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The final version of record is available at http://dx.doi.org/10.1109/TWC.2007.05502

Blind schemes for asynchronous CDMA systems on dispersive MIMO channels

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Abstract

The problem of blind detection in a dispersive Multiple-Input Multiple-Output (MIMO) Code Division Multiple Access (CDMA) channel is considered in this paper. Unlike previous studies, each user is assigned one spreading code to be employed on all of the transmit antennas, which poses a problem of data re-association at the receiver-end in the absence of prior information as to the channel state. Focusing on the differential Alamouti scheme, we propose a two-stage receive structure. The first stage performs a linear interference-blocking transformation, which allows user separation and channel equalization. The second stage is, instead, a novel differential Space-Time Block (STB) decoder suitable for frequency-selective channels. Interestingly, the proposed detector allows decoupling of the decisions on the transmitted symbols, while its blind implementation only requires a cubic (in the processing gain) complexity. A thorough performance assessment is undertaken to investigate, on one hand, the capability of acquiring the missing information, such as the system and the encoder timings, on the other hand, the interplay between the diversity gain provided by the MIMO structure of the communication system, and the additional co-channel interference that multiple transmit antennas produce in multi-path, multiple-access channels.

Index Terms

CDMA, MIMO channel, frequency-selective fading, blind detection, space-time coding/decoding.

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This paper was partly presented at the 13^{th} *European Signal Processing Conference,* Antalya, Turkey, Sept. 2005.

I. Introduction

The problem of blind detection on frequency-selective channels in asynchronous MIMO CDMA systems has been extensively addressed in the literature, and recent theoretical results on the efficiency loss tied to training in MIMO systems [1] have reinforced the interest in the topic. Additionally, the increasing interest in decentralized wireless networks - wherein peer-to-peer links are to be established with no channel state information at all - along with the need for fully random medium access protocols, high data rates and power-efficient transmission formats has brought up a number of research topics in the framework of CDMA communications: Among them we cite cooperative diversity [2], [3], possibly coupled with distributed space-time encoding, which represents the natural application of blind multi-user detection theory. If users are equipped with t transmit antennas, wide-band CDMA poses a challenging scenario due to the unavailability of many channel parameters. A number of studies have been published in recent times dealing with blind CDMA communications in dispersive MIMO channels. In particular, Direct-Sequence CDMA (DS/CDMA) systems have been considered in [4]–[8], while a Multi-Carrier DS/CDMA (MC-DS/CDMA) system has been studied in [9], [10]. All of these studies assume a binary differential Phase Shift Keying (PSK) transmission format and linear receivers, amenable to a blind implementation, to achieve Multi-Access Interference (MAI) elimination, Inter-Symbol Interference (ISI) removal and channel estimation, possibly in the presence of a space-time encoder to ensure the target bit error rate [11]. In these works, re-association, at the receiver-end, of demodulated data to the corresponding transmit antenna is easily done by assigning to each user a set of t spreading codes, one for each transmit antenna, which is exploited as a flag for data labeling. Such an inefficient use of the spreading codes is inevitable if the multi-antenna transmitter is a pure spatial multiplexer, but represents a resource waste if space-time encoding is performed, in that the code itself represents an additional signature that may be exploited for interference suppression and data re-association [12].

In this paper, we consider an asynchronous MC-DS/CDMA system over a MIMO dispersive channel, wherein each user transmits differentially STB encoded data [13], [14], using the same spreading code on all of the transmit antennas. We focus on the differential version of the Alamouti STB Code (STBC) [15], [16] and we assume that the receiver has prior knowledge of the spreading code of the user of interest only, but no other prior information. In this setup, blind decoding of the user of interest cannot be reduced to a blind deconvolution aimed at channel estimation and Rake-type detection of the interference-free observables as in [4]–[10]. We propose, instead, a different approach where a linear interference-blocking filter - amenable to a blind implementation - is employed to extract the spatial multiplex transmitted by the user of interest, while deferring to a subsequent stage the problem of exploiting the structure of the Alamouti STBC to perform data demodulation. The contributions of the present paper can be summarized as follows.

- At first, general conditions for blind linear MAI and ISI removal under the afore-mentioned single signature-per-user assignment are stated, whose compatibility with the differential Alamouti STBC is shown.
- Next, a novel differential decoder suitable for frequency-selective channels is derived, showing that: [a] its complexity is linear in the constellation size; [b] it can be implemented blindly (not even prior knowledge of the encoder timing is required), with a complexity cubic in the processing gain; [c] it subsumes as a special case the well-known incoherent differential receiver of [13], [16], corresponding to a synchronous, frequency-flat fading channel.
- Finally, a thorough performance assessment is carried on, in order to point out advantages and drawbacks of the newly proposed transmission format, and to elicit the inherent trade-offs, typical of multiple access systems, between spatial diversity advantage, multiple access and inter-symbol interference limitation, and multipath-induced diversity.

The rest of the paper is organized as follows. In the next section, the signal model is presented, while Section 3 is devoted to the receiver derivation. Section 4 contains the numerical analysis, while concluding remarks and hints for future developments form the object of Section 5.

Notation. In the following, $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote conjugate, transpose and conjugate transpose, respectively; $\mathcal{M}_{m\times n}(\mathbb{C})$ is the set of all the $m\times n$-dimensional matrices with complex-valued entries. $\mathrm{E}[\cdot]$ denotes statistical expectation. Column-vectors and matrices are indicated through boldface lowercase and uppercase letters, respectively; $\mathbf{A}(i,k)$ is the element at position (i,k) of the matrix $\mathbf{A}$, while $\mathbf{A}(i:k,\ m:n)$ is the submatrix obtained taking the rows from i to k and the columns from m to n of $\mathbf{A}$ (if rows or columns indices are omitted, all of the rows or all of the columns are addressed, respectively). $\mathbf{I}_n$ denotes the identity matrix of order n, while $\mathbf{O}_{m,n}$ is the $m\times n$-dimensional matrix with null entries; $\dim\{\mathcal{S}\}$ and $\mathrm{Im}(\mathbf{A})$ denote dimensionality of the vector space $\mathcal{S}$ and column span of the matrix $\mathbf{A}$, respectively; $\mathrm{tr}\{\mathbf{A}\}$, $\mathrm{rank}\{\mathbf{A}\}$ and $\mathbf{A}^\dagger$ denote trace, rank and *Moore-Penrose* generalized inverse of the matrix $\mathbf{A}$ [17], respectively. Finally, $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ denote the upper and lower integer part, respectively.

II. THE MODEL

We consider an asynchronous MC-DS/CDMA communication system wherein there are K active users equipped with t transmit antennas. The available bandwidth $2W$ is split up into M disjoint subcarriers; each subcarrier has a bandwidth B_{sc} and a guard band B_g is inserted between adjacent subcarriers to avoid inter-carrier interference. The bit stream of each user is differentially STB encoded by transmitting tb bits every t symbol intervals through a unitary $t \times t$ code matrix as in [13], [14]. At epoch $p = 0, \ldots, P-1$, the transmitted $t \times t$ code matrix for user $k = 0, \ldots, K-1$ is

$$\mathbf{\Sigma}_p^k = \begin{pmatrix} s_0^k(pt) & \ldots & s_0^k((p+1)t-1) \\ \vdots & & \vdots \\ s_{t-1}^k(pt) & \ldots & s_{t-1}^k((p+1)t-1) \end{pmatrix} = \mathbf{\Sigma}_{p-1}^k \mathbf{M}_p^k \in \mathcal{L}\ , \tag{1}$$

$\mathcal{L} \subseteq \mathcal{M}_{t\times t}(\mathbb{C})$ representing the set of all of the possible transmitted codewords, and $\mathbf{M}_p^k$ being a $t \times t$ unitary matrix carrying the new symbols to be transmitted at epoch p. To initialize the transmission, the transmitter sends the codeword $\mathbf{\Sigma}_0^k = \mathbf{M}_0^k$, the latter being an arbitrary message matrix. The transmission rate is $\mathcal{R} = b/T_s$, T_s being the duration of the symbol interval. In the following, we mostly focus on the Alamouti STBC [16], where $t = 2$ and

$$\mathbf{M}_p^k = \begin{pmatrix} \mu^k(2p) & -\mu^k(2p+1)^* \\ \mu^k(2p+1) & \mu^k(2p)^* \end{pmatrix}, \tag{2}$$

with $\{\mu^k(q),\ q = 0, \ldots, 2P-1\}$ representing the uncoded process, belonging to any constant modulus, half-energy constellation $\mathcal{A}$ with cardinality 2^b (for example a 2^b-PSK).

For the sake of readability, we pass over all of the standard mathematical developments leading to signal discretization: possible references linking the physical parameters to the discrete-time model can be found in [9], [10]. Assume that "0" is the user of interest; its unknown transmission delay τ^0 is regarded as the sum of a system delay $\tau_s^0 \in [0, T_s)$ tied to the channel, and an encoder delay $\tau_c^0 = n_{\tau_c^0} T_s$, with $n_{\tau_c^0} \in \{0, \ldots, t-1\}$. What matters here is that the discrete-time signal received at epoch $q = pt + u$,

$u = 0, \ldots, t-1$ and $p = 0, \ldots, P-1$, can be written as (see equation (12) in [10])

$$\mathbf{r}(q) = \underbrace{\sum_{i=0}^{t-1} s_i^0(q - n_{\tau_c^0})\mathbf{h}_i^0}_{\text{useful signal}} + \underbrace{\mathbf{z}(q)}_{\text{ISI+MAI}} + \underbrace{\mathbf{w}(q)}_{\text{noise}}$$

$$= \mathbf{d}(q) + \mathbf{z}(q) + \mathbf{w}(q) \ \in \ \mathbb{C}^{\overline{m}NQr}. \tag{3}$$

In (3), r is the number of receive antennas, Q is the oversampling factor, whereas $N = N_{sc}M$ is the overall processing gain, N_{sc} being the spreading factor along each subcarrier; $\overline{m} \geq \overline{L} = \big(2 + \lceil (L - Q - 1)/(N_{sc}Q) \rceil\big)$ is the length (expressed in symbol intervals) of the processing window, with L the sum of the maximum multi-path delay spread T_m of the channel and of the time duration of the convolution of the transmit and receive filters (expressed in multiples of the sampling rate $T_s/(N_{sc}Q)$). Finally, the $\overline{m}NQr$-dimensional vectors $\{\mathbf{h}_i^0, \ i = 0, \ldots, t-1\}$ represent the channel-modified signatures received at the base station, whereas $\mathbf{z}(q)$ and $\mathbf{w}(q)$ are the vector forms of the interference and of the additive Gaussian noise, respectively. Notice that, even if the thermal noise is assumed to be white with power spectral density $2\mathcal{N}_0$, using band-limited receive filters may introduce a known correlation among the noise samples.

As outlined in [9], [10], the unknown composite signatures $\mathbf{h}_i^0$ in equation (3) can be equivalently expressed as

$$\mathbf{h}_i^0 = \overline{\mathbf{S}}^0 \overline{\mathbf{g}}_i^0, \quad \text{for } i = 0, \ldots, t-1 \ , \tag{4}$$

where $\overline{\mathbf{g}}_i^0$ is a channel vector with a cluster of LMr consecutive non-zero entries whose position is dictated by the user delay τ_s^0, and whose length is determined by T_m and by the temporal extension of the transmit and receive filters. As to the matrix $\overline{\mathbf{S}}^0$, it is uniquely determined by the spreading code assigned to user "0". More generally, the composite spatial signatures appearing in (4) are formed as weighted sums of LMr consecutive columns of $\overline{\mathbf{S}}^0$, which can be cast in the following reduced-dimension matrix

$$\widetilde{\mathbf{S}} = \overline{\mathbf{S}}^0\big(:, n_{\tau_s^0}Mr{+}1 : (n_{\tau_s^0}{+}L)Mr \big) \ \in \mathcal{M}_{\overline{m}NQr \times LMr}(\mathbb{C}), \tag{5}$$

where $n_{\tau_s^0} = \lfloor \tau_s^0 Q/T_c \rfloor$, $T_c = T_s/N_{sc}$ being the chip interval. A blind system may solely rely upon knowledge of $\overline{\mathbf{S}}^0$, while all of the other quantities in (3) are unknown at the receiver, and so is the position of the first non-zero column contributing to the matrix $\widetilde{\mathbf{S}}$ in (5).

It is worthwhile pointing out here that the signal model considered above is truly general and subsumes, as the special case $M = 1$, the convectional DS/CDMA model adopted in [4]–[7], [18]–[20].

III. Detector design

Application of canonical strategies to decode the user of interest appears mathematically untractable and, in any case, would inevitably lead to an unaffordably complex system. In the sequel, we follow the sub-optimum approach of separating the interference-suppression stage and the decoding stage, emphasizing similarities and differences with known structures, as those presented in [4]–[9].

A. Blocking Stage

Moving back to (3), it is easily seen that, since each user is assigned only one spreading code and since the encoder timing is assumed unknown at the receiver, separation methods such as those suggested in [4]–[9] cannot be applied, and the blocking stage should extract the space-multiplexed signal $\mathbf{d}(q) = \sum_{i=0}^{t-1} s_i^0(q - n_{\tau_c^0})\mathbf{h}_i^0$ instead. To this end, we consider a classical Minimum Mean-Output-energy (MMOE) approach [21], which, for the present scenario, amounts to designing an interference-blocking matrix $\mathbf{D}(u) \in \mathcal{M}_{\overline{m}NQr\times LMr}(\mathbb{C})$ solving the following problem:

$$\begin{aligned} &\text{minimize} \quad \mathrm{E}\big[\ \| \ \mathbf{D}^H(u)\mathbf{v}(pt+u)\ \|^2\ \big], \\ &\text{subject to} \quad \det\!\big(\mathbf{D}^H(u)\widetilde{\mathbf{S}}\big) \neq 0, \quad \text{for } u = 0,\dots,t-1. \end{aligned} \tag{6}$$

Choosing, in (6), $\mathbf{v}(pt+u) = \mathbf{r}(pt+u)$ corresponds to minimizing the overall interference-plus-noise power, and will be referred to as Minimum-Mean-Square-Error (MMSE)-like in what follows; instead, setting $\mathbf{v}(pt+u) = \mathbf{d}(pt+u) + \mathbf{z}(pt+u)$ corresponds to minimizing the ISI-plus-MAI power, and will be referred to as Zero-Forcing (ZF)-like. We remark that, due to the structure imposed by the STBC, the received signal $\mathbf{r}(pt+u)$ in (3) is, in general, cyclostationary with period t and correlation matrices $\{\mathbf{R}_{\mathbf{rr}}(u) = \mathrm{E}[\mathbf{r}(pt+u)\mathbf{r}(pt+u)^H],\ u = 0,\dots,t-1\}$, whereby t different blocking matrices $\{\mathbf{D}(u),\ u = 0,\dots,t-1\}$ should be estimated. Evidently, if the structure of the STBC is such that the observations are Wide-Sense Stationary (WSS), $\mathbf{D}$ becomes time-invariant.

Requiring that the system be noise-limited is tantamount to requiring that, choosing $\mathbf{v}(pt+u) = \mathbf{d}(pt+u) + \mathbf{z}(pt+u)$, the interference be completely nullified. To this end, letting $\mathbf{R}_{\mathbf{vv}}(u) = \mathrm{E}[\mathbf{v}(pt+$

$u)\mathbf{v}^H(pt+u)]$, we give the following

Proposition 1: The filters

$$\mathbf{D}(u) = \left(\mathbf{R}_{\mathbf{vv}}(u) + \widetilde{\mathbf{S}}\widetilde{\mathbf{S}}^H\right)^{\dagger}\widetilde{\mathbf{S}}\,, \quad u = 1, \ldots, t-1, \tag{7}$$

solve the MMOE problem (6). Moreover, they are able to completely suppress (asymptotically for the MMSE solution) the overall interference if the following conditions are met:

C1. $\operatorname{Im}\left(\mathrm{E}[\mathbf{z}(pt+u)\mathbf{z}(pt+u)^H]\right) \cap \operatorname{Im}\left(\widetilde{\mathbf{S}}\right) = \emptyset$, for $u = 1, \ldots, t-1$.

C2. $\mathrm{E}\left[\mathbf{d}(pt+u)\mathbf{z}(pt+u)^H\right] = \mathbf{O}_{\overline{m}NQr,\overline{m}NQr}$, for $u = 1, \ldots, t-1$.

Proof: The proof is quite similar to that reported in [20]. □

It is at this point necessary to give a deeper insight into the implications of conditions [C1] and [C2] of Proposition 1. As to [C1], it is the generalization to the assumed scenario of the so-called *identifiability condition*, [6], [18], [19] and is equivalent to requiring:

$$\dim\left\{\operatorname{Im}\left(\mathrm{E}\left[\mathbf{z}(pt+u)\mathbf{z}(pt+u)^H\right]\right) \cup \operatorname{Im}\left(\widetilde{\mathbf{S}}\right)\right\} = \operatorname{rank}\left\{\mathrm{E}\left[\mathbf{z}(pt+u)\mathbf{z}(pt+u)^H\right]\right\} + \operatorname{rank}\{\widetilde{\mathbf{S}}\}; \tag{8}$$

whereby, in the worst case, we have:

$$\overline{m}NQr \geq Kt(\overline{m} + \overline{L} - 1) - t + LMr\ .$$

In other words, [C1] is a condition of non-saturation of the signal representation space, and leads to the following upper-bound to the maximum number of active users in the network:

$$K \leq K_{max} = \min\left\{\left\lfloor \frac{(\overline{m}NQ - LM)r + t}{(\overline{m} + \overline{L} - 1)t} \right\rfloor, \phi(N)\right\}, \tag{9}$$

where $\phi(N)$ is the number of available spreading codes, usually tied to the processing gain N. For a fixed N, this bound can be relaxed both enlarging the processing window size $\overline{m}T_s$ and, more effectively, increasing the number of receive antennas.

Condition [C2], instead, requires that the useful signal be uncorrelated with all of the interferers, i.e.

$$\mathrm{E}\left[\mathbf{d}(pt+u)\mathbf{z}_{MAI}(pt+u)^H\right] = \mathbf{O}_{\overline{m}NQr,\overline{m}NQr}\ , \tag{10a}$$

$$\mathrm{E}\left[\mathbf{d}(pt+u)\mathbf{z}_{ISI}(pt+u)^H\right] = \mathbf{O}_{\overline{m}NQr,\overline{m}NQr}\ , \tag{10b}$$

where $\mathbf{z} = \mathbf{z}_{MAI} + \mathbf{z}_{ISI}$, $\mathbf{z}_{MAI}$ and $\mathbf{z}_{ISI}$ representing the inter-symbol interference produced by the user "0" and the multiple-access interference generated by the remaining terminals, respectively. When [C1] is fulfilled, condition (10a) ensures blind linear separability among the active users, while condition (10b) guarantees blind linear ISI suppression [20]. Interestingly, (10a) is always satisfied provided that different users transmit independent bit-streams; condition (10b), instead, is equivalent to requiring that

$$\mathrm{E}\big[s_i^k(l)s_j^k(l+n)^*\big] = 0, \tag{11}$$

for $i, j = 0, \ldots, t-1$ and $n = -\overline{L}, \ldots, -1, 1, \ldots, \overline{m}-1$.

The task of using condition (11) *constructively*, i.e. of designing STBCs starting upon (11), appears challenging and is outside the scope of this paper. However the following considerations can be made:

- Compatibility of known differential STBCs with condition (11) can always be achieved by suitably interleaving the STB encoded symbols.
- For the differential Alamouti code, compatibility with condition (11) is achieved without interleaving, once the uncoded stream is an i.i.d. zero-mean proper process[1], as shown in the Appendix. This latter requirement is easily fulfilled by adopting a phase modulation with cardinality greater or equal to 4. Notice also that, in this case, the received signal is WSS, whereby $\mathbf{D}$ becomes time-invariant.

Due to its wide-spread application, and since it simplifies the design of the suppression stage, in the remaining part of this paper we focus only on the Alamouti code. Before proceeding, however, the following observations are in order:

- At the output of the blocking stage the noise is colored, whereby a noise-whitening transformation $\mathbf{W}$ is performed. The cascade of the filter (7) and $\mathbf{W}$ is denoted by $\mathbf{U} = \mathbf{D}\mathbf{W}$.
- Implementation of (7) requires knowledge of the matrix $\widetilde{\mathbf{S}}$, i.e. acquisition of the user timing; this problem is addressed in Section III-C.
- The blocking stage (7) may at most isolate the spatial multiplex $\mathbf{d}(q) = \sum_{i=0}^{t-1} s_i^0(q - n_{\tau_c^0})\mathbf{h}_i^0$ transmitted by the user of interest, which poses a problem of data reassociation at the receiver-end. To resolve this ambiguity a novel decoding strategy which exploits the additional ST signature provided by the underlying STBC is proposed and investigated in the following.

[1] A zero-mean complex process $m(n)$ is proper if its pseudo-correlation is identically zero, i.e., $\mathrm{E}\big[m(n)m(n-k)\big] = 0$.

B. Space-Time Decoding

Neglecting at the design stage the residual interference (which is rigorously true if a ZF-like criterion is adopted and asymptotically true at high SNRs in the MMSE-like case) and assuming differential Alamouti encoding, the signal at the output of the first stage can be written as

$$\mathbf{y}(q) = s_0^0(q - n_{\tau_c})\overline{\mathbf{h}}_0 + s_1^0(q - n_{\tau_c})\overline{\mathbf{h}}_1 + \mathbf{n}(q), \in \mathbb{C}^{LMr},$$

for $q = 0, \ldots, 2P-1$, where $\overline{\mathbf{h}}_i = \mathbf{U}^H \mathbf{S}^0 \mathbf{g}_i^0$ are the (unknown) filtered signatures of the 0-th user and $\mathbf{n}(q) = \mathbf{U}^H \mathbf{w}(q)$ is a white-noise vector. We stress that both the encoder delay $n_{\tau_c^0}$ and the user delay $n_{\tau_s^0}$ (which determines the matrix $\widetilde{\mathbf{S}}$ in (7)) are unknown and need to be estimated at some point. For the moment, we assume $n_{\tau_c^0} = 0$ and $n_{\tau_s^0}$ perfectly known, deferring to the next sub-section the problem of their estimation. The decoding problem can now be stated as follows. Relying on the signals

$$\mathbf{y}(2p+u) = s_0^0(2p+u)\overline{\mathbf{h}}_0 + s_1^0(2p+u)\overline{\mathbf{h}}_1 + \mathbf{n}(2p+u), \tag{12}$$

received in four consecutive signaling intervals, $u = -2, -1, 0, 1$, design a receive structure so as to recover the information symbols $\mu^0(2p)$ and $\mu^0(2p+1)$ conveyed in $\{s_i^0(2p+u),\ i = 0, 1,\ u = -2, -1, 0, 1\}$. Two different approaches are now discussed.

On one hand, neglecting the cross-correlation among the noise vectors $\{\mathbf{n}(2p+u),\ u = -2, -1, 0, 1\}$ and assuming the entries of the unknown equivalent channels $\{\overline{\mathbf{h}}_i,\ i = 0, 1\}$ in (12) to be independent and Rayleigh distributed, direct application of the results of [13] leads to the following differential detection rule (*strategy I*):

$$\begin{pmatrix} \widehat{\mu}^0(2p) \\ \widehat{\mu}^0(2p+1) \end{pmatrix} = \arg \min_{\nu_0, \nu_1 \in \mathcal{A}} \left\| \begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix} - \begin{pmatrix} \mathbf{y}^H(2p-2)\mathbf{y}(2p) + \mathbf{y}^H(2p+1)\mathbf{y}(2p-1) \\ \mathbf{y}^H(2p-1)\mathbf{y}(2p) - \mathbf{y}^H(2p+1)\mathbf{y}(2p-2) \end{pmatrix} \right\|. \tag{13}$$

A different decoding strategy can be, instead, derived by following a non-Bayesian approach. To be more specific, let us first re-write the observations (12) as follows:

$$\begin{pmatrix} \mathbf{y}(2p-2) \\ \mathbf{y}(2p-1)^* \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{h}}_0 & \overline{\mathbf{h}}_1 \\ \overline{\mathbf{h}}_1^* & -\overline{\mathbf{h}}_0^* \end{pmatrix} \begin{pmatrix} s_0^0(2p-2) \\ s_1^0(2p-2) \end{pmatrix} + \begin{pmatrix} \mathbf{n}(2p-2) \\ \mathbf{n}(2p-1)^* \end{pmatrix} \in \mathbb{C}^{2LMr},$$

$$\begin{pmatrix} \mathbf{y}(2p) \\ \mathbf{y}(2p+1)^* \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{h}}_0 & \overline{\mathbf{h}}_1 \\ \overline{\mathbf{h}}_1^* & -\overline{\mathbf{h}}_0^* \end{pmatrix} \begin{pmatrix} s_0^0(2p) \\ s_1^0(2p) \end{pmatrix} + \begin{pmatrix} \mathbf{n}(2p) \\ \mathbf{n}(2p+1)^* \end{pmatrix} \in \mathbb{C}^{2LMr},$$

where the structure of the Alamouti coding scheme has been exploited. In what follows we propose to parallel the design strategy usually adopted in common differential M-PSK with soft decoding, where *soft Maximum-Likelihood* (ML) estimates of the transmitted uncoded process are first derived and, then, the corresponding hard estimates are obtained by using a minimum distance classification rule [6], [18]. More precisely, since for the case at hand $\mathbf{M}_p^k = \mathbf{\Sigma}_{p-1}^{k\ H}\mathbf{\Sigma}_p^k$, soft ML estimates of $\mu^k(2p)$ and $\mu^k(2p+1)$ are given by

$$\begin{cases} \widetilde{\mu}^k(2p) = \widetilde{s}_0^k(2p)\widetilde{s}_0^k(2p-2)^* + \widetilde{s}_1^k(2p)\widetilde{s}_1^k(2p-2)^*\,, \\ \widetilde{\mu}^k(2p+1) = -\widetilde{s}_0^k(2p)\widetilde{s}_1^k(2p-2) + \widetilde{s}_1^k(2p)\widetilde{s}_0^k(2p-2)\,, \end{cases} \tag{14}$$

with $\widetilde{\mu}^k(2p+u)$ and $\widetilde{s}_i^k(2p+u)$ denoting soft ML estimates of the corresponding quantities. Since the noise vectors $\left(\mathbf{n}(2p)^T\mathbf{n}(2p+1)^H\right)^T$ and $\left(\mathbf{n}(2p-2)^T\mathbf{n}(2p-1)^H\right)^T$ are both zero-mean, circularly symmetric Gaussian distributed, with covariance matrix $\mathbf{I}_{2LMr}$, neglecting their cross-correlation, the ML estimates of the symbols transmitted at times $2p$ and $2p-2$ can be obtained separately as

$$\begin{pmatrix} \widetilde{s}_0^0(2p-2) \\ \widetilde{s}_1^0(2p-2) \end{pmatrix} = \frac{1}{h}\begin{pmatrix} \overline{\mathbf{h}}_0^H\mathbf{y}(2p-2) + \mathbf{y}^H(2p-1)\overline{\mathbf{h}}_1 \\ \overline{\mathbf{h}}_1^H\mathbf{y}(2p-2) - \mathbf{y}^H(2p-1)\overline{\mathbf{h}}_0 \end{pmatrix},$$

$$\begin{pmatrix} \widetilde{s}_0^0(2p) \\ \widetilde{s}_1^0(2p) \end{pmatrix} = \frac{1}{h}\begin{pmatrix} \overline{\mathbf{h}}_0^H\mathbf{y}(2p) + \mathbf{y}^H(2p+1)\overline{\mathbf{h}}_1 \\ \overline{\mathbf{h}}_1^H\mathbf{y}(2p) - \mathbf{y}^H(2p+1)\overline{\mathbf{h}}_0 \end{pmatrix},$$

which, upon elimination of the irrelevant scaling factor $h = \parallel \overline{\mathbf{h}}_0 \parallel^2 + \parallel \overline{\mathbf{h}}_1 \parallel^2$, can be plugged into the differential decoding rule (14), yielding the final test (*strategy II*):

$$\begin{aligned} \begin{pmatrix} \widehat{\mu}^0(2p) \\ \widehat{\mu}^0(2p+1) \end{pmatrix} = \arg\min_{\nu_0,\nu_1\in\mathcal{A}} \Bigg\| \begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix} &+ \\ &-\begin{pmatrix} \mathbf{y}^H(2p-2)\mathbf{R}_1\mathbf{y}(2p)+\mathbf{y}^H(2p+1)\mathbf{R}_1\mathbf{y}(2p-1) \\ \mathbf{y}^H(2p-1)\mathbf{R}_1\mathbf{y}(2p)-\mathbf{y}^H(2p+1)\mathbf{R}_1\mathbf{y}(2p-2) \end{pmatrix} \\ &+\begin{pmatrix} \mathbf{y}^H(2p-2)\mathbf{R}_2\mathbf{y}^*(2p+1)+\left(\mathbf{y}^H(2p)\mathbf{R}_2\mathbf{y}^*(2p-1)\right)^* \\ \mathbf{y}^H(2p-1)\mathbf{R}_2\mathbf{y}^*(2p+1)-\left(\mathbf{y}^H(2p)\mathbf{R}_2\mathbf{y}^*(2p-2)\right)^* \end{pmatrix} \Bigg\|, \end{aligned} \tag{15}$$

where $\mathbf{R}_1 = \overline{\mathbf{h}}_0\overline{\mathbf{h}}_0^H + \overline{\mathbf{h}}_1\overline{\mathbf{h}}_1^H \in \mathcal{M}_{LMr\times LMr}(\mathbb{C})$ and $\mathbf{R}_2 = \overline{\mathbf{h}}_0\overline{\mathbf{h}}_1^T - \overline{\mathbf{h}}_1\overline{\mathbf{h}}_0^T \in \mathcal{M}_{LMr\times LMr}(\mathbb{C})$. The matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ are channel-dependant and, thus, a priori unknown. However, we show in the next paragraph that they can be extracted from the received data.

Comparing (13) and (15), the following remarks are now in order:

- In both cases, the decisions on the two transmitted symbols have been decoupled, whereby the decoding complexity grows linearly with the constellation size.
- Both decoding strategies implement a kind of differential combining to remove the phase ambiguity due to the prior uncertainty as to the propagation channels. In the former rule, the realizations of $\{\overline{\mathbf{h}}_0, \overline{\mathbf{h}}_1\}$ do not come into play, but only the ensemble properties of the propagation channels are accounted for. Conversely, the latter strategy accounts for the actual realizations of $\{\overline{\mathbf{h}}_0, \overline{\mathbf{h}}_1\}$, which are contained in the weighting matrices $\mathbf{R}_1$, $\mathbf{R}_2$.
- Finally, for synchronous systems equipped with a single receive antenna and operating on frequency-flat channels, the observables become scalar quantities. As a consequence, (15) and (13) become equivalent, and they both coincide with the differential decoding rule in [13], [16].

C. Blind Implementation

To make the previous scheme blind, we have to show that all of the involved parameters can be extracted from the observations. We split the discussion in three parts, i.e.: [**a**] blind implementation of the blocking stage; [**b**] blind acquisition of the encoder delay; [**c**] blind implementation of the decoding rule.

As to [a], while $\mathbf{R}_{\mathbf{vv}}$ may be easily estimated starting upon $\mathbf{R}_{\mathbf{rr}} = \mathrm{E}[\mathbf{r}(q)\mathbf{r}^H(q)]$ [18], [20], estimation of the matrix $\widetilde{\mathbf{S}}$ in (5) requires more discussion. The problem here is the extraction of the parameter $n_{\tau_s^0}$, which in turn takes on values in the set $\{0, 1, \ldots, N_{sc}Q-1\}$. Thus, given the $N_{sc}Q$ candidate matrices

$$\widetilde{\mathbf{S}}_\ell = \overline{\mathbf{S}}^{0,0}\big(:, \ell Mr + 1 : (\ell + L)\, Mr\big), \tag{16}$$

$\ell = 0, 1, \ldots, N_{sc}Q-1$, as many blocking stages, $\{\mathbf{U}_\ell\}_{\ell=0}^{N_{sc}Q-1}$, can be formed and the following test can be performed [22]:

$$\mathbf{U} = \arg\max_{\mathbf{U}_\ell} \left\{\lambda_0\left(\mathbf{U}_\ell\right) + \lambda_1\left(\mathbf{U}_\ell\right)\right\}, \tag{17}$$

where $\{\lambda_i\left(\mathbf{U}_\ell\right),\ i = 0, 1\}$ denote the two largest eigenvalues of the matrix $\mathbf{U}_\ell^H \mathbf{R}_{\mathbf{rr}} \mathbf{U}_\ell$. Thus, among all of the $N_{sc}Q$ candidate blocking stages, each ensuring interference suppression, the one maximizing the output signal energy is picked up. A batch blind estimate of each matrix $\mathbf{U}_\ell$ requires an implementation complexity $\mathcal{O}\left((\overline{m}NQr)^3\right)$, since the inversion of a $\overline{m}NQr$-dimensional matrix is involved; the test in (17), instead, has a complexity $\mathcal{O}\left(N_{sc}Q(LMr)^3\right)$, since it involves $N_{sc}Q$ singular value decompositions of LMr-dimensional matrices. However, while the batch estimate of the blocking-matrix (7) has to be updated at the beginning of each new data packet (whose length depends upon the Doppler bandwidth), the symbol timing may be acquired *una tantum* at the beginning of the transmission and kept as far as the user is active.

Let us now move on to the problem [b] of recovering the encoder synchronism. Since the vectors $\{\mathbf{y}(q),\ \ q = 1, \ldots, 2P-1\}$ have to be processed in group of two in order to space-time decode the information transmitted at time p, there are two different ways of casting these vectors together: $\{\mathbf{y}(2p), \mathbf{y}(2p+1)\}$ if $\tau_c^0 = 0$ and $\{\mathbf{y}(2p+1), \mathbf{y}(2p+2)\}$ if $\tau_c^0 = T_s$. Indeed, the test to be solved is

$$\mathbf{y}(q) = \begin{cases} s_0^0(q)\overline{\mathbf{h}}_0 + s_1^0(q)\overline{\mathbf{h}}_1 + \mathbf{n}(q), & \text{if } \tau_c^0 = 0, \\ s_0^0(q-1)\overline{\mathbf{h}}_0 + s_1^0(q-1)\overline{\mathbf{h}}_1 + \mathbf{n}(q), & \text{if } \tau_c^0 = T_s. \end{cases} \tag{18}$$

The problem is that the above test is either composite, if the transmitted symbols and the fading vector are modeled as unknown parameters, or parametric, if the said parameters are assigned prior probability laws. Once again, we prefer to choose a sub-optimum approach, relying on the noticeable symmetry properties of the Alamouti STBC shown in the following

Observation 1: For the Alamouti code, the following relationship holds:

$$\mathbf{\Sigma}_p^0(:,1)\left(\mathbf{\Sigma}_p^0(:,2)\right)^T - \mathbf{\Sigma}_p^0(:,2)\left(\mathbf{\Sigma}_p^0(:,1)\right)^T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{19}$$

Furthermore, if the encoder delay is zero, according to (18), $\mathbf{\Sigma}_p^0$ is conveyed by $\mathbf{y}(2p)$ and $\mathbf{y}(2p+1)$; in this case from (19) we have

$$\mathrm{E}[\mathbf{y}(2p)\mathbf{y}^T(2p+1) - \mathbf{y}(2p+1)\mathbf{y}^T(2p)] = \overline{\mathbf{h}}_0\overline{\mathbf{h}}_1^T - \overline{\mathbf{h}}_1\overline{\mathbf{h}}_0^T = \mathbf{R}_2 \in \mathcal{M}_{LMr\times LMr}(\mathbb{C})$$

and $\mathrm{tr}\left\{\mathbf{R}_2\mathbf{R}_2^H\right\} = 2\big(\|\overline{\mathbf{h}}_0\|^2\|\overline{\mathbf{h}}_1\|^2 - |\overline{\mathbf{h}}_0^H\overline{\mathbf{h}}_1|^2\big) \geq 0$, where the equality holds with probability zero in our setup (i.e. for a dispersive channel). On the other hand, if the encoder delay is T_s, $\mathbf{\Sigma}_p^0$ is contained in $\mathbf{y}(2p+1)$ and $\mathbf{y}(2p+2)$ and we have

$$\mathrm{E}\big[\mathbf{y}(2p)\mathbf{y}^T(2p+1) - \mathbf{y}(2p+1)\mathbf{y}^T(2p)\big] = \mathbf{O}_{LMr,LMr}\ .$$

TABLE I
SUMMARY OF THE ALGORITHM

step 1:	*Estimate matrix* $\mathbf{R}_{\mathbf{vv}}$.	
(first packet only) **step 2**:	*Recover the symbol delay* $n_{\tau_s^0}$ *(and, thus, matrix* $\widetilde{\mathbf{S}}$*) through procedure (17).*	
step 3:	*Compute the blocking matrix* $\mathbf{U} = \mathbf{DW}$*, where* $\mathbf{D}$ *is given by (7) and* $\mathbf{W}$ *is the whitening filter.*	
(first packet only) **step 4**:	*Recover the encoder delay* τ_c^0 *through procedure (20).*	
	Strategy I	***Strategy II***
step 5:	*Adopt the decision rule in (13).*	*Estimate matrices* $\mathbf{R}_1$ *and* $\mathbf{R}_2$ *through (21).*
step 6:		*Adopt the decision rule in (15).*

Thus, exploiting the above results and forming the two sample estimates

$$\mathbf{F}_0 = \frac{1}{B}\sum_{n=0}^{B-1}\left[\mathbf{y}(2n)\mathbf{y}^T(2n+1) - \mathbf{y}(2n+1)\mathbf{y}^T(2n)\right],$$

$$\mathbf{F}_{T_s} = \frac{1}{B}\sum_{n=0}^{B-1}\left[\mathbf{y}(2n+1)\mathbf{y}^T(2n+2) - \mathbf{y}(2n+2)\mathbf{y}^T(2n+1)\right],$$

B denoting the estimation sample size, the desired non-parametric test takes on the form:

$$\widehat{\tau}_c^0 = \begin{cases} 0 & \text{if } \operatorname{tr}\left\{\mathbf{F}_0\mathbf{F}_0^H\right\} > \operatorname{tr}\left\{\mathbf{F}_{T_s}\mathbf{F}_{T_s}^H\right\}, \\ T_s & \text{otherwise.} \end{cases} \tag{20}$$

The test (20) has to be performed only once at the beginning of the transmission and has an implementation complexity $\mathcal{O}\left((LMr)^3\right)$, which is much less than that of the interference-blocking stage. Thus, the task of recovering the encoder synchronism does not add a significant burden to the overall system complexity.

Let us finally consider task [c]. For the decoding rule (13), no farther discussion is needed. Instead, when the decoding rule (15) is adopted, blind estimation of the matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ can be obtained as

$$\widehat{\mathbf{R}}_1 = 2\left(\mathbf{U}^H\widehat{\mathbf{R}}_{\mathbf{rr}}\mathbf{U} - \mathbf{I}_{LMr}\right), \tag{21a}$$

$$\widehat{\mathbf{R}}_2 = \arg\max_{\mathbf{F}\in\{\mathbf{F}_0,\mathbf{F}_{T_s}\}} \operatorname{tr}\left\{\mathbf{F}\mathbf{F}^H\right\}. \tag{21b}$$

Notice that both $\widehat{\mathbf{R}}_1$ and $\widehat{\mathbf{R}}_2$ have to be computed at the beginning of each data packet, entailing a complexity $\mathcal{O}\left(LMr(\overline{m}NQr)^2\right)$ and $\mathcal{O}\left((LMr)^3\right)$. In conclusion, the overall blind implementation complexity is approximatively $\mathcal{O}\left((\overline{m}NQr)^3\right)$, which is similar to that of popular blind linear blocking-stages for dispersive Single-Input-Multiple-Output (SIMO) CDMA systems (see [18]–[20], [22]), i.e., no extra complexity is added by the use of multiple transmit antennas.

For the reader's sake, the steps of the proposed blind decoding scheme are summarized in Table I.

IV. Numerical results

We consider a system with a total bandwidth constraint of $2W = 1.25N/T_s$ and $T_m = 3T_s/N$. The channel linking the i-th transmit antenna of the k-th user to the j-th receive antenna is modeled as $c^k_{i,j}(\tau) = \sum_{l=0}^{\nu} \alpha^k_{i,j,l}\delta\left(\tau - l/2W\right)$, with $\nu \simeq \lfloor 2WT_m \rfloor = 3$ [23]. Slow Rayleigh fading is assumed and the complex path gains are independently generated according to an exponentially decreasing profile, namely $E[|\alpha^k_{i,j,l}|^2] = 0.65, 0.25, 0.08, 0.02$ for $l = 0, 1, 2, 3$ and $\forall\, i, j, k$. The user and the encoder delays $\{\tau^k_s,\ k = 0, \ldots, K-1\}$ and $\{\tau^k_c,\ k = 0, \ldots, K-1\}$ are uniformly and independently generated in the interval $[0, T_s)$ and in the set $\{0, T_s, \ldots, (t-1)T_s\}$, respectively. We consider $M = 2$ subcarriers, separated by a guard band $B_g = 0.05B_{sc}$, resulting in a subcarrier bandwidth extension of $B_{sc} \simeq 0.61N/T_s$. Notice that, since the spacing between the subcarriers $\Delta f = (1 + 0.05)B_{sc} \simeq 0.64N/T_s$ exceeds the coherence bandwidth $B_c \simeq 1/T_m \simeq 0.33N/T_s$ of the channel, each subcarrier substantially experiences independent frequency-selective fading. At the transmitter/receiver side, raised cosine chip waveforms with roll-off factor 0.17, truncated to include the main lobe only, are employed. A minimum processing window size $\overline{m} = \overline{L} = 3$ and $Q = 1$ are selected. The processing gain is $N = 32$ and the spreading sequences are PN sequences of length 31 stretched out with a ± 1. The Alamouti code with a 4-PSK modulation format is adopted, giving a spectral efficiency of $\mathcal{R}/(2W) = 1.6/N$ bits/(sHz). The results are expressed as a function of the received *energy contrast* per symbol $\gamma = \mathcal{E}_s/\mathcal{N}_0$, $\mathcal{E}_s$ being the total received energy per symbol. Finally, an MMSE solution is adopted for the first stage and, for the sake of simplicity, the covariance matrix of the received signal is assumed perfectly estimated.

We begin by evaluating the synchronization performances of the proposed scheme. Fig. 1 shows the capability of the first stage to blindly select the correct sub-matrix $\widetilde{\mathbf{S}}$ from the set of candidate sub-matrices in (16), i.e. to recover the symbol delay $n_{\tau^0_s} = \lfloor \tau^0_s Q/T_c \rfloor$. The normalized estimation error $e_s = |\widehat{n}_{\tau^0_s} - n_{\tau^0_s}|/(N_{sc}Q)$ is reported versus K and for $\gamma = 14, 20$ dB; one receive antenna is employed and the Interference–to–Signal Ratio (ISR) is 15 dB. Figs. 2 and 3, instead, show the probability of

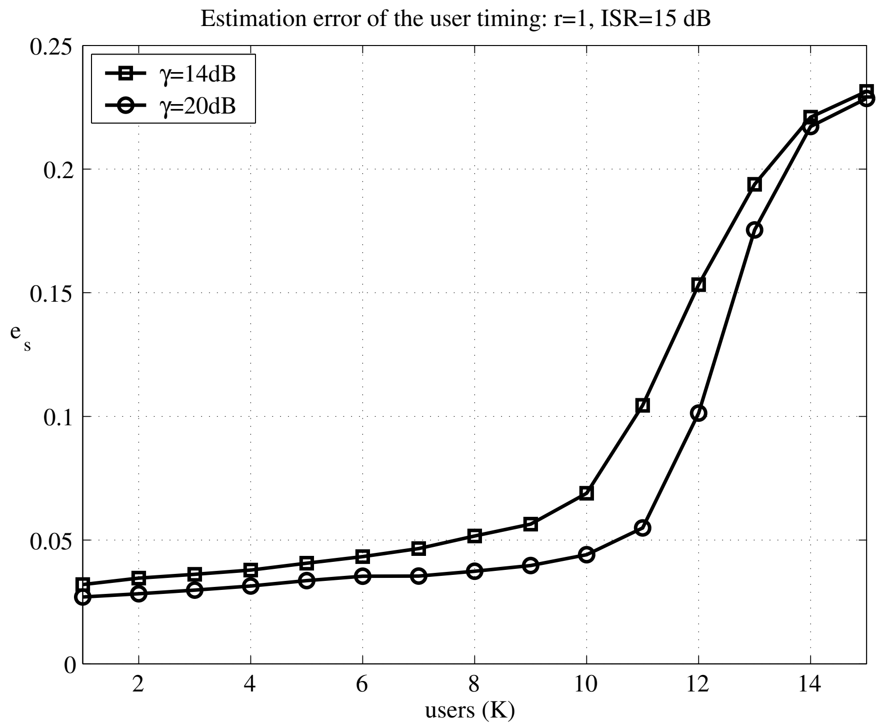

Fig. 1. Normalized estimation error of the user delay $e_s = |\widehat{n}_{\tau_s^0} - n_{\tau_s^0}|/(N_{sc}Q)$ versus the number of active users for $\gamma = 14, 20$ dB. System parameters: $N = 32$, $M = 2$, $Q = 1$, $t = 2$, $r = 1$, ISR=15 dB.

correct encoder synchronization, P_{sync}, versus the estimation-sample size B and the active users number K, respectively: one receive antenna, $M = 2$, ISR=15 dB and $\gamma = 14, 20$ dB are employed. Interestingly, as far as $K \leq K_{max} = 9$, the symbol delay is acquired with an estimation error within one chip; also, the P_{sync} is fairly close to one, even for moderate values of B.

Let us now move on to the illustration of the overall system performance in terms of Signal-to-Interference-plus-Noise Ratio (SINR) and Symbol-Error-Rate (SER). In Fig. 4, we report the SINR at the output of the interference-blocking stage versus K, once the user timing has been acquired. Both a power-controlled scenario (ISR=0 dB) and a severe near–far condition (ISR=15 dB) are investigated. In Fig. 5, instead, we report the SER versus K for the two decoding rules in (13) and (15), referred to as *strategy I* and *strategy II*, respectively. Remarkably, for the same output SINR, the proposed decoding rule (15) outperforms the one in (13), since more information about the useful signal subspace is exploited. Moreover, notice that, as long as $K \leq K_{max}$, the receiving scheme is substantially immune to the presence of strong interfering signals; instead, for $K > K_{max}$, the system performances (both SINR and

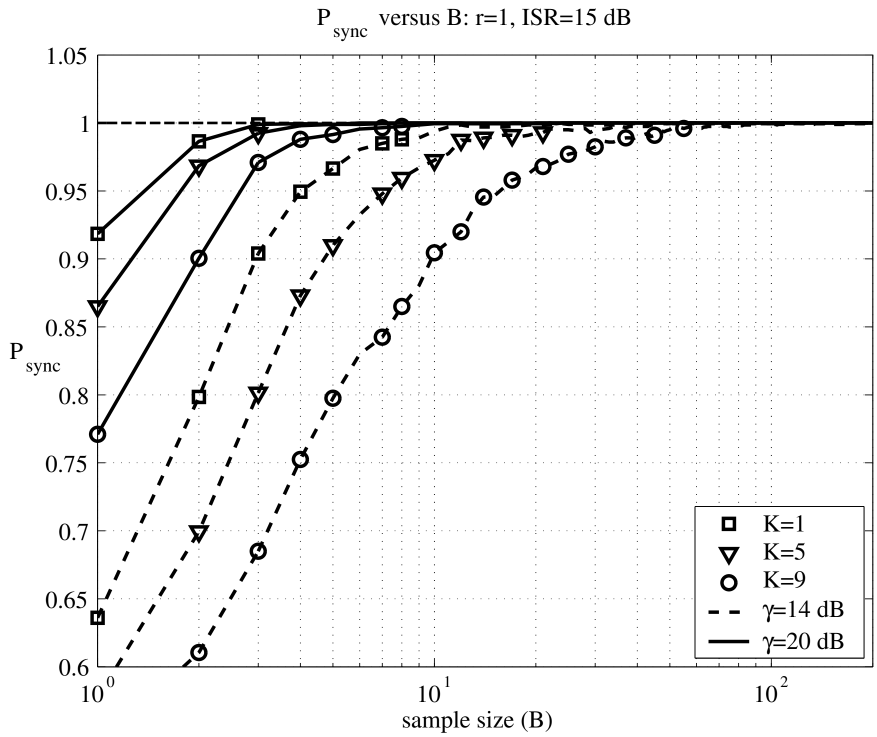

Fig. 2. Probability of correct encoder synchronization P_{sync} versus the estimation-sample size B, for $K = 1, 5, 9$. System parameters: $N = 32$, $M = 2$, $Q = 1$, $t = 2$, $r = 1$, ISR=15 dB and $\gamma = 14, 20$ dB.

SER) rapidly degrade in the absence of power control, in agreement with previous results in [7], [20].

Finally, we investigate the trade-off between the transmit spatial diversity provided by the use of a differential STB coding scheme, and the additional interference produced by the MIMO structure of the frequency-selective channel. To this end, we compare the performance of the proposed differentially Alamouti encoded MIMO MC/DS-CDMA scheme, with the performance of a blind SIMO MC/DS-CDMA system, where the same number of receive antennas is employed, but no transmit diversity is used (i.e., $t = 1$): in this latter case, a differential 4-PSK modulation format is adopted to maintain the same spectral efficiency, while the two-stage decoding strategy of [10] has been considered, i.e,

$$\mathbf{D} = \left(\mathbf{R_{rr}} + \widetilde{\mathbf{S}}\widetilde{\mathbf{S}}^H\right)^{-1}\widetilde{\mathbf{S}}, \tag{22a}$$

$$\widehat{\mu}^0(p) = \arg\min_{\nu \in \mathcal{A}} \left\| \nu - \mathbf{y}^H(p-1)\mathbf{R}_1\mathbf{y}(p) \right\|, \tag{22b}$$

with $\widetilde{\mathbf{S}}$ blindly estimated in a way similar to that in (16)-(17) and $\mathbf{R}_1 = \overline{\mathbf{h}}_0\overline{\mathbf{h}}_0^H$.

We start by noticing that, even if the transmit antennas of each user adopt the same spreading code,

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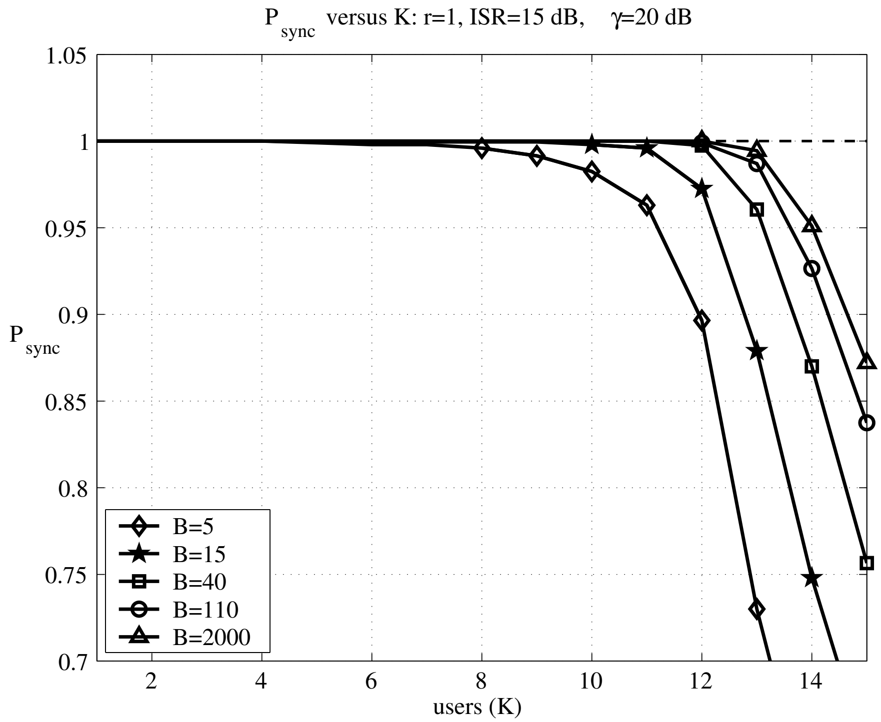

Fig. 3. Probability of correct encoder synchronization P_{sync} versus the number of active users, for different values of the estimation-sample size $B = 5, 15, 40, 110, 2000$. System parameters: $N = 32$, $M = 2$, $Q = 1$, $t = 2$, $r = 1$, ISR=15 dB and $\gamma = 20$ dB.

TABLE II
MAXIMUM USERS NUMBER GIVEN BY (9) FOR $r = 1, 2, 3, 4$

	r	1	2	3	4
K_{max}	MIMO ($t = 2$)	9	17	26	31
	SIMO ($t = 1$)	17	31	31	31

due to the dispersive nature of the channel and the usage of multiple receive antennas, the corresponding received signatures $\mathbf{h}_0^k$ and $\mathbf{h}_1^k$ in (4) are linearly independent with probability one. Thus, a first term of comparison comes immediately out: while providing extra diversity branches, the proposed system doubles the dimension of the interferers subspace with respect to a SIMO MC/DS-CDMA system, suffering a faster shortage of the interference-free directions (as highlighted in Table II, where the maximum users number given by (9) for $r = 1, 2, 3, 4$ is reported for both the MIMO and the SIMO schemes). The above intuition is confirmed by the results in Fig. 6, where we plot the SER of the two competing

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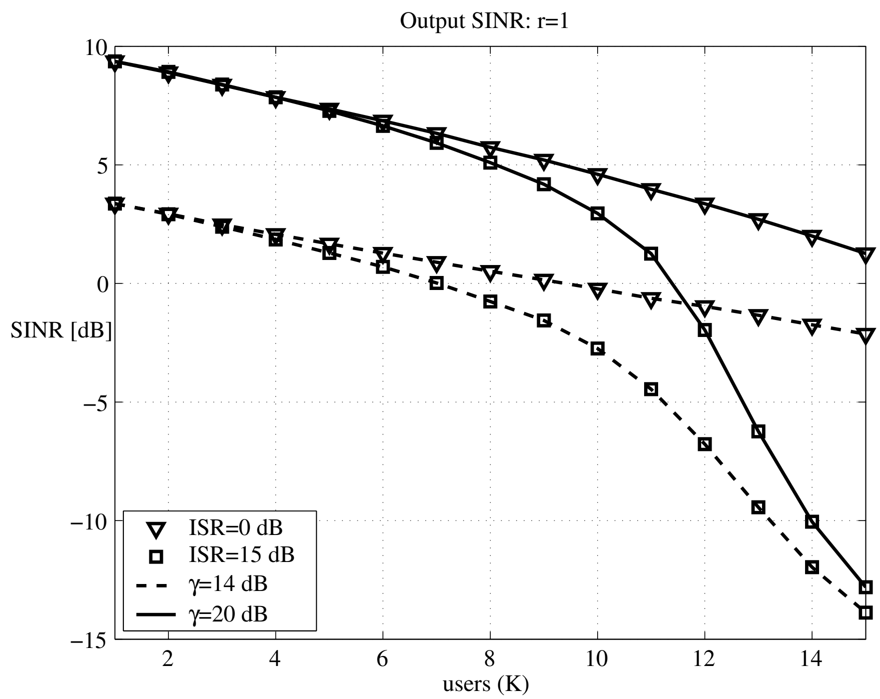

Fig. 4. SINR at the output of the first stage versus the number of active users, for ISR=0,15 dB. System parameters: $N = 32$, $M = 2$, $Q = 1$, $t = 2$, $r = 1$, $\gamma = 14, 20$ dB, $B = 2000$.

scenarios versus K: ISR=0, $M = 2$ and $\gamma = 14, 20$ dB are employed, while, for the sake of readability, only the SER of the decoding rule (15) is reported for the MIMO system. It can be seen that, while in lightly-loaded networks the MIMO system can take advantage of the additional transmit spatial diversity to outperform the single-transmit antenna system, on the other hand, as the number of users increases, the shortage of interference-free directions severely impairs the system performance, nullifying the transmit diversity advantage and suggesting - eventually - the use of only one transmit antenna. This effect is even more evident at low Signal-to-Noise-Ratios (SNRs), where the SNR gain provided by the transmit spatial diversity is still negligible, and, thus, does not compensate for the extra interference. Notice, however, that increasing the number of receive antennas, i.e. enlarging the signal representation space, has the beneficial effect of moving forward this limitation imposed by the user number. More generally we argue that, if the signal space can be made conveniently large so as to cope with the extra co-channel interference, the use of multiple transmit antennas becomes always beneficial regardless of the actual number of active users.

Finally, we comment on the special case where the channel is frequency-flat and only one receive

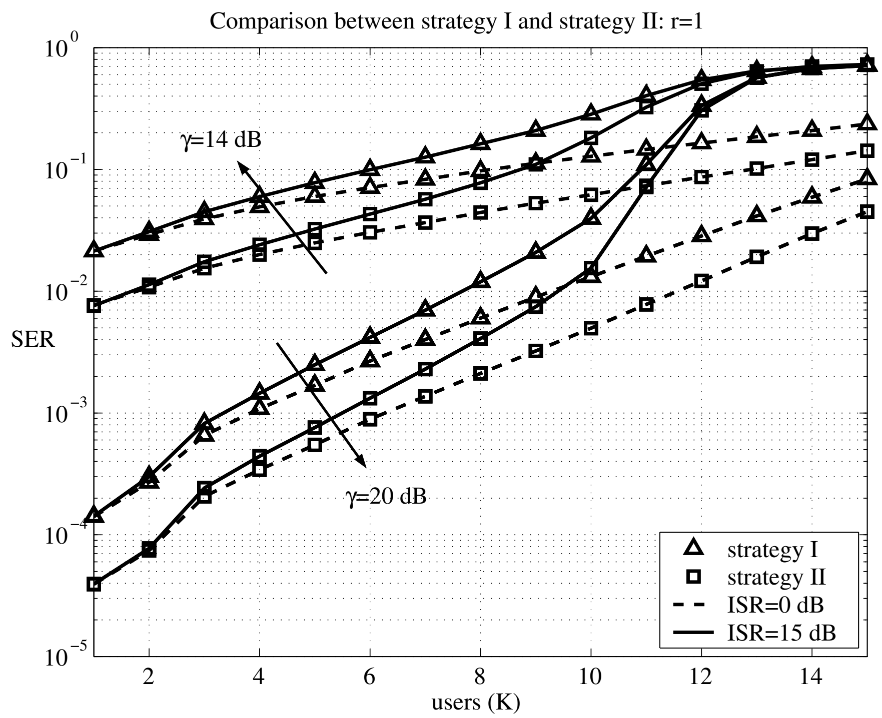

Fig. 5. SER versus the number of active users, for ISR=0,15 dB. Both the decoding rules in (13) and (15) – referred to as *strategy I* and *strategy II*, respectively – are considered. System parameters: $N = 32$, $M = 2$, $Q = 1$, $t = 2$, $r = 1$, $\gamma = 14, 20$ dB, $B = 2000$.

antenna is employed [12]. Indeed, in this case, $c_i^k(\tau) = \alpha_i^k \delta(\tau)$, with $\alpha_i^k \sim \mathcal{N}(0,1)$, $i = 0, 1$, and $\mathbf{h}_1^k = (\alpha_1^k/\alpha_0^k)\mathbf{h}_0^k$. This means that the channel-modified signatures are linearly dependant, whereby the dimension of the interferers subspace of a STB encoded multiple-input-single-output (MISO) system is equal to that of a single-input-single-output (SISO) system and the identifiability condition (8) gives a common upper bound to the maximum number of active users, i.e.

$$K \leq \min \left\{ \left\lfloor \frac{(\overline{m}NQ - LM) + 1}{(\overline{m} + \overline{L} - 1)} \right\rfloor, \phi(N) \right\}.$$

In this scenario, using an Alamouti encoded transmission is always beneficial, since the system can take advantage of extra transmit diversity branches, without incurring a faster shortage of the interference-free directions, as shown in Fig. 7.

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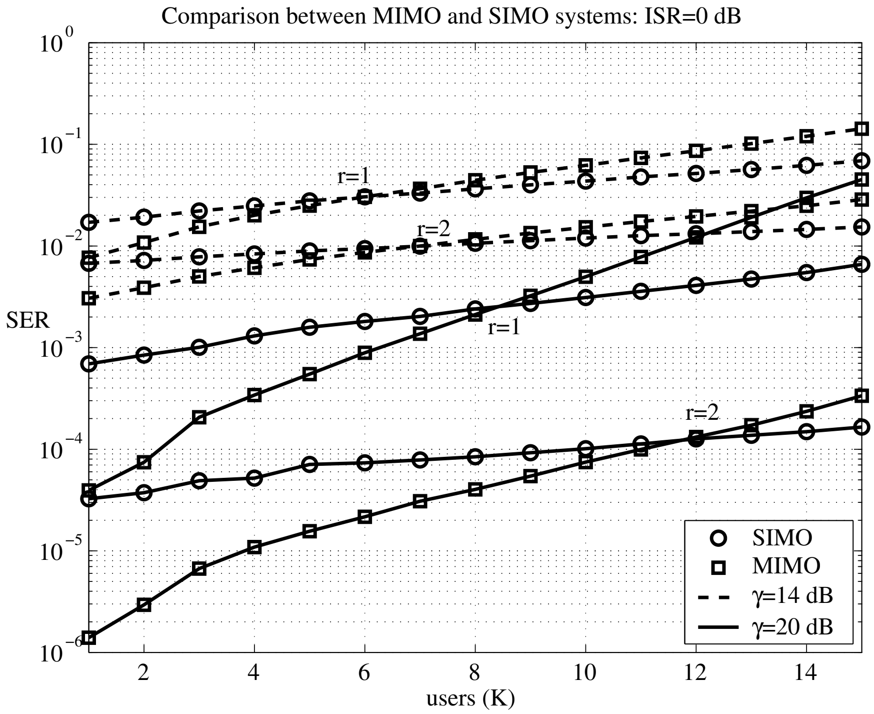

Fig. 6. SER versus the number of active users. For the SIMO system, the decoding strategy in (22) is adopted. For the MIMO system, differentially Alamouti encoding is assumed ($t = 2$), and the decoding rule (15) is employed. System parameters: $N = 32$, $M = 2$, $Q = 1$, $r = 1, 2$, ISR=0 dB, $\gamma = 14, 20$ dB, $B = 2000$.

V. Conclusions

In this paper, the problem of blind decoding in dispersive MC-DS/CDMA MIMO channels has been addressed. Each user is assigned one signature to be employed on all of the transmit antennas. This context rules out any form of uncoded transmission, since the association of the decoded symbols to the corresponding transmit antenna requires the availability of distinct signatures, labeling the different channels: in the proposed framework, those signatures are provided in the space-time domain through STBCs. Focusing on the differential Alamouti code, we have proposed a two-stage receive strategy, wherein the first stage performs a linear interference-blocking transformation, which allows user separation and ISI removal. The second stage is, instead, devoted to decode the underlying differential STBC: A new decoding rule suitable for frequency-selective channels has been introduced and discussed. Remarkably, the proposed receive structure can be blindly implemented with complexity cubic in the processing gain and allows decoupling of the decisions on the transmitted symbols.

The performance assessment has highlighted merits and drawbacks of the proposed system. As far as

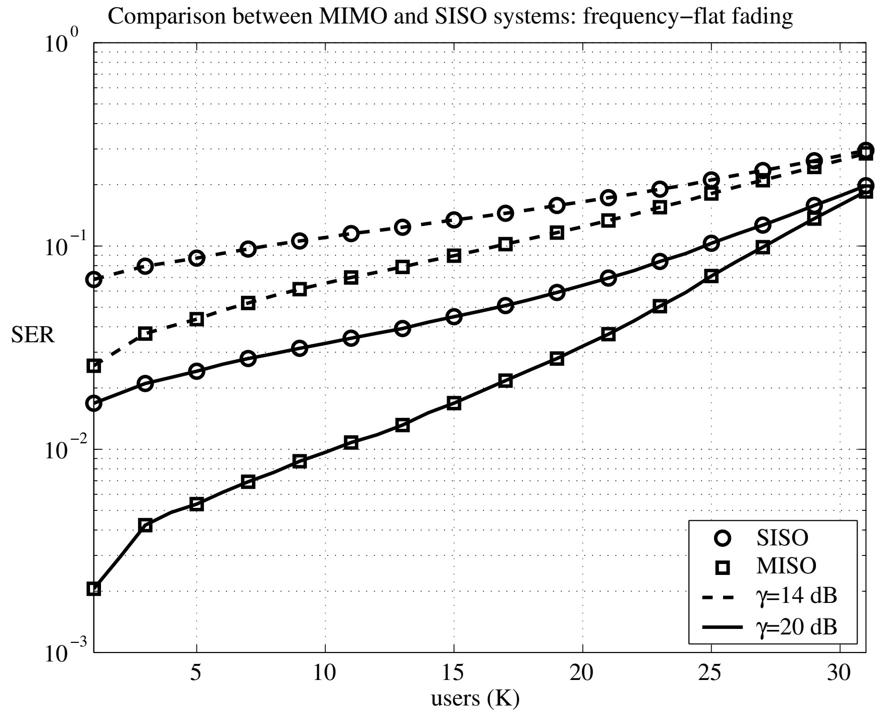

Fig. 7. SER versus the number of active users. For the SISO system, the decoding strategy in (22) is adopted. For the MISO system, differentially Alamouti encoding is assumed ($t = 2$), and the decoding rule (15) is employed. System parameters: $N = 32$, $M = 1$, $Q = 1$, $r = 1$, ISR=0 dB, $\gamma = 14, 20$ dB, frequency-flat fading and encoder delay τ_c^0 known.

the transmission efficiency is low (i.e., for number of active users conveniently smaller than the processing gain) using multiple transmit antennas is always beneficial, irrespective of the number of receive antennas. On the other hand, as the network load increases, the enhancement of co-channel interference may eat out the whole transmit diversity gain, unless the number of receive antennas is made conveniently large, so as to prevent any signal-space saturation.

A weakness of this study concerns user synchronization: indeed, even though *ad-hoc* schemes to acquire both the encoder and the channel delays have been devised, a systematic approach to the synchronizer design is still missing and forms the subject of current research. Likewise, some key problems, such as the structure of a globally optimum demodulator, as well as bounds on the achievable performance, are still open and under study.

APPENDIX

For the differential Alamouti code, we give the following.

Theorem A.1: Let $\mathbf{\Sigma}_p^k$ be the transmitted codeword defined as in (1), with $\mathbf{M}_p^k$ a 2×2 unitary information matrix given by (2). If $\{\mu^k(q),\}_{q=0}^{2P-1}$ is an i.i.d. zero-mean proper process belonging to a constant modulus, half energy constellation $\mathcal{A}$, then we have

$$\mathrm{E}\left[s_i^k(l)\right] = 0,$$

$$\mathrm{E}\left[s_i^k(l)s_j^k(m)^*\right] = \begin{cases} 1/2 & \text{if } (i,l) = (j,m) \\ 0 & \text{otherwise,} \end{cases}$$

for $i,j = 0,1,\ l,m = 0,\ldots,2P-1$, and, thus, the transmitted symbols $\left\{s_0^k(q), s_1^k(q)\right\}_{q=0}^{2P-1}$ satisfy condition [C1].

Proof: We will prove the theorem inductively. For $P = 1$, $\mathbf{\Sigma}_0^k = \mathbf{M}_0^k$ and the thesis is true since $\{\mu^k(q),\ q = 0,\ldots,2P-1\}$ is an i.i.d. zero-mean proper process with energy 1/2. Suppose now that the theorem holds for $P = n$, i.e. for the set $\left\{s_0^k(q), s_1^k(q)\right\}_{q=0}^{2n-1}$, and let us take the new codeword $\mathbf{\Sigma}_n^k = \mathbf{\Sigma}_{n-1}^k \mathbf{M}_n^k$. Since $\mathbf{\Sigma}_{n-1}^k$ is independent from $\mathbf{M}_n^k$ we have that:

$$\begin{aligned} \mathrm{E}\left[\mathbf{\Sigma}_n^k\right] &= \mathrm{E}\left[\mathbf{\Sigma}_{n-1}^k\right] \mathrm{E}\left[\mathbf{M}_n^k\right] = \mathbf{O}_{2,2}, \\ \mathrm{E}\left[\mathbf{\Sigma}_n^k s_i^k(q)^*\right] &= \mathrm{E}\left[\mathbf{\Sigma}_{n-1}^k s_i^k(q)^*\right] \mathrm{E}\left[\mathbf{M}_n^k\right] = \mathbf{O}_{2,2}, \end{aligned}$$

for $i = 0,1,\ q = 0,\ldots,2n-1$; furthermore

$$\begin{aligned} \mathrm{E}\left[|s_0^k(2n)|^2\right] = &\,\mathrm{E}\left[|\mu^k(2p)|^2\right] \mathrm{E}\left[|s_0^k(2n-2)|^2\right] \\ &+ 2\Re\Big\{\mathrm{E}\left[\mu^k(2p)\mu^k(2p+1)^*\right]\Big\}\Big\{\mathrm{E}\left[s_0^k(2n-2)s_0^k(2n-1)^*\right]\Big\} \\ &+ \mathrm{E}\left[|\mu^k(2p+1)|^2\right] \mathrm{E}\left[|s_0^k(2n-1)|^2\right] = 1/2, \end{aligned}$$

$$\mathrm{E}\left[|s_1^k(2n)|^2\right] = \mathrm{E}\left[|s_0^k(2n+1)|^2\right] = \mathrm{E}\left[|s_1^k(2n+1)|^2\right] = 1/2,$$

$$\begin{aligned} \mathrm{E}\left[s_0^k(2n)s_1^k(2n)^*\right] = &\,\mathrm{E}\left[|\mu^k(2n)|^2\right] \mathrm{E}\left[s_0^k(2n-2)s_1^k(2n-2)^*\right] \\ &+ \mathrm{E}\left[\mu^k(2n)\mu^k(2n+1)^*\right] \mathrm{E}\left[s_0^k(2n-2)s_1^k(2n-1)^*\right] \\ &+ \mathrm{E}\left[\mu^k(2n+1)\mu^k(2n)^*\right] \mathrm{E}\left[s_0^k(2n-1)s_1^k(2n-2)^*\right] \\ &+ \mathrm{E}\left[|\mu^k(2n+1)|^2\right] \mathrm{E}\left[s_0^k(2n-1)s_1^k(2n-1)^*\right] = 0, \end{aligned}$$

$$\mathrm{E}\left[s_0^k(2n+1)s_1^k(2n+1)^*\right] = 0,$$

$$\begin{aligned}\mathrm{E}\left[s_0^k(2n)s_1^k(2n+1)^*\right] =& \\ & - \mathrm{E}\left[\mu^k(2n)\mu^k(2n+1)\right]\mathrm{E}\left[s_0^k(2n-2)s_1^k(2n-2)^*\right] \\ & + \mathrm{E}\left[\left(\mu^k(2n)\right)^2\right]\mathrm{E}\left[s_0^k(2n-2)s_1^k(2n-1)^*\right] \\ & + \mathrm{E}\left[\left(\mu^k(2n+1)\right)^2\right]\mathrm{E}\left[s_0^k(2n-1)s_1^k(2n-2)^*\right] \\ & + \mathrm{E}\left[\mu^k(2n+1)\mu^k(2n)\right]\mathrm{E}\left[s_0^k(2n-1)s_1^k(2n-1)^*\right] = 0,\end{aligned}$$

$$\mathrm{E}\left[s_0^k(2n+1)s_1^k(2n)^*\right] = 0,$$

$$\begin{aligned}\mathrm{E}\left[s_0^k(2n)s_0^k(2n+1)^*\right] =& \\ & - \mathrm{E}\left[\mu^k(2n)\mu^k(2n+1)\right]\mathrm{E}\left[|s_0^k(2n-2)|^2\right] \\ & + \mathrm{E}\left[\left(\mu^k(2n)\right)^2\right]\mathrm{E}\left[s_0^k(2n-2)s_0^k(2n-1)^*\right] \\ & - \mathrm{E}\left[\left(\mu^k(2n+1)\right)^2\right]\mathrm{E}\left[s_0^k(2n-1)s_0^k(2n-2)^*\right] \\ & + \mathrm{E}\left[\mu^k(2n+1)\mu^k(2n)\right]\mathrm{E}\left[|s_0^k(2n-1)|^2\right] = 0,\end{aligned}$$

$$\mathrm{E}\left[s_1^k(2n)s_1^k(2n+1)^*\right] = 0.$$

Thus, the theorem holds for $P = n + 1$.

□

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