synchronization should be done. Once, timing information provided by the synchronization algorithm, one could exactly remove the prefix and, then, use FFT to extract the transmitted data. Besides, the channel estimation should also be done to obtain the channel impulse response. Then, with estimated channel response, the signal could be compensated by one-tap frequency domain equalizer to recover the fading effect. In the following section, a scheme with applying the regression method is proposed to estimate the frame timing and channel response.

### 3. Joint synchronization and channel estimation scheme

The regression method is a special case of the simple linear algorithm. The observations are assumed to be func-

tional related. The functional relation could be depended on one or more variables. In this paper, a linear multiple regression model is adopted. The received signal  $y$  could be expressed as

$$\underline{y}_{N \times 1} = S_{N \times L} \underline{h}_{L \times 1} + \underline{w}_{N \times 1}, \quad (7)$$

where  $\underline{y}$  denotes the vector of the received signal  $\underline{y}_{N \times 1} = [y_0 \ y_1 \ \dots \ y_{N-1}]'$ ,  $\underline{h}$  denotes the vector of the channel response  $\underline{h}_{N \times 1} = [h_0 \ h_1 \ \dots \ h_{L-1}]'$ ,  $\underline{w}$  denotes a vector of an Gaussian noise  $\underline{w}_{N \times 1} = [w_0 \ w_1 \ \dots \ w_{N-1}]'$ ,  $S$  is the matrix of the transmitted signal and  $\underline{A}'$  denotes the transpose operation of vector  $\underline{A}$ .

Based the solution in the regression model, if  $S'S$  is full rank and nonsingular, the unique solution of channel frequency response in Eq. (7) is

$$\underline{h} = (S'S)^{-1} S'y. \quad (8)$$

where  $(A)^{-}$  denotes a generalized inverse of  $A$ . If  $S'S$  is not full rank and singular, a set of solutions in Eq. (7) are

$$\underline{h} = (S'S)^{-} S'y + (I - (S'S)^{-} S'S)\underline{w}, \quad (9)$$

In this proposed scheme, with the known PN code added in the OFDM symbol, the transmitted signal could be written as

$$S = \begin{bmatrix} \underline{0}_{CR \times 1} & \underline{0}_{2 \cdot CR \times 1} & \dots & \underline{0}_{(\tau_{max}) \cdot CR \times 1} \\ \underline{c}_{N_{PN} \times 1} & \underline{c}_{(N_{PN} - CR) \times 1} & \dots & \underline{c}_{(N_{PN} - (\tau_{max}) \cdot CR) \times 1} \end{bmatrix}_{N_{PN} \times (\tau_{max} + 1)}, \quad (10)$$

where  $\underline{c}$  is a vector of the PN code, the length of PN code is  $N_{PN} = CR \cdot N_c$  chips,  $CR$  is the chip rate,  $N_c$  is the size of PN code before spreading and  $\tau_{max}$  denotes the maximum delay spread. The received signal  $\underline{Y}$  could be shown as

$$\underline{y}' = [y(m) \ y(m+1) \ y(m+2) \ \dots \ y(m+N_{PN}-1)]_{1 \times N_{PN}}, \quad m = 1, 2, \dots, N_{OB}. \quad (11)$$

where  $N_{OB}$  is the observation interval. Hence, with the Eq. (8), the vector of channel response  $\underline{h}$  could be obtained. The frame timing is chosen as the sample position with maximum absolute value among the first row of the matrix. When the frame start is found, the estimated channel impulse response is the vector located at the corresponding column. Then, at the OFDM receiver, the PN code and cyclic prefix could be retrieved in an OFDM symbol. Besides, with the estimated channel impulse response, the signal could be compensated through one-tap frequency domain equalizer.

In addition, with the known delay path time, the joint synchronization and channel estimation scheme could be simplified. Here, it is called as the joint synchronization and channel estimation scheme using conditional regression method (Cond. Regression Method). Under a two-ray channel with the known delay 0 and 0.8  $\mu$ s (in the simulation, one sample duration = 0.2  $\mu$ s), the transmitted data matrix could be designed as

$$S = \begin{bmatrix} \underline{0}_{N_{PN} \times 1} & \underline{0}_{N_{PN} \times 1} & \underline{0}_{N_{PN} \times 1} & \underline{0}_{N_{PN} \times 1} & \underline{0}_{4N_c \times 1} \\ \underline{c}_{(N_{PN} - 4N) \times 1} & & & & \end{bmatrix}. \quad (12)$$

The received signal could be expressed as

$$\underline{y}' = [y(m) \ y(m+1) \ y(m+2) \ \dots \ y(m+N_{PN}-1)]_{1 \times N_{PN}}, \quad m = 1, 2, \dots, N_{OB}. \quad (13)$$

With the Eq. (8), the vector  $\underline{h}$  could be obtained. The frame timing is chosen as the sample position with maximum absolute value among the first row of the matrix. When the frame start is found, the estimated channel impulse response is vector located at the corresponding column. Then, at the OFDM receiver, the PN code and cyclic prefix could be retrieved in an OFDM symbol. Besides, with the estimated channel impulse response, the signal could be compensated through one-tap frequency domain equalizer. Based on this proposed algorithm, the simulations are given in the following section.

#### 4. Simulation results

At the beginning, the simulations for the proposed frame synchronization scheme are performed under a two-ray Rayleigh channel (channel A). The simulation parameters are listed in Table 1. In the simulations, the number of subcarriers and the length of the cyclic prefix are assumed to be  $N = 1024$  and  $CP = 256$ , respectively. The number of samples in PN code before spreading was 35 samples with chip rate 1 or 4. In each simulation, it is assumed the carrier frequency to be 5 GHz, modulation scheme to be 16QAM, the number of pilots to be 32, the sample duration to be 0.2  $\mu$ s, the propagation delay to be 5.8  $\mu$ s (the 30th sample), the frame timing to be the position of the 31st sample in the case of chip rate 1, the frame timing to be the position of the 124th sample in the case of chip rate 4 and the period of PN code to be 2047 samples.

In Fig. 3, the performance is evaluated based on the missing probability of frame start for the Beek's estimation scheme (Beek) [7] and moving average (MA) estimation scheme [6] and proposed schemes (Regression Method). Fig. 3 shows that the missing probability of frame start

Table 1  
Simulation parameters

Parameters	Value
Modulation type	16 QAM
Required bandwidth	5 MHz
Data rate	20 Mbps
Carrier frequency	5 GHz
Number of subcarriers	1024
Sampling rate	5 MHz
Symbol duration	204.8 $\mu$ s
Samples in cyclic prefix	256
Duration of cyclic prefix	51.2 $\mu$ s
PN code	Gold code
Code period	2047
Chip rate (CR)	1 or 4
PN code size before spreading ( $N_c$ )	35
Propagation delay	5.8 $\mu$ s

with the proposed algorithm is excellent under a two-ray Rayleigh channel. In the figure, the frame timing could be exactly obtained with the proposed schemes at SNR = -10 dB.

Simulations for the proposed channel estimation scheme are performed under a four-ray (channel B) and six-ray (channel C) Rayleigh channel with maximum delay spread 5.8 μs (30 samples). In Figs. 4 and 5, the performance is evaluated based on the estimator mean square errors of overall channel response in frequency domain for least-

square (LS) and minimum mean square error (MMSE) with linear interpolation [13] and proposed schemes. The performance with the proposed scheme is better than that with the other channel estimation schemes. With the signal compensate, the bit error rate (BER) performance of the scheme with the perfect channel estimation in the OFDM symbol (shown as Perfect Est.), LS and MMSE with linear interpolation (shown as LS and MMSE), and the proposed channel estimation using regression method are shown in Figs. 6 and 7. In the figure, it shows that the BER perfor-

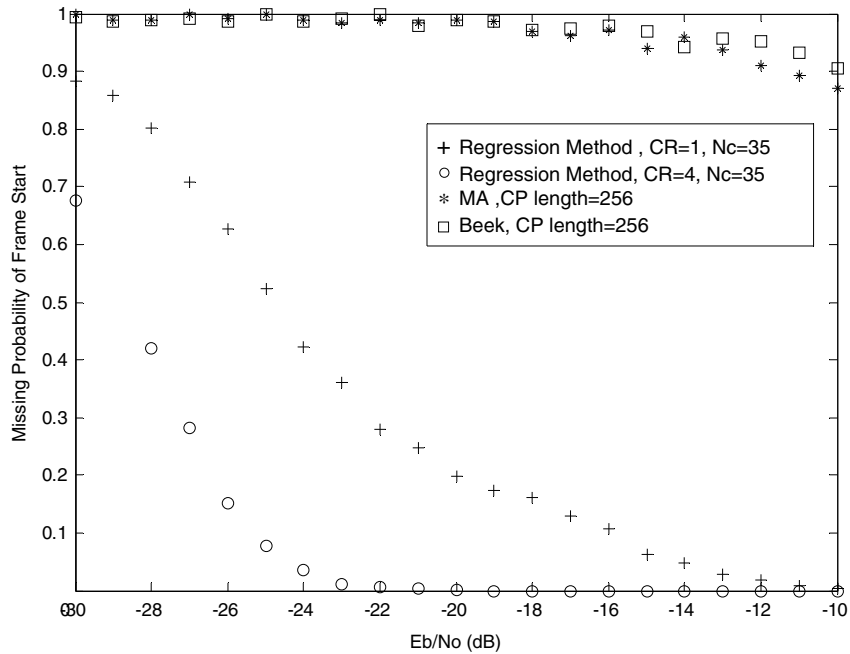


Fig. 3. Missing probability of frame start.

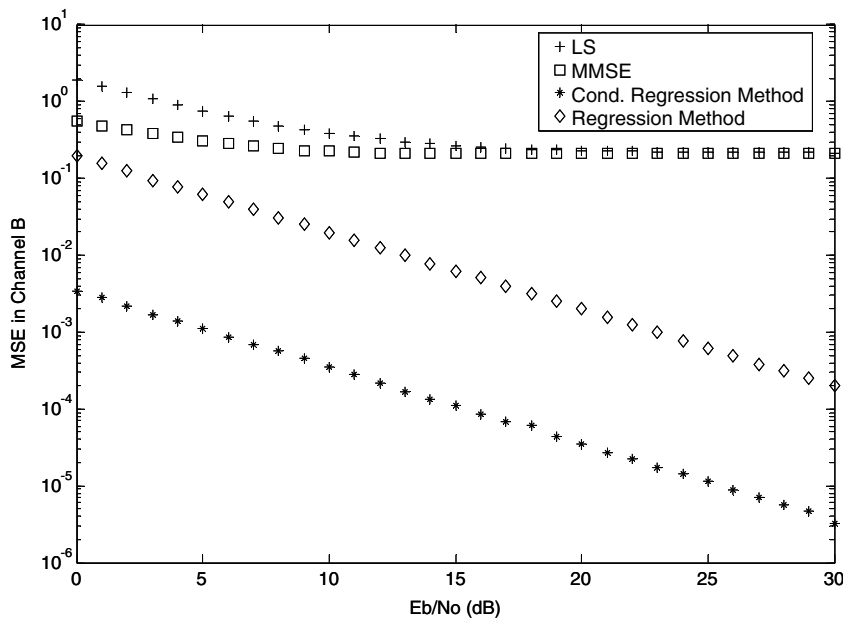


Fig. 4. Estimator MSE performance under Channel B.

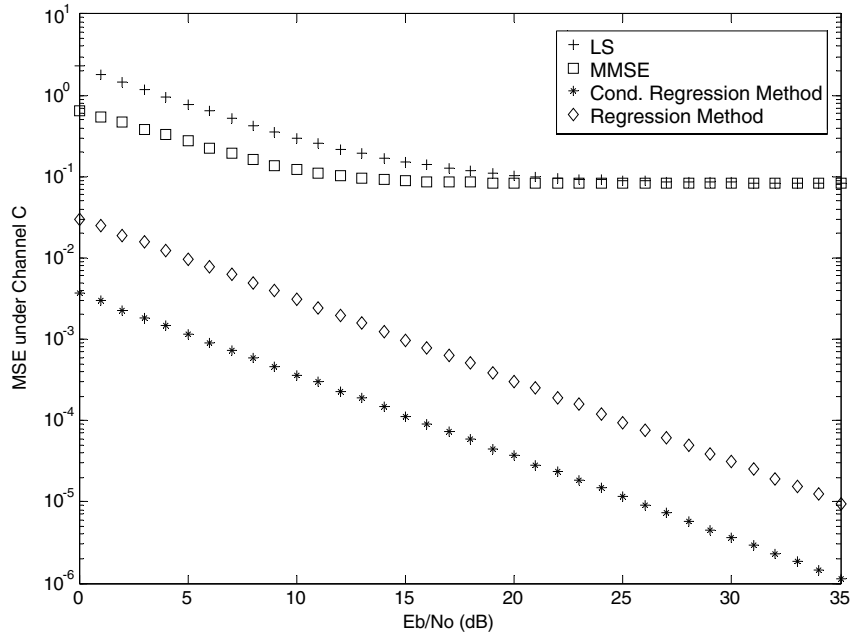


Fig. 5. Estimator MSE performance under Channel C.

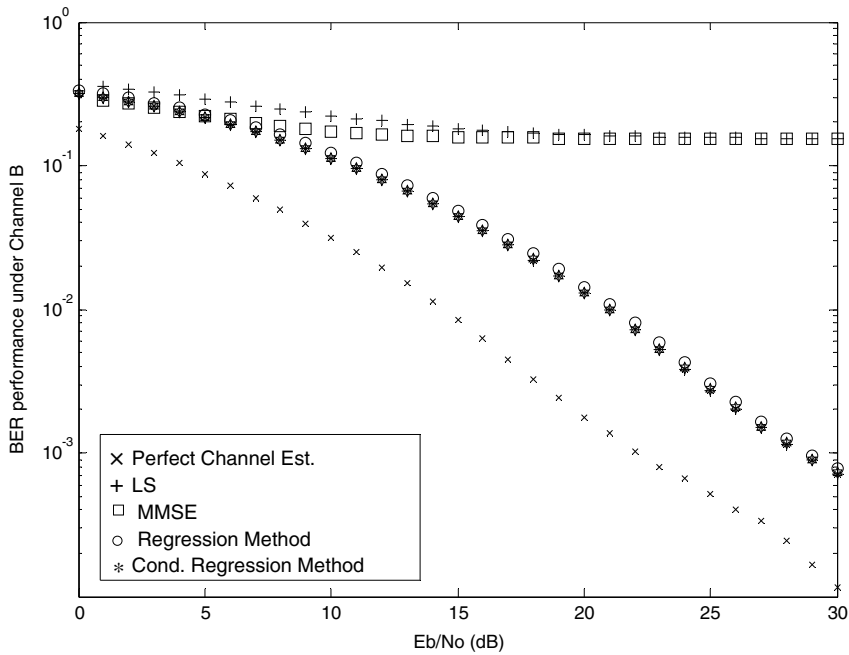


Fig. 6. BER performance under Channel B.

mance with the proposed schemes is better than that with other schemes.

**5. Conclusion**

In this paper, a joint frame synchronization and channel estimation scheme using the regression method is proposed in the OFDM systems. With the deigned frame format applied by the proposed scheme, the frame timing and the channel impulse response can be obtained with the

regression method. With the estimated channel information, a frequency domain one-tap equalizer is presented to achieve the signal compensation. Simulations for the proposed frame synchronization scheme are performed under a two-ray Rayleigh channel. The simulation result shows that the miss probability of frame start with the proposed algorithm is excellent when SNR is large than  $-10$  dB. Simulations for the proposed channel estimation scheme are performed over a four-ray and six-ray Rayleigh channel. The performance with the proposed schemes is

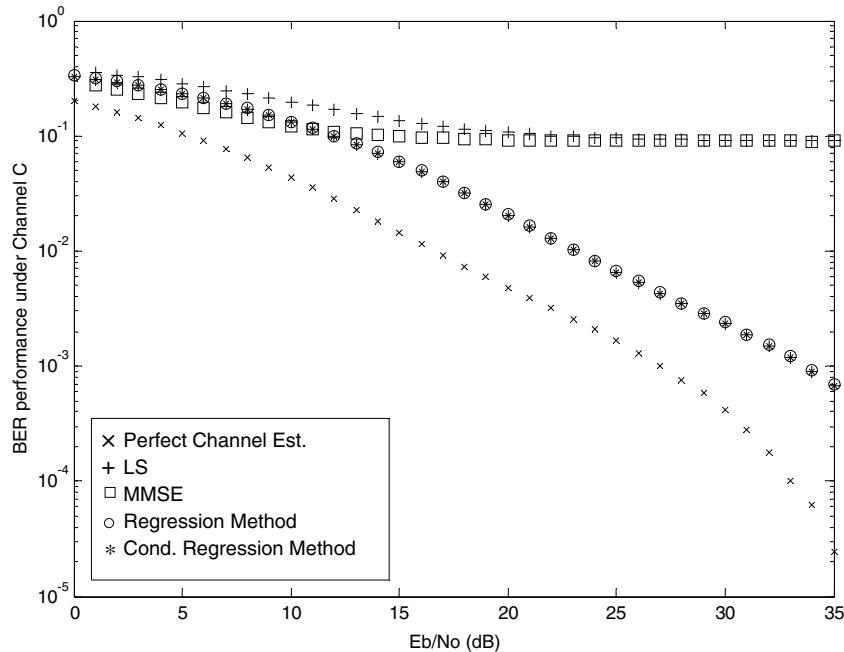


Fig. 7. BER performance under Channel C.

better than that with the other channel estimation schemes. With estimated channel response, the signal could be compensated by one-tap frequency domain equalizer to recover the fading effect.

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