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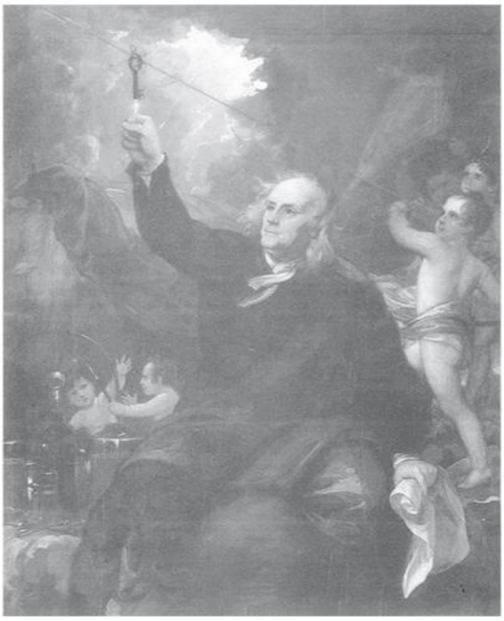

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Exponential synchronization for complex dynamical networks with sampled-data

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Abstract

This paper is concerned with the problem of exponential synchronization for a kind of complex dynamical networks (CDNs) with time-varying coupling delay and sampled-data. The sampling period considered here is assumed to be time-varying but bounded. A newly exponential synchronization condition is provided by using the Lyapunov method. Based on the condition, a set of sampled-data synchronization controllers is designed in terms of the solution to linear matrix inequalities (LMIs) that can be solved effectively by using available softwares. The derived results are theoretically and numerically proved to be less conservative than the existing results. Two numerical examples are introduced to show the effectiveness and improvement of the given results.

1. Introduction

In the past decades, there has been a highly focused interest in complex dynamical networks (CDNs), which have extensive applications in both science and engineering such as Internet, World Wide Web, food webs, and electric power grids [1]. It is well known that there are a lot of interesting and important collective phenomena in CDNs that can be

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described by coupled ordinary differential equations, such as self-organization, synchronization, and spatiotemporal chaos [2,3]. Among these phenomena, the synchronization phenomena have been intensively investigated in various different fields [4–12]. For example, in [4] the global synchronization of complex dynamical networks with network failures has been studied based on the framework of switching system, and the upper bounds of both the unavailability rate of the network and the frequency of network failures have been obtained to ensure the global synchronization of CDNs with network failures. In [6], a detailed analysis has been presented for the synchronization of CDNs with impulsive coupling based on the concept of average impulsive interval, and an unified synchronization criterion has been derived for directed impulsive dynamical networks, which takes into account two types of impulses simultaneously.

On the other hand, with the rapid advances in digital measurement and intelligent instrument, the analog signal processing methods are often replaced by digital signal processing methods to provide better reliability, accuracy and stable performance. Thus, sampled-data systems have attracted great attention, and many important and essential results have been reported in the literature over the past decades [13–17]. Very recently, the sampled-data synchronization control problem has been investigated for a class of general complex networks with time-varying coupling delays in [18], where a sufficient condition has been derived to ensure the exponential stability of the closed-loop error system based on the well known Jensen inequality, and the desired sampled-data feedback controllers have been designed. However, there is room for further investigation because the information of the involved delays has not been fully used. To be specific, the delay terms $\tau(t)$ and $\tau-\tau(t)$ with $0\le\tau(t)\le\tau$ are enlarged as τ, and the delay terms $d(t)$ and $p-d(t)$ with $0\le d(t)\le p$ are enlarged as p, that is, $\tau=\tau(t)+\tau-\tau(t)$ and $p=p(t)+p-p(t)$ are enlarged as 2τ and $2p$, respectively. It is clear that the aforementioned treatment may lead to a conservative result.

In this paper, the problem of sampled-data synchronization is studied for CDNs with time-varying coupling delay. The sampling period considered here is assumed to be time-varying but bounded. In the framework of the input delay approach [14], an exponential synchronization condition is proposed based on the LMI approach. The design method of the desired sampled-data synchronization controllers is provided. The derived results are theoretically and numerically proved to be less conservative than the existing results.

Notation: The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m\times n$ real matrices, respectively. The notation $X>Y$ ($X\ge Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive definite (positive semidefinite). I and 0 represent the identity matrix and a zero matrix, respectively. The superscript ‘‘T’’ represents the transpose, and diag$\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. For an arbitrary matrix B and two symmetric matrices A and C,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

denotes a symmetric matrix, where ‘‘$*$’’ denotes the term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Preliminaries

Consider the following CDN consisting of N identical coupled nodes with each node being an n-dimensional dynamical system:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} G_{ij} D x_j(t) + \sum_{j=1}^{N} G_{ij} A x_j(t-\tau(t)) + u_i(t), \quad i = 1,2,\ldots,N \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are, respectively, the state variable and the control input of the node i, $D=(d_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ and $A=(a_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ are constant inner-coupling matrices of the nodes, and $G=(G_{ij})_{N\times N}$ is the outer-coupling matrix of the network. If there is a connection between node i and node j $(i\neq j)$, then $G_{ij}=1$, otherwise, $G_{ij}=0$ $(i\neq j)$. The diagonal elements of matrix G are defined by

$$G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij}, \quad i=1,2,\ldots,N \tag{2}$$

The scalar $\tau(t)$ denotes the time-varying delay satisfying

$$0 \le \tau(t) \le \mu, \quad \dot{\tau}(t) \le v \tag{3}$$

where $\tau > 0$ and $v > 0$ are known constants. $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector-valued function and satisfies the following sector-bounded condition [19]:

$$[f(x)-f(y)-U(x-y)]^{\mathrm{T}}[f(x)-f(y)-V(x-y)] \le 0, \quad \forall x,y \in \mathbb{R}^n \tag{4}$$

where U and V are constant matrices of appropriate dimensions.

Let $e_i(t) = x_i(t) - s(t)$ be the synchronization error, where $s(t) \in \mathbb{R}^n$ is the state trajectory of the unforced isolate node $\dot{s}(t) = f(s(t))$. Then, the error dynamics of CND (1) can be obtained as follows:

$$\dot{e}_i(t) = g(e_i(t)) + \sum_{j=1}^{N} G_{ij} D e_j(t) + \sum_{j=1}^{N} G_{ij} A e_j(t-\tau(t)) + u_i(t), \quad i=1,2,\ldots,N \tag{5}$$

where $g(e_i(t)) = f(x_i(t)) - f(s(t))$.

The control signal is assumed to be generated by using a zero-order-hold (ZOH) function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$. Therefore, the state feedback controller takes the following form:

$$u_i(t) = K_i e_i(t_k), \quad t_k \le t < t_{k+1}, \ i=1,2,\ldots,N \tag{6}$$

where K_i is sampled-data feedback controller gain matrix to be determined, $e_i(t_k)$ is discrete measurement of $e_i(t)$ at the sampling instant t_k, $\lim_{k\to+\infty} t_k = +\infty$. It is assumed that $t_{k+1} - t_k = h_k \le p$ for any integer $k \ge 0$, where $p > 0$ represents the largest sampling interval.

By substituting Eq. (6) into Eq. (1), we obtain

$$\begin{aligned}\dot{e}_i(t) = {} & g(e_i(t)) + \sum_{j=1}^{N} G_{ij} D e_j(t) + \sum_{j=1}^{N} G_{ij} A e_j(t-\tau(t)) \\ & + K_i e_i(t-d(t)), \quad i=1,2,\ldots,N \end{aligned} \tag{7}$$

where $d(t) = t - t_k$. It can be seen that

$$0 \le d(t) \le p \tag{8}$$

It is clear that Eq. (7) can be rewritten as

$$\dot{e}(t)=\overline{g}(e(t))+(G\otimes D)e(t)+(G\otimes A)e(t-\tau(t))+Ke(t-d(t)) \tag{9}$$

where $K=\mathrm{diag}\{K_1,K_2,\ldots,K_N\}$, and

$$e(t)=\begin{bmatrix} e_1(t)\\ e_2(t)\\ \vdots\\ e_N(t)\end{bmatrix},\quad \overline{g}(e(t))=\begin{bmatrix} g(e_1(t))\\ g(e_2(t))\\ \vdots\\ g(e_N(t))\end{bmatrix}$$

Before proceeding further, the following lemma and definition are given.

Lemma 1 (*Shu and Lam [20]*). *For any matrix $W>0$, scalars γ_1 and γ_2 satisfying $\gamma_2>\gamma_1$, a vector function $\omega:[\gamma_1,\gamma_2]\to\mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$(\gamma_2-\gamma_1)\int_{\gamma_1}^{\gamma_2}\omega(\alpha)^{\mathrm{T}}W\omega(\alpha)\,\mathrm{d}\alpha\geq\left[\int_{\gamma_1}^{\gamma_2}\omega(\alpha)\,\mathrm{d}\alpha\right]^{\mathrm{T}}W\left[\int_{\gamma_1}^{\gamma_2}\omega(\alpha)\,\mathrm{d}\alpha\right] \tag{10}$$

Definition 1. The CDN (1) is said to be exponentially synchronized if the error system (9) is exponentially stable, i.e., there exist two constants $\alpha>0$ and $\beta>0$ such that

$$\|e(t)\|^2\leq\alpha e^{-\beta t}\sup_{-\max\{\mu,p\}\leq\theta\leq 0}\|e(\theta)\|^2 \tag{11}$$

The aim of this paper is to design a set of sampled-data controllers (6) to ensure the exponential synchronization of the complex network (1), that is, we are interested in designing the gain matrices K such that the error system (9) is exponentially stable.

3. Main results

In this section, we first give a sufficient condition, which ensures error system (9) to be exponentially stable. Then, we propose a design method of the sampled-data controllers for CND (1). Before presenting the main results, for the sake of presentation simplicity, we denote:

$$\overline{U}=\frac{(I_N\otimes U)^{\mathrm{T}}(I_N\otimes V)}{2}+\frac{(I_N\otimes V)^{\mathrm{T}}(I_N\otimes U)}{2}$$

$$\overline{V}=-\frac{(I_N\otimes U)^{\mathrm{T}}+(I_N\otimes V)^{\mathrm{T}}}{2}$$

Theorem 1. *The system (9) is exponentially stable if there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $Z_1>0$, $Z_2>0$, $Z_3>0$, and a scalar $\lambda>0$ such that*

$$Z_2+(1-\nu)Z_3>0 \tag{12}$$

$$\begin{bmatrix}\Xi_1-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 & \mathcal{Y}^{\mathrm{T}}\\ * & -Z\end{bmatrix}<0 \tag{13}$$

$$\begin{bmatrix} \Xi_1 - p^{-1}\Delta_1^{\mathrm{T}} Z_1 \Delta_1 - \mu^{-1}\Theta_2^{\mathrm{T}} Z_2 \Theta_2 & \mathcal{Y}^{\mathrm{T}} \\ * & -Z \end{bmatrix} < 0 \tag{14}$$

$$\begin{bmatrix} \Xi_1 - p^{-1}\Delta_2^{\mathrm{T}} Z_1 \Delta_2 - \Theta_1^{\mathrm{T}} \mathcal{U} \Theta_1 & \mathcal{Y}^{\mathrm{T}} \\ * & -Z \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} \Xi_1 - p^{-1}\Delta_2^{\mathrm{T}} Z_1 \Delta_2 - \mu^{-1}\Theta_2^{\mathrm{T}} Z_2 \Theta_2 & \mathcal{Y}^{\mathrm{T}} \\ * & -Z \end{bmatrix} < 0 \tag{16}$$

where

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & PK + p^{-1}Z_1 & 0 & \Xi_{14} & 0 & P - \lambda\overline{V} \\ * & -2p^{-1}Z_1 & p^{-1}Z_1 & 0 & 0 & 0 \\ * & * & -Q_1 - p^{-1}Z_1 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \mu^{-1}Z_2 & 0 \\ * & * & * & * & -Q_2 - \mu^{-1}Z_2 & 0 \\ * & * & * & * & * & -\lambda I \end{bmatrix}$$

$$\Xi_{11} = P(G \otimes D) + (G \otimes D)^{\mathrm{T}} P + Q_1 + Q_2 + Q_3 - \lambda\overline{U} - p^{-1}Z_1 - \mathcal{U}$$
$$\Xi_{14} = P(G \otimes A) + \mathcal{U}$$
$$\Xi_{44} = -(1-\nu)Q_3 - \mathcal{U} - \mu^{-1}Z_2$$
$$Z = pZ_1 + \mu Z_2 + \mu Z_3$$
$$\mathcal{U} = \mu^{-1}(Z_2 + (1-\nu)Z_3)$$
$$\mathcal{Y} = [Z(G \otimes D) \;\; ZK \;\; 0 \;\; Z(G \otimes A) \;\; 0 \;\; Z]$$
$$\Delta_1 = [I \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0]$$
$$\Delta_2 = [0 \;\; I \;\; -I \;\; 0 \;\; 0 \;\; 0]$$
$$\Theta_1 = [I \;\; 0 \;\; 0 \;\; -I \;\; 0 \;\; 0]$$
$$\Theta_2 = [0 \;\; 0 \;\; 0 \;\; I \;\; -I \;\; 0]$$

Proof. Consider the following Lyapunov functional for the system (9):

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{17}$$

where

$$V_1(t) = e(t)^{\mathrm{T}} Pe(t) + \int_{t-p}^{t} e(s)^{\mathrm{T}} Q_1 e(s) ds + \int_{t-\mu}^{t} e(s)^{\mathrm{T}} Q_2 e(s) + \int_{t-\tau(t)}^{t} e(s)^{\mathrm{T}} Q_3 e(s) \, ds$$

$$V_2(t) = \int_{-p}^{0} \int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_1 \dot{e}(s) \, ds \, d\theta$$

$$V_3(t) = \int_{-\mu}^{0} \int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_2 \dot{e}(s) \, ds \, d\theta + \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_3 \dot{e}(s) \, ds \, d\theta$$

Taking the derivative of Eq. (17) along the solution of system (9) yields

$$\dot{V}_1(t) \le 2e(t)^{\mathrm{T}} P\dot{e}(t) + e(t)^{\mathrm{T}}(Q_1 + Q_2 + Q_3)e(t) - e(t-p)^{\mathrm{T}} Q_1 e(t-p)$$

$$-e(t-\mu)^{\mathrm{T}} Q_2 e(t-\mu) - (1-v) e(t-\tau(t))^{\mathrm{T}} Q_3 e(t-\tau(t)) \tag{18}$$

$$\dot{V}_2(t) = p\dot{e}(t)^{\mathrm{T}} Z_1 \dot{e}(t) - \int_{t-p}^{t} \dot{e}(s)^{\mathrm{T}} Z_1 \dot{e}(s)\, \mathrm{d}s \tag{19}$$

$$\begin{aligned}\dot{V}_3(t) \le\ & \mu\dot{e}(t)^{\mathrm{T}} Z_2 \dot{e}(t) + \mu\dot{e}(t)^{\mathrm{T}} Z_3 \dot{e}(t) \\ & - \int_{t-\mu}^{t} \dot{e}(s)^{\mathrm{T}} Z_2 \dot{e}(s)\, \mathrm{d}s - (1-v) \int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}} Z_3 \dot{e}(s)\, \mathrm{d}s\end{aligned} \tag{20}$$

On the other hand, denoting $\varphi(t) = (p-d(t))/p$ [21], then we have that $0 \le \varphi(t) \le 1$ and $d(t) = (1-\varphi(t))p$. According to Lemma 1, we get that

$$\begin{aligned}
-\int_{t-p}^{t} \dot{e}(s)^{\mathrm{T}} Z_1 \dot{e}(s)\, \mathrm{d}s &= -p \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s - p \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s \\
&= -d(t) \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s - (p-d(t)) \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s \\
&\quad -(p-d(t)) \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s - d(t) \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s \\
&\le -d(t) \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s - \varphi(t) d(t) \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s \\
&\quad -(p-d(t)) \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s - (1-\varphi(t))(p-d(t)) \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}} p^{-1} Z_1 \dot{e}(s)\, \mathrm{d}s \\
&\le -\int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}}\, \mathrm{d}s p^{-1} Z_1 \int_{t-d(t)}^{t} \dot{e}(s)\, \mathrm{d}s - \varphi(t) \int_{t-d(t)}^{t} \dot{e}(s)^{\mathrm{T}}\, \mathrm{d}s p^{-1} Z_1 \int_{t-d(t)}^{t} \dot{e}(s)\, \mathrm{d}s \\
&\quad - \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}}\, \mathrm{d}s p^{-1} Z_1 \int_{t-p}^{t-d(t)} \dot{e}(s)\, \mathrm{d}s\, \mathrm{d}s \\
&\quad -(1-\varphi(t)) \int_{t-p}^{t-d(t)} \dot{e}(s)^{\mathrm{T}}\, \mathrm{d}s p^{-1} Z_1 \int_{t-p}^{t-d(t)} \dot{e}(s)\, \mathrm{d}s\, \mathrm{d}s \\
&= \begin{bmatrix} e(t) \\ e(t-d(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -p^{-1} Z_1 & p^{-1} Z_1 \\ * & -p^{-1} Z_1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-d(t)) \end{bmatrix} - \varphi(t) \delta(t)^{\mathrm{T}} \Delta_1^{\mathrm{T}} p^{-1} Z_1 \Delta_1 \delta(t) \\
&\quad + \begin{bmatrix} e(t-d(t)) \\ e(t-p) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -p^{-1} Z_1 & p^{-1} Z_1 \\ * & -p^{-1} Z_1 \end{bmatrix} \begin{bmatrix} e(t-d(t)) \\ e(t-p) \end{bmatrix} - (1-\varphi(t)) \delta(t)^{\mathrm{T}} \Delta_2^{\mathrm{T}} p^{-1} Z_1 \Delta_2 \delta(t)
\end{aligned} \tag{21}$$

where

$$\delta(t) = [e(t)^{\mathrm{T}}\ \ e(t-d(t))^{\mathrm{T}}\ \ e(t-p)^{\mathrm{T}}\ \ e(t-\tau(t))^{\mathrm{T}}\ \ e(t-\mu)^{\mathrm{T}}\ \ \overline{g}(e(t))^{\mathrm{T}}]^{\mathrm{T}}$$

Similarly, denoting $\rho(t) = (\mu-\tau(t))/\mu$, then we have that $0 \le \rho(t) \le 1$ and $\tau(t) = (1-\rho(t))\mu$. According to Lemma 1, we get that

$$-\int_{t-\mu}^{t} \dot{e}(s)^{\mathrm{T}} Z_2 \dot{e}(s)\, \mathrm{d}s - (1-v) \int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}} Z_3 \dot{e}(s)\, \mathrm{d}s$$

$$
\begin{aligned}
&= -\mu \int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\mathcal{U}\dot{e}(s)\,\mathrm{d}s - \mu \int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\mu^{-1}Z_2\dot{e}(s)\,\mathrm{d}s \\
&= -\tau(t)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\mathcal{U}\dot{e}(s)\,\mathrm{d}s - (\mu-\tau(t))\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\mathcal{U}\dot{e}(s)\,\mathrm{d}s \\
&\quad -(\mu-\tau(t))\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\mu^{-1}Z_2\dot{e}(s)\,\mathrm{d}s - \tau(t)\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\mu^{-1}Z_2\dot{e}(s)\,\mathrm{d}s \\
&\le -\tau(t)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\mathcal{U}\dot{e}(s)\,\mathrm{d}s - \rho(t)\tau(t)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\mathcal{U}\dot{e}(s)\,\mathrm{d}s \\
&\quad -(\mu-\tau(t))\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\mu^{-1}Z_2\dot{e}(s)\,\mathrm{d}s - (1-\rho(t))(\mu-\tau(t))\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\mu^{-1}Z_2\dot{e}(s)\,\mathrm{d}s \\
&\le -\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\,\mathrm{d}s\,\mathcal{U}\int_{t-\tau(t)}^{t} \dot{e}(s)\,\mathrm{d}s - \rho(t)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}\,\mathrm{d}s\,\mathcal{U}\int_{t-\tau(t)}^{t} \dot{e}(s)\,\mathrm{d}s \\
&\quad -\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\,\mathrm{d}s\,\mu^{-1}Z_2\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\dot{e}(s)\,\mathrm{d}s - (1-\rho(t))\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\,\mathrm{d}s\,\mu^{-1} \\
&Z_2\int_{t-\mu}^{t-\tau(t)} \dot{e}(s)^{\mathrm{T}}\dot{e}(s)\,\mathrm{d}s \\
&= \begin{bmatrix} e(t) \\ e(t-\tau(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\mathcal{U} & \mathcal{U} \\ * & -\mathcal{U} \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \end{bmatrix} - \rho(t)\delta(t)^{\mathrm{T}}\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1\delta(t) \\
&\quad + \begin{bmatrix} e(t-\tau(t)) \\ e(t-\mu) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -\mu^{-1}Z_2 & \mu^{-1}Z_2 \\ * & -\mu^{-1}Z_2 \end{bmatrix} \begin{bmatrix} e(t-\tau(t)) \\ e(t-\mu) \end{bmatrix} \\
&\quad -(1-\rho(t))\delta(t)^{\mathrm{T}}\Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2\delta(t)
\end{aligned} \tag{22}
$$

On the other hand, based on Assumption 1, we have that any $\lambda > 0$

$$
y(t) = \lambda \begin{bmatrix} e(t) \\ \overline{g}(e(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \overline{U} & \overline{V} \\ * & I \end{bmatrix} \begin{bmatrix} e(t) \\ \overline{g}(e(t)) \end{bmatrix} \le 0 \tag{23}
$$

Thus,

$$
\begin{aligned}
\dot{V}(t) &\le \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) - y(t) \\
&= \delta(t)^{\mathrm{T}}\varphi(t)(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1 - \rho(t)\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 - (1-\rho(t))\Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2)\delta(t) \\
&\quad + \delta(t)^{\mathrm{T}}(1-\varphi(t))(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2 - \rho(t)\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 - (1-\rho(t))\Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2)\delta(t) \\
&= \delta(t)^{\mathrm{T}}\varphi(t)\rho(t)(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1 - \Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1)\delta(t) \\
&\quad + \delta(t)^{\mathrm{T}}\varphi(t)(1-\rho(t))(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1 - \Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2)\delta(t) \\
&\quad + \delta(t)^{\mathrm{T}}(1-\varphi(t))\rho(t)(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2 - \Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1)\delta(t) \\
&\quad + \delta(t)^{\mathrm{T}}(1-\varphi(t))(1-\rho(t))(\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2 - \Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2)\delta(t)
\end{aligned} \tag{24}
$$

By using the Schur complement, we can find from Eqs. (13)–(16) that there exists a scalar $\alpha > 0$ such that

$$
\Xi_1 + \mathcal{Y}^{\mathrm{T}}Z\mathcal{Y} - p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1 - \Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 < -\alpha I \tag{25}
$$

$$\Xi_1+\mathcal{Y}^{\mathrm{T}}Z\mathcal{Y}-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2<-\alpha I \tag{26}$$

$$\Xi_1+\mathcal{Y}^{\mathrm{T}}Z\mathcal{Y}-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1<-\alpha I \tag{27}$$

$$\Xi_1+\mathcal{Y}^{\mathrm{T}}Z\mathcal{Y}-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\Theta_2^{\mathrm{T}}\mu^{-1}Z_2\Theta_2<-\alpha I. \tag{28}$$

Thus,

$$\dot{V}(t)\le-\alpha\|e(t)\|^2. \tag{29}$$

Applying the similar method of [22], we can find that the system (9) is exponentially stable. This completes the proof. □

Based on Theorem 1, we can obtain the design method of the desired sampled-data controllers to ensure the CDN (1) exponentially synchronized.

Theorem 2. *The CDN (1) is exponentially synchronized by controllers of the form (6) if there exist matrices* $P=\mathrm{diag}\{P_1,P_2,\ldots,P_N\}>0,\ Q_1>0,\ Q_2>0,\ Q_3>0,\ Z_1>0,\ Z_2>0,\ Z_3>0,$ $X=\mathrm{diag}\{X_1,X_2,\ldots,X_N\}$ *and a scalar* $\lambda>0$ *such that Eq. (12) and the following LMIs hold*:

$$\begin{bmatrix}\hat{\Xi}_1-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 & \hat{\mathcal{Y}}^{\mathrm{T}}\\ * & -2P+Z\end{bmatrix}<0 \tag{30}$$

$$\begin{bmatrix}\hat{\Xi}_1-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\mu^{-1}\Theta_2^{\mathrm{T}}Z_2\Theta_2 & \hat{\mathcal{Y}}^{\mathrm{T}}\\ * & -2P+Z\end{bmatrix}<0 \tag{31}$$

$$\begin{bmatrix}\hat{\Xi}_1-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 & \hat{\mathcal{Y}}^{\mathrm{T}}\\ * & -2P+Z\end{bmatrix}<0 \tag{32}$$

$$\begin{bmatrix}\hat{\Xi}_1-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\mu^{-1}\Theta_2^{\mathrm{T}}Z_2\Theta_2 & \hat{\mathcal{Y}}^{\mathrm{T}}\\ * & -2P+Z\end{bmatrix}<0 \tag{33}$$

where

$$\hat{\Xi}_1=\begin{bmatrix}\Xi_{11} & X+p^{-1}Z_1 & 0 & \Xi_{14} & 0 & P-\lambda\overline{V}\\ * & -2p^{-1}Z_1 & p^{-1}Z_1 & 0 & 0 & 0\\ * & * & -Q_1-p^{-1}Z_1 & 0 & 0 & 0\\ * & * & * & \Xi_{44} & \mu^{-1}Z_2 & 0\\ * & * & * & * & -Q_2-\mu^{-1}Z_2 & 0\\ * & * & * & * & * & -\lambda I\end{bmatrix}$$

$$\hat{\mathcal{Y}}=[P(G\otimes D)\ \ X\ \ 0\ \ P(G\otimes A)\ \ 0\ \ P]$$

and the other parameters follow the same definitions as those in Theorem1. Furthermore, the desired controllers gain matrices are given by

$$K_i=P_i^{-1}X_i,\quad i=1,2,\ldots,N \tag{34}$$

Proof. Define matrix $J=\mathrm{diag}\{I,I,I,I,I,I,PZ^{-1}\}$ and $X=PK$. Then, pre- and post-multiplying Eqs. (13)–(16) with J and J^{T}, respectively, we obtain that Eqs. (13)–(16) are equivalent to

$$\begin{bmatrix} \hat{\Xi}_1-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 & \hat{\mathcal{Y}}^{\mathrm{T}} \\ * & -PZ^{-1}P \end{bmatrix}<0 \tag{35}$$

$$\begin{bmatrix} \hat{\Xi}_1-p^{-1}\Delta_1^{\mathrm{T}}Z_1\Delta_1-\mu^{-1}\Theta_2^{\mathrm{T}}Z_2\Theta_2 & \hat{\mathcal{Y}}^{\mathrm{T}} \\ * & -PZ^{-1}P \end{bmatrix}<0 \tag{36}$$

$$\begin{bmatrix} \hat{\Xi}_1-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\Theta_1^{\mathrm{T}}\mathcal{U}\Theta_1 & \hat{\mathcal{Y}}^{\mathrm{T}} \\ * & -PZ^{-1}P \end{bmatrix}<0 \tag{37}$$

$$\begin{bmatrix} \hat{\Xi}_1-p^{-1}\Delta_2^{\mathrm{T}}Z_1\Delta_2-\mu^{-1}\Theta_2^{\mathrm{T}}Z_2\Theta_2 & \hat{\mathcal{Y}}^{\mathrm{T}} \\ * & -PZ^{-1}P \end{bmatrix}<0 \tag{38}$$

Noting $Z>0$, we have $-PZ^{-1}P\leq -2P+Z$. Thus, it is clear that if Eqs. (30)–(32) hold, then (35)–(38) hold, which implies that Eqs. (13)–(16) hold. This completes the proof. □

Remark 1. It is noted that the sampled-data synchronization problem has been solved for CDN (1) in Theorem 2, and the desired controllers can be obtained when LMIs (12) and (30)–(32) are feasible. It is noted that in [18] the delay terms $\tau(t)$ and $\tau-\tau(t)$ are enlarged as τ, and the delay terms $d(t)$ and $p-d(t)$ are enlarged as p, that is, $\tau=\tau(t)+\tau-\tau(t)$ and $p=p(t)+p-p(t)$ are enlarged as 2τ and $2p$, respectively. It is clear this treatment cannot make full use of the information on the involved delays $d(t)$ and $\tau(t)$, and may lead to a conservative result. Different from [18], we introduce two scalars $\varphi(t)$ and $\rho(t)$ in Eqs. (21) and (22) to take full advantage of the information on the involved delays $d(t)$ and $\tau(t)$. In fact, it is easy to find that if Theorems 1 and 2 of [18] hold, our given results also hold, that is, the results proposed in this paper have *theoretically* less conservatism than [18].

Remark 2. It should be pointed out that the given results can be extended to more general CDNs with external disturbances, uncertainties, and time-delay in the control input in accordance with standard flow of robust control theory. For example, when time-delay in the control input is considered, we can get the following error system:

$$\dot{e}(t)=\overline{g}(e(t))+(G\otimes D)e(t)+(G\otimes A)e(t-\tau(t))+Ke(t-d(t))$$

where

$$p_1\leq d(t)\leq p_2$$

For the above given system, we choose the following Lyapunov functional:

$$V(t)=V_1(t)+V_2(t)+V_3(t)$$

where

$$V_1(t)=e(t)^{\mathrm{T}}Pe(t)+\int_{t-\mu}^{t}e(s)^{\mathrm{T}}Q_2e(s)+\int_{t-\tau(t)}^{t}e(s)^{\mathrm{T}}Q_3e(s)\,\mathrm{d}s$$

$$V_2(t)=\int_{-p_1}^{0}\int_{t+\theta}^{t}\dot{e}(s)^{\mathrm{T}}Z_1\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta+\int_{t-p_1}^{t}e(s)^{\mathrm{T}}Q_1e(s)\,\mathrm{d}s$$

$$+\int_{t-p_2}^{t} e(s)^{\mathrm{T}} Q_2 e(s)\,\mathrm{d}s + \int_{-p_2}^{-p_1}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_4 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta$$

$$V_3(t) = \int_{-\mu}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_2 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}} Z_3 \dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Then, the results can be obtained by using the similar methods.

4. Numerical examples

In this section, two numerical examples are given to illustrate the validness of our results.

Example 1. Chua's circuit is considered as the isolated node of the dynamical network, which is described by the following equation:

$$\begin{cases} \dot{x}_1 = \sigma_1(-x_1 + x_2 - u(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\sigma_2 x_2 \end{cases}$$

where $\sigma_1 = 10$, $\sigma_2 = 14.87$, and $u(x_1)) = -0.68x_1 + 0.5(-1.27 + 0.68)(|x_1 + 1| - |x_1 - 1|)$. It can be calculated that in Eq. (4)

$$U = \begin{bmatrix} 2.7 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}$$

The inner-coupling matrices are given as $D=0$ and

$$A = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$$

and the outer-coupling matrix

$$G = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The time-varying delay is chosen as $\tau(t) = 0.03 + 0.01\sin(t)$, which implies $\mu = 0.04$ and $v = 0.01$, and the controller gains

$$K_1 = K_2 = K_3 = \begin{bmatrix} -12 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -12 \end{bmatrix}$$

Based on Theorem 1 in our paper, we can find that the maximum value of sampling period $p = 0.0711$.

Example 2. Consider CND (1) with three nodes [18]. The outer-coupling matrix is assumed to be $G=(G_{ij})_{N\times N}$ with

$$G=\begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & 1\\ 1 & 1 & -2\end{bmatrix}$$

The time-varying delay is chosen as $\tau(t)=0.2+0.05\sin(10t)$. A straightforward calculation gives $\mu=0.25$ and $\nu=0.5$. The nonlinear function f is taken as

$$f(x_i(t))=\begin{bmatrix}-0.5x_{i1}+\tanh(0.2x_{i1})+0.2x_{i2}\\ 0.95x_{i2}-\tanh(0.75x_{i2})\end{bmatrix}$$

It can be found that f satisfies Eq. (8) with

$$U=\begin{bmatrix}-0.5 & 0.2\\ 0 & 0.95\end{bmatrix},\quad V=\begin{bmatrix}-0.3 & 0.2\\ 0 & 0.2\end{bmatrix}$$

(1) The inner-coupling matrices are given as $D=0$ and

$$A=\begin{bmatrix}0.5 & 0\\ 0 & 0.5\end{bmatrix}.$$

By applying Theorem 2 of [18], the maximum value of sampling period $p=0.5409$. While using Theorem 2 in our paper, the maximum value of sampling period $p=0.5573$. Thus, our result has less conservatism than the existing one. Moreover, the gain matrices of the desired controllers can be obtained as follows:

$$K_1=\begin{bmatrix}-0.4201 & -0.1614\\ 0.0001 & -1.1698\end{bmatrix}$$
$$K_2=\begin{bmatrix}-0.4201 & -0.1614\\ 0.0001 & -1.1698\end{bmatrix}$$
$$K_3=\begin{bmatrix}0.1221 & -0.2073\\ -0.0024 & -1.0093\end{bmatrix}$$

Using the above parameters, the state trajectories of the error system (9) are given in Fig. 1, and the control inputs $u_i(t)$ are shown in Fig. 2, where $x_1(0)=[3\ \ -2]^{\mathrm{T}}$, $x_2(0)=[2\ \ 5]^{\mathrm{T}}$, $x_3(0)=[-5\ \ 6]^{\mathrm{T}}$, $s(0)=[3\ \ 2]^{\mathrm{T}}$.

(2) The inner-coupling matrices are given as

$$D=\begin{bmatrix}0.3 & 0\\ 0 & 0.3\end{bmatrix},\quad A=\begin{bmatrix}0.4 & 0\\ 0 & 0.4\end{bmatrix}$$

According to Theorem 2 with $p=0.05$, we can get the corresponding controller parameters:

$$K_1=\begin{bmatrix}-0.4330 & -0.2071\\ -0.0631 & -1.5627\end{bmatrix}$$

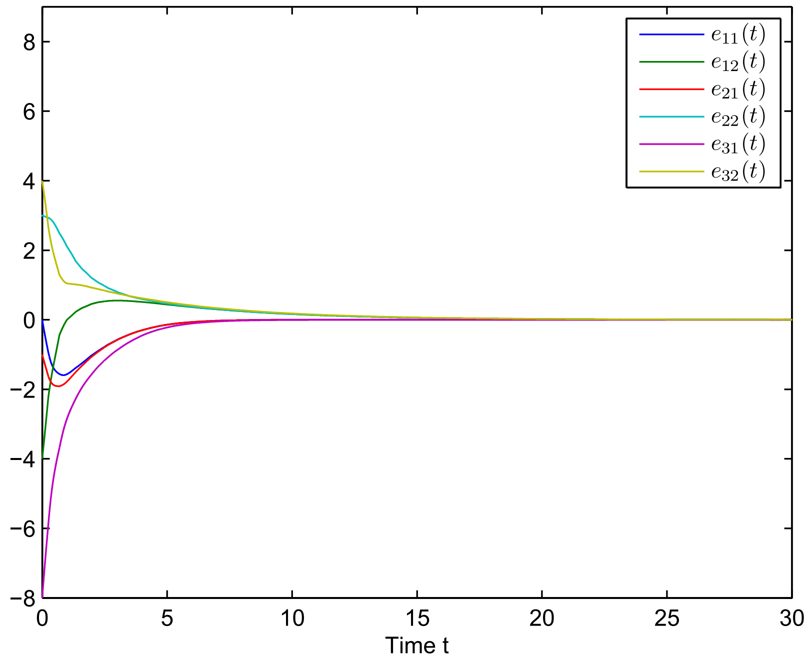

Fig. 1. State trajectories of the error system (9).

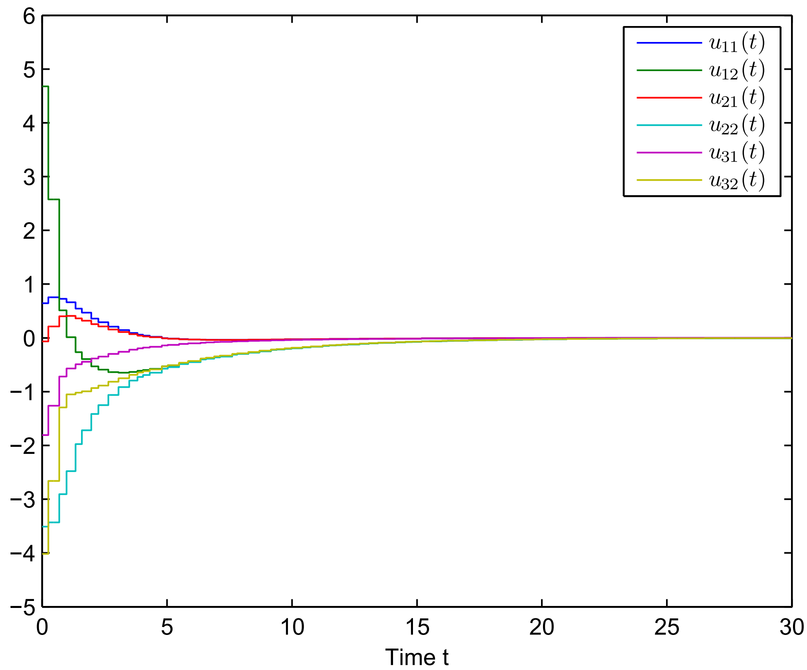

Fig. 2. Responses of the control inputs $u_i(t)$.

$$K_2 = \begin{bmatrix} -0.4330 & -0.2071 \\ -0.0631 & -1.5627 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -0.1631 & -0.1744 \\ -0.0355 & -1.0803 \end{bmatrix}$$

Under the above parameters, the state trajectories of the error system (9) are given in Fig. 3, and the control inputs $u_i(t)$ are shown in Fig. 4, where $x_1(0) = [-5 \;\; 6]^T$, $x_2(0) = [3 \;\; 4]^T$, $x_3(0) = [-2 \;\; 5]^T$, $s(0) = [-3 \;\; 4]^T$.

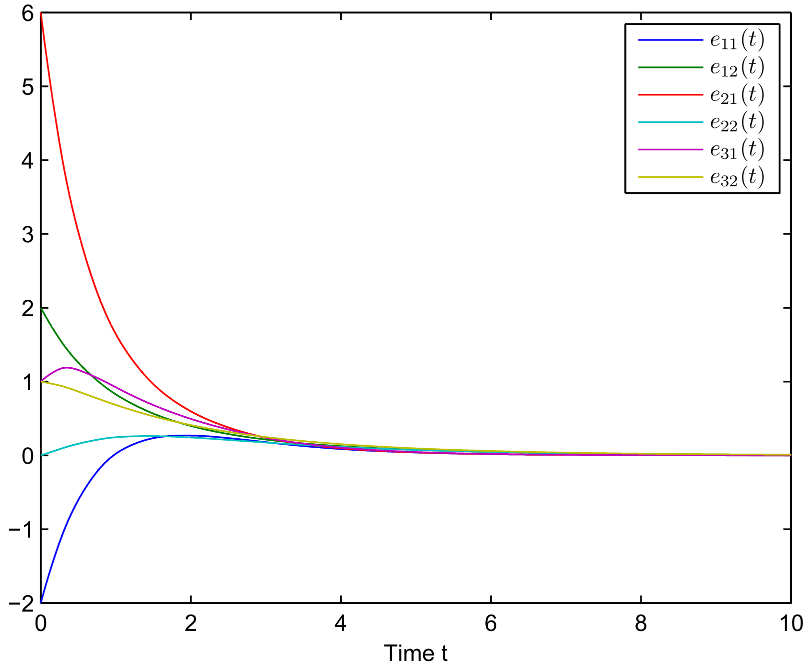

Fig. 3. State trajectories of the error system (9).

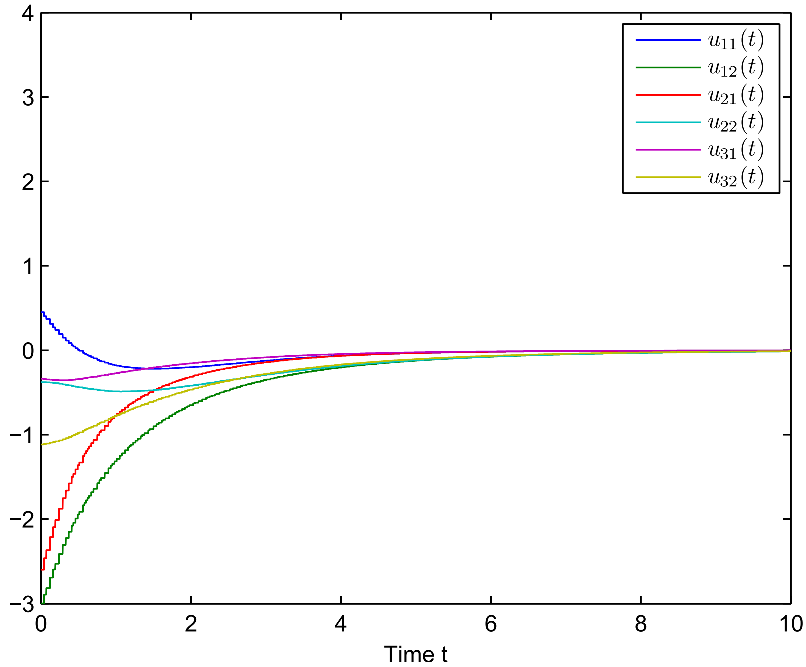

Fig. 4. Responses of the control inputs $u_i(t)$.

5. Conclusions

In this paper, the sampled-data synchronization problem has been solved for a kind of CDNs with time-varying coupling delay. The sampling period considered here is assumed to be time-varying but bounded. By combining the LMI approach, a newly exponential synchronization condition has been proposed. A set of sampled-data controllers has been designed. The derived results are theoretically and numerically proved to be less conservative than existing results. Two illustrative examples and their simulation results have been given to illustrate the effectiveness and less conservatism of the proposed methods.

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