

# A MODEL PREDICTIVE CONTROL APPROACH TO NETWORKED SYSTEMS

Guofeng Zhang, Xiang Chen, and Tongwen Chen

**Abstract**—This paper addresses the problem of controller design for networked control systems regulated by a network data transmission scheme proposed in [23]. The plant under the transmission constraint is first formulated as a mixed logical dynamical system, then model predictive control (MPC) based on the mixed-integer programming is applied to derive the controller. It is shown that the non-convexity feature of this class of networked control systems rules out piecewise affine controllers that are designable for linear convex problems. Nevertheless, controller design is still feasible due to the special nature of the data transmission strategy, i.e., only a small number of logic values is involved. An example is given to illustrate the strength of the proposed approach.

**Keywords**— model predictive control, networked control systems, non-convex mixed-integer programming, mixed logical dynamical systems, hybrid systems

## I. INTRODUCTION

In recent years, secure high-speed communication networks have obtained rapid development ([1], [2]). With these newly-developed renderers, network-based control becomes possible. The insertion of a communication channel into a control loop has many conspicuous advantages, for examples, wire reduction, low cost and easy installation and maintenance, etc. Due to these potential merits, networked control systems have been built in various fields like automotive control ([3], [4]), aircrafts manufacturing ([5], [6]), and robotic controls ([7], [8]). However, since the encoded system output, controller output and other information are transmitted through communication networks usually shared by many clients, data traffic congestion is always unavoidable. This often manifests in the form of time delays, packet loss, and other undesirable effects on the control systems. How to compensate these undesirable effects has become a major subject of research in the control community as well as several closely related disciplines such as communication. To date, many types of network communication schemes and control strategies have been proposed to deal with this issue. In general, these methods fall into three categories that are elaborated below.

The first category simply models a networked control system as a control system with bounded deterministic time

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G. Zhang is with the School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan, China, 610054. gfzhang@ee.uestc.edu.cn

X. Chen is with the Department of Electrical and Computer Engineering, University of Windsor, Windsor, Ontario, Canada N9B 3P4. xchen@uwindsor.ca

T. Chen is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4. tchen@ece.ualberta.ca

delays. For instance, a so-called try-once-discard (TOD) protocol is recently proposed and studied extensively in [9], [10], [11], [12], where an upper bound of sensor-to-controller time delays induced by the network is derived, for which exponential stability of the closed-loop system is guaranteed. This idea is then generalized by Netic and Teel ([13], [14]) to develop a set of Lyapunov UGES (Uniformly Globally Exponentially Stable) protocols in the  $L^p$  framework. In [15], based on the assumption of bounded time delays and packet dropouts, a robust  $H_\infty$  control problem is formulated and studied for networked control systems. Unfortunately, none of these papers have addressed the issues of controller-to-actuator delays. A possible reason might be that a controller designed a priori is incapable of predicting and then handling time delays from controller to actuator. Generally speaking, the approaches in this category are quite conservative due to their intrinsic limitations in plant modeling, as has been widely recognized.

The preceding conservativeness has motivated the development of the second category of methods, where network time delays and packet dropouts are modeled as random processes, in particular Markov chains. By this modeling, some specific features of these random processes can be employed to guide controller design in order to achieve desired system performance. For example, in [3], by assuming time delays as Markov chains, a jumped linear system is constructed via state augmentation. Necessary and sufficient conditions for zero-state mean-square exponential stability have been derived for this system. In [16], both sensor-to-controller and controller-to-actuator time delays are modeled as independent white-noise with zero mean and unit variance, and consequently a (sub)optimal stochastic control problem is formulated and studied. A similar approach is adopted in [17] where sensor-to-controller and controller-to-actuator time delays are supposed to behave according to Markov chains respectively. The plant to be controlled may not be stable necessarily, which adds much difficulty to control if there are unknown delays from a controller to an actuator. By augmenting the system to obtain a delay-free system and then with resort to LMI techniques, a sufficient condition is derived that guarantees the closed-loop system is stochastically stable. Based on this, a time-varying controller is constructed. It is worth mentioning that time-varying controllers are always necessary when there are delays from a controller to an actuator.

The above two categories of methods deal with network effects *passively* in that they only consider the influence of the network on the control systems, leaving aside interac-

tion between control systems and communication network. This latter consideration is quite natural and of course very important, and thus has inspired the third category of methodologies that address the tradeoff between data transmission rate and performance of control systems. Some interesting work has already been done along this direction. For instance, in order to minimize bandwidth utilization, Goodwin *et al.* [18] propose to use signal quantization to reduce the size of the transmitted data and solve the problem via a moving horizon technique. The adoption of moving horizon techniques is natural as it can effectively deal with constraints induced by quantization. In [19], the effect of quantization error, quantization time and propagation time on the containability, a weaker stability concept, of networked control systems is studied. The tradeoff of data rate and desirable control objectives is considered in [20] with emphasis on observability and stabilizability under finite network bandwidth constraints. A necessary condition is established on the rate of the communication channel for asymptotic observability and stabilizability of an unstable linear discrete-time system. More specifically, the rate must be bigger than the summation of the logarithms of modules of the unstable system poles. Then, these results are further generalized to the study of control over noisy channels in [21]. For the LQG optimal control of an unstable scalar system over an additive white Gaussian noise (AWGN) channel, it is reported in [22] that the achievable data transmission rate is governed by the Bode sensitivity integral formula, thereby establishing the equivalence between feedback stabilization through an analog communication channel and a communication scheme based on feedback and thus unifying the design of control systems and communication channels.

In [23], a new data transmission strategy is proposed aiming at reducing network traffic congestion. The basic idea is the following: By adding constant deadbands to both a controller and a plant to be controlled, signals will be sent *only* when necessary. By designing the deadbands carefully, a tradeoff between control performance and reduction of network data transmission rate can be achieved. This network transmission strategy is suitable for fitting a control network into an integrated communication network composed of control and data networks, so as to fulfill the need for a new breed geared toward total networking. Seamless integration of control systems into communication networks is of course very appealing as depicted by Raji [24]; and at the same time is fundamentally important as a fundamental future direction in control research in an information-rich world [25]. Essentially speaking, under the network data transmission strategy proposed in [23], in an integrated network composed of data and control networks, it is requested that the network provide sufficient communication bandwidth upon the request of control systems. As a payoff, control systems will save network resources by deliberately dropping packets while without degrading system performance severely. This is a crucial tradeoff. On the one hand, control signals are normally time critical, hence the priority should be given to them whenever requested; on the other hand, due to

one characteristic of control networks, namely, small packet size but frequent packet flows, it is somewhat troublesome to manage because it demands frequent transmissions. The proposed scheme aims to relieve this burden for the whole communication network.

In this paper we continue the study of this networked control system. The plant under the transmission constraint is first formulated as a mixed logical dynamical system, then model predictive control (MPC) based on the mixed-integer programming is applied to derive the controller. An example show that control design based on this new formulation is effective.

This paper is organized as follows: The proposed network protocol is presented in Section 2. In Section 3, the networked system is converted into a mixed logical dynamical system and its property is analyzed. One example is given in Section 4 to illustrate the effectiveness of this configuration. Some concluding remarks are given in Section 5.

## II. THE PROPOSED NETWORK TRANSMISSION STRATEGY

In [23], a new data transmission strategy is proposed, which is briefly reviewed here. Consider the feedback system shown in Fig. 1, where  $G$  is a discrete-time system of the

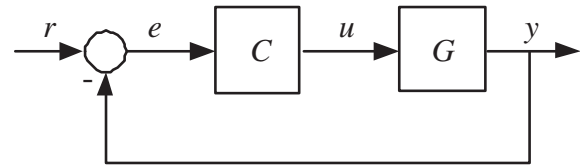


Fig. 1. A typical feedback system

form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned}$$

with the state  $x \in \mathbb{R}^n$ , the input  $u \in \mathbb{R}^m$ , the output  $y \in \mathbb{R}^p$ , and the reference input  $r \in \mathbb{R}^p$ , respectively;  $C$  is a stabilizing controller:

$$\begin{aligned} x_d(k+1) &= A_d x_d(k) + B_d e(k), \\ u(k) &= C_d x_d(k) + D_d e(k), \\ e(k) &= r(k) - y(k), \end{aligned}$$

with its state  $x_d \in \mathbb{R}^{n_c}$ . Let  $\xi = \begin{bmatrix} x \\ x_d \end{bmatrix}$ . Then, the closed-loop system from  $r$  to  $e$  is described by

$$\begin{aligned} \xi(k+1) &= \begin{bmatrix} A - BD_d C & BC_d \\ -B_d C & A_d \end{bmatrix} \xi(k) + \begin{bmatrix} BD_d \\ B_d \end{bmatrix} r(k), \\ e(k) &= \begin{bmatrix} -C & 0 \end{bmatrix} \xi(k) + r(k). \end{aligned}$$

Now, we add some nonlinear constraints on both  $u$  and  $y$ . Specifically, consider the system shown in Fig. 2. The

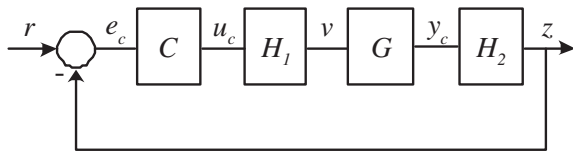


Fig. 2. A constrained feedback system

nonlinear constraint  $H_1$  is defined as follows: for given  $\delta_1 > 0$ , take  $v(-1) = 0$ ; and for  $k \geq 0$ , let

$$\begin{aligned} v(k) &= H_1(u_c(k), v(k-1)) \\ &= \begin{cases} u_c(k), & \text{if } \|u_c(k) - v(k-1)\|_\infty > \delta_1, \\ v(k-1), & \text{otherwise.} \end{cases} \end{aligned}$$

Similarly,  $H_2$  is defined as follows: for given  $\delta_2 > 0$ , take  $z(-1) = 0$ ; for  $k \geq 0$ , let

$$\begin{aligned} z(k) &= H_2(y_c(k), z(k-1)) \\ &= \begin{cases} y_c(k), & \text{if } \|y_c(k) - z(k-1)\|_\infty > \delta_2, \\ z(k-1), & \text{otherwise.} \end{cases} \end{aligned}$$

In [26] *adjustable* deadbands are proposed to reduce network traffics, where the closed-loop system with deadbands is modeled as a perturbed system, and its exponential stability follows that of the original system [27]. The constraints,  $\delta_1$  and  $\delta_2$  proposed here, are fixed. We have observed in [23] that the stability of the system shown in Fig. 2 is fairly complicated and only local stability can be obtained. However, the main advantage of fixed deadbands is that it will reduce network traffics more effectively. Furthermore, the stability region can be scaled as large as desired (at least for low order systems).

At this point, one can see that in the framework of communication networks containing both data and control networks, this proposed data transmission strategy is likely to provide sufficient communication bandwidth upon the request of control networks used by the control systems. As a payoff, the control systems will try to save network resources by deliberately dropping packets without deteriorating control performance severely. This consideration is well tailored to the requirement of control networks in general. On the one hand, control signals are normally time critical, hence the priority should be given to them whenever requested; on the other hand, due to the characteristics of control networks, namely, small packet size but frequent packet flows, it is somewhat troublesome to manage because it demands frequent transmissions. Our scheme aims to relieve this burden for the whole communication network.

### III. MAIN RESULTS

In this section, the network-based control problem raised in Section II is converted to an optimization problem of a mixed logical dynamical system. This re-configuration enables us to use optimization techniques recently developed for predictive control of hybrid systems to do the controller design that takes into account both control performance and reduction of network data transmission rate. Here we only

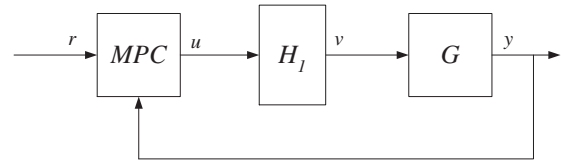


Fig. 3. Networked-based MPC control system

consider the network traffic from control to actuator. More concretely, consider the configuration in Fig. 3, where the system  $G$  is given by

$$\begin{aligned} x(k+1) &= Ax(k) + Bv(k), \\ y(k) &= Cx(k), \end{aligned} \quad (1)$$

in which

$$\begin{aligned} v(k) &= H_1(u(k), v(k-1)) \\ &= \begin{cases} u(k) & \text{if } |u(k) - v(k-1)| > \delta, \\ v(k-1) & \text{otherwise.} \end{cases} \end{aligned} \quad (2)$$

For ease of presentation, define

$$z(k) := v(k-1).$$

Then the system composed of (1)-(2) becomes

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} &= \begin{bmatrix} Ax(k) + Bu(k) \\ u(k) \end{bmatrix} \\ &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} u(k), \\ y(k) &= Cx(k), \end{aligned} \quad (3)$$

if  $|u(k) - z(k)| > \delta$ ; otherwise,

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} &= \begin{bmatrix} Ax(k) + Bz(k) \\ z(k) \end{bmatrix} \\ &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} B \\ I \end{bmatrix} z(k), \\ y(k) &= Cx(k). \end{aligned} \quad (4)$$

Denote

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ I \end{bmatrix},$$

and define

$$\begin{aligned} \xi(k) &:= u(k) - z(k), \\ X &= [x^T \ z^T]^T, \end{aligned} \quad (5)$$

where the superscript “ $T$ ” stands for the transpose. Then the system composed of (3)-(4) is equivalent to

$$X(k+1) = \begin{cases} \bar{A}X(k) + \bar{B}u(k) & \text{if } |\xi| > \delta, \\ \bar{A}X(k) + \bar{B}u(k) - \bar{B}\xi(k) & \text{otherwise.} \end{cases} \quad (6)$$

System (6) is a switched system under a logical law. In general, it is not easy to control such systems even if the consideration of network traffic rate reduction is neglected. Next we convert this logical law to a logical value. To do that, let us first recall some Boolean connectives as listed in Table I. By means of these Boolean connectives, the *literal*

TABLE I  
BOOLEAN ALGEBRA CONNECTIVES

$\wedge$	and
$\vee$	or
$\sim$	not
$\rightarrow$	implies
$\leftrightarrow$	if and only if
$\oplus$	exclusive or

$|\xi(k)| > \delta$  can be associated with a logical value  $\gamma(k)$  via

$$[|\xi(k)| > \delta] \leftrightarrow [\gamma(k) = 1]. \quad (7)$$

In terms of (7), system (6) can be transformed to

$$\begin{aligned} X(k+1) &= (\bar{A}X(k) + \bar{B}u(k))\gamma(k) \\ &\quad + (\bar{A}X(k) + \bar{B}u(k) - \bar{B}\xi(k))(1 - \gamma(k)) \\ &= \bar{A}X(k) + \bar{B}u(k) - \bar{B}\xi(k)(1 - \gamma(k)). \end{aligned} \quad (8)$$

Define

$$\eta(k) := \xi(k)(1 - \gamma(k)). \quad (9)$$

Then system (8) is equivalent to

$$\begin{aligned} X(k+1) &= (\bar{A}X(k) + \bar{B}u(k))\gamma(k) \\ &\quad + (\bar{A}X(k) + \bar{B}u(k) - \bar{B}\xi(k))(1 - \gamma(k)) \\ &= \bar{A}X(k) + \bar{B}u(k) - \bar{B}\eta(k). \end{aligned} \quad (10)$$

Define an upper and a lower bound of  $\xi$  to be

$$M = \max(\xi), \quad m = \min(\xi). \quad (11)$$

these bounds are usually specified by the practical consideration, and they facilitate the derivation of linear inequalities that will serve as constraints in the optimization problem. It is easy to show that

$$\eta(k) \leq M(1 - \gamma(k)), \quad (12)$$

$$\eta(k) \geq m(1 - \gamma(k)), \quad (13)$$

$$\eta(k) \leq \xi(k) - m\gamma(k), \quad (14)$$

$$\eta(k) \geq \xi(k) - M\gamma(k). \quad (15)$$

Furthermore, define

$$[\xi(k) > \delta] \leftrightarrow [\alpha(k) = 1], \quad [\xi(k) < -\delta] \leftrightarrow [\beta(k) = 1].$$

Consequently,

$$\{[\alpha(k) = 1] \vee [\beta(k) = 1]\} \leftrightarrow [\gamma(k) = 1],$$

which is equivalent to

$$\begin{cases} \alpha(k) \leq \gamma(k), \\ \beta(k) \leq \gamma(k), \\ \gamma(k) \leq \alpha(k) + \beta(k). \end{cases} \quad (16)$$

Also, it is easy to verify that

$$[\xi(k) > \delta] \leftrightarrow [\xi(k) - \delta > 0] \leftrightarrow [\alpha(k) = 1]$$

if and only if

$$\begin{cases} \xi(k) - \delta < (M - \delta)\alpha(k), \\ \xi(k) - \delta > \epsilon + (m - \delta - \epsilon)(1 - \alpha(k)). \end{cases} \quad (17)$$

where  $\epsilon$  is a positive number which is sufficiently small. Similarly,

$$[\xi(k) < -\delta] \leftrightarrow [\xi(k) + \delta < 0] \leftrightarrow [\beta(k) = 1]$$

if and only if

$$\begin{cases} \xi(k) + \delta < (M + \delta)(1 - \beta(k)), \\ \xi(k) + \delta > \epsilon + (m + \delta - \epsilon)\beta(k). \end{cases} \quad (18)$$

Keeping in mind that we are now studying a network-based control problem, hence in addition to stability and performance of the control systems, network transmission must also be taken into account. In light of this, we define a new state variable

$$\omega(k) = 1 - \gamma(k). \quad (19)$$

Note that when  $\omega(k) = 0$ , there is one network data transmission; otherwise, no data transmission. The overall state of the system therefore becomes

$$\tilde{X} = [X^T \ \omega]^T. \quad (20)$$

Now we have reconfigured the system composed of  $G$  and  $H_1$  as a mixed logical dynamical (MLD) system given by (5) and (9)-(20). For convenience, we hereafter denote it by  $\Sigma$ . It is clear that the state of  $\Sigma$  is  $\tilde{X}$  that is composed of  $x$ ,  $z$  and  $\omega$ , the input of  $\Sigma$  is  $u$ , the output of  $\Sigma$  is  $y$ , and  $\xi$ ,  $\eta$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  are all auxiliary variables. It can be easily verified that this mixed logical dynamical system well-posed. However, due to the switching law defined in (6) that enters implicitly into the mixed logical dynamical system via  $\xi$ , system  $\Sigma$  is essentially *non-convex*.

Next we study the following problem: How to design a controller for system  $\Sigma$  such that the closed-loop system has satisfactory control performance and at the same time the network communication scheme can reduce network traffic effectively. Observe that the logic law has been converted to linear inequalities constraints, hence we are motivated to seek a desirable control law using MPC techniques. In general, it is not easy to find an appropriate prediction control law for an MLD system because the system is essentially nonlinear and integer variables are involved. Fortunately some effective tools have been developed recently ([28], [29]) based on mixed-integer algorithms ([30]). Hereafter we will borrow this idea to reduce our controller design problem to a problem of predictive control.

Suppose the objective of control is to force the output  $y$  to track a reference signal  $y_r$ . Also let  $X_r$ ,  $u_r$  be desired references of the state and input respectively. Then at the current sampling instant  $k$ , the predictive control problem can be formulated as follows:

$$\begin{aligned} P(N_r, N_p, X(k)) : \min_{\mathcal{U}} & \left\{ \|Q_x(X(k+N|k) - X_r)\|_p \right. \\ & \left. + \sum_{i=0}^{N_p-1} \|Q_y(y(k+i|k) - y_r)\|_p + \|Q_x(X(k+i|k) - X_r)\|_p \right\} \end{aligned}$$

$$+ \|Q_u(u(k+i|k) - u_r)\|_p + |Q_\omega(\omega(k+i|k) - 1)|\}$$

subject to

$$\begin{aligned}
X(k|k) &= X(k), \\
X(k+i+1|k) &= \bar{A}X(k+i|k) + \bar{B}u(k+i|k) \\
&\quad - \bar{B}\eta(k+i|k), \\
\omega(k+i+1|k) &= 1 - \gamma(k+i|k), \\
y(k+i|k) &= Cx(k+i|k), \\
\xi(k+i|k) &= u(k+i|k) - z(k+i|k), \\
\eta(k+i|k) &\leq M(1 - \gamma(k+i|k)), \\
-\eta(k+i|k) &\leq -m(1 - \gamma(k+i|k)), \\
\eta(k+i|k) &\leq \xi(k+i|k) - m\gamma(k+i|k), \\
-\eta(k+i|k) &\leq -\xi(k+i|k) + M\gamma(k+i|k), \\
\alpha(k+i|k) &\leq \gamma(k+i|k), \\
\beta(k+i|k) &\leq \gamma(k+i|k), \\
\gamma(k+i|k) &\leq \alpha(k+i|k) + \beta(k+i|k), \\
\xi(k+i|k) - \delta &< (M - \delta)\alpha(k+i|k), \\
-\xi(k+i|k) - \delta &< -\epsilon - (m - \delta - \epsilon)(1 - \alpha(k+i|k)), \\
\xi(k+i|k) + \delta &< (M + \delta)(1 - \beta(k+i|k)), \\
-\xi(k+i|k) - \delta &< -\epsilon - (m + \delta - \epsilon)\beta(k+i|k),
\end{aligned}$$

where  $\mathcal{U} = \{u(k|k), u(k+1|k), \dots, u(k+N-1|k)\}$  is a future input sequence to be determined by solving the above optimization problem. Furthermore, when  $p = \infty$ , we define

$$\begin{aligned}
&\|Q_x(X(k+N|k) - X_r(k+N|k))\|_p \\
&:= \|Q_x(X(k+N|k) - X_r(k+N|k))\|_\infty.
\end{aligned}$$

Others are defined similarly. Weighting matrices satisfy

$$Q_x \geq 0, Q_y \geq 0, Q_u \geq 0, Q_\omega > 0.$$

Because  $\omega$  reflects the transmission rate, it is separated from other state variables. Moreover,  $Q_\omega$  must be a strictly positive number. The bigger  $Q_\omega$  is, the more severe the demand is on the network transmission. So consideration of network traffic is integrated into the above optimization explicitly.

Here the control and prediction horizon are set equal. In the current literature of MPC, an infinite prediction horizon MPC problem is always assumed to be solvable when a finite horizon MPC problem is to be dealt with. This key observation has enabled many effective approaches to solving various MPC problems. Unfortunately, due to the very nature of the problem studied here, i.e., control performance with transmission rate reduction, the corresponding infinite prediction horizon MPC problem formulated above is *not* solvable. This is stated as the following theorem.

**Theorem 1:** If  $G$  is unstable, the set of  $X(k)$  such that the optimization problem  $P(N, N, X(k))$  with  $N = \infty$  admits finite solutions is of zero Lebesgue measure.

**Proof.** For a given  $X(k)$ , suppose that the optimization problem  $P(N, N, X(k))$  with  $N = \infty$  has a solution. Then it has finite cost. Therefore, there is a time  $K_0$  such that

$v(k) = v(K_0)$  for all  $k \geq K_0$ , i.e., there will be no more new input update. Consequently,

$$\begin{aligned}
X(k+1) &= AX(k) + Bv(k), \\
y(k) &= Cx(k), \quad (\forall k \geq K_0),
\end{aligned} \tag{21}$$

which yields

$$X(K_0 + L) = A^L X(K_0) + \sum_{i=K_0}^{L-1} A^{L-i} Bv(K_0), \tag{22}$$

for any integer  $L > 0$ . Suppose the problem under consideration is to regulate the state to the origin, namely, drive  $X(K_0 + L) \rightarrow 0$  as  $L \rightarrow \infty$ . Denote by  $E_{K_0}$  the set of  $X(K_0)$  such that  $X(K_0 + L)$ , governed by (22), tends to zero as  $L$  goes to  $\infty$ . Then, since  $G$  is unstable, i.e., the matrix  $A$  is unstable, the Lebesgue measure,  $m(E_{K_0})$ , is zero. Note that  $K_0$  is a non-negative integer. For convenience, denote  $E_{K_0}$  by  $E_i$  when  $K_0 = i$ . Thus the union of  $E_i$  for  $i$  from 0 to  $\infty$  contains all  $X(K_0)$  such that  $P(N, N, X(k))$  with  $N = \infty$  is solvable. Observe that  $m(E_i) = 0$  for each non-negative integer  $i$ , thus

$$m(\cup(E_i)) \leq \sum_{i=0}^{\infty} m(E_i) = 0,$$

which indicates that the set of all  $X(K)$  such that  $P(N, N, X(k))$  with  $N = \infty$  is solvable has Lebesgue measure 0.  $\square$

#### IV. AN EXAMPLE

In this section, an example is given to illustrate the effectiveness of the approach proposed in Section III. The first example is a scalar case whose dynamics have been found unexpectedly complex. Here we show that desirable control can be achieved.

**Example 1.** Consider the following system:

$$\begin{aligned}
\Sigma_1 : \quad x(k+1) &= ax(k) + bv(k), \\
y(k) &= x(k),
\end{aligned}$$

with  $v(-1) \in \mathbb{R}$  without loss of generality, and for  $k \geq 0$ ,

$$v(k) = \begin{cases} u(k), & \text{if } |u(k) - v(k-1)| > \delta, \\ v(k-1), & \text{otherwise,} \end{cases}$$

The chaotic dynamics of system  $\Sigma_1$  have been studied in detail in [23]. Now we discuss its control problem. Specifically we address the following tracking problem.

Following the development in Section III, we define the following optimization index:

$$\begin{aligned}
\min_{\mathcal{U}} \sum_{i=0}^N |Q_y(y(k+i|k) - y_r(k+i))| \\
+ \|Q_x(x(k+i|k) - x_r(k+i))\|_\infty
\end{aligned}$$

where

$$\begin{aligned}
y_r &\equiv 1, \quad x_r = [1 \quad 1 \quad 1]^T, \\
Q_y &= 110, \quad Q_x = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\end{aligned}$$

By using propositional logic, system  $\Sigma_1$  can be written into a HYSDEL (Hybrid System Description Language) code (see Appendix). Then by running this code a mixed logical dynamical model is obtained (However it can not be converted to a piece-wise affine form, due to the nonconvexity of the constraints). Choose  $\delta = 0.04$ , and  $N = 2$ . Take and initial condition  $(x(0), v(-1)) = (0.5, -10)$ . Now we study two cases. Case 1 is with  $a = 0.9$  and  $b = -0.3$  and case 2 is with  $a = 1.2$  and  $b = -0.3$ . Hence the original system in case 1 is stable while that in case 2 is not. If we desire that the output  $y$  tracks a sinusoidal signal,  $0.3 * \sin((0 : T_{stop})'/5)$ , where  $T_{stop}$  is the simulation time (here it is set to be 150), then Fig. 4 is obtained. In case 1,

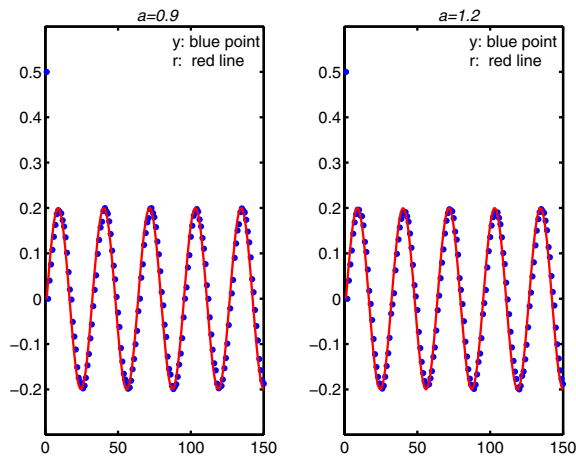


Fig. 4. Tracking of a sinusoidal signal

the transmission rate is 66.67% while that of case 2 is 74%. We conclude that tracking is achieved while the network transmission rate is also reduced to a certain degree. When the system is excessively unstable, for example,  $a = 20$ , the optimization process with a prediction horizon  $N = 2$  generates a sequence of input  $u(k + i|k)$  which makes all  $\omega(k + i + 1|k)$  equal to 0. Note that the zero value of  $\omega$  indicates the successful network transmission. By using a longer prediction horizon, say,  $N = 8$ , some values of  $\omega$  are 1, i.e., the controller based on the prediction control can still reduce some network traffic. However, when  $a = 50$ , the prediction  $N$  will have to be extremely big.

## V. CONCLUSIONS

We have studied a network-based control problem originated from a newly proposed network data transmission scheme. By re-formulating the system into a mixed logical dynamical system, we are able to use some recently developed optimization tools to achieve desired control performance as well as reduce network traffic. Our example has demonstrated the effectiveness of this new treatment.

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