Cognitive Radio in a Frequency-Planned Environment: Some Basic Limits

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***Abstract*—The objective of this work is to assess some fundamental limits for opportunistic spectrum reuse via cognitive radio in a frequency-planned environment. We present a first-order analysis of the signal-to-noise-and-interference situation in a wireless cellular network, and analyze the impact of cognitive users starting to transmit. Two main conclusions emerge from our study. First, obtaining any substantial benefits from opportunistic spatial spectrum reuse in a frequency-planned network without causing substantial interference is going to be very challenging. Second, the cognitive users need to be more sensitive, by orders of magnitude, than the receivers in the primary system, especially if there is significant shadow fading. This latter problem can be alleviated by having cognitive users cooperate, but only if they are separated far apart so that they experience independent shadowing.**

***Index Terms*—Wireless communications, cognitive radio.**

I. Introduction

THERE is a widespread conception that radio spectrum is underutilized and that there are many "holes" in the spectrum. This notion is well supported by recent measurements [1], which show that there are substantial geographical areas where the spectral activity is low in a specific frequency band, and that there are bands for which the duty cycle is very low at specific geographical locations. The idea of exploiting such spectrum holes has led to the concept of *cognitive radio* [2]–[4], with which licensed spectrum dedicated to a primary network would be available for reuse by secondary (cognitive) devices, provided that these cognitive users do not create harmful interference for the primary system. This requires the cognitive radios to be far from the primary system and transmit at fairly low signal levels. In particular, it has been argued that the cognitive radios shall be permitted to transmit if they cannot "hear" any primary transmission [5]. This requires the cognitive radios to be able to detect the presence of very weak primary signals, a problem which is known to be fundamentally hard [6].

In this paper we examine the fundamental limitations for the deployment of cognitive radios when the primary system is a frequency-planned (e.g., cellular) network. This extends previous work [7], [8], which has presented such considerations for the special case that the primary system operates in a noise-limited environment (i.e., there is only a *single* primary base station). This extension is fundamental because the notion of a cognitive radio being "far from a primary base station" only has a practical meaning when its distance to the primary base station is related to the distance between two primary base stations. Also, primary systems found in reality will have multiple primary base stations, because no operator will buy spectrum which is *not* reused geographically. (It is only a matter of what the reuse factor is.) Additionally, while previous work modeled shadow fading only by incorporating it as an additional fading margin, we model it as random and then consider an operating point that corresponds to a specific outage probability ϵ for the primary system. We also consider the impact of pilot orthogonality among the primary base stations.

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II. Model and Preliminaries

We consider a primary wireless system based on a standard hexagon-type frequency reuse model. Throughout, we are concerned only with the downlink. Figure 1 shows the system, including the main primary base station (BS_0) and the first tier of co-channel interferers (BS_1–BS_6). BS_0–BS_6 transmit omni-directionally with power P_{prim}.[1] The nominal radius of the cell served by BS_0 is r and the distance to the first tier of interferers is $D = \sqrt{3n}r$ where $1/n$ is the frequency reuse factor for the primary system [9] (we refer to the number n itself as the frequency reuse).[2] Primary users outside the cell of BS_0 are served by other primary base stations operating at other frequencies, not shown in the figure. The base station BS_0 is located at the origin, and BS_1–BS_6 are located at the coordinates $x_{\text{BS},k} = D\cos([k-1]\pi/3)$, $y_{\text{BS},k} = D\sin([k-1]\pi/3)$. We model propagation via a distance-dependent path-loss and lognormal shadow fading [9]. We define the channel gain function to be a random function of a distance x,

$$\rho(x) \triangleq x^{-\alpha} \cdot 10^{\chi/10}, \quad \chi \sim N(0,\sigma) \tag{1}$$

where α is the path loss exponent and σ is the standard deviation of the lognormal fading [dB]. The function $\rho(x)$ measures how much a signal attenuates when it travels a distance of x. For example, a measurement of the power received by a primary receiver located x meters from BS_0 will

[1] "Transmit power" is defined as the power received by a receiver at unit distance from the transmitter. All antennas are assumed to have the same gain. We deal only with power ratios, so physical power units will be immaterial for the analysis.

[2] The cell radius r cancels out in all results. We can take it as fixed, and independent of n, without loss of generality.

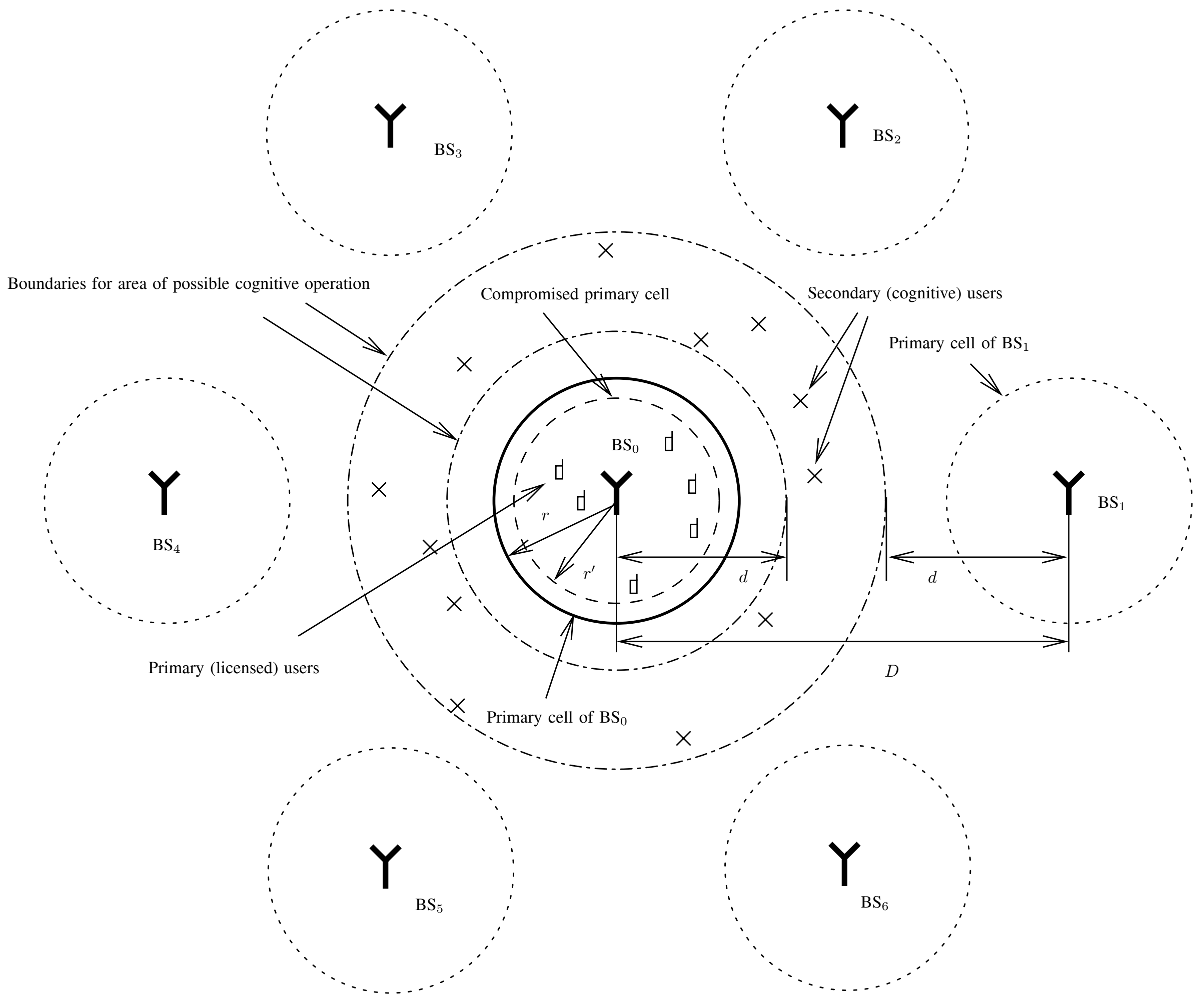


Fig. 1. Illustration of the system under consideration. With n-reuse frequency planning for the primary system, the inter-BS distance, D, is $D = \sqrt{3n}r$ where r is the primary cell radius.

be a realization of $P_{\text{prim}} \cdot \rho(x)$. We shall use the convention that each time $\rho(\cdot)$ appears in an expression, a new independent realization of χ is drawn. (That is, $\rho(x)+\rho(x)$ means the sum of two independent realizations of $\rho(x)$ but $2\rho(x)$ means one realization multiplied by two.) As a consequence, the shadow fading is assumed to be uncorrelated between different base stations and between different geographical locations.

For future reference we also note that the total co-channel interference power that a primary receiver, located at (x, y), experiences from the surrounding primary base stations BS_1–BS_6 is given as $P_{\text{prim}} \cdot \mu(x, y)$, where

$$\mu(x,y) \triangleq \sum_{k=1}^{6} \rho\left(\sqrt{(x_{\text{BS},k} - x)^2 + (y_{\text{BS},k} - y)^2}\right)$$

III. Analysis

A. Nominal Operating Point for Primary System

We first consider the primary system, without cognitive users. A primary user located at (x, y) will experience the following signal-to-interference-plus-noise ratio:

$$\mathsf{SINR}_{\text{prim},(x,y)} = \frac{P_{\text{prim}}\rho\left(\sqrt{x^2+y^2}\right)}{P_{\text{prim}}\mu(x,y) + N} \tag{2}$$

where N is the receiver noise floor. In general the primary system (without cognitive users) will operate either in the noise-limited regime or in the interference-limited regime. We quantify this operation condition in terms of the following *dimensionless* ratio:

$$\psi \triangleq \frac{P_{\text{prim}}}{N}\bar{\mu} \qquad \text{where} \qquad \bar{\mu} \triangleq E[\mu(r\cos(\phi), r\sin(\phi))]$$

and where the expectation is taken over the lognormal components and over ϕ (uniform over $[-\pi, \pi]$). The quantity ψ basically measures the ratio between the average received co-channel interference power at the primary cell border, and the noise power. If $\psi > 1$, then co-channel interference dominates over noise (on the average), and vice versa. When $\psi = \infty$ the system is completely interference limited. Then increasing P_{prim} does not increase performance, only increasing the reuse n can help. When $\psi = 0$ the system is completely noise limited. Then increasing n does not help, only increasing P_{prim} will improve performance. Systems found in reality and which operate in licensed spectrum will most likely have a value of ψ in the interval $[-10, 10]$ dB. The reason is that for $\psi > 10$ dB, the operator would essentially waste transmit power, i.e., the same system performance would be obtained even if P_{prim} were reduced. For $\psi < -10$ dB, the frequency plan is

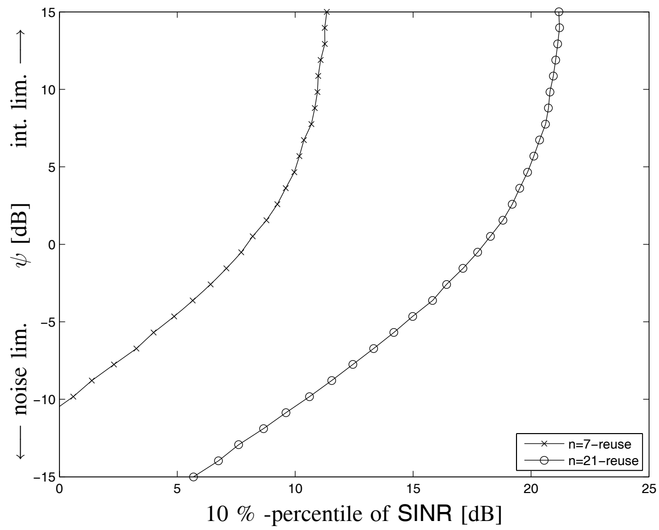


Fig. 2. 10 % -percentile of the SINR at a random point inside the cell, without cognitive users, for some different primary-system operating points ψ and different primary frequency reuse (n). (The operating point ψ was defined as the ratio between the average received co-channel interference power at the primary cell border, and the noise power.) Here the path loss exponent is $\alpha = 4$, and the standard deviation of the lognormal fading is $\sigma = 6$ dB.

flawed in the sense that frequencies can be reused much more densely with no loss of performance, i.e. licensed spectrum is wasted. One can think of ψ as a dimensionless, normalized, transmit power. Basically, it measures the proportion of noise to interference in the primary system. When $\psi = 0$ this proportion is 1:0, and when $\psi = \infty$ it is 0:1. Note that $N = P_{\text{prim}}\bar{\mu}/\psi$, and hence that the total interference-plus-noise power at the primary cell border is $P_{\text{prim}}\bar{\mu}(1 + 1/\psi)$. It follows that $\mathsf{SINR}_{\text{prim},(x,y)} = \rho\left(\sqrt{x^2+y^2}\right)/(\mu(x,y) + \bar{\mu}/\psi)$.

We define the *ϵ-operating point* $\mathsf{SINR}_{\text{prim},\epsilon}$ of the primary system to be the SINR that a user, who is randomly located in the primary cell [i.e., located at a coordinate (x, y) which is uniformly distributed over the circle $x^2 + y^2 < r^2$], experiences with probability $1 - \epsilon$:

$$\text{Prob}\big(\mathsf{SINR}_{\text{prim},(x,y)} < \mathsf{SINR}_{\text{prim},\epsilon}\big) = \epsilon$$

Figure 2 shows $\mathsf{SINR}_{\text{prim},\epsilon}$ for $\epsilon = 10\%$, and for some different primary frequency plans, as a function of ψ. In Figure 2 we chose $\alpha = 4$ and $\sigma = 6$ dB, common values for outdoor propagation [9]. A typical design point is, at $\epsilon = 10\%$, $\mathsf{SINR}_{\text{prim},10\%} = 10$ dB. For example cellular telephony based on TDMA/FDMA is often planned this way. From Figure 2 we see that this is achieved with a frequency plan $n = 7$, for realistic values of ψ.

B. Introducing Cognitive Users

We consider now the introduction of secondary (cognitive) users into the network in Figure 1. We assume that these users are uniformly distributed inside the ring between the two circles centered at BS_0 and with radii d and $D - d$, respectively. We call this circular ring the *area of cognitive operation*. We will assume that there are in total N_{cogn} cognitive users and that each one transmits with power P_{cogn}. Thus the aggregate (total) power from the cognitive users is $N_{\text{cogn}}P_{\text{cogn}}$. These users will introduce interference which on the average will bring $\mathsf{SINR}_{\text{prim},\epsilon}$ below its nominal value. Therefore, to tolerate cognitive users one will need to compromise the cell border of the primary cell to a radius r', $r' < r$.[3] (The associated coverage loss is about $(r^2 - r'^2)/r^2 \approx 1 - 2r'/r$ for the primary cell.) We call r' the *compromised cell radius* of the primary system. This phenomenon was quantified for the special case of a single primary base station, i.e., a purely noise limited system (and no shadow fading) in [7], [8].

The question we ask now is: Given $N_{\text{cogn}}P_{\text{cogn}}$, r', and ψ, how large is the size of the cognitive area, and under what conditions is it nonempty? More precisely, we want to determine the feasible values for d. Let (x_m, y_m) be the coordinate of the mth cognitive (secondary) user. A primary receiver at location (x, y) now experiences the *compromised* SINR in (3); on top of the next page. We can define the ϵ-operating point for the compromised system, $\mathsf{SINR}_{\text{compr},\epsilon}$, based on the values of $\mathsf{SINR}_{\text{compr},(x,y)}$ inside the compromised cell with radius r', instead of r. Specifically, $\mathsf{SINR}_{\text{compr},\epsilon}$ is defined via

$$\text{Prob}\big(\mathsf{SINR}_{\text{compr},(x,y)} < \mathsf{SINR}_{\text{compr},\epsilon}\big) = \epsilon$$

where the coordinate (x, y) is uniformly distributed over the circle $x^2 + y^2 < r'^2$. We can then investigate for which values of d it holds that

$$\mathsf{SINR}_{\text{prim},\epsilon} \leq \mathsf{SINR}_{\text{compr},\epsilon} \tag{4}$$

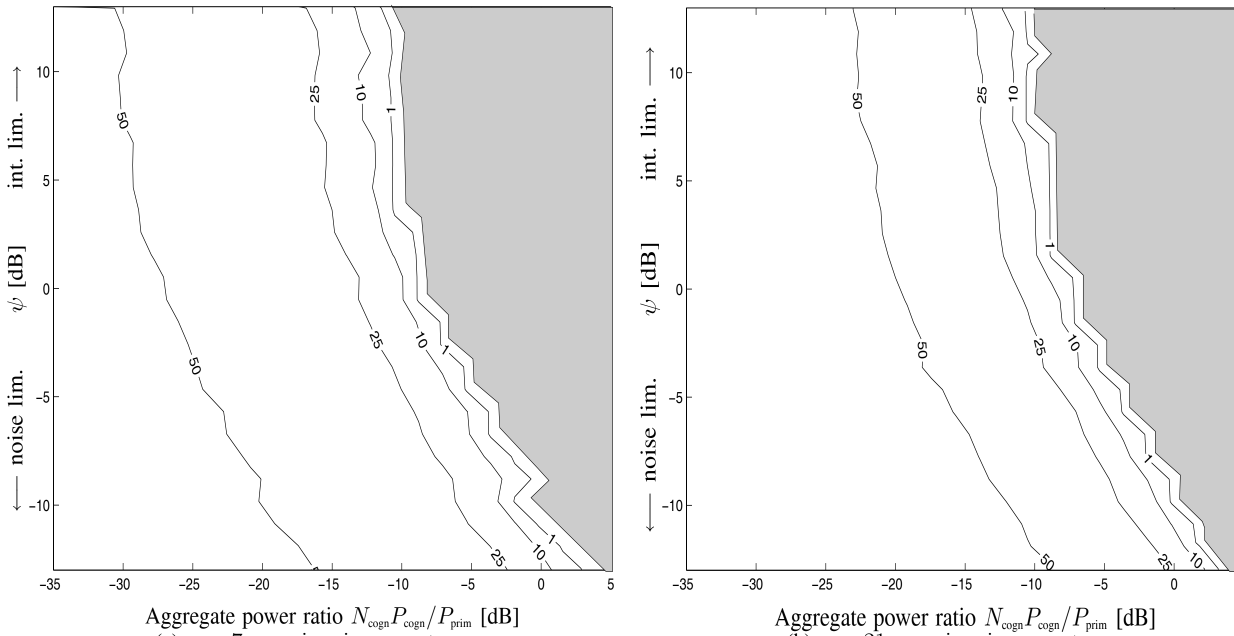
for a specific ϵ (say, 10%). If no value of d, $d < D/2$, can result in (4) being satisfied we say that the configuration $(\psi, N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}})$ is *unfeasible*. Otherwise, we can determine the smallest possible radius (equality in (4)), say d^*, of the inner circle that defines the cognitive operation area and then compute the *relative* size of this area, obtained as

$$A(N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}}, \psi) = \frac{(D - d^*)^2 - (d^*)^2}{D^2} = 1 - 2\frac{d^*}{D}$$

where $0 \leq d^* \leq D/2$. We write $A(N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}}, \psi)$ to stress that this is a function of the aggregate power ratio $(N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}})$ and of the primary operating point (ψ). Note that $0 \leq A \leq 1$. If $A = 0$, then cognitive users are nowhere permitted. In the other extreme, with $A = 1$, then they would be allowed everywhere.

To illustrate these relations we first consider the performance of two systems, with $n = 7$ and 21-reuse, respectively, and take $\alpha = 4$, $\sigma = 6$ dB. Furthermore, we assume $\epsilon = 10\%$ and we set the acceptable reduction in primary coverage to $r'/r = 0.95$. (This choice was used also in [7]. It corresponds to a reduction of power of $0.95^{-4} \approx 1$ dB in a pure noise limited primary-only system. It further corresponds to about 10% coverage loss for the primary cell served by BS_0.) This is actually a large value and it is debatable whether such a reduction would be acceptable, so in this sense our result will provide an optimistic bound. Figures 3 shows the result, in terms of contour plots of the two-dimensional function $A(N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}}, \psi)$. The resulting *forbidden region*, that is,

[3] As in, e.g., [7] we measure the "cost" of allowing cognitive users as a loss in geographic coverage. Other possible measures include power and/or bandwidth loss, or equivalently, a rate-loss [10] for primary users. A first-order translation between these different phenomena can easily be established (based on relevant capacity expressions).



(a) $n = 7$-reuse in primary system (b) $n = 21$-reuse in primary system

Fig. 3. Contour plot of the relative size of the area of cognitive operation, as function of the aggregate power ratio ($N_{cogn}P_{cogn}/P_{prim}$) and of the primary system operating point (ψ). Here the path-loss exponent is $\alpha = 4$, the standard deviation of the lognormal fading is $\sigma = 6$ dB, the percentile of interest is $\epsilon = 10\%$ and the ratio between the nominal and the compromised cell radius is $r'/r = 0.95$. The unit for $A(N_{cogn}P_{cogn}/P_{prim}, \psi)$ used in the figure is "percent." (Note that A is a *relative* area.) In the upper right area (shaded), $A(N_{cogn}P_{cogn}/P_{prim}, \psi)$ is zero and in this "forbidden region," cognitive operation is not possible at all. The results are obtained by evaluating the size of the permissible area over a grid of the parameters $N_{cogn}P_{cogn}/P_{prim}, \psi$ and then visualising the result as a "contour plot". The reason for the curve edges being somewhat jagged is simply the finite resolution of the parameter grid.

$$\mathsf{SINR}_{\text{compr},(x,y)} = \frac{\rho\left(\sqrt{x^2+y^2}\right)}{\mu(x,y) + \frac{P_{\text{cogn}}}{P_{\text{prim}}}\sum_{m=1}^{N_{\text{cogn}}}\rho\left(\sqrt{(x_m-x)^2+(y_m-y)^2}\right) + \bar{\mu}/\psi} \tag{3}$$

the set of unfeasible points $(N_{cogn}P_{cogn}/P_{prim}, \psi)$ resulting in $A(N_{cogn}P_{cogn}/P_{prim}, \psi) = 0$, is shaded.

It is interesting to examine how the ratio r'/r affects the size of the forbidden region. Figure 4 illustrates this for a reuse $n = 12$ frequency plan, $\alpha = 4$, $\sigma = 6$ dB and some different values of r'/r. (Note that $r'/r = 0.9$ corresponds to a coverage loss of almost 20%, or equivalently, a reduction of primary power with $0.9^{-4} \approx 2$ dB, in a pure noise-limited primary-only system. The corresponding numbers for $r'/r = 0.99$ are 2% coverage loss and 0.2 dB reduction in power.)

From Figures 3–4 we can conclude the following:

(i) If a reasonable size of the cognitive operation area is desired (say 25%), then the aggregate cognitive user power $N_{cogn}P_{cogn}$ must be 1–2 orders of magnitude less than the primary power P_{prim}. For example, if there are $N_{cogn} = 100$ cognitive users that transmit simultaneously, then each of them must transmit with a power less than -35 dB below P_{prim}. This is not impossible if P_{prim} is of the order of kW (TV broadcasting, for example) and P_{cogn} is in the order of a few hundred mW (sensor node, or WLAN card, for instance). The conclusion regarding power scaling is in agreement with earlier studies (of single-primary-base station networks), e.g., [8].

(ii) For a fixed $N_{cogn}P_{cogn}/P_{prim}$, noise-limited primary systems are more tolerant to cognitive users. This is not surprising. Keeping all other parameters fixed, decreasing ψ (equivalently, decreasing the primary transmit power) corresponds to decreasing the sum of noise and interference in the system and this increases the size of the area where cognitive radios can exist. (In the limit $\psi \to 0$ we will have the case in [7].)

(iii) There is a forbidden region for $(N_{cogn}P_{cogn}/P_{prim}, \psi)$ where the size of the area of cognitive operation is zero. This corresponds to the case when (4) has no solution w.r.t. d, $d < D/2$.

(iv) The ratio r'/r has a major effect on the size of the forbidden region. For example, in Figure 4, increasing r'/r from 0.9 to 0.99 increases the size of the forbidden region (in the $N_{cogn}P_{cogn}/P_{prim}$-dimension) by about 10 dB for all ψ. This effect can also be approximately quantified analytically as follows. Let us neglect shadow fading [set $\chi = 0$ in (1)] and consider the noise limited regime ($\psi \ll 1$, i.e. primary co-channel interference negligible at the cell border). The area of cognitive operation is empty precisely in the limit when a single cognitive user at $(D/2, 0)$ cannot exist. In this limit a primary user at position $(r, 0)$ in the absence of cognitive users shall experience the same SINR as a primary user at position $(r', 0)$ when there is a single cognitive user at $(D/2, 0)$. Thus $\rho(r)/(\bar{\mu}/\psi) = \rho(r')/((P_{cogn}/P_{prim})\rho(D/2 - r') + \bar{\mu}/\psi)$. For two different compromised cell radii, say r'_1 and r'_2,

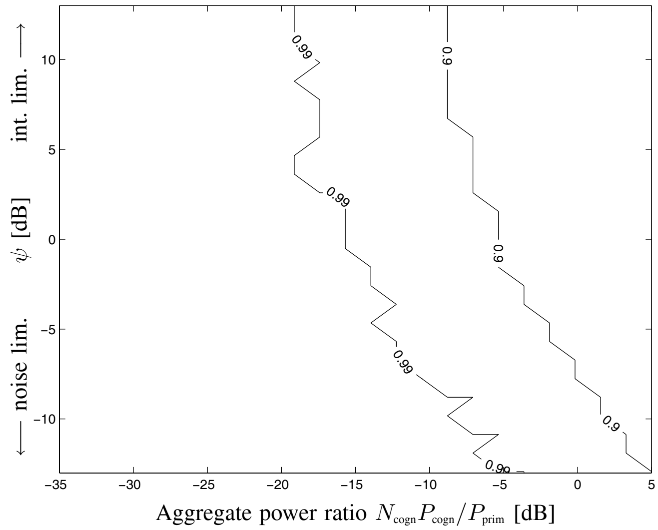


Fig. 4. The forbidden (upper right) region, as a function of the primary system operating point (ψ) and the aggregate power ratio ($N_{\text{cogn}}P_{\text{cogn}}/P_{\text{prim}}$) for n = 12-reuse, $\alpha = 4$, $\sigma = 6$ dB, $\epsilon = 10\%$ and the following different values of the compromised cell radius: $r'/r = 0.9$ and $r'/r = 0.99$.

and two corresponding cognitive powers $P_{\text{cogn},1}, P_{\text{cogn},2}$, we can solve for the ratio $P_{\text{cogn},1}/P_{\text{cogn},2}$:

$$\frac{P_{\text{cogn},1}}{P_{\text{cogn},2}} = \frac{1-\frac{\rho(r_1')}{\rho(r)}}{1-\frac{\rho(r_2')}{\rho(r)}}\frac{\rho(D/2-r_2')}{\rho(D/2-r_1')} \tag{5}$$

For example with $n = 12$, $\alpha = 4$, $r_1' = 0.9r$ and $r_2' = 0.99r$, the ratio $P_{\text{cogn},1}/P_{\text{cogn},2}$ is about 12 dB, consistent with Figure 4.

Space limitations prohibit us to show more extensive numerical results. However, we have made the following observations. Plotting results for smaller ϵ tends to shift the curves to the left. Changing σ does not have a major effect, except for when $\epsilon \ll 10\%$. Decreasing α tend to shift the level curves in Figure 3 to the right (i.e., lead to somewhat more "optimistic" results).

C. Requirements on the Radios of the Cognitive Users

A cognitive user will need to listen to the primary system and determine whether it is far enough away from a primary base station to begin transmitting. To analyze this, we assume that the cognitive user knows the signaling format used by the primary system, and that it knows what pilot signals are being used. It looks for these pilots to determine whether a primary base station is present within a certain distance. As discussed in [7], this scenario is more favorable than considering the signals from the primary base station to be random. We assume that the cognitive user can detect the primary pilots if it experiences a SINR which is no lower than the SINR seen by a primary user at the cell border minus an extra sensitivity margin, say Δ.

The value of Δ will depend on whether the pilots used by BS_0–BS_6 completely overlap one another in the signal space, or not. With some abuse of terminology we will say that the pilots can be either orthogonal or nonorthogonal, because in practice non-overlapping pilots are usually designed as orthogonal, e.g., based on different pseudo-noise sequences. If the pilots are orthogonal and the cognitive user has some knowledge about their structure, then she may suppress primary co-channel interference when trying to detect the pilot signal transmitted by a specific primary base station. (This can be done simply by projecting the received signal onto the orthogonal complement of the space spanned by all other pilot signals.) By contrast, for nonorthogonal (fullspread) pilots, all dimensions of the signal space are equally affected by interference during the detection. That is, when detecting pilots from the primary BS_0, the cognitive device cannot suppress interfering pilots from other primary base stations.

To gain some insight, we will first do a simple closed-form analysis that neglects shadow-fading. It is clear that the worst location for the cognitive user is at the distance $d = D/2$ from BS_0. We consider first the situation with nonorthogonal pilots. A cognitive user at the location $(D/2, 0)$ experiences $\mathsf{SINR}_{(D/2,0)} = \rho(D/2)/(\mu(D/2,0)+\mu(r,0)/\psi)$. [We use $\bar{\mu} \approx \mu(r,0)$ here. In interpreting the equations in this section, set $\chi = 0$ in (1).] We now define Δ_{nonortho} as the ratio between the SINR experienced by a primary user at the cell border (say at location $(r,0)$) and the SINR experienced by the cognitive device:

$$\Delta_{\text{nonortho}} \triangleq \frac{\mathsf{SINR}_{(r,0)}}{\mathsf{SINR}_{(D/2,0)}} = \frac{\frac{\rho(r)}{\mu(r,0)+\mu(r,0)/\psi}}{\frac{\rho(D/2)}{\mu(D/2,0)+\mu(r,0)/\psi}} \tag{6}$$

From (6) we obtain the following bounds (which are tight in the limit):

$$\Delta^{\psi\to\infty}_{\text{nonortho}} \triangleq \frac{\mu(D/2,0)}{\rho(D/2)}\frac{\rho(r)}{\mu(r,0)} \le \Delta_{\text{nonortho}} \le \frac{\rho(r)}{\rho(D/2)} \triangleq \Delta^{\psi\to 0} \tag{7}$$

Numerical values are shown in Table I, using $\chi = 0$ and $\alpha = 4$ in (1). Apparently the extra sensitivity requirements depend relatively little on whether the primary system is noise- or interference-limited. Consider next the case when the primary base stations use orthogonal pilots. The cognitive user can then suppress interfering pilots when listening to a particular base station, while the SNR is not affected.[4] We have the corresponding definition and bound

$$\Delta_{\text{ortho}} \triangleq \frac{\mathsf{SINR}_{(r,0)}}{\mathsf{SNR}_{(D/2,0)}} = \frac{\frac{\rho(r)}{\mu(r,0)+\mu(r,0)/\psi}}{\frac{\rho(D/2)}{\mu(r,0)/\psi}} = \frac{\rho(r)}{\rho(D/2)}\frac{1}{1+\psi} \le \frac{\rho(r)}{\rho(D/2)} = \Delta^{\psi\to 0} \tag{8}$$

As expected, we see that orthogonality of the pilots matters significantly only for interference limited systems, and that it can only make the outlook better. However, the reduction in

[4]To see this, let the received signal be $\boldsymbol{y} = \boldsymbol{c}+\boldsymbol{i}+\boldsymbol{n}$, where $\boldsymbol{c}$ is a signal vector associated with the signal of interest to detect, $\boldsymbol{i}$ is interference and $\boldsymbol{n}$ is noise. Assume that the signal and noise are zero mean and uniformly spread over the signal space: $E[\boldsymbol{c}\boldsymbol{c}^H] = C\cdot\boldsymbol{I}$, $E[\boldsymbol{n}\boldsymbol{n}^H] = N\cdot\boldsymbol{I}$, but that $\boldsymbol{i}$ lies in a known subspace of smaller dimension. Let $\Pi_{\boldsymbol{i}}^{\perp}$ be a projector onto the orthogonal complement of the space of possible $\boldsymbol{i}$ vectors, so that $\Pi_{\boldsymbol{i}}^{\perp}\boldsymbol{i} = 0$. Then the SNR in $\boldsymbol{y}$ is the same as that in $\Pi_{\boldsymbol{i}}^{\perp}\boldsymbol{y}$: $E[\|\Pi_{\boldsymbol{i}}^{\perp}\boldsymbol{c}\|^2]/E[\|\Pi_{\boldsymbol{i}}^{\perp}\boldsymbol{n}\|^2] = E[\|\boldsymbol{c}\|^2]/E[\|\boldsymbol{i}\|^2]$. The integration time may be longer when interference suppression is performed, though.

TABLE I
BOUNDS ON HOW MUCH MORE SENSITIVE THAN THE PRIMARY RECEIVERS THAT A THE COGNITIVE DEVICE MUST BE, QUANTIFIED VIA THE VARIABLE Δ [SEE (7)–(8)]. NO SHADOW FADING, PATH-LOSS EXPONENT $\alpha = 4$.

Primary cell plan	Lower bound, $\Delta^{\psi\to 0}$ (noise limited)	Upper bound, $\Delta^{\psi\to\infty}_{\text{nonortho}}$ (interference limited)
$n = 3$-reuse	7 dB	10 dB
$n = 7$-reuse	14 dB	19 dB
$n = 12$-reuse	19 dB	24 dB
$n = 21$-reuse	24 dB	29 dB

Δ due to this orthogonality, $1/(1+\psi)$, is marginal for most realistic values of ψ.

The analysis indicates that cognitive users must be orders of magnitude more sensitive than the users in the primary system. Shadow fading is going the make this even worse. To understand the magnitude of the difficulties involved, we consider the following example. Suppose the cognitive user can determine that it is in the permissible area (associated with BS_0) whenever its SINR, or SNR, is below a specific threshold, say η (i.e., it can perform an error-free estimation of SINR or SNR and compare this to a predefined threshold). As before we take this threshold η to be a factor Δ below the average SINR for a primary user at the primary cell border:

$$\eta \triangleq \Delta \cdot E\left[\frac{\rho(\sqrt{x^2+y^2})}{\mu(x,y)+\bar{\mu}/\psi}\right] \quad (9)$$

In (9), the expectation is taken over coordinates (x, y) at the primary cell border, i.e., randomly distributed subject to $\sqrt{x^2+y^2} = r$. We are now interested in the *probability of misjudgment*, i.e., the probability that the cognitive user is actually *outside* the permissible area when its SINR or SNR is below the threshold. This is given by (for orthogonal and nonorthogonal pilots, respectively)

$$\text{Prob}^{\text{nonortho}}_{\text{miss}} \triangleq \text{Prob}\left(\sqrt{x^2+y^2} < d \,\middle|\, \frac{\rho(\sqrt{x^2+y^2})}{\mu(x,y)+\bar{\mu}/\psi} < \eta\right)$$
$$\text{Prob}^{\text{ortho}}_{\text{miss}} \triangleq \text{Prob}\left(\sqrt{x^2+y^2} < d \,\middle|\, \frac{\rho(\sqrt{x^2+y^2})}{\bar{\mu}/\psi} < \eta\right) \quad (10)$$

where (x, y) is uniformly random in the circle $\sqrt{x^2+y^2} \leq D/2$. Figure 5 shows $\text{Prob}_{\text{miss}}$ as a function of Δ for some different operating points, and for $A = 1 - 2d/D = 25\%$ size of the cognitive area and $n = 7$-reuse. (When $\sigma = 0$, the only source of randomness is the location of the cognitive user. When $\sigma = 0$, $A \to 0$, and $\psi \to 0, \infty$ respectively, the asymptotes of the curves will be those predicted in Table I.) It is clear that when choosing Δ, a very large margin must be added to the values in Table I to account for fading effects, even if a relatively large number of position misjudgments can be tolerated. Pilot orthogonality certainly helps in the interference limited case, but the fundamental difficulty is still the shadow fading.

In the above example we have assumed that the cognitive devices will be able to perfectly detect whether SINR $< \eta$, or SNR $< \eta$, respectively. This is not possible in practice, and hence our predictions are probably somewhat overoptimistic.

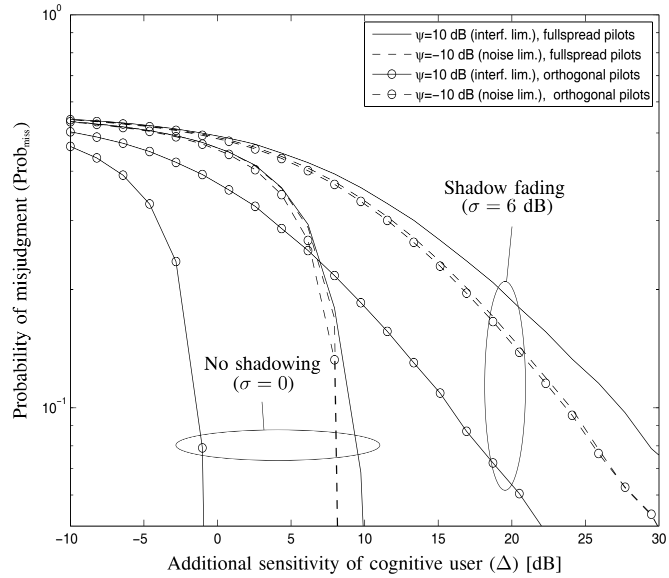


Fig. 5. The probability that the cognitive user is located outside the permissible area, when it determines that the primary SINR, respectively SNR, is below the threshold η [defined as a factor Δ under the average SINR for a primary user at the primary cell border], for $n = 7$-frequency reuse, path loss exponent $\alpha = 4$, and a relative size of the forbidden region $A = 25\%$.

In practice the cognitive device might listen to multiple base stations, i.e., not only BS_0, and this would on the other hand improve the situation somewhat. Additionally, several cognitive radios could cooperate on the detection.

IV. CONCLUSIONS

We have analyzed the limits for deployment of cognitive devices in a frequency-planned network. The models we used have been fairly simple but they do capture the most relevant physical phenomena (path loss, and shadow fading). The results are quite discouraging. If cognitive devices are to be introduced then this will require that (i) the cognitive devices that are tolerated to transmit simultaneously in a given licensed band are few in numbers, since the aggregate power $N_{\text{cogn}}P_{\text{cogn}}$ scales proportionally with the number of devices; (ii) such cognitive devices transmit with extremely low power, typically 30 dB below the primary transmitter, at least; and (iii) the cognitive nodes have very sensitive radio receivers, i.e. radios that can detect signals at very low SINR or SNR, more precisely of the order 20–30 dB more sensitive than the primary radios. Appropriate design of pilots can alleviate this problem somewhat, but it does not change the situation fundamentally in realistic networks. Hence in practice it will probably be necessary that cognitive users cooperate on spectrum measurements, perhaps by forming a sensor network, or that the primary system is made somewhat tolerant to occasional interference from the secondary users.

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