Cooperative navigation in robotic swarms

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***Abstract*—We present a cooperative navigation algorithm for robotic swarms. Its purpose is to let a robot find a given target robot, while being guided by the other robots of the swarm. The system is based on wireless communication: the robots forward messages containing navigation information over the ad hoc network among them, and the searching robot uses this information to find its target. We study the algorithm in two different scenarios. In the first scenario, a single searching robot needs to find a single target, while all other robots are involved in tasks of their own. We show that the communication based navigation system allows the robots of the swarm to guide the searching robot without the need to adapt their own movements. In the second scenario, we study collective navigation: all robots of the swarm need to navigate back and forth between two targets. We show that in this case, the proposed navigation algorithm gives rise to synergies in robot navigation, and lets the swarm self-organize into a robust dynamic structure. This self-organization improves navigation efficiency, and lets the swarm find shortest paths in cluttered environments. We test our system both in simulation and on real robots.**

I. INTRODUCTION

In this paper, we study cooperative navigation in swarm robotics. In general, robot navigation is the task of finding a collision-free path for a robotic system between one state and another [1]. For autonomous mobile robots, this usually involves the availability of a map, which can possibly be built by the robot itself [2]. Sometimes, the use of a map can be avoided, e.g., by fitting the environment with a network of communication nodes, which guides the robot [3]. In multi-robot and swarm systems [4], map-less navigation can be obtained if robots cooperatively help in each other's navigation.

Most work in the context of cooperative swarm robotics navigation is based on indirect communication between robots, and is inspired by the foraging behavior of certain types of ants in nature [5], [6], [7], [8], [9], [10], [11]. This behavior relies on *stigmergic communication*, which is a form of indirect communication through local modification and sensing of the environment. Specifically, ants moving between the nest and a food source leave a chemical substance, called pheromone, in the environment, which attracts other ants and guides them to the food. The interesting aspect is that the collective process of pheromone laying and following reinforces the most efficient paths, so that eventually the shortest path appears as a consequence of the swarm's collective actions [12], [13]. This is an example of *emergent self-organized behavior*, meaning that the swarm organizes without outside control, due to local interactions between individual agents [14]. An important difficulty with the use of this pheromone-based navigation model in robotics is the practical implementation of the indirect communication, in terms of a satisfactory artificial replacement for the chemical pheromone used by ants. Also, this approach assumes a problem setup where all robots of the swarm move back and forth between two target locations, and cannot easily be generalized to other scenarios.

In this work, we propose a new approach for navigation in swarm robotics, based on direct communication between robots. We use the following general problem description. The swarm is deployed in a confined area. A robot S of the swarm needs to navigate to a given target robot T, which is outside the range of its sensors and communication devices. T announces its presence with periodic wireless message broadcasts. We investigate how S can find T through cooperative support from the other robots in the swarm. An important aspect is that these other robots are involved in tasks that are independent of the navigation of S. They do not adapt their movements to guide S in its navigation task, but they do offer help through communication. Note that the presented problem description is very general. This is because the behavior of the remaining robots of the swarm does not depend on the navigation of S to T. In fact, these robots may be involved in any task of their own, including a different navigation task, to another target T', or even to the same target T. Depending on the behavior of the different robots, a variety of scenarios can be obtained, including the earlier cited problem setup where all robots move back and forth between two targets.

Our algorithm is based on mobile wireless network communication. Each robot A coming in communication range of a target robot T, and receiving its periodic broadcasts, stores information about T in a local data structure, which we call a *navigation table*. This information consists of a sequence number, indicating the relative age of the message, and a distance value, which is an estimate of the navigation distance to T. As A moves around, it updates the information in its navigation table, and periodically broadcasts it to neighboring robots. This way, navigation information can travel through the (possibly intermittently connected) network formed among the swarm of robots. A searching robot S receiving new navigation information from a robot B, compares this new information to previously received navigation information, and moves towards B's location if the new information is better. This way, S moves from robot to robot towards the target, somehow similar to how a data packet follows a route through a mobile ad hoc network (MANET) [15], [16].

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The proposed algorithm is relatively simple, but very powerful and versatile. When applied in different scenarios, it can give rise to different swarm-level movement patterns, while each time providing efficient navigation. We study two scenarios in particular, which we refer to as *single robot navigation* and *collective navigation*.

In the single robot navigation scenario, a single robot S needs to find a single target robot T, which remains static. An example application of this scenario could be that T is indicating a place where a certain task needs to be performed, and S has the capabilities required for this task. All other robots of the swarm execute random movements, expressing that they are involved in other tasks, which are independent of S's navigation. The goal is to show that using the proposed algorithm, they can offer support to S's navigation without having to adapt their own movements. We investigate the performance of the system with varying swarm sizes, environments, and random movement patterns. We show that the approach is efficient, scalable, and robust to robot failures.

The collective navigation problem is essentially the earlier cited problem setup, frequently studied in swarm robotics, where all robots of the swarm navigate back and forth between two targets T and T'. We show that this problem can also be tackled using the simple communication based navigation scheme we propose. Compared to the single robot navigation problem of the first scenario, we show that collective navigation gives rise to synergies, improving navigation performance. In particular, the concurrent execution of communication based navigation by all robots lets the swarm self-organize, and a collective movement pattern emerges in the swarm behavior. This self-organized movement improves navigation efficiency and is robust with respect to the swarm size. Moreover, it allows to find shortest paths in cluttered environments. This means that collective navigation based on our communication-based system has similar properties to ant-inspired pheromone-based navigation, while avoiding the problem of how to implement stigmergic communication. Besides showing a new approach for collective navigation, this is also an example of the general applicability of our simple navigation system.

Our system relies on wireless message communication between robots to find paths for navigation. To make this approach feasible, we require some specific properties from the robots' wireless communication device. First of all, the device should provide only line-of-sight communication, so that communication links can be related to obstacle-free paths. Second, the device should be able to link received messages to relative position information (angle and distance) about their sender, so that robots can follow paths detected through communication. Similar requirements were formulated in [17], where a network of embedded communication nodes is used to guide a single robot to a target. Similar to that work, we address these requirements using an infrared range-and-bearing (IrRB) communication system, of which implementations exist for various robots [18], [19], [20], [21]. While most results presented in this paper were obtained through simulation, we present in Section V an implementation of our algorithm on real robots, using the IrRB system.

The rest of this paper is organized as follows. In Section II, we describe the communication aided navigation algorithm. In Section III, we study the working of this algorithm in the scenario of single robot navigation. In Section IV, we investigate the scenario of collective navigation: we study how the system self-organizes, and how it is able to find shortest paths. After that, in Section V we describe the implementation of our system on real robots, and in Section VI we discuss related work. Some of the work presented here appeared earlier in conference papers [22], [23].

II. Communication aided navigation

In this section, we explain the communication aided navigation system. We first describe the details of the algorithm executed by the robots. Then, we take a look at the swarm as a whole and explain how the joint execution of the proposed algorithm by the robots can support effective navigation.

A. The navigation algorithm

The navigation system we propose is loosely based on routing algorithms used in MANETs. Using wireless communication, the robots of the swarm form a MANET among them. The general idea is to build up navigation information through communication in this MANET, and use it to guide a searching robot from hop to hop to its target, similar to how routing information is gathered in a MANET and used to forward data packets to their destination. All robots in the swarm maintain a table with navigation information about all known target robots. The information about a target T contains an estimate of the navigation distance to T, as well as an indication of the relative age of the information. Each robot periodically broadcasts the content of its table to its neighbors, which update their table based on the received information. This way, navigation information spreads throughout the swarm via wireless communication. Robots also update the distance estimates in their table based on their own movements, using odometry information. This way, navigation information can travel between parts of the MANET which are not connected through wireless communication, by being carried on board of the mobile robots, as is common in the area of delay tolerant networking (DTNs) [24], [25]. This is important to let the algorithm operate both in dense and sparse robot swarms. To navigate to a given target robot T, a searching robot S continuously monitors all received navigation information, and moves each time to the neighbor that sent it the best information (where the quality of navigation information is defined based on its distance and age, as explained below). This way, S moves between the robots of the swarm until it reaches T. In what follows, we describe different aspects of this system in detail.

a) Navigation tables and message broadcasts.: The navigation information about a target T present in a robot A's navigation table consists of a sequence number $s(T)$, indicating the relative age of the information, and a distance $d(A,T)$, indicating the distance traveled by the information between T and A. Since navigation information can only travel via line-of-sight wireless communication or on board of moving robots, $d(A,T)$ is an estimate for the navigation distance between A and T. At the start of swarm deployment, all robots have an empty table. When a robot T becomes a target robot (i.e., it discovers a target location and starts announcing it), it puts an entry about itself in its table. In this entry, both the sequence number $s(T)$ and the distance $d(T,T)$ are set to 0. At periodic intervals, robots broadcast the content of their table to neighbors. When T broadcasts the information about itself, it first increases sequence number $s(T)$ in its table by 1. The distance $d(T,T)$ is broadcast without modification. Another robot A broadcasting information about T does not modify $s(T)$, so that the sequence number marks the relative time when the information left T. The use of sequence numbers to mark the relative age of messages was inspired by MANET routing protocols such as DSDV [26]. The size of each robot's navigation table, and hence of its update messages, depends only on the number of targets in the environment. If bandwidth is limited, robots select a subset of targets to send updates about, in a round-robin fashion.

b) Processing received broadcasts.: A robot B receiving a broadcast from A processes the entries for all targets T in the message. It reads the received sequence number $s'(T)$ and distance $d'(A,T)$ from the message. On the basis of $d'(A,T)$, it calculates a new estimate for its own distance to T, $d'(B,T)$, by adding the distance $d(B,A)$ between itself and A (as measured at message reception with the IrRB communication system). Then, B compares the new values, $s'(T)$ and $d'(B,T)$, to the information about T in its own table, $s(T)$ and $d(B,T)$. The new information is considered better if either $s'(T) > s(T)$ (the new information is more recent), or $s'(T) = s(T)$ and $d'(B,T) < d(B,T)$ (the new information indicates a shorter path). In that case, the information in the table is replaced by the new information.

c) Updating distance estimates.: If B moves around without receiving new updates about T, the distance $d(B,T)$ in its table needs to be updated for it to remain an estimate of the navigation distance to T. Therefore, as B is moving, it measures its moved distance through odometry, and adds this to $d(B,T)$. This way, $d(B,T)$ grows and remains a measure of the distance traveled by the navigation information. The direction of B's movement is not taken into account, so that $d(B,T)$ is not necessarily the shortest distance to T. However, it is an upper bound of the shortest obstacle-free path (since B per definition moved over an obstacle-free path). Using this mechanism, the navigation system can work in sparsely connected swarms: navigation information can bridge gaps in network connectivity by traveling on board of moving robots.

d) Using the received messages for navigation.: A searching robot S moves towards the location of the neighbor from which it receives the best navigation information about its target T. The information $s(T)$ and $d(A,T)$, received from a neighbor A, is considered better than the information $s'(T)$ and $d'(B,T)$, received from a neighbor B, if $s(T) > s'(T)$ (A's information is more recent), or if $s(T) = s'(T)$ and $d(A,T) < d'(B,T)$ (A's navigation distance to T is less than B's). In case A's information is the best, S stores $s(T)$ and $d(A,T)$ as $s^*(T)$ and $d^*(T)$ respectively, and also A's relative location L_A, as measured by the IrRB system at the moment of message reception. It moves towards L_A using odometry. Note that S does not adapt its goal in case A moves: only A's location L_A at the moment of reception of the navigation information is important. Any newly received navigation information (either from A again, or from another neighbor) is compared to $s^*(T)$ and $d^*(T)$. If the information received from a neighbor C is better, S moves towards C's location L_C. This can happen either before S had reached its previous goal L_A, or after that. In the former case, S just abandons its previous goal in favor of the new one. In the latter case, S is faced with a period in which it has no direction to go to (between the arrival at L_A and the reception of the new information). In this case, we consider two possible strategies: S can either wait statically at L_A, or start performing random movements until new information is received. We refer to the former strategy as *navigation with stopping* (NwS), and to the latter as *navigation with random* (NwR); we compare the two strategies in Section III. The repeated moves let S follow the best navigation information through the network. When S eventually receives a message directly from T, it goes straight to T and finishes the search. Finally, we point out that we let the searching robot S approach any location (be it that of another robot A or of the target T) from the right. This is to avoid collisions head-on between robots (especially useful when two searchers move towards each other, as in the scenario of Section IV).

B. The system's dynamics

The proposed navigation algorithm lets a searching robot S move towards the location of neighbors that have information about its target T that is better than what S had previously received, where "better" information means either more recent information (higher sequence number), or information that has traveled over a shorter path from T (lower estimated distance). Here, we discuss how such moves can bring S closer to T.

The issue is relatively straightforward in scenarios where robot density is high and the swarm forms a connected MANET including S and T. In this case, the periodic local broadcasting of messages by the robots of the swarm lets each new message from T (each new sequence number) flood the MANET. Flooding spreads as an expanding ring from T, and new navigation information reaches S first over the shortest path through the network. Such flooding mechanisms are the same as those used by reactive MANET routing algorithms to define the shortest path for data forwarding (see, e.g. [27]). Hence, when S moves towards the most recent navigation information, it follows the shortest path available for data routing in the MANET. Since T is continuously sending new messages (with increasing sequence numbers), the path followed by S is constantly adapted to changes in the MANET topology. The correspondence between the shortest path for data routing and the shortest path for navigation depends on the density and spread of robots in the environment (see examples in Figure 1).

When we consider scenarios where the robot distribution is sparser, the MANET formed among the swarm may no longer be connected. At this point, a new message sent out by T does not immediately flood throughout the swarm: to

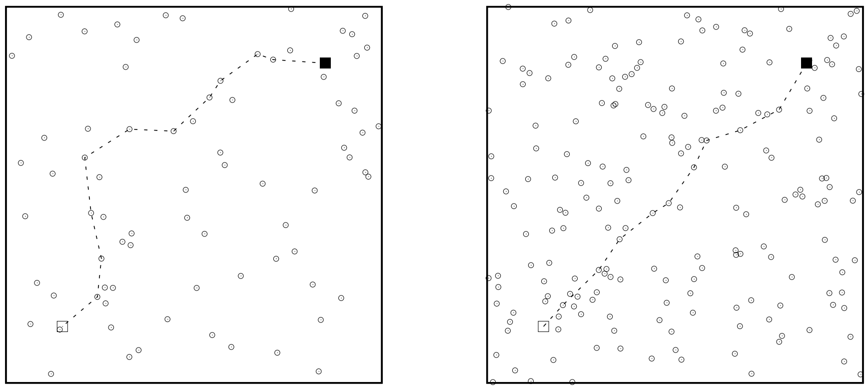

Fig. 1. Shortest communication path between two robots in a MANET. The area is 20×20 m^2, and the communication range is 3 m. The searcher is placed at the bottom left and the target at the top right. The correspondence with the shortest path for navigation (the straight line) depends on robot placement and density: we show an example with 80 robots (left) and one with 200 robots (right).

reach disconnected parts of the MANET, a message needs to be carried there by mobile robots. This means that message spreading depends on a combination of robot mobility and message communication. Several studies investigated message spreading in sparsely connected MANETs [28], [29], [30], [31], [32]. In case robot density is not extremely sparse, so that robots can communicate with others relatively frequently, new messages spread from T in an expanding wave-like propagation [31], [32]. Such propagation is similar to the form of spreading obtained through flooding (but slower, as part of the spreading is based on robots carrying the message away from T). As a consequence, if S goes towards the most recent information (or the information that has traveled the shortest distance), it moves into the direction of the expanding wave, and can therefore be expected to make steps in T's direction.

In the case of very sparse swarms, robots only occasionally meet each other. In this situation, robot mobility is the main factor defining information spreading: each robot A that meets T picks up a new message and carries it around the environment. If A does not meet any other robot, its sequence number $s(T)$ and distance estimate $d(A,T)$ are defined by respectively the time when A met T, and the total length of the movements made by A since then. When S meets A, it moves towards A if A's navigation information is better than what S has received before. Whether this effectively brings S closer to T depends on the relationship between the time/distance that A has traveled from T, and its real distance to T. This obviously depends on the movement patterns followed by A. Nevertheless, several studies in the MANET literature have shown that in general, for most reasonable mobility patterns, there is a positive correlation between the travel time/distance and the actual distance [33], [34]. This positive correlation has been used to support message forwarding, e.g., based on node encounter histories [33], [35].

To investigate more in detail the properties of this correlation and its dependence on the number of robots in the network, we performed simulation tests considering both one and multiple moving robots (the specific characteristics of the robot models and of the simulation environment are discussed in the next section). In the first set of experiments, we placed a target robot T in the middle of an uncluttered environment of 20×20 m^2, and let a single other robot A move according to a *random direction mobility* model (see Section III for details about the simulator and the mobility model). We did 10 such tests of 10000 s each. At each time step of 0.1 s, we measured the difference between the sequence number on board of A and the most recent sequence number sent out by T. We call this the *sequence number gap*. It is the relative age of the information on board of A, and measures the elapsed time since A last encountered T. We also measured at each time step the real distance between A and T. In Figure 2, we plot the average sequence number gap against the real distance. The graph shows that the sequence number gap is on average an increasing function of the distance: when A has a lower sequence number gap, it has a higher probability of being closer to T. This means that if a searching robot S moves towards a robot announcing a newer sequence number, it will, in expected value, move closer to the target. However, it must be noted that the curve in Figure 2 levels out at high distances from T; also, it has a large standard deviation (not shown here to keep the figure readable). This means that the information is quite unreliable: many of S's moves will still go in a wrong direction. The situation improves dramatically when we increase the swarm size. We performed the same tests with 20 randomly moving robots. In this case, we get in the earlier described situation where the swarm is not extremely sparse, and information spreads both through mobility and communication: the robots update each other's sequence number when they meet, and new sequence numbers spread faster through the area, according to a wave-like propagation. This makes the information much more reliable. As shown in Figure 2, we get a much smoother, almost linear relation between the sequence number gap and the distance to T.

We also performed these same experiments using the estimated navigation distance $d(A,T)$, rather than the sequence number. This gives very similar behavior, as shown in Figure 2. This means that both parts of the navigation information, the sequence number and the estimated navigation distance, are positively correlated to the actual distance to the target, and are therefore both useful navigation measures. In our

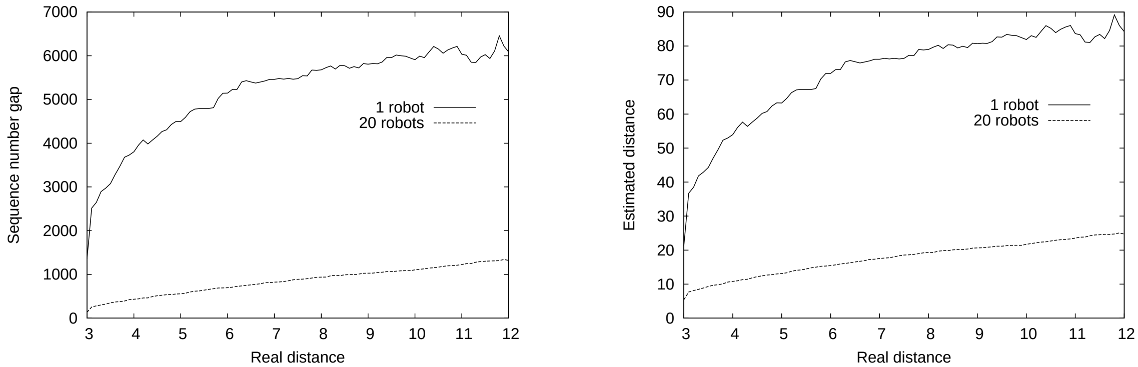

Fig. 2. We plot the navigation information (y-axis) against the distance from the target (x-axis): on the left the sequence number gap and on the right the estimated distance. We plot data for the case of 1 robot and 20 robots. See main text for explanation.

algorithm we use the sequence number and the estimated distance in combination, because this gives the best results. One could, however, also use them separately, e.g., to get a simpler system, which uses less communication bandwidth.

III. Single robot navigation

In this section, we focus on the single robot navigation scenario. As explained in Section II, the scenario consists of a robot S searching for a static target robot T. All other robots of the swarm are involved in tasks of their own, and perform movements that are unrelated to the navigation of S. To obtain such independent movements, we use random mobility patterns. Using the communication-based navigation system, the robots of the swarm can support S's navigation to T without the need to adapt their own movements.

We investigate the performance of the communication-based navigation system under varying conditions, using experiments performed in simulation. In what follows, we first describe the simulator and the robots we used in these experiments. After that, we study the system in an uncluttered environment, to show its basic working. Next, we investigate the influence of the movement patterns of the robots of the swarm, performing tests with varying mobility models. Then, we study cluttered environments, and show that the system can work even in highly complex environments, such as mazes. Finally, we investigate situations where two paths of different length are available, and show that our algorithm has a preference for the shortest path.

A. The robots and the simulator

All tests presented in this and in the next section are executed using a simulated model of the *foot-bot*, a small ground robot developed within the Swarmanoid project [36] (http://www.swarmanoid.org) on the basis of the marXbot platform [21]. The tests with real robots, presented in Section V, use this same robot.

The foot-bot is shown in Figure 3. It has a diameter of about 15 cm and it is about 20 cm high. It moves on the ground using a combination of tracks and wheels, for increased stability. It is quite a powerful robot, carrying various sensors and actuators, including two cameras, a rotating distance scanner, a gripper,

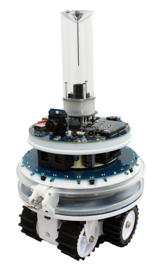

Fig. 3. The foot-bot robot developed within the Swarmanoid project.

etc. For the work presented here, two of these are particularly relevant: the infrared proximity sensors, and the IrRB module. The proximity sensors detect obstacles at a range of a few centimeters. We use them as virtual bumpers, to let robots turn away from nearby obstacles. The IrRB module [20], [21] provides local line-of-sight communication. It sends messages of 10 bytes, and has a capacity of 10 messages per second (so robots can broadcast an update every 0.1 s). Its maximum range can be of more than 5 m, but was limited to 3 m here, in order to be able to do tests in smaller environments.

As simulation tool, we use *ARGoS* [37], a physics-based simulator for heterogeneous multi-robot systems. Being developed within the Swarmanoid project, ARGoS contains reliable physics models of this robot. It also comes with a middleware for controlling the real robots, so that any code written for the simulator can be ported unchanged to the robots.

B. Tests in an uncluttered environment

We use an uncluttered closed area of $20{\times}20$ m^2. The robots are placed in the area according to a uniform random distribution. One of the robots is a target and remains static. A second robot needs to navigate to this target. The remaining robots move according to a *random direction mobility model with fixed speed* [38]. This model is defined as follows: choose a direction θ uniformly from $]-\pi, \pi]$, turn towards θ, choose a time t from an exponential distribution with fixed average (set to 10 s here), move forward for this time t, and then repeat this process. We use a forward speed of 0.15 m/s, both for the searching and the randomly moving robots. We vary the number of robots in the swarm, from 2 (0 randomly

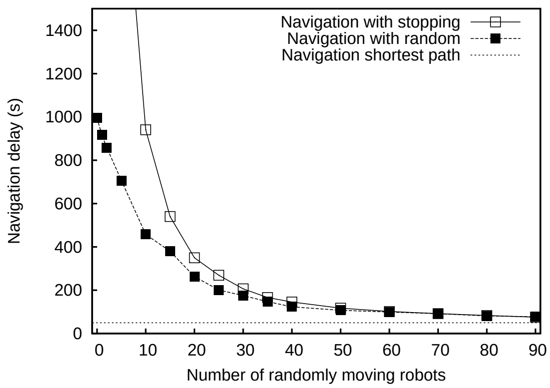

Fig. 4. Experimental results in an uncluttered environment for a single searching robot and an increasing number of randomly moving robots. In the figure, the results for NwS and NwR (the two different strategies used to deal with the temporary absence of navigation information) are reported together with the performance that would be obtained by following the shortest path.

moving robots) up to 92 (90 randomly moving). For each data point, we make 500 independent test runs (this high number is needed because the random initial positions of searcher and target induce a high variance). We measure the time between the start of each test and the moment the searching robot comes in range of the target.

The results are shown in Figure 4. We compare the two variants of the navigation system presented in Section II, navigation with stopping (NwS) and navigation with random (NwR), which differ in the strategy used by the searching robot when it does not have any navigation information (respectively, waiting for new information, or performing a random movement according to the random direction model). The results show a large difference in performance between the two strategies for low numbers of robots. This is because the communication network is sparse, and navigation information spreads slowly from the target, so that the searcher often falls without information. In the extreme case with 0 randomly moving robots, navigation with stopping can never reach the target. Navigation with random, on the other hand, does find the target, through random search. The expected time for a randomly moving agent to find a static target within a given environment is normally referred to as the expected *hitting time*, ET [39]. For many mobility models, including the random direction model used here, ET can be calculated analytically [39]. In our case, ET can be considered an upper bound for the performance of the navigation with random strategy. It is interesting to note that even a very low number of randomly moving robots in the environment gives an improvement in the navigation delay compared to ET. This confirms that even in very sparse swarms, the navigation information on board of randomly moving robots can be useful to guide the searcher, as explained in Section II-B.

For larger swarm sizes, performance improves for both strategies. This is on the one hand because the improved connectivity in the swarm makes the navigation information more reliable, as pointed out in Section II-B, and on the other hand because information reaches the searcher more frequently. The latter also means that the searcher finds itself less often without navigation information, so that the difference between the two strategies decreases. For the highest numbers of robots, performance gets close to the time needed to cross the expected straight line distance between the searcher's initial position and the target. This is indicated in Figure 4 as "Delay navigation shortest path". This gives a lower bound for the expected navigation time. The good performance for large swarm sizes shows both the *efficiency* and *scalability* of the system. It is also interesting to note the graceful degradation of the system's performance as the number of robots decreases. This indicates that the navigation system is *robust* with respect to failure or loss of robots in the swarm.

C. Tests with different movement patterns

The performance of our system depends on the movement patterns of the robots of the swarm: this defines for a large part how and where navigation information spreads. Here we carry out experiments in the same uncluttered environment used in Section III-B, using different mobility models. We use the random waypoint model (RWP) [40] and the restricted random waypoint model (RRWP) [41].

Under RWP, each robot randomly chooses a location in the environment to move to, and chooses a speed. It moves to the chosen destination with the chosen speed, and then waits there for a fixed pause time, before choosing a new destination and speed. RWP has very different statistical properties compared to the earlier used random direction model [42], [43]. E.g., it lets robots make longer straight movements (since robots can choose any location in the area to move to), it leads to a non-uniform stationary distribution of robots over the area, etc. We use RWP with a fixed speed of 0.15 m/s and a pause time of 10 s. We vary the swarm size from 2 up to 75. The results are shown in Figure 5 left. They are very similar to the ones obtained with random direction movement in Section III-B, showing that the differences between the mobility models has a limited impact on the performance of the navigation algorithm.

RRWP is a variation of RWP, in which robots can choose their destinations only from pre-defined destination areas in the environment. A fixed *roaming probability* p defines whether a robot picks its new destination from its current destination area or from a different one (roaming). To define the destination areas, we overlay the environment with a grid of $3{\times}3$ cells, where each cell is a different destination area (so, in our experiments, each point in the environment is part of exactly one destination area). We vary p between 10^{-4} and 1. For low values of p, robots remain mainly within their cell, so that we get almost exclusively local robot movements. In this case, navigation information rarely spreads between cells by being carried on board of robots, but rather through communication between robots near the cell boundaries. This has as a side effect that if one or more cells fall without robots, information may not be able to travel between target and searcher for long periods of time. Instead, for high values of p, robots are forced to leave their cell often, so that they execute long movements through the environment and bring navigation information around quickly. We believe that the different movement patterns obtained this way cover a large

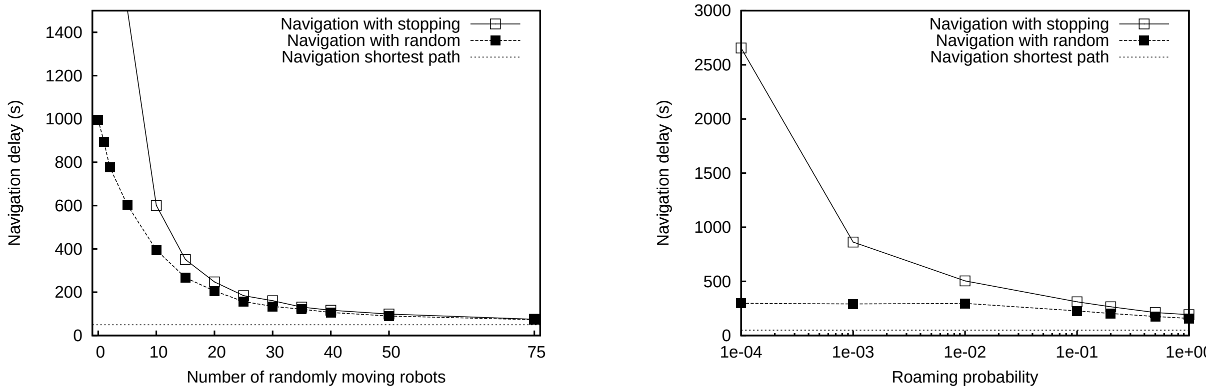

Fig. 5. Experimental results with different random mobility models for a single searching robot. The left figure refers to the case when the other robots in the swarm move according to the random waypoint model, and shows the results for an increasing number of robots. The right figure refers to the case when the other robots move according to the restricted random waypoint model. In this case, a swarm of 25 robots is considered and the results shows the impact of using different roaming probabilities. In both the two figures, the results for NwS and NwR are reported together with the performance that would be obtained by following the shortest path.

Fig. 6. Layout of cluttered environments used in our experiments. Left: a simple environment; right: a maze. The area is $20{\times}20$ m^2 in both cases.

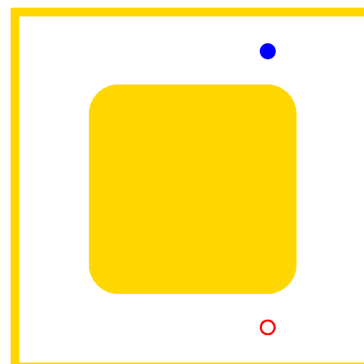

Fig. 8. Test setup for shortest path testing. The searcher starts from the bottom (indicated by the red circle). The target is placed above (indicated by the blue disk). The area is $14{\times}14$ m^2.

variety of possible behaviors of the robots of the swarm. E.g., the different cells may represent different parts of a factory, where robots perform mainly local movements around assigned work stations, or they could refer to different areas in a warehouse, where robots perform long range movements to bring goods around. We use a swarm with 25 randomly moving robots, which is a relatively sparse setup, in which the MANET is normally not connected. The results are shown in Figure 5. The very bad results for the navigation with stopping strategy at low values of p are due to the earlier mentioned effect that cells can fall without robots for a long time, effectively stopping the spreading of navigation information. However, for the navigation with random strategy, these negative effects are rather limited. For larger values of p, we note that the higher mobility of robots improves the performance for both algorithms, though, again, the effect is limited for the navigation with random strategy. We can conclude from these results that if the situation is such that information can flow from target to searcher between the robots, the actual movement patterns of the intermediate nodes does not matter much.

In the following, all tests are executed with the randomly moving robots following the random direction mobility model (see Section III-B).

D. Tests in cluttered environments

Since our navigation algorithm looks for obstacle free paths (see Section II), it deals naturally with cluttered environments. We did experiments in the two environments shown in Figure 6. Again, we deploy the swarm according to a uniform random distribution, and we measure the time needed for the searcher to reach the target. The results are shown in Figure 7. As can be expected, navigation delays get higher as the environment gets more complex, with the highest values measured in the maze. Also, a larger swarm is needed to bring this delay down, and the system has more difficulties to reach the time required to travel over the shortest path. Nevertheless, we get the same trends in performance as in uncluttered environments, and with a large enough swarm, the system guides a searching robot to its target efficiently.

We point out that our navigation system can also deal with dynamic obstacles. We do not report results here, due to lack of space, but it is clear that a reactive approach such as the one presented here has advantages in dynamic environments compared to, e.g., map based navigation systems.

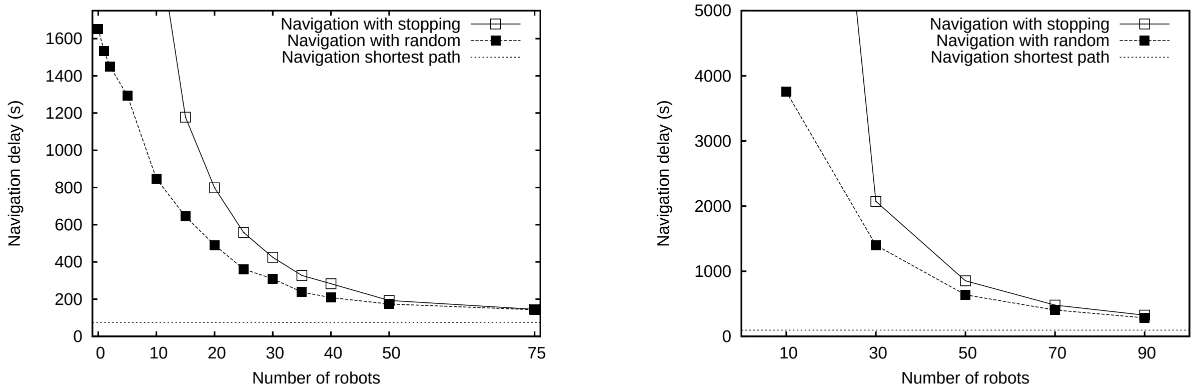

Fig. 7. Experimental results in the cluttered environments of Figure 6 for a single searching robot and an increasing number of randomly moving robots. The left plot refers to Figure 6 (left), and the right one to Figure 6 (right). In both the two figures, the results for NwS and NwR are reported together with the performance that would be obtained by following the shortest path.

E. Shortest path

In cluttered environments, a searcher may have several possible paths available to move to its target. Since our algorithm lets a searcher move in the direction from where it receives the navigation information that has traveled the shortest time or the shortest distance (see Section II-B), it should have a preference for the shortest path. We consider the scenario of Figure 8 to test this property. The target is placed in the upper part of the arena, and the searcher in the bottom part. There are two paths between them: a long one of 24 m, and a short one of 12 m. We do tests with increasing swarm sizes, from 3 robots (1 searcher, 1 target and 1 randomly moving robot) up to 72 (70 randomly moving robots). The results are shown in Figure 9. We show how often the searcher chooses the short path (as a fraction of the total number of tests), and we show the time needed for navigation. The results show that the navigation algorithm has a clear preference for the shortest path. Also, this preference leads to lower navigation delays (for the NwR strategy, we plotted the navigation delay separately for the tests in which respectively the short or the long path was chosen).

One striking element in these results is that the probability of choosing the short path is related to the swarm size, and that this relationship is different for the two navigation strategies. To explain this, let us first look at the results for the largest and smallest swarm sizes. In the scenarios with largest swarm size (70 randomly moving robots), navigation information travels primarily through multi-hop message forwarding between robots. The swarm is well connected, and navigation information travels equally quickly in all directions from the target T. Since the distance to be covered is less over the short path, the information reaches the searcher S faster this way, letting S prefer the short path. Since S rarely finds itself without navigation information, the behavior and performance are identical for both navigation strategies.

The situation is very different for the smallest swarm size (1 randomly moving robot). Here, navigation information only travels by being carried on board of the single randomly moving robot A. Under the navigation with random strategy, the influence of A is rather limited, and S finds T mainly through random search. Under the navigation with stopping strategy, on the other hand, S moves only when A brings it a new sequence number. This means that each time A moves from T to S, S makes a step towards T, where the step size depends on the communication range. Whether this step is towards the short or the long path, depends on which path was used by A to reach S. To analyze this behavior, we model it as a random walk in a one-dimensional, discrete state space, as shown in Figure 10: S starts in an initial state i, and moves in discrete steps either to the left or to the right. The walk ends when S reaches either state 0 (which means S reached T over the long path) or state N (S reached T over the short path). Since we use a communication range of 3 m, we set $N = \frac{24+12}{3} = 12$ and $i = \frac{24}{3} = 8$. The model shown in Figure 10 corresponds to a well-known problem in probability theory, called the gambler's ruin problem [44]. The probability for an agent starting in i to end up in N, rather than in 0, is known to be:

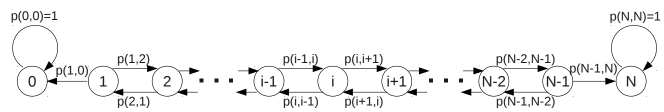

Fig. 10. State space representing the movement of the searching robot.

$$P_N(i) = \frac{1 + \sum_{m=2}^{i} \prod_{k=1}^{m-1} \frac{p(k,k-1)}{p(k,k+1)}}{1 + \sum_{m=2}^{N} \prod_{k=1}^{m-1} \frac{p(k,k-1)}{p(k,k+1)}}. \tag{1}$$

We first use this formula to model the behavior of the randomly moving robot A. In this case, the transition probabilities between states are all equal $p(i, i+1) = p(i, i-1) = 0.5$, and equation 1 simplifies to $P_N(i) = \frac{i}{N}$: the probability of choosing the short path depends linearly on the difference in path length. This behavior of A can be compared to the movement of S in the navigation with random strategy (since S moves mainly randomly), where the fraction of runs using the short path is 0.67 (a very close fit, given that $\frac{i}{N} = \frac{8}{12}$). For the navigation with stopping strategy, the transition probabilities depend precisely on the probability of the randomly moving robot A to reach S either over the short or the long path, so that $p(i, i{+}1) = \frac{i}{N}$ and $p(i, i{-}1) = 1 - \frac{i}{N}$.

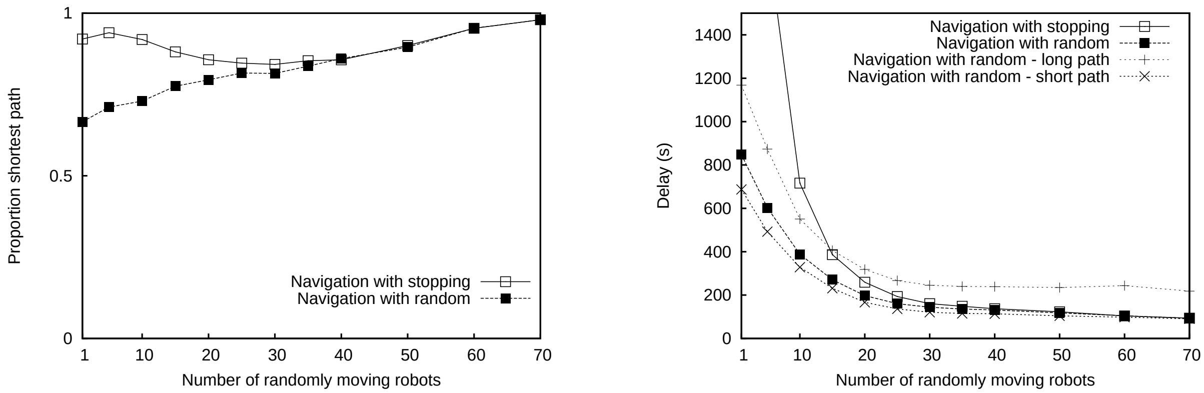

Fig. 9. Experimental results in the cluttered environment of Figure 8 for a single searching robot and an increasing number of randomly moving robots. We show the fraction of runs in which the searcher uses the short path (left), and the average time needed for navigation when taking either the long or the short path (right). Results are reported for both the NwR and NwS strategies.

Plugging this in equation 1, we get $P_N(i) = 0.89$, which is very close to the observed performance of 0.92.

Scenarios with intermediate swarm sizes fall in between these two extremes. We make a distinction between intermediate large swarm sizes (40–60 randomly moving robots) and intermediate small swarm sizes (5–35 randomly moving robots). For intermediate large swarm sizes, the performance of both navigation strategies is identical. This means that S rarely finds itself without navigation information, which is an indication that there is usually a connected route between S and T in the MANET, over which information flows continuously. However, due to the lower robot density compared to the largest swarm size, network connectivity may be less than perfect. As a consequence, the connected communication route sometimes only exists over one of the two navigation paths, and S may occasionally be attracted towards the long path. For intermediate small swarm sizes, the performance differs between the two navigation strategies, indicating that S regularly finds itself without navigation information. This is because at low densities, a MANET falls apart into smaller connected clusters [45], such that information cannot flow continuously. However, compared to the case of very small swarm sizes (e.g., the case with only 1 randomly moving robot), the presence of connected clusters has an important consequence. It means that whenever S meets a robot with navigation information, it immediately also finds a number of other robots with similar information, so that it moves longer into the same direction before finding itself again without information. In the context of the state space shown in Figure 10, this could roughly be modeled by using less states (because each step of S in a given direction will normally go on for longer than the communication range). E.g., if we assume a step size of 6 m, we could use the same model with $N = 6$ and $i = 4$, while keeping the same transition probabilities of $p(i, i+1) = \frac{i}{N}$. This gives a result of $P_N(i) = 0.81$. This preference for the short path is lower than in the case of the navigation with stopping strategy with only 1 randomly moving robot (0.89), but higher than the case of the navigation with random strategy with only 1 randomly moving robot (0.67). This explains why navigation with random always improves the preference for the short path with increasing swarm sizes, while navigation with stopping first decreases this preference, and only later increases it (when end-to-end connected routes appear).

IV. Collective navigation

In the collective navigation problem. there are two targets present in the environment, T and T', and all robots of the swarm navigate back and forth between them. As pointed out in Section I, this is a common task in swarm robotics. To follow swarm terminology, we refer to the two target locations as nest and food source. We use only the "navigation with random" strategy, as this gives the best performance. Our goal is to show that our communication based navigation algorithm can also be used in this scenario. However, the observed performance and properties of robot navigation are different compared to the single robot navigation scenario, due to the specific characteristics of the collective navigation scenario. In particular, the collective execution of the same behavior by all robots lets the swarm self-organize and show coordinated behavior. This self-organization improves the performance and efficiency of navigation.

In what follows, we first investigate the behavior of the system in uncluttered environments, to study the self-organized behavior of the swarm. After that, we study the same behavior in cluttered environments. Finally, we investigate what happens when two paths of different lengths are available between nest and food source: we show that the self-organized behavior lets the swarm select one of the two paths, with strong preference for the shortest.

A. Self-organized behavior in an uncluttered environment

We first use the setup shown in Figure 11. Two robots, indicating the nest and food source, are placed in opposite corners of the arena, at a distance of about 20 m. All other robots are placed according to a uniform random distribution. Half of these robots initially go to the food source, the other half to the nest. A robot that has reached its target (i.e., food source or nest) starts moving towards the other target. A robot is said to have reached a target when it comes within 0.5

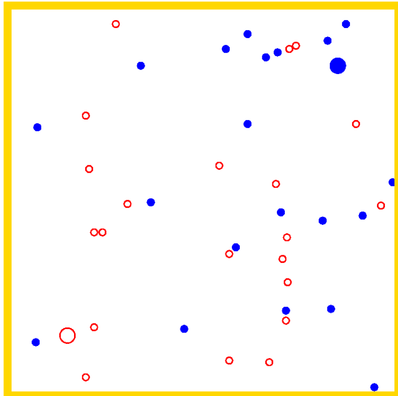

Fig. 11. Setup for collective navigation experiments. The area is 20×20 m^2. The target robots (food and nest) are located in the top-right corner (food robot, indicated with a big blue disk) and in the bottom-left corner (nest robot, indicated with a big red circle). Initially, half of the robots (indicated with circles) go to the nest source, the other half (indicated with disks) go to the food source.

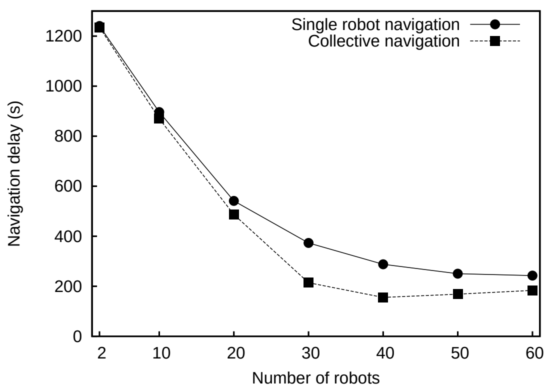

Fig. 12. Experimental results for collective navigation in the uncluttered scenario of Figure 11. The figure reports the observed navigation delays for an increasing number of robots for the case when all robots in the swarm go back and forth between nest and food source (collective navigation), and when only one robot is going back and forth between nest and food, while the other robots of the swarm move according to the random direction mobility model (single robot navigation).

m of it. We vary the total number of searching robots in the swarm from 2 up to 60. We perform 50 independent test runs of 5000 s for each setup. We measure the average time needed by robots to move from one target to the other. We compare to experiments with the same numbers of robots, but where only one robot is going back and forth between nest and food source, while the other robots of the swarm are moving according to the random direction mobility model (as in the single robot navigation experiments of Section III).

The results of these tests are shown in Figure 12. For both single robot navigation and collective navigation, performance improves as the number of robots increases, since navigation information spreads more easily in densely connected swarms (see Section III). However, for the collective navigation scenario, the performance improves faster (with 30 robots, navigation delay of collective navigation is about half of that of single robot navigation). This is due to cooperation. When a robot moving towards the food source (and hence

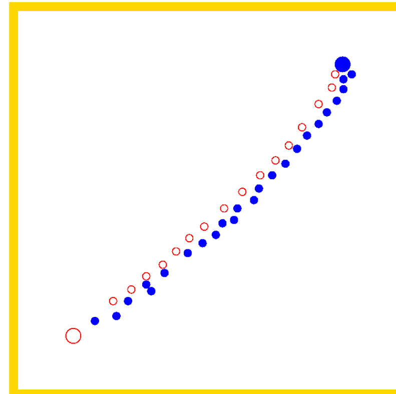

Fig. 13. Collective navigation after 300 s of simulation: a self-organized dynamic chain has formed, with part of the robots going to the food source (red circles) and the others going to the nest source (blue disks).

coming from the nest) and a robot navigating towards the nest meet, they can give each other navigation information about their respective targets. Moreover, if a group of robots moving towards the same target are in communication range from each other, new information received by any of them spreads throughout the whole group, and they simultaneously move in the same direction. These two effects make robots form clusters moving in opposite directions. When there are enough robots, such clusters can become large enough to cover the whole distance between nest and food source. At that point, the swarm organizes into a stable structure, which we refer to as a *dynamic chain*. Figure 13 illustrates this behavior for a typical run of collective navigation with 40 robots. It is this behavior which causes the strong improvement in performance between 20 and 30 robots in Figure 12. For larger swarms (50 and 60 robots), congestion of robots near the target locations leads to a slight decrease in performance.

The dynamic chain is an example of emergent *self-organized* behavior: the swarm shows organization at the global level that emerges from local interactions between individual robots. In what follows, we investigate when this self-organization arises and how stable it is. To do this, we first need a measure for self-organization. Several authors use *entropy* to measure self-organization in the context of swarm robotics [46], [47]. If X is a random variable which can take M different states, its entropy $H(X)$ is defined as

$$H(X) = -\sum_{i=1}^{M} p_i \log_2(p_i), \tag{2}$$

where p_i is the probability that X is in state i (here, we refer to Shannon's information entropy [48]). Strictly speaking, this is a measure of order (or disorder), rather than self-organization: the more a system is ordered, the more you can find it in a limited subset of its possible states, and the lower the entropy. In principle, self-organization is more than just an increase in order, and different measures for self-organization have been proposed [49]. For us, however, it is sufficient to measure whether there is increased order in the behavior of the robots, so we stick with entropy.

To calculate the entropy $H(X)$, we need a discrete variable

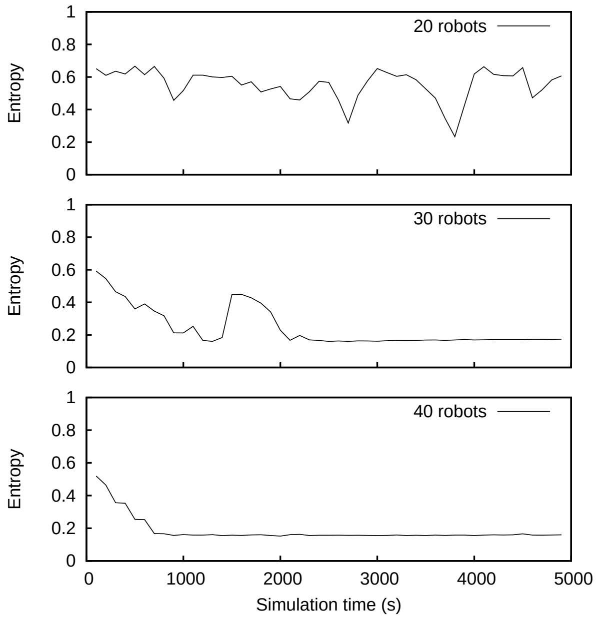

Fig. 14. Evolution of the hierarchic social entropy $S(R)$ over the course of an example test run for 20, 30, and 40 robots

X that characterizes the swarm behavior. In [50], [47] the authors use the orientation of the robots, discretized into four bins; the entropy based on this variable indicates to what extent the robots face the same direction. In our case, this measure can be used (once the chain is formed, robots face in similar directions), but it is quite noisy, especially when there is congestion (robots turn to avoid each other). What we really want to measure is whether the robots move in a low number of connected clusters; that is, whether there is order in their physical locations. To do this, we turn to *hierarchic social entropy* [51], which proposes an entropy measure for a group of robots characterized by a multi-dimensional variable. In our case, this multi-dimensional variable will be the location coordinates of each robot. The idea behind hierarchic social entropy is to first cluster the robots using hierarchic clustering based on a distance threshold h: a robot is added to a cluster if it is within distance h from all robots in the cluster. The division of robots into clusters gives a discrete variable X on the basis of which entropy is calculated (the clusters form the M different states for X, and the probabilities p_i are defined by the number of robots in each cluster). Obviously, X depends on the threshold h: if $h = 0$, each robot is in a cluster of its own, and entropy is maximal, while if $h = \infty$, all robots fall in a single cluster, and entropy is 0. Therefore, the notation $H(R, h)$ is used to refer to the entropy of a group of robots R using clustering distance h. The hierarchic social entropy $S(R)$ is then defined by integrating $H(R, h)$ over all values of h:

$$S(R) = \int_0^\infty H(R, h) dh. \tag{3}$$

We use $S(R)$ based on the location coordinates of the robots to analyze the behavior of the swarm. Compared to the definition of S(R) in [51], we introduce one change, related to the clustering: we use single linkage clustering, which means that a robot is added to a cluster if it is within distance h from any robot of that cluster. Single linkage clustering can find long stretched clusters [52], which enables it to detect the chaining behavior of the swarm. In Figure 14, we show the evolution of $S(R)$ over the course of example test runs with 20, 30 and 40 robots; we calculate $S(R)$ at every time-step of 0.1 s, and average it per 100 s of simulation. When the robots of the swarm move close together, there is a drop in entropy. When the dynamic chain forms, entropy stays low for an extended amount of time. All runs with 20 and 40 robots display patterns similar to the ones shown here: for 20 robots, the chain never forms, while for 40 robots it forms quickly and remains for the whole duration of the simulation. With 30 robots, varying patterns have been observed. In some runs, including the example here, the chain forms after a while. In other runs, it does not form. Interestingly, when it does form, it usually stays for the whole test duration. This suggests that the chain is stable with respect to perturbations.

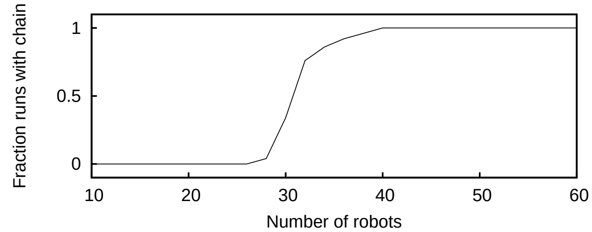

Fig. 15. Fraction of test runs for collective navigation in which a stable dynamic chain forms.

In Figure 15, we study the *stability* of the chain. For increasing numbers of robots, we perform each time 50 test runs, and measure in which fraction of those runs a stable dynamic chain appears. We consider the chain stable if for the last 1000 s of the test $S(R)$ remains below 0.2. The graph shows a clear phase transition around 30 robots: with less robots, the system never self-organizes, with more it always does. Such phase transitions are typical of self-organizing systems in physics, and have also been observed in swarm robotics [46]. They indicate that within a given range of a control parameter, the self-organizing behavior is robust and takes place independently of perturbations in the system (e.g., loss of robots due to failures, or the arrival of new robots).

Finally, in Figure 16, we show how frequently the targets are reached by robots. This indicates how many items the swarm could *transport* between the two locations. Increasing the swarm size, one could expect a sub-linear performance improvement, because more robots can transport proportionally more items (linear improvement), but there is also increased congestion. In our system, increased swarm size also gives more cooperation, which leads to a super-linear increase in performance between 10 and 40 robots (dotted lines in the figure illustrate for each swarm size the extrapolated performance in case of linear improvement). For more robots, congestion makes the performance growth decrease.

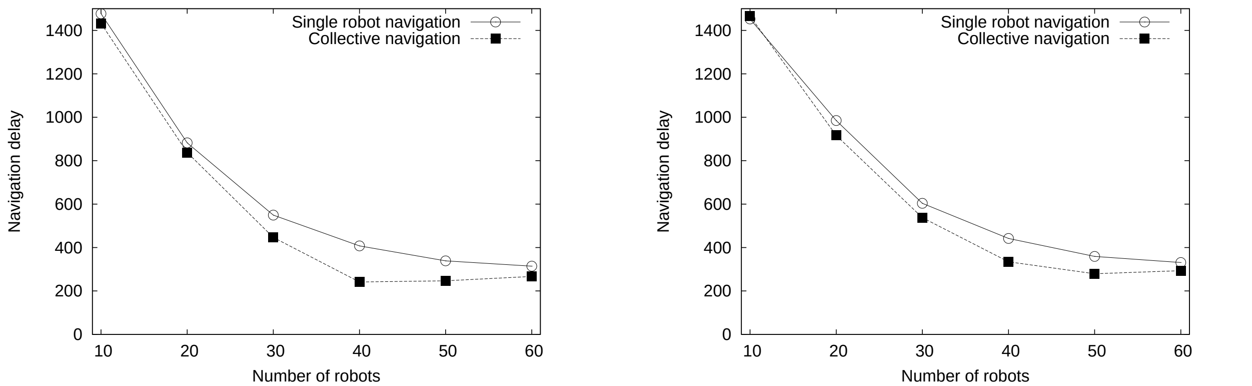

Fig. 18. Experimental results in the cluttered environments of Figure 17. The left plot refers to Figure 17 (left), and the right one to Figure 17 (right). The figures report the observed navigation delays for an increasing number of robots for the case when all robots in the swarm go back and forth between nest and food source (collective navigation), and when only one robot is going back and forth between nest and food, while the other robots of the swarm move according to the random direction mobility model (single robot navigation). The NwR strategy has been used in all the reported experiments.

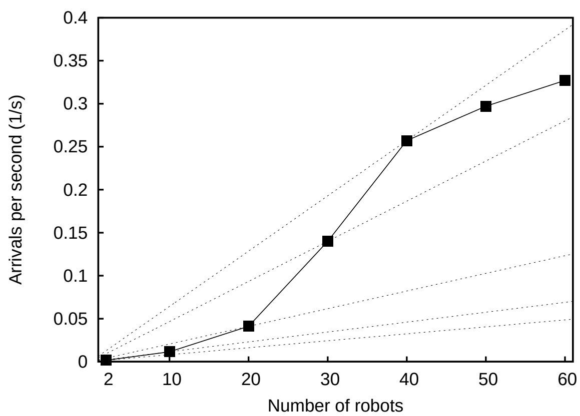

Fig. 16. Transport performance of the swarm: frequency with which the target robots are reached by the other robots of the swarm as a function of the number of robots in the swarm. The dotted lines show, for each swarm size $s \in \{2, 10, 20, 30, 40, 50, 60\}$, the expected performance in case of a linear speedup in the improvement of the transport performance (indicated by the straight line between the values corresponding to swarm size equal to 1 and swarm size equal to s).

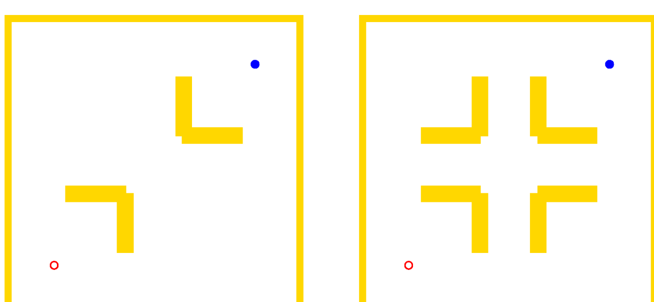

Fig. 17. Layout of cluttered environments used for collective navigation experiments. The area is 20×20 m^2 in both cases.

B. Cluttered environments

We now deploy our system in the cluttered environments shown in Figure 17. The nest and food source are placed in the same locations as in the uncluttered environment, but now obstacles have been placed between them. We compare again single robot navigation and collective navigation. We report the average navigation delay in experiments with varying swarm sizes in Figure 18. As in the case without obstacles studied in Section IV-A, collective navigation is more efficient than single robot navigation. However, as the environment gets more complex, its advantage gets smaller. This is because the swarm has more difficulties to form and maintain the dynamic chain around the obstacles. We illustrate this in Figure 19, where we show the evolution of the hierarchic social entropy over time for a typical test run with 40 robots in the scenario of Figure 17 (right). The entropy is low for certain stretches of time, indicating that the dynamic chain is formed, but also goes up again, showing that the chain gets lost sometimes. These results show that the self-organized behavior works in the presence of obstacles, but that it has difficulties when the environment becomes too complex. In such environments, the navigation algorithm still works, but it looses the advantage obtained through self-organization, and the performance becomes comparable to that obtained in single robot navigation.

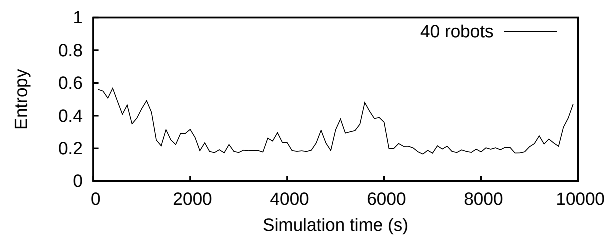

Fig. 19. Evolution of the hierarchic social entropy $S(R)$ over the course of an example test run for 40 robots in the environment of Figure 17 (right).

C. Shortest path finding

As in the case of single robot navigation, we investigate the behavior of the system in case two paths of different length are available between nest and food source. We use the environment of Figure 8, where we now place a nest and a food source in the locations that were previously used for searcher and target. The two locations are connected by a short path of length $d_s = 12$ and a long path of length $d_l = 24$.

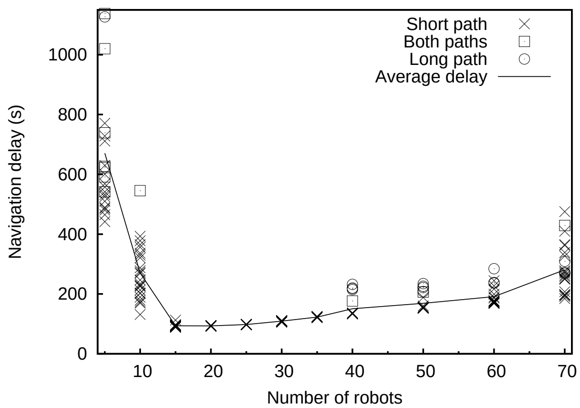

Fig. 20. Shortest path finding performance in the cluttered environment of Figure 8. The figure shows the navigation delay versus number of robots for each individual test, as well as the average per swarm size (25 tests per swarm size). The choice of path in each test is shown by the point symbols.

We vary the swarm size from 5 to 70 robots, and perform 25 tests of 5000 s for each size. We measure the average time needed for a robot to navigate between the target locations. We also observe which path each robot takes to reach its target: the short or the long one. We combine this per test run, to calculate the percentage of robots using the short path, p_s. If $p_s > 66\%$, we say the swarm uses the short path, if $p_s < 33\%$ it uses the long path, and otherwise it uses both.

Figure 20 shows the result of each individual test, as well as the average per swarm size. As for the case of single robot navigation, the robots have a preference for the short path. Also, this preference leads to efficient navigation, as those runs that use the short path usually experience a lower navigation delay (with exception for swarm size 70, where congestion starts to play a major role).

It is interesting to observe the evolution of the preference for the short path for increasing swarm sizes. For swarm size 5, the preference for the short path is rather modest; to be precise, in 67% of the test runs, the swarm uses the short path, which is comparable to the case of single robot navigation with the same number of robots (see Section III-E). For 10 robots, the preference is already much stronger, but it is starting from swarm size 15 that the results start to look different: in all runs, all robots always use the short path, and navigation delay is very low and equal over all runs. This *highly efficient navigation behavior* is due to the self-organized formation of the dynamic chain. On the one hand, we observe here the same improvement of navigation efficiency as in uncluttered environments (see Section IV-A). On the other hand, there is also a second effect, namely that the dynamic chain makes the collective navigation lock onto one of the paths: once the swarm forms the chain on one path, it will normally not change to the other path anymore. Between 15 and 30 robots, there are enough robots to form the chain over the short path, but not over the long path. This makes the swarm always choose the short path. Starting from 35 robots, the chain can also be formed over the long path (verified in separate tests not shown here), and we start to observe this from swarm size 40. While the robots' general preference for the short path normally makes the chain form there, fluctuations due to the robots' random initial distribution, or due to collisions and congestion, let the chain occasionally choose the long path. Such amplification of fluctuations is a typical phenomenon in self-organizing systems in nature [53]. We also conducted tests moving the targets so as to reduce the difference between d_s and d_l (swarm size 50). This led to proportional changes in the number of runs choosing the short path.

V. Implementation on real robots

We implemented the communication based navigation system on real foot-bots [21]. Since this is the robot used as model in the simulation experiments, the robot characteristics (IrRB capacity, robot speed, etc.) are the same as described in Section III.

In a first experiment, we used an arena of 10×4 m^2, which is largely uncluttered, except for a wall of 1.4 m on the side. Figure 21 shows a photograph of the arena, as well as an image of how it was reproduced in simulation. We placed a source and target robot in this arena, in the locations of the two robots shown in the figure. Due to the small size of the arena, we limited the communication range of the IrRB system to 2.5 m. We carried out tests similar to the ones reported in Figure 12: we compare single robot navigation (1 searcher, all other robots perform random movements) and collective navigation (all robots are searchers) in tests with increasing swarm sizes (from 1 moving robot, up to 10). For each swarm size, we run one single long experiment of 30 minutes, in which the searching robot(s) go back and forth many times. We also reproduce the same experiments in simulation. We report the average time needed for navigation between the source and target. The results are shown in Figure 22. Both in reality and in simulation, the data show the same trend as in the earlier results of Figure 12: navigation delay improves with increasing numbers of robots, but for collective navigation, there is a faster increase thanks to the chain formation. This chain formation was also observed visually by us

Moreover, although there are some quantitative differences between the results from simulation and those with the real robots, the trends are qualitatively the same, and it can generally be said that the results from simulation are quite reliable.

In a second experiment, we did tests similar to the ones of Section IV-C. We used an arena of 3.10×4.35 m^2, with in the middle an obstacle of 0.75×1.75 m^2. The target and source were placed on either side of the obstacle, at about two thirds along the long edge of the arena, such that a long and a short path were available among them. Figure 23 shows a photograph of the arena, as well as an image of how it was reproduced in simulation. Due to the small size of the arena, we restricted the communication range of the robots to 1.5 m. We ran tests with increasing numbers of moving robots, from 1 up to 8, for both single robot navigation and collective navigation, and reproduced the same tests in simulation. Each test lasted 40 minutes, but for the collective navigation, we split this up into 4 times 10 minutes. This is because the chain formation makes the robots' choice for

Fig. 21. Arena used in the real robot experiments: photograph (left) and as reproduced in simulation (right). The photograph was taken from the position of the camera icon in the right image. In this image, the circle and the disk symbols indicate respectively the position of food and nest robots.

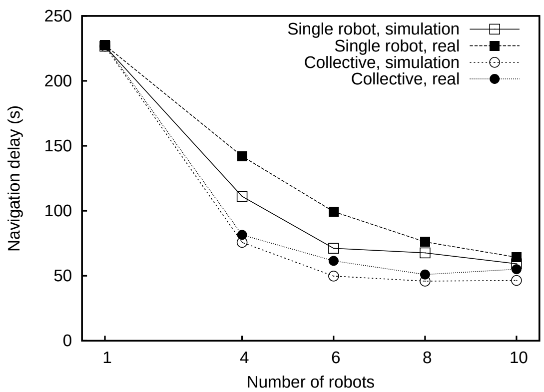

Fig. 22. Experimental results for single and collective navigation using real robots in the scenario of Figure 21 (left). The average navigation delay is reported for an increasing number of robots. The results obtained in simulation considering the equivalent scenario of Figure 21 (right) are also reported to show the good correspondence between simulation and real robot behavior.

either the short or the long path stable for long time, such that consecutive trips between source and target can not be considered as independent samples; in single robot navigation, on the other hand, the correlation between consecutive trips of the searching robot is limited, such that all trip times gathered during a single run of 40 minutes can therefore give enough independent test samples. The results are shown in Figure 24. We report the average delay needed to move between source and target, and the fraction of robots following the short path. The correspondence between simulation and real robots is good, qualitatively, although quantitatively there are some differences. The trip time results show that the difference between single robot and collective navigation is limited in this case, due to the small arena, which allows less space to get improved times, and which gives more congestion. On the other hand, the choice for the shortest path shows a strong difference between the two navigation scenarios: while single robot navigation leads to a preference for the short path that increases linearly with the number of robots, collective navigation has a faster increasing preference for the short path, due to the chain formation (compare to Figure 20).

Finally, we point out that in previous work [22], we implemented the navigation algorithm on e-puck robots [54], fitted with an IrRB communication board [19]. The capacities of

Fig. 23. Arena used in the real robot experiments with obstacle: photograph (left) and as reproduced in simulation (right). The circle and the disk symbols in the right image indicate the position of the food and the nest robots.

these robots and their IrRB system are limited compared to those of the foot-bots: each robot could send only 2 bytes per second, with significant packet loss, and very noisy range and bearing estimates. Nevertheless, the navigation system worked fairly well. We refer to [22], where we report results from these tests. Finally, in some other tests using the foot-bot robots, reported in [23], we observed frequent robot failures, and we tested adding robots to and removing robots from the swarm, with the navigation algorithm adapting easily to such changes. All these tests show the general robustness, adaptivity and scalability of the algorithm.

Videos of these experiments as well as of similar experiments used for the paper [23] can be seen in on-line supporting material at:
`http://www.idsia.ch/~gianni/IeeeR/videos.html`.

VI. Related work

In this paper, we have presented an algorithm for communication based cooperative navigation in swarm robotics. The works closest related to ours are situated in the areas of communication based navigation, and cooperative navigation in swarm robotics.

Several works are related to ours because of the way they use communication to guide navigation. One setup that has been studied extensively over the past few years is to fit the environment with a network of wireless communication nodes, which guide a single robot to a target [55], [3]. The communication nodes may be wireless sensor nodes, which sense the local environment and take this sensed information into account when planning a path [56], or sensor-less nodes, which use only communication for path planning [17]. Many of these approaches use communication links to define obstacle-free paths, e.g., using infrared communication [3], so that they can use network routing algorithms to define navigation paths. An important difference with our approach is that most of these works assume that the communication network that guides the mobile robot is static and embedded in the environment; they do not foresee the possibility that mobile robots guide each other's navigation. Some works do use mobile robots, e.g., to deploy the static communication nodes [57], or to fill gaps in the sensor network [58]. The approach closest to ours is [59], where a navigating robot gets support to move around obstacles from a few mobile explorer robots, using line-of-sight communication. Different from our work, however, these

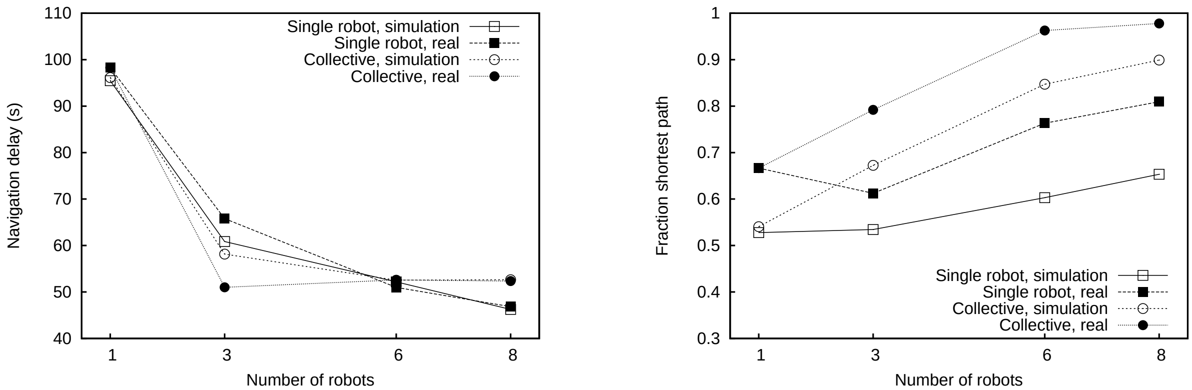

Fig. 24. Experimental results for single and collective navigation using real robots in the cluttered scenario of Figure 23 (left). In the left figure, the observed average delay for the navigation between source and target is reported for an increasing number of robots. The right figure shows the fraction of robots following the short path. The results obtained in simulation considering the equivalent scenario of Figure 23 (right) are also reported to show the good correspondence between simulation and real robot behavior.

few explorer robots are dedicated to support the single robot's navigation task. The authors do not consider the possibility that a whole swarm of mobile robots guide each other's navigation, where each robot may be involved in a task of its own and is not dedicated to support the navigation of the others.

Within the context of swarm robotics, most work on cooperative navigation is based on indirect stigmergic communication [5], [6], [7], [60], [61], [8], [62], [9], [10], [63], [11], [64], rather than on direct communication as in our algorithm. This approach is typically inspired by the behavior of certain types of ants, where individual ants mark their paths using a chemical substance, called pheromone, and follow these pheromone trails to find their way between the nest and a food source [13]. The joint pheromone laying and following actions of the ants of a colony reinforce the most efficient paths, and lets the swarm as a whole self-organize to find shortest paths [12]. An important problem for the application of such approaches in robotics is how to physically implement pheromone. A common solution is to mark the trail with a chain of robots [5], [8], [9], [11]. Compared to our system, this has the disadvantage that some of the robots remain static and cannot take part in navigation. Moreover, the system is vulnerable to failures of robots in the chain, making it less robust. Other approaches include the use of alcohol [60], [10], phosphorescent paint [63], or light encoding of pheromone using an overhead projector [62], [61], which are interesting, but are rather hard to detect and follow reliably or to implement in a general context. A general disadvantage of all these pheromone-inspired swarm navigation algorithms is that they crucially assume that all robots move between two targets. Our algorithm can also work in this situation, with properties that are similar to other swarm navigation methods; in particular, it lets the swarm self-organize, and displays emergent shortest path finding behavior, as shown in Section IV. However, it is also very general and usable in a wide range of different situations. We have illustrated the single robot navigation task in Section III, but one can understand that also many other scenarios could easily be addressed with this algorithm.

Finally, we describe a number of cooperative swarm navigation algorithms that do not implement pheromone-based navigation. In [65], the authors propose a method based on direct communication, partially inspired by the bee waggle dance: robots inform each other about the way to a target by exchanging a list of landmarks, in the form of waypoint coordinates. Like pheromone-based methods, however, also this approach assumes that all robots of the swarm navigate back and forth between two targets. Also in [66], robots use direct communication to help each other navigate between a nest and a food source. Here, the robots exchange the estimated position of targets (nest or food source), and a robot searching a target can move directly towards the indicated location. However, since the only used navigation information are target locations, the method would not be able to indicate obstacle-free paths in cluttered environments. In [47], the authors address the collective navigation problem with neuro-evolution. Interestingly, they find a swarm level behavior that is similar to our dynamic chain, though based on very different individual robot behavior (using visual feedback, robots turn around in local dynamic chains; these chains merge and grow and may eventually include the targets). However, this behavior was not designed to generalize to scenarios that are radically different from that for which it was developed, namely a collective navigation scenario in an uncluttered environment. Finally, in [67], the authors propose a navigation method inspired by trophallaxis, which is the behavior of social insects to pass food to each other. In this method, the food corresponds to navigation information, which is exchanged through local direct communication. The authors evaluate their method in the context of a foraging task performed by a large swarm of simulated robots. It is not clear whether this method would be applicable in other contexts, such as the single robot navigation task, or in small swarms, and whether it would be usable on real robots.

VII. CONCLUSIONS AND FUTURE WORK

We have presented a navigation system for robotic swarms. It is a simple and flexible algorithm that can be used in different contexts. We have first shown how it allows robots of a swarm to guide a single robot to a target, without the need to adapt their own movements. Then, we have investigated how the system can be used for collective navigation between two targets, a common task in swarm robotics. We have shown that cooperation improves navigation performance, and that when enough robots are present, the swarm self-organizes into a dynamic structure that supports efficient navigation and is robust to perturbations and robot failures. Moreover, we have shown that collective navigation has a preference for short paths, similar to pheromone mediated navigation in ant colonies. In tests with real robots, we have shown the feasibility of the approach.

In future work, we will first investigate the performance of the current system in more complex scenarios. We will investigate single robot navigation with different, realistic robot movement patterns, and study the dynamic chain behavior in complex cluttered environments. Also, we will perform more extensive tests with real robots to confirm all results from simulation. After that, we will integrate this system in other scenarios of swarm deployment, e.g., where the swarm performs tasks in service of humans. Many such scenarios require navigation. Moreover, the swarm communication we use for navigation can be extended to carry more information, e.g., for task allocation, planning, etc.

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