Delay Analysis of Wireless Nakagami Fading Channels

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Abstract—In this paper we analyze the delay performance of a single user with perfect channel state information transmitting data over a wireless fading channel. We consider a dynamic resource allocation policy that maximizes average capacity by adapting transmission power, as well as a policy that adapts instantaneous bandwidth. Nakagami fading is assumed, and the impact of the fading parameter, m, as well as the channel quality in terms of signal-to-noise ratio on the mean queuing delay of transmitted data is analyzed. We show that resource allocation policies that achieve superiority over competing strategies in terms of maximizing average capacity do not necessarily result in superior delay performance.

Index Terms—Queuing analysis, fading channels, Information rates, dynamic resource allocation, average delay

I. INTRODUCTION

Data communications in wireless networks generally take place over fading channels with time-varying characteristics. The extent to which the dynamic nature of the wireless medium impacts the Quality of Service (QoS) of transmitted data depends on factors such as the severity of the fading channel and the resource allocation policy being employed to adapt to the time-varying channel. This is in contrast to wired point-to-point links where QoS is exclusively a function of the data traffic arrival statistics and the fixed capacity of the transmitter. In the wired case, QoS attributes, such as delay performance, can usually be studied by using appropriate queuing models and analyses. The time-varying wireless channel, on the other hand, poses a challenge in terms of queuing analysis and performance evaluation.

The measurement of achievable performance of wireless communications over fading channels has historically been relegated to the realm of information theory, where channel capacity is the figure of merit. The delay component that accounts for the time that data spends in a transmit buffer, as well as other measures of QoS, are typically decoupled from the information theoretic problem, and often times simply ignored. This separation is reasonable for wired links where a constant transmit data rate can be assumed, but results in an inability to capture the important relationship between physical layer behavior and higher layer performance in a wireless network. Some of these issues are discussed in [1] where the authors consider a system in which transmit power is adapted according to channel state information as well as buffer occupancy. A dynamic programming approach is employed to explore the tradeoffs associated with average transmit power and average delay. A further application of this general approach to cross-layer design methodologies is considered in [2].

A number of authors (see [3], [4], and [5] for example) have used queuing theoretic techniques in an attempt to approximate the delay induced on data by wireless channels. However, simple models that express mean delay as a function of the wireless fading environment, including the presence of outages, do not exist. Such models would be helpful in coming to a fuller understanding of the tradeoffs associated with dynamic resource allocation strategies, as well as in performing OoS provisioning functions such as call admission and traffic policing in wireless networks. Toward this end, a capacity model for wireless channels is proposed in [6] where the authors provide a framework for translating physical layer wireless channel attributes to an available channel capacity that could potentially be useful to higher layer protocols for QoS provisioning. Ultimately, a great deal more needs to be done in order to provide the necessary linkage between network QoS behavior and wireless channel dynamics, particularly in the presence of adaptive resource allocation policies at the physical layer.

In this paper, we present a queuing model for the delay analysis of a single user wireless fading channel with outages. The proposed model utilizes a two priority M/G/1 queue in which link outages are approximately modeled by highpriority customers. Nakagami-*m* fading is assumed, and the impact of the fading parameter, *m*, as well as the channel quality in terms of signal-to-noise ratio, on the mean queuing delay of transmitted data is analyzed. We consider a dynamic resource allocation policy that maximizes average capacity by adapting transmission power, as well as a policy that adapts instantaneous bandwidth. We show that resource allocation policies that achieve superiority over competing strategies in terms of maximizing average capacity do no necessarily result in superior delay performance.

II. LINK MODEL

We investigate a single user channel subject to time varying, slow, flat fading with additive white Gaussian noise (AWGN) at the receiver. The fading is modeled as Nakagami-*m* [7], which has been shown to be a suitable model for a number of wireless environments (see [8] and [9], for example). Under this type of fading, the SNR is gamma distributed; therefore the probability density function (pdf) of the SNR is given by

$$f(\gamma) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\gamma^{m-1}}{\Gamma(m)} e^{-\gamma \frac{m}{\bar{\gamma}}},$$

where *m* is the Nakagami fading parameter, $\Gamma(\cdot)$ is the Gamma function, and $\bar{\gamma}$ is the expected value of the SNR random variable, γ .

At low SNR, the data transmission rate is reduced to a level such that transmission may not be justified. Hence, links are considered to be in outage when the SNR is below a predetermined threshold, denoted γ_{th} and referred to as the SNR threshold. When in an outage, no data may be transmitted over the link.

In the face of outages and time varying fading, adaptive resource allocation strategies have been developed to more efficiently use the wireless resource. Typically, the performance of resource allocation policies is measured in terms of mean capacity and mean spectral efficiency. Unfortunately, in a wireless network these metrics may not be ideal measures of link performance, because links, rather than data, are their frame of reference. A more practical measure of link performance is delay, which measures the experience of data in the system. The delay experienced by data influences application performance and the overall utility of the network. Because delay is composed of transmit time and time waiting in transmit buffers, we develop and utilize a queuing analysis to measure the mean delay on links for various resource allocation strategies.

III. QUEUING MODEL

The fading model determines, in part, the time varying channel capacity. Thus, the link can be modeled as a variable rate queue, where the queue represents the transmit buffer and the service rate is equal to the time varying capacity. We assume Poisson arrivals. The service time, S, is a function of SNR and the specific resource allocation policy, and hence must take a general form in our analysis, resulting in an M/G/1 model. Link outages are modeled through the introduction of high-priority "outage customers" that receive head of line priority, thereby approximating the delay experienced by data in the presence of an outage. The mean delay experienced by data in this priority queue is given by [10]

$$\bar{d} = \frac{\lambda E\{S^2\}}{2(1-\rho_o)\left[1-(\rho_o+\rho_d)\right]} + E\{S\},$$
(1)

where λ is the total arrival rate of outage customers and data customers, S is the overall distribution of service times for both classes of customers, ρ_o is the utilization due to outage customers, and ρ_d is the utilization due to data customers. The total arrival rate is simply the sum of the mean arrival rates of outage and data customers. Since both data and outages arrive according to Poisson arrival processes, there exists an implicit assumption that the interarrival times of outages are independent and identically distributed (iid).

The utilization due to an outage, ρ_o , is the proportion of time the server spends serving outage customers, and is equivalent to the probability the link is in an outage, which is given by

$$\rho_o = \int_0^{\gamma_{\rm th}} f(\gamma) \, d\gamma.$$

As noted above, the service time distribution, S, is a function of both the fading and the particular resource allocation scheme. In this paper we investigate several resource allocation strategies and their impact on mean spectral efficiency and mean delay.

IV. RESOURCE ALLOCATION STRATEGIES

For each resource allocation strategy considered, it is assumed that perfect, zero delay channel state information is available at the transmitter and receiver. Three resource allocation strategies are investigated: static resource allocation, adaptive power allocation, and adaptive bandwidth allocation. We also provide results for the additive white Gaussian noise (AWGN) channel for comparison.

A. Static Resource Allocation

In the case of static resource allocation (SRA) the link bandwidth and power are predetermined and are not adapted in response to fading. Therefore, the instantaneous capacity is given by

$$c_{\text{SRA}}(t) = \begin{cases} 0 & \gamma(t) < \gamma_{\text{th}} \\ B \log_2(1+\gamma(t)) & \gamma(t) \ge \gamma_{\text{th}}, \end{cases}$$
(2)

where B is the bandwidth of the wireless channel. Consequently, the mean capacity under SRA is

$$E\{C_{SRA}\} = \int_{\gamma_{th}}^{\infty} B \log_2(1+\gamma) f(\gamma) \, d\gamma.$$
(3)

Following the pattern set out in [11], an expression for (3) can be derived. By defining $\xi = 1 + \gamma_{\text{th}}$ and

$$\beta(x,q) = \frac{B}{\ln 2\Gamma(x)} e^{-q\frac{x}{\overline{\gamma}}},$$

the general expression for (3) becomes

$$E\left\{C_{SRA}\right\} = \beta(m, \gamma_{th}) \sum_{k=1}^{m} \frac{(m-1)!}{(m-k)!} \left(\frac{m}{\bar{\gamma}}\right)^{m-k} \\ \left[\gamma_{th}^{m-k} \ln \xi + e^{\xi \frac{m}{\bar{\gamma}}} \sum_{j=0}^{m-k} \binom{m-k}{j} \gamma_{th}^{j} \xi^{m-k-j} \right. \\ \left. \Gamma(m-k-j+1) \Gamma\left(k+j-m, \xi \frac{m}{\bar{\gamma}}\right) \right],$$

$$(4)$$

where $\Gamma(\cdot, \cdot)$ designates the upper incomplete gamma function [12].

In Rayleigh fading, (4) simplifies to

$$\mathbf{E}\left\{C_{\mathrm{SRA}}\right\} = \beta(1,\gamma_{\mathrm{th}}) \left[\ln\xi + e^{\frac{\xi}{\bar{\gamma}}} E_1\left(\frac{\xi}{\bar{\gamma}}\right)\right],\tag{5}$$

where $E_1(\cdot)$ denotes the exponential integral [12].

The following functions, η and ζ , are defined in order to facilitate the derivation of the capacity and service time pdfs under an SRA policy:

$$\eta(x,q) = \left(\frac{m}{\bar{\gamma}}\right)^m \frac{\ln 2}{B\Gamma(m,x)} q^{m-1} e^{-q\frac{m}{\bar{\gamma}}},$$
$$\zeta(x) = 2^{\frac{x}{B}} - 1.$$

and

The pdf of the SRA capacity random variable is given by

$$f(c_{\text{SRA}}) = \begin{cases} \rho_o & c_{\text{SRA}} = 0\\ 0 & 0 < c_{\text{SRA}} < B \log_2 \xi\\ \eta(0, \zeta(c_{\text{SRA}})) 2^{\frac{c_{\text{SRA}}}{B}} & c_{\text{SRA}} \ge B \log_2 \xi. \end{cases}$$
(6)

The probability density function of the SRA service time random variable, $S_{\text{SRA}} = C_{\text{SRA}}^{-1}$, is given by

$$f(s_{\text{SRA}}) = \eta\left(\gamma_{\text{th}}\frac{m}{\bar{\gamma}}, \zeta\left(\frac{1}{s_{\text{SRA}}}\right)\right) 2^{\frac{1}{s_{\text{SRA}}B}}\frac{1}{s_{\text{SRA}}^2}, \quad (7)$$

from which the first and second moments can be determined, and used in (1) to compute mean delay.

B. Adaptive Power Allocation

Adaptive power allocation (APA) strategies have been studied for extending battery life in wireless nodes and optimizing capacity of wireless links. In [13] the authors derive the APA scheme that maximizes the mean capacity of a link, subject to an average power constraint. As described in [13], the instantaneous capacity of an APA link is

$$c_{\text{APA}}(t) = \begin{cases} 0 & \gamma(t) < \gamma_{\text{th}} \\ B \log_2\left(\frac{\gamma(t)}{\gamma_0}\right) & \gamma(t) \ge \gamma_{\text{th}}, \end{cases}$$
(8)

where γ_0 is a cutoff SNR that ensures the average power allocation is equal to the average power constraint. γ_0 may or may not be equal to γ_{th} . In order to achieve the optimal power allocation, γ_0 and γ_{th} must be equal. For the purposes of this analysis, however, γ_{th} is greater than γ_0 so that the mean service time can be computed (otherwise the service time is infinite when $\gamma(t) = \gamma_0$).

The mean capacity may be found by

$$E\{C_{APA}\} = \int_{\gamma_{th}}^{\infty} B \log_2\left(\frac{\gamma}{\gamma_0}\right) f(\gamma) \, d\gamma \tag{9}$$

Following the pattern set out in [11], the following general expression for (9) can be derived:

$$E \{C_{APA}\} = \beta(m, \gamma_{th})(m-1)! \sum_{k=0}^{m-1} \frac{1}{k!} \left[\ln\left(\frac{\gamma_{th}}{\gamma_0}\right) \left(\gamma_{th}\frac{m}{\bar{\gamma}}\right)^k + e^{\gamma_{th}\frac{m}{\bar{\gamma}}} \Gamma\left(k, \gamma_{th}\frac{m}{\bar{\gamma}}\right) \right].$$

$$(10)$$

In Rayleigh fading, this simplifies to

$$\mathbf{E}\left\{C_{\mathrm{APA}}\right\} = \beta(1,\gamma_{\mathrm{th}}) \left[\ln\left(\frac{\gamma_{\mathrm{th}}}{\gamma_{0}}\right) + e^{\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}} E_{1}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}}\right) \right].$$
(11)

The mean capacity may also be found directly from the pdf of the APA capacity random variable, which, in Nakagami fading, is

$$f(c_{\text{APA}}) = \begin{cases} \rho_o & c_{\text{APA}} = 0\\ 0 & 0 < c_{\text{APA}} < B \log_2\left(\frac{\gamma_{\text{th}}}{\gamma_0}\right)\\ \eta\left(0, \nu(c_{\text{APA}})\right) \nu(c_{\text{APA}}) & c_{\text{APA}} \ge B \log_2\left(\frac{\gamma_{\text{th}}}{\gamma_0}\right), \end{cases}$$
(12)

where the ν function is defined as

$$\nu(x) = \gamma_0 2^{\frac{*}{B}}.$$

The probability density function of the APA service time random variable is given by

$$f(s_{\text{APA}}) = \eta \left(\gamma_{\text{th}} \frac{m}{\bar{\gamma}}, \nu \left(\frac{1}{s_{\text{APA}}} \right) \right) \nu \left(\frac{1}{s_{\text{APA}}} \right) \frac{1}{s_{\text{APA}}^2}, \quad (13)$$

from which the first and second moments can be derived and substituted into (1).

C. Adaptive Bandwidth Allocation

Adaptive bandwidth allocation (ABA) schemes attempt to modify the link bandwidth so that it is efficiently utilized. Any unallocated bandwidth could potentially be utilized by other nodes in a wireless network. In [14] the authors derive a bandwidth allocation scheme that maximizes mean capacity of a link without outages, subject to an average bandwidth constraint. Here, we consider for the first time a bandwidthadaptive link with outages. The resulting bandwidth allocation strategy is

$$B(\gamma(t)) = \frac{B}{\bar{\gamma} - \dot{\gamma}}\gamma(t), \qquad (14)$$

where

$$ar{\gamma} - \dot{\gamma} = \int_{\gamma_{\mathrm{th}}}^{\infty} \gamma f(\gamma) \, d\gamma.$$

In Nakagami fading, this quantity becomes

$$\bar{\gamma} - \dot{\gamma} = \bar{\gamma}Q\left(m+1, \gamma_{\rm th}\frac{m}{\bar{\gamma}}\right)$$

where $Q(\cdot, \cdot)$ is the regularized upper incomplete gamma function [15].

In an ABA channel, the instantaneous capacity is given by

$$c_{\rm ABA}(t) = \begin{cases} 0 & \gamma(t) < \gamma_{\rm th} \\ B(\gamma(t)) \log_2 \left(1 + \gamma(t) \frac{\bar{B}}{B(\gamma(t))} \right) & \gamma(t) \ge \gamma_{\rm th}, \end{cases}$$

where \bar{B} is the mean bandwidth of the channel, and the $\frac{B}{B(\gamma(t))}$ term is due to the fact that $\gamma(t)$ is measured relative to some reference bandwidth (in this case \bar{B}). By performing substitutions and simplifying, the instantaneous capacity equation can be expressed as

$$c_{\text{ABA}}(t) = \begin{cases} 0 & \gamma(t) < \gamma_{\text{th}} \\ \frac{\bar{B}}{\bar{\gamma} - \dot{\gamma}} \gamma(t) \log_2[1 + (\bar{\gamma} - \dot{\gamma})] & \gamma(t) \ge \gamma_{\text{th}}. \end{cases}$$

Therefore, the mean capacity of the ABA channel is

$$E\{C_{ABA}\} = \bar{B}\log_2[1 + (\bar{\gamma} - \dot{\gamma})], \qquad (15)$$

which approaches the Shannon limit as γ_{th} approaches 0 or as $\bar{\gamma}$ approaches ∞ . The mean spectral efficiency can be found by taking the mean of the instantaneous spectral efficiencies, and is given by

$$\mathbf{E}\left\{\frac{C_{\text{ABA}}}{B(\gamma)}\right\} = \log_2[1 + (\bar{\gamma} - \dot{\gamma})]Q\left(m, \gamma_{\text{th}}\frac{m}{\bar{\gamma}}\right).$$
(16)

In Nakagami fading, the ABA capacity pdf is given by

$$f(c_{ABA}) = \begin{cases} F(\gamma_{th}) & c_{ABA} = 0\\ 0 & 0 < c_{ABA} < \frac{m\gamma_{th}}{\bar{\gamma}\kappa} \\ \kappa^m \frac{c_{ABA}^{m-1}}{\Gamma(m)} e^{-c_{ABA}\kappa} & c_{ABA} \ge \frac{m\gamma_{th}}{\bar{\gamma}\kappa}, \end{cases}$$
(17)

where $\kappa = \frac{m(\bar{\gamma} - \dot{\gamma})}{\bar{\gamma}\bar{B}\log_2[1 + (\bar{\gamma} - \dot{\gamma})]}$. The pdf of the service time random variable, in Nakagami fading with outages, is given by

$$f(s_{ABA}) = \frac{\kappa^m}{\Gamma\left(m, \gamma_{\text{th}} \frac{m}{\bar{\gamma}}\right)} \frac{1}{s_{ABA}^{m+1}} e^{-\frac{\kappa}{s_{ABA}}}.$$
 (18)

The first moment of the ABA service time random variable can be easily derived from the pdf. It is

$$E\{S_{ABA}\} = \kappa \frac{\Gamma\left(m-1, \gamma_{th} \frac{m}{\bar{\gamma}}\right)}{\Gamma\left(m, \gamma_{th} \frac{m}{\bar{\gamma}}\right)}.$$
(19)

Similarly, the second moment of the ABA service time random variable is

$$E\left\{S_{ABA}^{2}\right\} = \begin{cases} \frac{\kappa^{2}}{\Gamma\left(m,\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right)}\frac{\bar{\gamma}_{\rm th}}{\gamma_{\rm th}}E_{2}\left(\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right) & \text{for } m = 1\\ \frac{\kappa^{2}}{\Gamma\left(m,\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right)}E_{1}\left(\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right) & \text{for } m = 2\\ \frac{\kappa^{2}}{\Gamma\left(m,\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right)}\Gamma\left(m-2,\gamma_{\rm th}\frac{m}{\bar{\gamma}}\right) & \text{for all other } m \end{cases}$$

$$(20)$$

where $E_2(\cdot)$ is a special case of the E_n function defined in [12].

V. RESULTS

We investigate the performance of the resource allocation strategies in terms of mean spectral efficiency and mean delay in Rayleigh fading (m = 1). In obtaining the results, a signalto-noise ratio threshold of 3 dB was used.

In Fig. 1 we plot the spectral efficiency of the three resource allocation policies discussed earlier. As illustrated in [13], SRA and APA mean spectral efficiency converge but do not approach the mean spectral efficiency of the AWGN channel. The mean spectral efficiency of ABA, on the other hand, approaches that of the AWGN channel at large mean SNRs. At lower mean SNRs, ABA is less efficient spectrally than APA and SRA. In terms of mean capacity, however, ABA outperforms the other resource allocation strategies, as shown in Fig. 2 for Rayleigh fading. Therefore, ABA offers higher potential mean capacity at the cost of lower spectral efficiency on low mean SNR links.

Figs. 1 and 2 reveal that manipulating bandwidth to overcome poor channel conditions, even according to an algorithm that maximizes mean capacity, can be wasteful. It is somewhat



Fig. 1. Mean spectral efficiency of various resource allocation strategies in Rayleigh fading (m = 1) with $\gamma_{\text{th}} = 3 \text{ dB}$.



Fig. 2. Mean capacity of various resource allocation strategies in Rayleigh fading with $\bar{B} = B = 50 \text{ kHz}$ and $\gamma_{\text{th}} = 3 \text{ dB}$.

surprising that the benefit of adapting bandwidth to maximize mean capacity only manifests itself (in terms of spectral efficiency) at higher mean SNR. This is because, for any γ_1 , $B(\gamma_1)$ for a link with mean SNR $\bar{\gamma}_1$ is greater than $B(\gamma_1)$ for a link with mean SNR $\bar{\gamma}_2$ provided $\bar{\gamma}_2 > \bar{\gamma}_1$, equivalent B, and equivalent γ_{th} . Therefore, on low mean SNR links, constant bandwidth resource allocation strategies are more spectrally efficient than adaptive bandwidth allocation strategies.

In Figs. 3 and 4 we consider delay performance. We measure delay in terms of the mean delay in system-that is, the mean queuing delay in addition to the mean service time—given above in (1).

Fig. 3 illustrates the effect of fading intensity on mean delay for ABA. Naturally, as the fading becomes less severe with increasing m, the mean delay performance improves. The magnitude of the improvement, however, is less for larger values of m, as the mean delay performance approaches the optimal (the AWGN channel).

Fig. 4 shows the mean delay of various resource allocation



Fig. 3. Mean delay of adaptive bandwidth allocation in Nakagami fading of various severities with $\bar{B} = 50 \text{ kHz}$, $\bar{\gamma} = 20 \text{ dB}$, and $\gamma_{\text{th}} = 3 \text{ dB}$.



Fig. 4. Mean delay of various resource allocation strategies in Rayleigh fading with $\bar{B} = B = 50 \text{ kHz}$, $\bar{\gamma} = 20 \text{ dB}$, and $\gamma_{\text{th}} = 3 \text{ dB}$.

strategies in Rayleigh fading. The performance of SRA is indistinguishable from that of APA at the chosen bandwidth. Both SRA and APA are close to the AWGN channel in terms of delay at low utilization, and remain close to the AWGN channel performance even at high utilizations. ABA, on the other hand, performs worse than the other allocation strategies considered here. ABA provides a higher mean capacity than the other resource allocation strategies, yet APA and SRA experience much lower delay over the same channels. This is due to the fact that, as shown in [14], the instantaneous bandwidth follows the SNR in order to maximize mean capacity. Under Nakagami-m fading, this results in a service time distribution $f(S_{ABA})$ that is much less favorable from a queuing perspective than those for APA and SRA. This yields the interesting observation that mean capacity and mean spectral efficiency are not always the best predictors of QoS performance, particularly in the presence of adaptive resource allocation.

VI. CONCLUSIONS

We have presented an approach to analyzing delay over a flat fading channel with outages under various resource allocation strategies. The model is capable of capturing the impact of the Nakagami fading parameter, m, on delay performance. Our results indicate that commonly used information theoretic measures of wireless performance are not always the best indicators of achievable network QoS. A bandwidth-adaptive channel that achieves superior channel capacity performance over other strategies was shown to experience substantially worse packet delays when compared to these same strategies. However, these results are applicable only to a single user, and we are investigating the spectrum sharing aspects of such bandwidth adaptive techniques and their potential to outperform other allocation strategies in a multi-user environment.

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