Engineering Warp Drives

Brice N. Cassenti[1] and Harry I. Ringermacher[2]

[1]*United Technologies Research Center, 411 Silver Lane, MS 129-73, East Hartford, CT 06108*
(860) 610-7460 (voice), (860) 610-7526 (fax), cassenbn@utrc.utc.com
[2]*General Electric Company, Corporate Research and Development, Bldg. KW, D254, PO Box 8, Schenectady, NY 12301*
(518) 387-5469 (voice), (518) 387-5752 (fax), ringerha@crd.ge.com

Abstract. Warp drives are widely perceived by the public to be the propulsion system of choice in the not too distant future. Engineers and scientists usually ignore the possibility because of the apparently insurmountable obstacles required to create a warp drive. The promise and the problems of warp drives are examined in three parts: 1) a review of some of the current literature, 2) a summary of the physics, and 3) the engineering developments required to develop warp drives.

INTRODUCTION

Warp drives are a popular device used in science fiction to circumnavigate the speed of light as the maximum possible speed. This restriction follows from the special theory of relativity. Relieving it allows unlimited access to the galaxy and the universe. But the physics of warp drives is not well understood and the engineering prospects appear to place their development well into the future.

The existing literature consists mostly of the description of the physics of spacetime associated with wormholes and is not suited to developing actual systems, but does establish the possibility. A summary of the physics of wormholes can be found in an excellent book by Visser (1996). Millis (1996) has described several conceptual approaches to develop warp drives. A general description of general relativity and space warps can be found in Forward (1989), and includes descriptions of rotating and charged black holes. A description of the effects of magnetic fields on spacetime can be founds in Maccone (1995), and Landis (1997). The most directly applicable work on warp drives can be found in Alcubierre (1994) and Van Den Broeck (1999).

This paper first summarizes the Theory of General Relativity, and then applies the equations of General Relativity to motion near the surface of the earth. The next section consists of a detailed discussion of the Alcubierre warp drives, and is followed by an extension of General Relativity that includes electromagnetic forces (Ringermacher, 1994 and Ringermacher, 1997). The last two sections consist of a brief description of the engineering aspects and conclusions.

SUMMARY OF GENERAL REALTIVITY

The Theory of Special Relativity (Einstein, 1905) was proposed to reconcile experimental and theoretical conflicts that existed in physics at the beginning of the twentieth century. Experimental evidence indicated that the speed of light in a vacuum was not a function of the speed of the source or the observer. For this to remain true in all constant velocity reference frames the interval, $d\tau$ below, must remain constant, where

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 . \tag{1}$$

In eq. (1) c is the speed of light, t is the time, and x, y, z are the Cartesian coordinates of a point. Equation (1) can be written in a more general form by setting

CP504, *Space Technology and Applications International Forum–2000*, edited by M. S. El-Genk

$$ds = cd\tau\,,\ dx^0 = cdt\,, dx^1 = dx\,,\ dx^2 = dy\,,\ dx^3 = dz\ \ , \tag{2}$$

equation (1) can now be written as

$$ds^2 = \sum_{\mu=0}^{3} g_{\mu\nu} dx^{\mu} dx^{\nu} = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{3}$$

where superscripts are for contravariant tensors and subscripts are for covariant, The tensor $g_{\mu\nu}$ is the metric, and the last equality in eq. (3) indicates repeated indices (one contravariant and one covariant) are summed unless otherwise noted. For eq. (2) the metric is given by

$$g_{00} = 1, \text{ and } g_{11} = g_{22} = g_{33} = -1 \tag{3'}$$

All other components of the metric vanish for the interval given by eq. (1). In relativity the metric is symmetric (i.e., $g_{\mu\nu}=g_{\nu\mu}$). Although Einstein's original papers on Special Relativity are easy to read, good physical insight can obtained from the text by Taylor and Wheeler (1966).

General Relativity is an extension of eqs. (3) and (3') to spacetimes that are not flat (Einstein, 1956, and Bergmann, 1976), where the metric cannot be transformed to (3') by any coordinate transformation. Such spacetimes are Riemannian and posses an intrinsic curvature, $R_{\mu\nu\sigma}{}^{\lambda}$ (the Riemannian tensor), given by:

$$R_{\mu\nu\sigma}{}^{\lambda} = \frac{\partial \Gamma^{\lambda}_{\sigma\nu}}{\partial x^{\mu}} - \frac{\partial \Gamma^{\lambda}_{\sigma\mu}}{\partial x^{\nu}} + \Gamma^{\lambda}_{\tau\mu}\Gamma^{\tau}_{\sigma\nu} - \Gamma^{\lambda}_{\tau\nu}\Gamma^{\tau}_{\sigma\mu}\,, \tag{4}$$

where the connection, $\Gamma^{\sigma}_{\mu\nu}$, is given by

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left(\frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} + \frac{\partial g_{\lambda\mu}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}}\right), \tag{5}$$

and,

$$g^{\lambda\sigma} g_{\sigma\mu} = \delta^{\lambda}{}_{\mu} = \begin{cases} 1 & \lambda = \mu \\ 0 & \lambda \neq \mu \end{cases} \ . \tag{6}$$

The connection is sometimes referred to as the Riemann-Christoffel symbol of the second kind, and vanishes only if the metric is constant.

The Theory of General Relativity does not use the rank four Riemann curvature tensor directly but instead uses the rank two Ricci tensor, $R_{\mu\nu}$, given by:

$$R_{\nu\sigma} = R_{\lambda\nu\sigma}{}^{\lambda} = \frac{\partial \Gamma^{\lambda}_{\sigma\nu}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\lambda}_{\sigma\lambda}}{\partial x^{\nu}} + \Gamma^{\lambda}_{\tau\lambda}\Gamma^{\tau}_{\sigma\nu} - \Gamma^{\lambda}_{\tau\nu}\Gamma^{\tau}_{\sigma\lambda} = R_{\sigma\nu}\,. \tag{7}$$

The equations of motion are found by 'minimizing' the interval, *ds*, between two events as

$$\frac{d^2 x^{\sigma}}{ds^2} + \Gamma^{\sigma}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} = \frac{e}{mc^2} F^{\sigma\mu}\frac{dx_{\mu}}{ds}, \tag{8}$$

where $x_\mu = g_{\mu\nu} x^\nu$, e is the charge on the particle, m is the mass of the particle, and $F^{\sigma\mu}$ contains the electromagnetic forces. In Cartesian coordinates :

$$F^{\sigma\mu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z & H_y \\ E_y & H_z & 0 & -H_x \\ E_z & -H_y & H_x & 0 \end{bmatrix} \tag{9}$$

where (E_x, E_y, E_z) is electric field intensity, and (H_x, H_y, H_z) is the magnetic field intensity.

The equations for the fields can now be written as

$$G_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu} \tag{10}$$

where the Einstein tensor, $G_{\mu\nu}$, is given by:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \quad . \tag{11}$$

R is the Ricci scalar and is given by

$$R = g^{\mu\nu} R_{\mu\nu} \tag{12}$$

$T_{\mu\nu}$, is the energy density tensor, which can be found from the relation:

$$T_{\mu\nu} = (p + \rho c^2) \frac{dx_\mu}{ds} \frac{dx_\nu}{ds} - p g_{\mu\nu} + \frac{1}{4\pi^2} \left(\frac{1}{4} g_{\mu\nu} F_{\sigma\lambda} F^{\sigma\lambda} - F_{\mu\lambda} F_\nu^\lambda \right) \quad , \tag{13}$$

where p is the pressure of the matter, ρ is the density of matter, and G is the gravitational constant.

MOTION NEAR THE SURFACE OF THE EARTH

The motion of a particle near the surface of the earth can provide some insight into the conservation of momentum and energy in relativity theory. In special relativity the momentum and energy are the components of a four dimensional vector exactly analogous to the coordinates x, y, z, and t of an event. The length (i.e. the interval) of the vector is invariant with a coordinate system rotation or translation (including systems with a relative velocity) and is given by

$$m_0^2 c^4 = E^2 - p_x^2 c^2 - p_y^2 c^2 - p_z^2 c^2 \quad . \tag{14}$$

The quantity $m_0 c^2$ is the rest mass energy, where m_0 is the rest mass, and is an invariant. Equation (14) connects the energy, E, which contains the kinetic energy, and the momentum, $\vec{p} = (p_x, p_y, p_z)$. Hence, energy and momentum are components that can be transformed into one another by a coordinate system translation.

For a particle near the surface of the earth the metric is given by (e.g. see Cassenti and Ringermacher, 1996),

$$ds^2 = \left(1 - \frac{2g_0R^2}{c^2} + \frac{2g_0R^2z}{c^2R^2}\right)d(ct)^2 - \left(1 + \frac{2z}{R}\right)dx^2 - \left(1 + \frac{2z}{R}\right)dy^2 - \left(1 + \frac{2g_0R^2}{c^2} - \frac{2g_0R^2z}{c^2R^2}\right)dz^2 \quad .(15)$$

where z is the altitude, R is the radius to the surface of the earth, and g_0 is the acceleration of gravity at the surface of the earth. Using the metric in eq. (15), the equations of motion become:

$$\frac{d^2t}{d\tau^2} + \frac{2g_0}{c^2}\frac{dt}{d\tau}\frac{dz}{d\tau} = 0,\ \frac{d^2x}{d\tau^2} = 0,\ \frac{d^2y}{d\tau^2} = 0,\ \text{and}\ \frac{d^2z}{d\tau^2} + g_0\left(\frac{dt}{d\tau}\right)^2 - \frac{g_0}{c^2}\left(\frac{dz}{d\tau}\right)^2 = 0 \quad . \quad (16)$$

These equations can be integrated once to yield

$$\frac{dt}{d\tau} = 1 + \frac{g_0R^2}{c^2} + \frac{v^2}{2c^2} - \frac{g_0R^2z}{c^2R},\ \frac{dx}{d\tau} = 0,\ \frac{dy}{d\tau} = 0,\ \text{and}\ \frac{dz}{d\tau} = -g_0\frac{dt}{d\tau} \quad , \quad (17)$$

where $\left(\frac{dz}{cd\tau}\right)^2 << 1$, $\left(\frac{z}{R}\right)^2 << 1$ and the initial conditions were taken as $\frac{dt}{d\tau} = 1$ and $\frac{dx}{d\tau} = \frac{dy}{d\tau} = \frac{dz}{d\tau} = 0$. Making use of eq. (15) and neglecting terms that are small relative to one, the first and last of eqs. (17) become:

$$\frac{1}{2}v^2 + g_0z = \frac{E}{m_0} = const.\ \text{and}\ v + g_0t = \frac{P}{m_0} = const. \quad , \quad (18)$$

where $v=dz/dt$. Using the momentum $p=m_0v$ eqs.(18) can be written as

$$\frac{p^2}{2m_0} + m_0g_0z = E\ \text{and}\ p + m_0g_0t = P \quad . \quad (19)$$

Equations (19) are the conservation of energy and momentum. The common explanation is to note that the center of mass of the earth and the particle continues to move at a constant velocity thus conserving the energy and momentum, but eqs.(19) indicate a different interpretation. Momentum and energy are transferred locally from the gravitational field to the particle and then changes in the field due to the particle are propagated to the source (i.e., the earth). With this interpretation the source still cannot create a change in spacetime that also changes the total momentum and its energy. But note that if only a local field is necessary to transfer energy and momentum then a warp drive is possible. The local field may then propagate the back to other sources such as the vacuum fluctuations or the universe as a whole.

ALCUBIERRE WARP DRIVE SOLUTION

Alcubierre (1994) has proposed a metric that appears to allow travel faster than the speed of light. The metric proposed by Alcubierre results in the interval

$$ds^2 = c^2dt^2 - (dx - v_s(t)f(r_s)dt)^2 - dy^2 - dz^2, \tag{20}$$

where $v_s(t) = \dfrac{dx_s}{dt}$ is the source speed, $r_s = \sqrt{[x - x_s(t)]^2 + y^2 + z^2}$, and

$$f(r_s) = \frac{\tanh[\sigma(r_s + R)] - \tanh[\sigma(r_s - R)]}{2\tanh(\sigma R)}. \tag{21}$$

The parameters σ and R are constants. Note that as $\sigma \to \infty$

$$f(r_s) \to \begin{cases} 1 & -R \le r_s \le R \\ 0 & otherwise \end{cases}.$$

Alcubierre shows that the space behind the source is expanding while the space is contracting ahead of the source. An observer outside the source region can see the source as moving faster than then speed of light. The metric when analyzed does not satisify the Weak Energy Condition and the Strong Energy Condition (see Visser, 1996). Van Den Broeck (1999a) modified the metric proposed by Alcubierre to

$$ds^2 = c^2dt^2 - B^2(r_s)(dx - v_s(t)f(r_s)dt)^2 - dy^2 - dz^2. \tag{22}$$

The proper choice for $B(r_s)$ greatly reduces the energy requirements and satisfies the Strong Energy Condition.

THE ELECTRODYNAMIC CONNECTION

The Alcubierre warp drive requires a method for distorting the space. Exotic matter is needed but electromagnetic fields could provide a more realistic method for distorting the spacetime. This requires a more direct connection between gravitational and electromagnetic forces and would make the development of the necessary fields clearer. Ringermacher has proposed a change to the connection (Ringermacher, 1994), and the associated field equations (Ringermacher, 1997) that creates a geometry that can include electromagnetic forces. The modified connection, $\tilde{\Gamma}^{\sigma}_{\mu\nu}$, is the sum of the connection, $\Gamma^{\sigma}_{\mu\nu}$, in eq.(5) and terms containing the electromagnetic fields, $F^{\sigma\mu}$, in eq.(9), and reads

$$\tilde{\Gamma}^{\sigma}_{\mu\nu} = \Gamma^{\sigma}_{\mu\nu} - \frac{e}{2mc^2} g^{\sigma\lambda}[v_\mu F_{\lambda\nu} + v_\nu F_{\lambda\mu} - v_\lambda F_{\mu\nu}]. \tag{23}$$

Note that $F_{\mu\nu} = g_{\mu\sigma} g_{\nu\lambda} F^{\sigma\lambda}$. Using the connection in eq.(23), the resulting field equations (with $T_{\mu\nu}=0$) can be written as (see Ringermacher, 1997)

$$G_{\mu\nu} = \frac{e}{mc^2} v^\sigma (F_{\nu\sigma;\mu} + F_{\mu\sigma;\nu} + g_{\nu\sigma} j_\mu + g_{\mu\sigma} j_\nu - 2g_{\mu\nu} j_\sigma), \tag{24}$$

where

$$F_{\mu\nu;\sigma} = \frac{\partial F_{\mu\nu}}{\partial x^\sigma} - \Gamma^{\lambda}_{\mu\sigma} F_{\lambda\nu} - \Gamma^{\lambda}_{\nu\sigma} F_{\lambda\mu}, \tag{25}$$

$$j_{\mu} = \begin{Bmatrix} c\rho_e \\ -j_x \\ -j_y \\ -j_z \end{Bmatrix} \quad . \tag{26}$$

ρ_e is the charge density and $\vec{j}$ is the current density in a flat spacetime. Since the theory includes currents which are higher order contributions than the energy densities, and, hence, experimental tests of the theory appear to be feasible (Ringermacher, 1998).

CONCLUSIONS

A warp drive would, not only, allow travel faster than the speed of light but warping spacetime could also reduce or cancel gravity, provide artificial gravity, and allow for propulsion without the use of a propellant. The first steps must include an examination of the physics in detail, and any extensions of physics that can unite electromagnetism with matter and gravity. If a source cannot pick up, or dump, momentum to the universe, or the vacuum, then it may not be possible to construct warp drives, and if electromagnetism cannot be readily used to warp space then the construction of a warp drive will require enormous energy densities over significant volumes. Hence, if the physics works out, the best path to a warp drive includes using the vacuum as source of momentum and applying electromagnetic fields to warp the spacetime.

REFERENCES

Alcubierre, M., "The Warp Drive: Hyper-Fast Travel Within General Relativity," Classical and Quantum Gravity, 11, pp. L73-L77,(1994).

Bergmann, P.G., *Introduction to the General Theory of Relativity*, New york, Dover Publications, 1976.

Cassenti, B.N., and H.I. Ringermacher, "The 'How To' of Antigravity," Paper No. AIAA-95-2788, presented at the 32nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference, Lake Buena Vista, FL, July, 1-3, (1996).

Einstein, A.,"On the Electrodynamics of Moving Bodies" and "Does the Inertia of a Body Depend on its Energy Content?," Annalen Der Physick, 17, (1905). See *The Principle of Relativity,* New York, Dover Publications, Inc. , 1952.

Einstein, A., *The Meaning of Relativity,* Princeton, The Princeton University Press, 1956.

Forward, R.L., "Space Warps: A Review of On Form of Propulsionless Transport;" Journal of the British Interplanetary Society, 42, pp. 533-542, (1989).

Landis, G.A., "Magnetic Wormholes and the Levi-Civita Solution to the Einstein Equations," Journal of the British Interplanetary Society, 50, pp. 155-157, (1997).

Maccone, C., "Interstellar Travel Through Magnetic Wormholes," Journal of the Interplanetary Society, 48, pp. 453-458, (1995).

Millis, M.G., "The Challenge to Create the Space Drive," Journal of the British Interplanetary Society, (1996).

Ringermacher, H.I., "An Electrodynamic Connection," Classical and Quantum Gravity, 11, pp. 2383-2394, (1994).

Ringermacher, H.I. , et al " Search for the Effects of an Electrostatic Potential on Clocks in the Frame of Reference of a Charged Particle," presented at the Breakthrough Propulsion Physics Workshop, Cleveland, August 12-14, (1997).

Ringermacher, H.I., and B.N. Cassenti, "An Experiment to Verify a Relativistic Field Theory Based on an Electrodynamic Connection," Paper No. AIAA-98-3136, presented at the 34th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 13-15, (1998).

Taylor, E.F., and A. Wheeler, *Spacetime Physics*, San Francisco, W.H. Freeman and Company, 1966.

Van Den Broeck, C., "A 'warp drive' with reasonable total energy requirements," to be published in General Relativity and Quantum Cosmology, (1999).

Visser, M., *Lorentzian Wormholes - From Einstein to Hawking*, Woodbury, New York, American Institute of Physics, 1996 .