2013 IEEE/RSJ International Conference on
Intelligent Robots and Systems (IROS)
November 3-7, 2013. Tokyo, Japan

Projection-Based Force Reflection Algorithms for Teleoperated Rehabilitation Therapy

S. Farokh Atashzar, Ilia G. Polushin, Rajni V. Patel

***Abstract*— The problem of designing of a haptics-enabled teleoperated rehabilitation system in the presence of communication delays is addressed. In a teleoperated rehabilitation system, communication delays introduce phase shift which may result in the task inversion phenomenon. To overcome the task inversion, a new type of projection-based force reflection algorithm is proposed which is suitable for assistive/resistive therapy in the presence of irregular communication delays. Additionally, algorithms for augmented therapy are introduced which combine the projection-based force reflection with a delay-free local virtual therapist. A small-gain design is developed which guarantees stability of the proposed schemes for both assistive and resistive modes of the therapy. Simulations and experimental results are presented which confirm the improvement achieved by the proposed methods.**

I. INTRODUCTION

Telerobotic and haptic technologies have been developed intensively over the last two decades. One emerging application of these technologies is haptic enabled robotic rehabilitation [1]. Currently, there are more than 5 million stroke victims alive in the United States alone [2]; over 300,000 patients in America survive each year after experiencing a stroke [1]. Haptics-enabled robotic rehabilitation therapy substantially intensifies the recovery process for these patients, while also allowing therapists to quantify the severity of the disorder and to reduce the recovery period [1], [3], [4]. A number of commercially available haptic-enabled robotic rehabilitation systems utilize a concept of virtual therapist [1], [4], [5]. The virtual therapist is an interactive virtual reality game-like environment which can provide the patient with assistive/resistive/coordinated forces during execution of simple tasks. However the conventional haptics-enabled virtual therapist systems suffer from the lack of direct contribution/supervision of the therapist in the rehabilitation procedure or difficulty in providing appropriate assistive/resistive/coordinated forces. Research has shown that the key to effective and progressive therapy is the ability to modify therapy exercises considering the progress of the rehabilitation. To deal with this issue, in [6], a performance-based adaptation law for adjusting the level of assistance/resistance is proposed. However, it is difficult, if not impossible, to design an adaptation law that duplicates the expertise of a skilled therapist. Robotics-assisted cooperative therapy as proposed in [7] uses the therapists actions in manipulating one end of a virtual object while the other end is manipulated by the patient. This differs from assistive/resistive/coordinated rehabilitation where the therapist provides assistive/resistive/coordinated forces while the patient tries to track a trajectory. In order to provide appropriate expertise of resistive/assistive/coordinative therapy and to include the therapist in the rehabilitation procedure, we have proposed a bilateral haptics-enabled teleoperated rehabilitation architecture in [8]. We have therefore used the term "teleoperated rehabilitation" rather than the more conventional term "tele-rehabilitation" as used in [9]. In the proposed architecture, a one-master/one-slave structure of a teleoperated rehabilitation system was considered, where the patient moves the master manipulator, while the therapist interacts with the slave. It was pointed out that, during the assistive therapy, the therapist acts as a non-passive "environment" adding energy to the teleoperated rehabilitation system, which makes it difficult to apply conventional passivity-based methods [10] for stabilization of such a system. Instead, in [8], a small-gain approach to stabilization of a teleoperated rehabilitation system in the presence of communication delays was developed.

Although the small-gain design approach guarantees stability during both assistive and resistive therapy, however, the presence of significant communication delays may create additional problems which can lead to substantial performance deterioration of teleoperated rehabilitation therapy. Specifically, the delay in the communication channel may bring a phase shift to the force feedback signal that can cause the assistive actions of the therapist to become resistive, and *vice versa*. Below, this phenomenon is called task inversion. To overcome the task inversion while guaranteeing overall stability, we introduce new versions of the projection-based force reflection algorithms for both assistive and resistive types of therapy. We also propose new algorithms for augmented therapy which intelligently combine the assistive/resistive forces generated by the hu-

This research was supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada under Discovery Grant RGPIN1345 (R.V. Patel) and under a Collaborative Research and Development Grant # CRDPJ 411603-10 (R. V. Patel). The equipment used for this work was acquired through infrastructure grants from the Canada Foundation for Innovation awarded to the London Health Sciences Centre (Canadian Surgical Technologies & Advanced Robotics (CSTAR)) and to The University of Western Ontario (R.V. Patel). The project leader is R. V. Patel. The authors are with the Department of Electrical and Computer Engineering, Western University, London, ON, N6A 5B9, Canada, and with CSTAR, London, ON, Canada. E-mails: {satashza,ipolushi,rvpatel}@uwo.ca.

978-1-4673-6357-0/13/$31.00 ©2013 IEEE

man therapist with assistive/resistive actions of a local virtual therapist depending on the level of task deterioration caused by the existence of communication delays. A design approach is developed based on small-gain considerations which guarantees stability of the teleoperated rehabilitation system with the proposed algorithms. Simulations as well as experimental results are presented which confirm the improvement achieved by the proposed methods.

The paper is organized as follows. The mathematical model of a teleoperated rehabilitation system is introduced in Section II. Section III defines resistive and assistive types of therapy and describes how the communication delay may result in the task inversion phenomenon. Section IV introduces a modification of the projection-based force reflection algorithms which is suitable for teleoperated rehabilitation systems and, in particular, allows to overcome the task inversion phenomenon. In Section V, an augmented teletherapy approach is described which is essentially based on projection-based force reflection augmented with a local virtual therapist. Simulations and experimental results are presented in Section VI. All proofs are omitted due to space constraints.

II. Teleoperated rehabilitation system

We address a bilateral teleoperator system designed for rehabilitation purposes, where the master manipulator is moved by a patient, while the slave interacts with a therapist. For simplicity, we address the case of linear 1-DOF master and slave manipulators; the results presented, however, can be extended to a higher DOF case in a straightforward manner. The master manipulator is described in the Laplace domain according to the following equation,

$$Z_m(s)V_p(s) = u_{cm}(s) + F_p(s), \tag{1}$$

where $Z_m(s)$ is the impedance of the master manipulator, $V_p(s)$ is the master velocity which is also the velocity of the patient's hand, $u_{cm}(s)$ is the master control input, and $F_p(s)$ is the force applied by the patients hand. The slave manipulator, on the other hand, is described by a similar equation, as follows

$$Z_s(s)V_{th}(s) = u_{cs}(s) - F_{th}(s), \tag{2}$$

where $Z_s(s)$ is the impedance of the slave manipulator, $V_{th}(s)$ is the velocity of the slave/therapist's hand, $u_{cs}(s)$ is the slave control input, and $F_{th}(s)$ is the force exerted on the slave manipulator by the therapist's hand. The dynamics of the patient's hand are described according to the formula

$$F_p(s) = -Z_p(s)V_p(s) + F_p^*(s), \tag{3}$$

where $Z_p(s)$ is the impedance of the patient's hand, while $F_p^*(s)$ is the active force which is voluntarily generated by the patient's muscle system with the purpose of moving the master device. The dynamics of the therapist have similar structure and are described as follows,

$$F_{th}(s) = Z_{th}(s)V_{th}(s) + F_{th}^*(s), \tag{4}$$

where $Z_{th}(s)$ is the impedance of the therapist's hand and $F_{th}^*(s)$ are the active component of the forces that are generated voluntary by the therapist. However, since the primary purpose of the therapist's voluntary actions is to perform therapy by either helping the patient to move the device (assistive therapy) or suppressing the motion by applying resistive forces (resistive therapy), it is natural to assume that the voluntary component of the therapist's forces $F_{th}^*(s)$ has the form

$$F_{th}^*(s) = K_{th}(s)V_{th}(s), \tag{5}$$

where $K_{th}(s)$ is a transfer function that depends on the type of therapy. It is worth to mention that, since the transfer functions $Z_m(s)$, $Z_s(s)$, $Z_p(s)$, $Z_{th}(s)$ represent passive impedances of mechanical systems (master and slave manipulators and patient's and therapist's hands, respectively), all these transfer functions are positive real [11]. A transfer function $H(s)$ is called *positive real* if its frequency response satisfies $H(j\omega) \geq 0$ for all $\omega \in (-\infty, +\infty)$; passivity of an LTI system is equivalent to the positive realness of its transfer function [11]. The master (patient) site and the slave (therapist) site exchange information over communication channels according to the formulae

$$\hat{v}_p(t) := v_p\left(t - \tau_f(t)\right), \quad \hat{f}_{th}(t) := f_{th}\left(t - \tau_b(t)\right), \tag{6}$$

where $v_p(t) := \mathcal{L}^{-1}\left[V_p(s)\right]$ and $f_{th}(t) := \mathcal{L}^{-1}\left[F_{th}(s)\right]$ are the inverse Laplace transforms of $V_p(s)$ and $F_{th}(s)$, respectively, and $\tau_f(\cdot)$, $\tau_b(\cdot)$ are the communication delays in the forward (from patient to therapist) and the backward (from therapist to patient) directions, respectively. The control algorithms for master and slave devices are defined according to the following formulae

$$u_{cm}(s) := Z_m(s)V_p(s) - F_r(s), \tag{7}$$

$$u_{cs}(s) := F_{th}(s) + Z_s(s)\hat{V}_p(s), \tag{8}$$

where $\hat{V}_p(s) := \mathcal{L}\left[\hat{v}_p(t)\right]$, $F_r(s)$ is the force reflected to the motors of master (patient's) manipulator. It is worth noting that the control algorithm (7), (8) with $F_r(s) = \mathcal{L}\left[\hat{f}_{th}(t)\right]$ allows to achieve the perfect transparency (*i.e.*, $F_{th}(s) = F_p(s)$, $V_{th}(s) = V_p(s)$) in the absence of communication delays ($\tau_f(t) \equiv \tau_b(t) \equiv 0$) (see [12], [8]). Substituting (7), (8) into (1), (2), and taking into account (3), (5), the closed-loop dynamics of the master and the slave subsystems are obtained as follows

$$V_p(s) = Z_p^{-1}(s) \cdot \left[F_p^*(s) - F_r(s)\right], \tag{9}$$

$$F_{th}(s) = \left[Z_{th}(s) + K_{th}(s)\right]\hat{V}_p(s), \tag{10}$$

where $\hat{V}_p(s) := \mathcal{L}\left[\hat{v}_p(t)\right]$, and the force reflection term $F_r(s)$ is designed below.

III. Assistive vs. Resistive Therapy

In teleoperated rehabilitation systems, therapist (real or virtual) performs therapy by interacting with a patient over distance using a teleoperator system. Depending on the type of therapy, the therapist can either resists the motion generated by the patient by applying forces against the direction of the movement, or assist the patient to execute a desired movement by applying aiding forces. These two types of therapy admit an obvious interpretation in terms of the properties of mathematical model of the therapist. Specifically, during resistive therapy, the therapist applies forces which are directed against the velocity of the movement generated by the patient, thus effectively dissipating the energy produced by the patient. Thus, during resistive therapy, the therapist acts as a passive dynamical system; in terms of the mathematical model of the therapist,

$$F_{th}(s) = \left[Z_{th}(s) + K_{th}(s)\right] V_{th}(s) \tag{11}$$

the latter means that the transfer function of the therapist's actions $Z_{th}(s)+K_{th}(s)$ is positive real. On the other hand, during assistive therapy, the therapist generates forces that aid the patient's efforts by adding energy to the movement generated by the patient. In this case, therapist acts as an active mechanical system, which implies that $Z_{th}(s)+K_{th}(s)$ is negative real.

Such a co-existence of different therapy modes presents certain challenge for the control design of the teleoperated rehabilitation system. Specifically, most of the existing design methods aim to guarantee stability under the assumption that a teleoperator system interacts with a passive environment. In the case of a teleoperated rehabilitation system, the role of the environment is played by the therapist. As explained above, passivity of the therapist is a reasonable assumption in the case of resistive therapy; however, in the case of assistive therapy, the therapist is necessarily non-passive. This problem was addressed in the authors' previous work [8], where the small gain approach was adopted for the stability analysis of a teleoperated rehabilitation system; moreover, a control design method was presented that guarantees stability of a teleoperated rehabilitation system simultaneously in both assistive and resistive modes. One additional advantage of the small gain design is that it doesn't impose phase constraints to guarantee the stability which essentially implies that the system remains stable in the presence of communication delays, both constant and time-varying.

However, although the stability of a teleoperated rehabilitation system can be guaranteed in both assistive and resistive mode by using the small gain design approach of [8], there still exists at least one potential problem that may arise in such a teleoperated rehabilitation system in the presence of significant communication delays. Specifically, as mentioned in the introduction, the delay in the communication channel creates phase lag which may result in the task inversion phenomenon. The task inversion introduces disturbances into the interaction process between the therapist and the patient; as a result, if sufficiently pronounced and not treated properly, it may effectively defeat the purpose of the rehabilitation therapy.

IV. Projection-based force reflection for assistive/resistive therapy

To overcome the task inversion phenomenon, a method is proposed below which is based on an appropriate development of the projection-based force reflection principle. The projection-based force reflection algorithms were introduced into force-reflecting teleoperator systems in [13] and subsequently developed in [14], [15]. The goal pursued by introduction of these algorithms was to separate the "interaction" and the "momentum-generating" components of the reflected force and consequently reflect these components with different gains to the motor of the master device, specifically, emphasizing the former while attenuating the latter. This method allows for accurate representation of the slave-environment contact forces to the operator while suppressing the master motion that may be generated by these forces, which makes it suitable for teleoperation on passive and predominantly static environments. The teleoperated rehabilitation therapy, on the other hand, is predominantly a dynamic process where the environment (therapist's hand) is moving synchronously with the patient's hand while applying forces that depend on the direction of the movement and the type of therapy (assistive or resistive). This, in particular, implies that the component of the reflection force to be suppressed during teleoperated rehabilitation therapy is not the "momentum-generating" component as in the traditional force reflecting teleoperation, but rather the force component that opposes the specific type of therapy (*i.e.*, the resistive component during assistive therapy, and *vice versa*). This fact calls for the development of new types of projection-based force reflection algorithms where the reflected force depends on the direction (velocity) of movement as well as the type of therapy (assistive or resistive). The new type of projection-based algorithms proposed in this paper can be described in terms of an auxiliary force signal $F_v(s)$ which is defined according to the formula

$$F_v(s) := H_s(s)V_p(s), \tag{12}$$

where $H_s(s)$ is a positive real transfer function which otherwise can be chosen arbitrarily by the designer subject to the norm constraints defined below. The projection based force reflection algorithm for resistive therapy is defined as follows,

$$f_r = \hat{\phi}^r := \underset{[0,1]}{\mathrm{Sat}} \left\{ \frac{\hat{f}_{th}^T f_v}{\max\left\{|f_v|^2, \epsilon\right\}} \right\} f_v, \tag{13}$$

where $f_r(t) := \mathcal{L}^{-1}\left[F_r(s)\right]$, $f_v(t) := \mathcal{L}^{-1}\left[F_v(s)\right]$, $\hat{f}_{th}(t)$ is the therapist's force as received at the patient's side according to (6), and $\epsilon > 0$ is a small positive constant. The projection-based force reflection algorithm for assistive therapy, on the other hand, has the form

$$f_r := \hat{\phi}^a := \underset{[-1,0]}{\mathrm{Sat}} \left\{ \frac{\hat{f}_{th}^T f_v}{\max\{|f_v|^2, \epsilon\}} \right\} f_v. \qquad (14)$$

Remark 1. The above formulated projection-based force reflection algorithms (12), (13) and (12), (14) can be given the following explanation. First, since $H_s(s)$ in (12) is a positive real transfer function, a force reflection signal of the form $f_r = f_v$ would resist the motion of the master manipulator. The force signal f_v represents the maximum resistive force possible in the teleoperated rehabilitation system under consideration. The projection based algorithm (12), (13), therefore, calculates the projection of the delayed therapist forces $\hat{f}_{th}$ onto the positive direction of the signal $f_v(t)$. The upper saturation limit at 1 in (13) guarantees that the forces reflected to the patient's hand doesn't exceed $f_v(t)$ in the magnitude, which allows to guarantee the stability of the overall system. The lower saturation limit at 0, on the other hand, ensures that the component of the resistive therapist's forces that has become assistive due to the task inversion phenomenon (*i.e.*, because of the phase shift generated by the delay in the communication channel, see Section III) is cancelled out from the force reflecting signal. A similar explanation can be given to the algorithm (12), (14) for assistive therapy, which calculates the projection of the delayed therapist forces $\hat{f}_{th}$ onto the negative direction of the signal $f_v(t)$. In this case, the saturation limit at 0 guarantees that the resistive component of the delayed therapist forces is cancelled out from the force reflecting signal $f_r(t) := \mathcal{L}^{-1}[F_r(s)]$. Overall, the proposed force reflection algorithms enable us to overcome the task inversion phenomenon while, as we will see below, guaranteeing stability of the overall system. •

Now, let us address the problem of stability of the teleoperated rehabilitation system with the projection-based force reflection described above. The stability result presented below is valid under the following assumptions on the communication delay functions $\tau_f(t)$, $\tau_b(t)$ in (6).

Assumption 1. [15], [16] The communication delays $\tau_f, \tau_b : \mathbb{R} \to \mathbb{R}_+$ are Lebesgue measurable functions with the following properties:

i) there exist $\tau_* > 0$ and a piecewise continuous function $\tau^* \colon \mathbb{R} \to \mathbb{R}_+$ satisfying $\tau^*(t_2) - \tau^*(t_1) \le t_2 - t_1$, such that the inequalities $\tau_* \le \min\{\tau_f(t), \tau_b(t)\} \le \max\{\tau_f(t), \tau_b(t)\} \le \tau^*(t)$ hold for all $t \ge 0$;

ii) $t - \max\{\tau_f(t), \tau_b(t)\} \to +\infty$ as $t \to +\infty$. •

The Assumption 1 is satisfied in any real-life communication networks unless the communication is completely lost on a semi-infinite time interval. In particular, the fulfilment of this assumption does not depend on the characteristics of the communication channel, such as bandwidth and information loss percentage. Next, by $\|H(s)\|_1$ let us denote the 1-norm of a transfer function $H(s)$ defined according to the formula $\|H(s)\|_1 := \int_0^{+\infty} |h(\tau)|\, d\tau$, where $h(t) := \mathcal{L}^{-1}[H(s)]$ is the corresponding impulse response function. The following statement is valid.

Theorem 1. Consider a teleoperated rehabilitation system (9), (10), (6) with a force reflecting algorithm described by either (12), (13) or (12), (14). Suppose the communication delay functions $\tau_f(t)$, $\tau_b(t)$ satisfy Assumption 1. If

$$\|H_s(s)\|_1 \cdot \left\|Z_p^{-1}(s)\right\|_1 < 1, \qquad (15)$$

then the trajectories of the teleoperated rehabilitation system are bounded and convergent. •

V. Augmented assistive/resistive teleoperated rehabilitation therapy

The projection-based force reflection algorithms presented in the previous section allow us to overcome the task inversion phenomenon by cancelling out the unwanted assistive (in the case of resistive therapy) or resistive (in the case of assistive therapy) component of the reflected force. In the case of significant communication delays or substantially irregular communication, however, a situation is possible where a major part of the reflected force is cancelled, which may decrease the efficiency of the teleoperated rehabilitation therapy. Below, a scheme is proposed where a cancelled component of the reflection force is substituted with an appropriate artificial component generated locally at the patient's site. The proposed algorithm for resistive tele-therapy has the form

$$\begin{aligned} f_r = \hat{\phi}^R_{Aug} &:= \underset{[0,1]}{\mathrm{Sat}} \left\{ \frac{f_r^T f_v}{\max\{|f_v|^2, \epsilon\}} \right\} f_v \\ &+ \frac{\beta}{2} \left(1 - \mathrm{sign}\left\{ \frac{f_r^T f_v}{\max\{|f_v|^2, \epsilon\}} \right\}\right) f_v, \end{aligned} \qquad (16)$$

where $\beta \in [0, 1]$ is the gain of the virtual therapist. A similar algorithm for assistive tele-therapy is given by

$$\begin{aligned} f_r = \hat{\phi}^A_{Aug} &:= \underset{[-1,0]}{\mathrm{Sat}} \left\{ \frac{f_r^T f_v}{\max\{|f_v|^2, \epsilon\}} \right\} f_v \\ &- \frac{\beta}{2} \left(1 + \mathrm{sign}\left\{ \frac{f_r^T f_v}{\max\{|f_v|^2, \epsilon\}} \right\}\right) f_v. \end{aligned} \qquad (17)$$

The stability of the teleoperated rehabilitation system augmented by a virtual therapist is described by the following result.

Theorem 2. Consider a teleoperated rehabilitation system (9), (10), (6) with an augmented projection-based force reflecting algorithm described by either (12), (16) or (12), (17). Suppose the communication delay functions $\tau_f(t)$, $\tau_b(t)$ satisfy Assumption 1. Then, the system is stable (more precisely, bounded and convergent) if (15) holds. •

VI. Simulations and Experimental Results

In this section, we briefly present samples of simulations and experimental results in support of the theory outlined in the previous sections. First, we present samples of experimental evaluation of the proposed projection-based force reflection algorithm (12), (14). To this end, a Phantom Omni haptic device from Sensible Inc. was used as the master robot, and a simulated assistive/resistive actions of the therapist were implemented in Matlab/Simulink environment using Quarc 2.2 software. Only results related to assistive therapy are presented; similar results for resistive therapy are omitted because of space constraints. In the results presented below, the transfer function of the (assistive) therapist is $Z_{th}(s) + K_{th}(s) = -40$ N·s/m. The communication delay in each direction is $\tau_f = \tau_b = 0.05 + 0.01 \sin 30t$ s, and $H_s(s) = 10$ N·s/m in (12). In this experiment, the patient attempts to move the device along the trajectory which is approximately a sinusoidal function of time; the resulting force response of the therapist and the forces reflected to the patient's hand are shown in Figure 1, left plot. It can be seen that the projection-based force reflection algorithm keeps the overall system stable by decreasing the magnitude of the therapist's forces. A magnified view of the therapist's forces and the forces reflected to the patient's hand are presented in Figure 1, right plot, where also a scaled patient's velocity is shown. It is clearly visible that the projection-based algorithm eliminates the force response of the therapist entirely during the period of time when the therapist force is directed against the patient's velocity, thus overcoming the task inversion phenomenon. The stabilizing

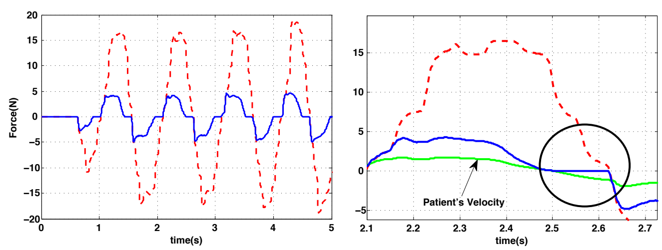

Fig. 1: Experimental results: Response of the teleoperated rehabilitation system with the force reflection algorithm (12), (14). Left plot: therapist's force (dashed red line) vs. reflected force (solid blue line). Right plot: magnified view of a part of the response, therapist's force (dashed red line), reflected force (solid blue line), and scaled patient's velocity (solid green line)

properties of the proposed projection-based force reflection algorithm are demonstrated in Figure 2. In this experiment, the projection-based force reflection algorithm is switched off after $t = 5$ s; it can be seen that the assistive therapy becomes unstable almost immediately after the proposed controller is switched off.

In order to demonstrate the advantages of the augmented therapy algorithms presented in Section V, simulations of the teleoperated rehabilitation system were performed in the presence of large communication delays. Specifically, in the simulations presented below, the round trip time (RTT) communication delay is equal 2 s. This large communication

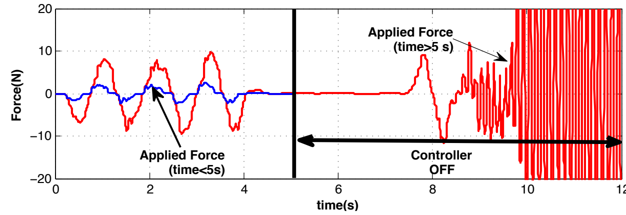

Fig. 2: Experimental results: Response of the teleoperated rehabilitation system with the force reflection algorithm (12), (14); the force reflection algorithm is switched off after $t = 5$ s which results in instability.

delay introduces a phase shift that may result in the task inversion phenomenon. The following three algorithms are compared in terms of their performance during assistive therapy: i) the small-gain controller developed in [8]; ii) the projection-based force reflection algorithm for assistive therapy presented in Section IV, and iii) the augmented assistive therapy algorithm described in Section V. In order to compare the performance of these algorithms, the external forces applied by the patient should be exactly the same in every case; this was the main reason why simulations were chosen over the experiments in this study. In the simulations below, the following parameters are used: $Z_p(s) = 80 + 5s^{-1}$, $Z_{th}(s)+K_{th}(s) = -400-25s^{-1}$, $H_s(s) = 35+2s^{-1}$, and the external forces applied by the patient are $f_p^*(t) = 60 \sin(0.7t)$ s. In every case described below, the therapists does not interact with the slave device during the first 40 s of the simulations, and starts to assist the patient's movement at $t = 40$ s. The results of simulations of the small-gain controller are shown in Figure 3. It can be seen from these Figure that, although the small-gain comptroller guarantees stability in the presence of large communication delays, the task inversion phenomenon clearly occurs in this case. Specifically, because of the phase shift in the communication channel, the assistive actions of the therapist become resistive as can be clearly seen in Figure 3, bottom plot. As a result, the magnitude of the patient's movement is decreased after $t = 40$ s.

The results of the simulations of the teleoperated rehabilitation system with the projection-based force reflection algorithm (12), (14) are illustrated in Figure 4. It can be clearly seen in these Figures that the projection-based force reflection algorithm overcomes the task inversion phenomenon by eliminating the resistive component of the reflected forces. However, the effect of the assistive actions of the therapist in this case is largely unnoticeable since the therapist's forces are almost entirely eliminated. Finally, the simulation results for the case of the augmented assistive force reflection algorithm (12), (17) are shown in Figure 5. It can be seen that in this case the assistive actions of the therapist are successfully restored by the local delay-free augmented therapy algorithm, which leads to a clear increase in the magnitude of the patient's movement.

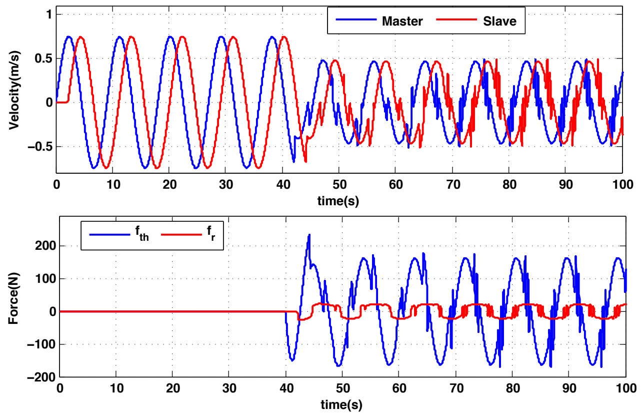

Fig. 3: Simulation results, assistive therapy, RTT delay 2 s, small gain controller [8]. Top plot: therapist's velocity (red line) vs. patient's velocity (blue line). Bottom plot: therapist's forces (blue line) vs. reflected forces (red line).

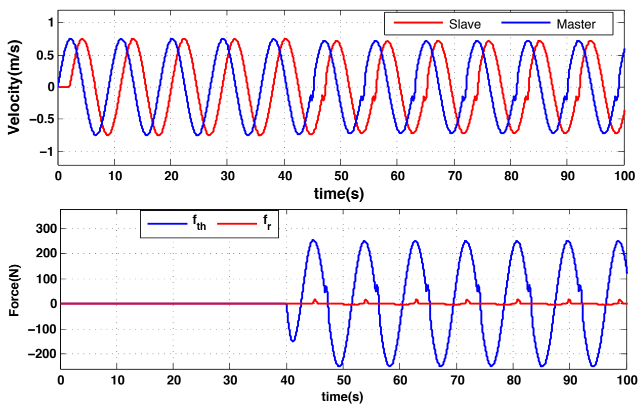

Fig. 4: Simulation results, assistive therapy, RTT delay 2 s, projection-based force reflection algorithm (12), (14). Top plot: therapist's velocity (red line) vs. patient's velocity (blue line). Bottom plot: therapist's forces (blue line) vs. reflected forces (red line).

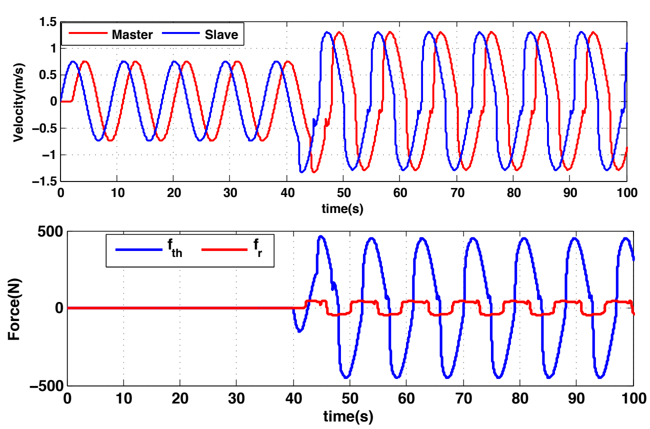

Fig. 5: Simulation results, assistive therapy, RTT delay 2 s, augmented force reflection (12), (17). Top plot: therapist's velocity (red line) vs. patient's velocity (blue line). Bottom plot: therapist's forces (blue line) vs. reflected forces (red line).

References

[1] H. I. Krebs and N. Hogan, "Therapeutic robotics: A technology push," *Proceedings of the IEEE*, vol. 94, no. 9, pp. 1727–1738, 2006.

[2] "Heart Disease and Stroke Statistics – 2011 Update. A Report From the American Heart Association." http://circ.ahajournals.org/content/123/4/e18.

[3] H. I. Krebs, L. Dipietro, S. Levy-Tzedek, S. E. Fasoli, A. Rykman-Berland, J. Zipse, J. A. Fawcett, J. Stein, H. Poizner, A. C. Lo, B. T. Volpe, and N. Hogan, "A paradigm shift for rehabilitation robotics," *Engineering in Medicine and Biology Magazine, IEEE*, vol. 27, pp. 61–70, Aug. 2008.

[4] R. Colombo, F. Pisano, S. Micera, A. Mazzone, C. Delconte, M. C. Carrozza, P. Dario, and G. Minuco, "Robotic techniques for upper limb evaluation and rehabilitation of stroke patients," *IEEE Transactions on Neural Systems and Rehabilitation Engineering*, vol. 13, pp. 311–324, Sept. 2005.

[5] H. I. Krebs, M. Ferraro, S. P. Buerger, M. J. Newbery, A. Makiyama, M. Sandmann, D. Lynch, B. T. Volpe, and N. Hogan, "Rehabilitation robotics: pilot trial of a spatial extension for MIT-Manus.," *Journal of NeuroEngineering and Rehabilitation*, vol. 1, Oct. 2004.

[6] H. I. Krebs, J. J. Palazzolo, L. Dipietro, M. Ferraro, J. Krol, K. Rannekleiv, B. T. Volpe, and N. Hogan, "Rehabilitation Robotics: Performance-Based Progressive Robot-Assisted Therapy," *Autonomous Robots*, vol. 15, pp. 7–20, July 2003.

[7] C. R. Carignan and P. A. Olsson, "Cooperative control of virtual objects over the internet using force-reflecting master arms," in *Proceedings of the 2004 IEEE International Conference on Robotics and Automation*, (New Orleans, LA), pp. 1221–1226, 2004.

[8] S. F. Atashzar, I. G. Polushin, and R. V. Patel, "Networked teleoperation with non-passive environment: Application to tele-rehabilitation," in *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2012)*, (Vilamoura, Algarve, Portugal), pp. 5125–5130, Oct. 2012.

[9] G. G. Demiris, C. L. Shigaki, and L. H. Schopp, "An evaluation framework for a rural home-based telerehabilitation network.," *Journal of Medical Systems*, vol. 29, pp. 595–603, Dec. 2005.

[10] P. F. Hokayem and M. W. Spong, "Bilateral teleoperation: A historical survey," *Automatica*, vol. 42, no. 12, pp. 2035–2057, 2006.

[11] B. Brogliato, R. Lozano, B. Maschke, and O. Egeland, *Dissipative Systems Analysis and Control. Theory and Applications*. Springer, second ed., 2007.

[12] E. Naerum and B. Hannaford, "Global transparency analysis of the lawrence teleoperator architecture," in *IEEE International Conference on Robotics and Automation*, (Kobe, Japan), pp. 4344–4349, 2009.

[13] I. G. Polushin, P. X. Liu, and C.-H. Lung, "A force-reflection algorithm for improved transparency in bilateral teleoperation with communication delay," *IEEE/ASME Transactions on Mechatronics*, vol. 12, no. 3, pp. 361–374, 2007.

[14] I. G. Polushin, P. X. Liu, and C.-H. Lung, "Projection-based force reflection algorithm for stable bilateral teleoperation over networks," *IEEE Transactions on Instrumentation and Measurement*, vol. 57, no. 9, pp. 1854–1865, 2008.

[15] I. G. Polushin, P. X. Liu, and C.-H. Lung, "Stability of bilateral teleoperators with generalized projection-based force reflection algorithms," *Automatica*, vol. 48, no. 6, pp. 1005–1016, 2012.

[16] I. G. Polushin, H. J. Marquez, A. Tayebi, and P. X. Liu, "A multichannel IOS small gain theorem for systems with multiple time-varying communication delays," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 404–409, 2009.

[17] A. R. Teel, "Connections between Razumikhin-type theorems and the ISS nonlinear small gain theorem," *IEEE Transactions on Automatic Control*, vol. AC-43, pp. 960–964, July 1998.

[18] I. G. Polushin, S. N. Dashkovskiy, A. Takhmar, and R. V. Patel, "A small gain framework for networked cooperative force-reflecting teleoperation," *Automatica*, vol. 49, no. 2, pp. 338 – 348, 2013.