Capacity of DS/CDMA Communication Systems with Optimum Spectral Overlap

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***Abstract*—We consider frequency overlapped DS/CDMA (FO/CDMA) communication systems where a number of DS/CDMA systems share frequency bands with adjacent systems. We analyze the multiple-access interference from adjacent systems with arbitrary amount of frequency overlap and compare the capacity of FO/CDMA with that of single wide-band CDMA (WCDMA). The optimum amount of overlap increases with the number of overlapped systems. However the maximum capacity is obtained when the optimum number of systems are overlapped by half null-to-null bandwidth. It is shown that the FO/CDMA yields higher capacity in the case of relaxed bit error rate requirement and low decay rate of multipath intensity profile. Otherwise, WCDMA is superior.**

***Index Terms*— FO/CDMA, overlap, wide-band CDMA (WCDMA).**

I. Introduction

Spectrally overlapping system was proposed by Schilling and Pickholtz [1]. They showed the potential of the spectrally overlapping scheme and analyzed the capacity in additive white Gaussian noise (AWGN) channels, considering the interference from adjacent bands negligible. When DS/CDMA systems are overlapped, the interference from adjacent bands for a given carrier spacing was numerically evaluated by spectral density ratio in [2]. The signal-to-noise ratio including pulse shaping filter is obtained by numerical integration in [3].

In this letter, we consider frequency overlapped DS/CDMA (FO/CDMA) communication systems where a number of DS/CDMA systems share their frequency bands with adjacent systems. The spectrum of FO/CDMA system is shown in Fig. 1 where DS/CDMA systems are partially overlapped. In [4], the optimum overlap between two DS/CDMA systems was considered. We find the optimum amount of overlap for general FO/CDMA and compare the capacity of FO/CDMA with that of single wide-band CDMA (WCDMA) in frequency selective Rayleigh fading channel.

II. System Description

We consider S DS/CDMA systems are overlapped in frequency bands. K active users are assumed to be uniformly distributed into S DS/CDMA systems. The bandwidth of each DS/CDMA system is identical, and each user is assumed to use a uniquely assigned random signature sequence.

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We define the amount of overlap α as

$$\alpha \triangleq \frac{B_o}{B_n} \tag{1}$$

where B_o is the overlapped bandwidth between adjacent systems and B_n is the null-to-null bandwidth of each system. For example, $\alpha = 0$ corresponds to the nonoverlapped FD/CDMA while $\alpha = 1$ corresponds to WCDMA.

We assume the overall bandwidth is fixed. Then, the configuration of FO/CDMA system is clearly determined by two parameters, S and α. The processing gain of FO/CDMA system N is given by

$$N = \frac{N^*}{\alpha + (1-\alpha)S} \tag{2}$$

where superscript * refers to WCDMA. Note that setting $S = 1$ or $\alpha = 1$ in (2) gives $N = N^*$.

The fading channel between kth transmitter and receiver pair is modeled as a multipath tapped delay line [5, p. 730], namely

$$h_k(\tau) = \sum_{l=1}^{L} \beta_{k,\,l}\delta(\tau - lT_c)\exp(j\varphi_l) \tag{3}$$

where $\beta_{k,\,l}$ and $\varphi_{k,\,l}$ are the tap gain and the phase associated with a path of delay lT_c, respectively. We assume that a RAKE receiver has L taps and perfectly knows the fading amplitudes and phases of all resolvable multipaths. We assume that the tap gains are independent Rayleigh random variables, and the tap phases are independent and uniformly distributed over $[0,\,2\pi]$. The number of resolvable multipath components L is given by

$$L = \lfloor T_m/T_c \rfloor + 1 \tag{4}$$

where T_m is multipath delay spread.

When the overall bandwidth is fixed, the number of resolvable paths, chip duration, and processing gain are related by

$$\frac{L^*}{L} = \frac{T_c}{T_c^*} = \frac{N^*}{N} = \alpha + (1-\alpha)S. \tag{5}$$

The signal-to-noise ratios of the different resolved paths are assumed to decrease exponentially with increasing path delay, that is

$$E[\beta_{k,\,l}^2] \triangleq \Omega_l = \Omega_0 \exp\left[-\frac{\delta}{L^*}\frac{T_c}{T_c^*}l\right], \qquad 1 \le l \le L \tag{6}$$

where Ω_0 is normalized to make $\sum_{l=1}^{L} E[\beta_{k,\,l}^2] = 1$ and the parameter δ reflects the rate at which decay occurs. Large L^* means small path-strength decay rate. T_c/T_c^* reflects the relation of the path-strength decay rate between WCDMA and FO/CDMA systems.

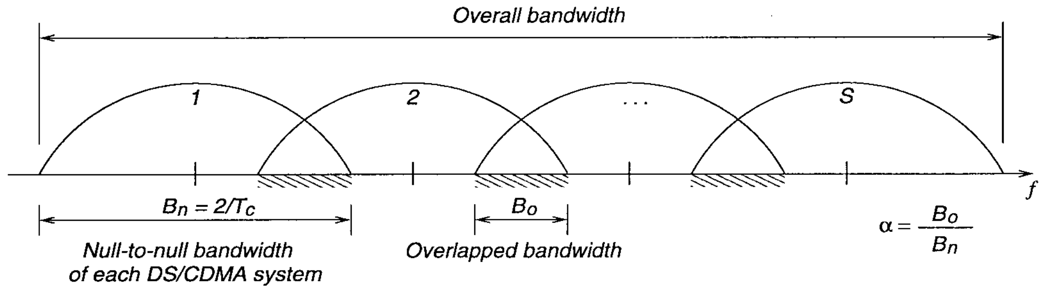

Fig. 1. Spectrum of FO/CDMA systems.

III. Performance Analysis

The input signal to RAKE receiver of the desired user 1 is given by

$$r(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sqrt{2P}\beta_{k,\,l}b_k(t-\tau_k-lT_c) \cdot a_k(t-\tau_k-lT_c)\cos(2\pi f_k t+\varphi_{k,\,l})+n(t) \tag{7}$$

where P is the average transmitter power and $b_k(t)$ and $a_k(t)$ are data and random signature signal, respectively. The delay τ_k is uniformly distributed over $[0,\, T_s]$, where T_s is the information bit duration ($= NT_c$); f_k is the carrier frequency of user k; and $n(t)$ is AWGN with mean zero and double-sided power spectral density $N_0/2$. We assume the desired user 1 sent "1" as a data bit, then the RAKE receiver output of the desired user can be written as

$$\begin{aligned} g(T_s) &= \int_0^{T_s}\sum_{l=1}^{L} r(t+(l-1)T_c)\beta_{1,l}a_1(t)\cos(2\pi f_1 t+\varphi_l)dt \\ &= \sqrt{P/2}T_s\left[\sum_{l=1}^{L}\beta_{1,l}^2+\sum_{k=2}^{K}\kappa_{k,1}(\tau_k,\,\Delta f_{k,1})\right]+\overline{N} \end{aligned} \tag{8}$$

where $\kappa_{k,\,1}(\tau_k,\,\Delta f_{k,\,1})$ is the multiple access interference term from the kth user signal with carrier spacing $\Delta f_{k,\,1}$ ($=|f_k-f_1|$). We first obtain the bit error rate (BER) conditioned on $A \triangleq \sum_{l=1}^{L}\beta_{1,\,l}^2$ and then average it to calculate the average BER. $\overline{N}$ is a Gaussian random variable with zero mean and variance of $(N_0T_s/4)\sum_{l=1}^{L}\beta_{1,\,l}^2$.

Conditioned on the random variable A, the mean of $\kappa_{k,\,1}$ can be easily shown to be zero, and the variance of $\kappa_{k,\,1}$ is given by [4]

$$\mathrm{var}[\kappa_{k,\,1}(\tau_k,\,\Delta f_{k,\,1})|A] = \frac{\Lambda(\Delta f_{k,\,1}T_c)}{3N}\sum_{l=1}^{L}\beta_{1,\,l}^2 \tag{9}$$

where

$$\Lambda(x) \triangleq \frac{6}{(2\pi x)^2}\left[1-\frac{\sin(2\pi x)}{2\pi x}\right]. \tag{10}$$

$\Lambda(x)$ corresponds to the desired to adjacent multiple-access interference ratio in AWGN channel, which is shown in Fig. 2. By the L'Hospital's rule, it can be shown that

$$\lim_{\Delta f_{k,1}\to 0}\Lambda(\Delta f_{k,\,1}T_c) = 1 \tag{11}$$

which shows that the analysis is consistent with [6, eq. (16)]. When $\Delta f_{k,\,1}T_c = 1$, the interference power from an adjacent carrier user amounts to about 15% of that from an identical carrier user, and 4% when $\Delta f_{k,\,1}T_c = 2$.

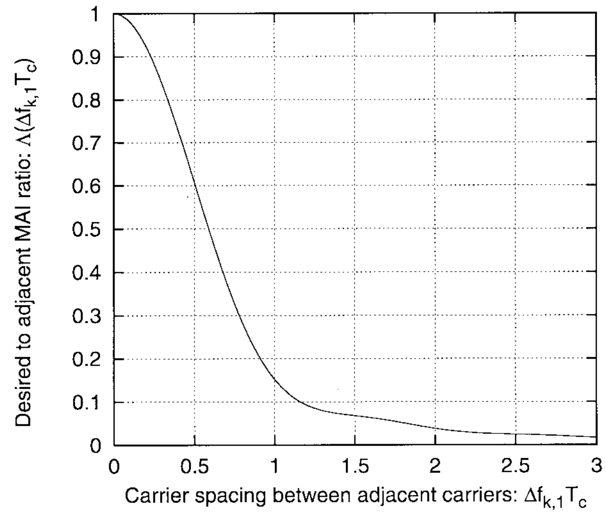

Fig. 2. Desired to adjacent multiple-access interference ratio versus carrier spacing between two adjacent carriers in AWGN channel.

We assume each user experiences an independent fading. Then $\kappa_{k,\,1}$'s are statistically independent and the variance of the multiple access interference is the sum of variance of each $\kappa_{k,\,1}$. From (8) and (9), the conditional BER ρ for a given A is given as

$$\rho(\gamma_s) = Q\left(\sqrt{2\gamma_s}\right) \tag{12}$$

where

$$\gamma_s = \left[\frac{N_0}{\overline{E}_b}+\sum_{k=2}^{K}\frac{2\Lambda(\Delta f_{k,\,1}T_c)}{3N}\right]^{-1}\sum_{l=1}^{L}\beta_{1,\,l}^2 \triangleq \sum_{l=1}^{L}\gamma_l.$$

We can get the probability density function of γ_s as in [5]

$$p(\gamma_s) = \sum_{l=1}^{L}\frac{\pi_l}{\overline{\gamma}_l}e^{-(\gamma_s/\overline{\gamma}_l)}, \qquad \gamma \ge 0 \tag{13}$$

where

$$\pi_l = \prod_{\substack{i=1\\ i\neq l}}\frac{\overline{\gamma}_l}{\overline{\gamma}_l-\overline{\gamma}_i}$$

$$\overline{\gamma}_l = \Omega_l\left[\frac{N_0}{\overline{E}_b}+\sum_{k=2}^{K}\frac{2\Lambda(\Delta f_{k,1}T_c)}{3N}\right]^{-1}.$$

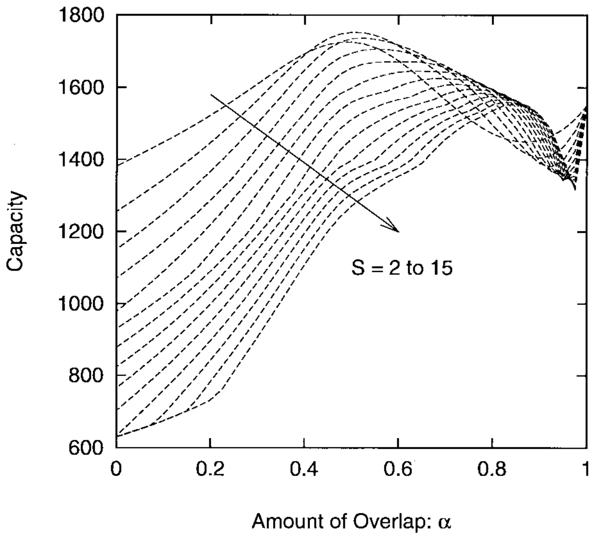

Fig. 3. Capacity versus $\overline{E}_b/N_0 = 10$ dB; $L^* = 12$, $\delta = 2$, $P_{b_{\rm req}} = 10^{-1}$, $N^* = 1000$.

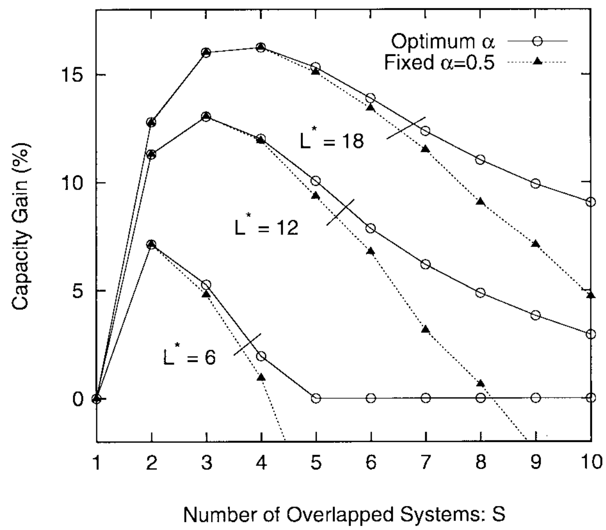

Fig. 4. Capacity gain of FO/CDMA over WCDMA versus S: $\overline{E}_b/N_0 = 10$ dB; $P_{b_{\rm req}} = 10^{-1}$, $\delta = 2$, $N^* = 1000$.

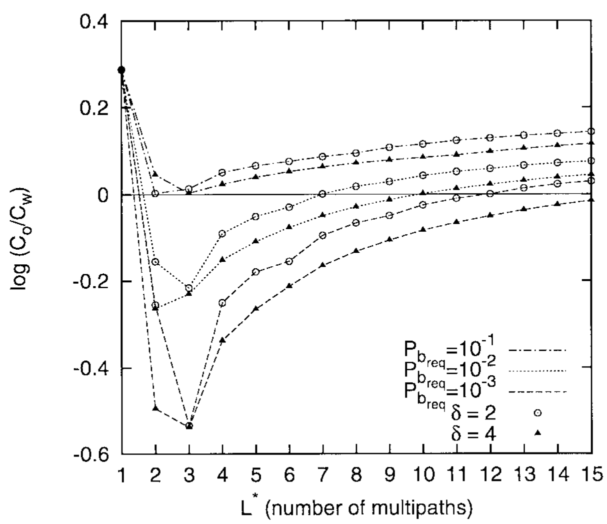

Fig. 5. Capacity versus L^*: C_w capacity of WCDMA) C_o (capacity of FO/CDMA with optimum S for $S \geq 2$), $\overline{E}_b/N_0 = 25$ dB, $\alpha = 0.5$, $N^* = 1000$

By averaging (12) over (13), the average BER, P_b, is obtained as

$$P_b = \frac{1}{2}\sum_{l=1}^{L}\pi_l\left[1-\sqrt{\frac{\overline{\gamma}_l}{1+\overline{\gamma}_l}}\right]. \tag{14}$$

IV. Numerical Results and Discussion

The performance is measured by capacity, which is defined as the maximum number of admissible users satisfying a BER requirement. Fig. 3 indicates that the optimum α that maximizes the capacity increases with the number of overlapped systems S. However, the maximum capacity is attained when α is near 0.5. Also, the capacity of nonoverlapped system ($\alpha = 0$) considerably decreases as S increases. In general, adjacent multiple access interference increases as the amount of spectral overlap (α) increases, but the RAKE diversity gain (number of taps) also increases because of the increased bandwidth in each system. Fig. 3 indicates that the RAKE diversity gain more than compensates the capacity loss due to the adjacent multiple access interference.

Fig. 4 shows the capacity gain of FO/CDMA over WCDMA versus S for several values of L^*. The capacity curves have a peak and then decreases as S increases. This is due to the reduced RAKE diversity gain for large S, caused by the bandwidth reduction in each system. We also find that the optimum S and capacity gain increase as L^* increases. From Figs. 3 and 4, we can induce that the optimum α increases with increasing S, but the maximum peak capacity is obtained when the optimum number of systems are overlapped by half null-to-null bandwidth.

From Fig. 5, it is seen that FO/CDMA is preferable in the case of relaxed BER requirement and low δ. Otherwise, WCDMA yields higher capacity. Note FO/CDMA yields about 33% higher capacity than WCDMA for frequency nonselective fading channel ($L^* = 1$).

Note the number of taps required in the RAKE receiver of FO/CDMA decreases as S increases. Thereby the receiver complexity also decreases, which depends on T_c and the number of RAKE receiver taps.

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