On the Average Output SNR in Selection Combining With Three Correlated Branches Over Nakagami-m Fading Channels

Dimitris A. Zogas, *Student Member, IEEE*, George K. Karagiannidis, *Member, IEEE*, and Stravos A. Kotsopoulos

***Abstract*—An exact and rapidly converging infinite series for the average output signal-to-noise ratio in a triple selection diversity system, over correlated Nakagami-m fading channels, is presented. Numerical results are presented to illustrate the proposed approach and to point out the effect of the fading correlation to the performance of the combiner, as well as the improvement achieved by the triple selection combining compared to the dual diversity case.**

***Index Terms*—Correlated fading, Nakagami-m fading channels, selection diversity, signal-to-noise ratio (SNR).**

I. Introduction

WIRELESS communications are characterized by various impairments such as fading and multipath. Diversity is a very common technique to compensate for the channels impairments and is usually implemented by using an antenna array consisted of two or more receiving antennas. The most popular diversity techniques are selection combining (SC), equal gain combining (EGC), and maximal ratio combining (MRC). Among these types of diversity combining, SC is the least complicated technique since the processing is performed only on one of the diversity branches. Traditionally, in SC the combiner chooses the branch with the highest signal-to-noise ratio (SNR), which corresponds to the strongest signal, if equal noise power is assumed among the branches. The performance analysis of SC, assuming independent channel fading, has been studied extensively in the literature. However, independent fading assumes antenna elements be placed sufficiently apart, which is not always realized in practice due to insufficient antenna spacing when diversity is applied in compact terminals. In this kind of terminal, the fading among the channels is correlated, resulting in a degradation of the diversity gain obtained [1].

In order to study the effectiveness of any modulation scheme and the type of diversity used, it is required to evaluate the systems performance over the channels conditions. Bit-error rate (BER) evaluation gives a good indication of the system's performance, but it does not provide information about the type of errors, for example, it does not give incidents of bursty errors. Evaluating the probability of outage is another means to judge the system's performance. An outage event is specified by a specific number of bit errors occurring in a given transmission. However, probably the most common and best understood performance criterion of a digital communication system is the SNR, which often is measured at the output of the receiver. It is the easiest to evaluate and serves as an excellent indicator of the systems fidelity. In the context of a communication system subject to fading impairment, the appropriate performance criterion is average SNR, where the word average refers to the statistical averaging over the probability distribution of the fading [1], [2].

In reviewing the literature, there are few approaches for the evaluation of the average output SNR of a SC receiver. In [3], the average output SNR of dual SC over correlated Nakagami-m fading channels is investigated, while in [4] the average SNR of a generalized diversity selection combining scheme in independent and identically distributed (i.i.d.) Rayleigh channels is derived. To the best of the authors' knowledge, no analytical study investigating the average output SNR of triple SC over correlated Nakagami-m fading channels has been reported in the literature, due to the nonexistence of a useful formulation for the trivariate Nakagami-m probability density function (pdf) or moment generation function (mgf).

In this letter, based on a formula for the trivariate Nakagami-m pdf, recently proposed by the authors in [5], an exact and rapidly converging infinite series for the average output SNR of a triple SC receiver over correlated Nakagami-m fading channels has been derived. Also, the improvement achieved by the triple SC compared to the corresponding dual diversity case is presented. The remainder of this letter is organized as follows: In Section II, the average output SNR of the triple SC is extracted in a closed form, while in Section III numerical results are presented to illustrate the proposed formulation. Finally, some concluding remarks are offered in Section IV.

II. Average Output SNR of the Triple SC

The Nakagami-m model is used in modeling various propagation channels, which are characterized by multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves. It provides optimum fits to collected data in indoor and outdoor mobile radio environments and it is used in many wireless communications applications [6].

If r is a Nakagami-m variable, then its corresponding pdf is described by

$$f_r(r) = \frac{2r^{2m-1}}{\Gamma(m)\Omega^m} exp\left(\frac{-r^2}{\Omega}\right), \quad r \geq 0 \tag{1}$$

whereas $\Gamma(\cdot)$ is the Gamma function, $\Omega = \overline{r^2}/m$, with $\overline{r^2}$ being the average signal power and m the inverse normalized variance of r^2 which must satisfy $m \geq 1/2$, describing the fading severity [6].

Manuscript received May 25, 2002; revised September 23, 2002; accepted December 19, 2002. The editor coordinating the review of this paper and approving it for publication is A. Swami.

D. A. Zogas and S. A. Kotsopoulos are with the Electrical and Computer Engineering Department, University of Patras, Patras, Greece (e-mail: zogas@space.noa.gr; kotsop@ee.upatras.gr).

G. K. Karagiannidis is with the Institute for Space Applications and Remote Sensing, National Observatory of Athens, 15236 Athens, Greece (e-mail: gkarag@space.noa.gr).

Digital Object Identifier 10.1109/TWC.2003.821219

Recently, the authors in [5] presented an approach to the n-variate (joint) Nakagami-m pdf and cumulative distribution function (cdf) with an arbitrary correlation matrix $\mathbf{\Sigma}$ of the underlying Gaussian process. According to this approach, the trivariate ($n = 3$) Nakagami-m pdf can be extracted in two steps:

Step 1) The entries of the correlation matrix $\mathbf{\Sigma}$ are approximated with the elements of a Green's matrix $\mathbf{C}$, in order $\mathbf{W} = \mathbf{C}^{-1}$ being tridiagonal [5], [7], i.e.,

$$\mathbf{W} = \mathbf{C}^{-1} = \begin{bmatrix} p_{1,1} & p_{1,2} & 0 \\ p_{1,2} & p_{2,2} & p_{2,3} \\ 0 & p_{2,3} & p_{3,3} \end{bmatrix}. \tag{2}$$

Step 2) The trivariate Nakagami-m pdf can be written as

$$\begin{aligned} f_{r_1,r_2,r_3}(r_1,r_2,r_3) = {} & \frac{|\mathbf{W}|^m |p_{1,2}\, p_{2,3}|^{-(m-1)}}{2^{m-1}\Gamma(m)} r_1^m r_2 r_3^m \\ & \times e^{-\frac{1}{2}\left(p_{1,1}{r_1}^2 + p_{2,2}{r_2}^2 + p_{3,3}{r_3}^2\right)} \\ & \times I_{m-1}\left(|p_{1,2}| r_1 r_2\right) \\ & \times I_{m-1}\left(|p_{2,3}| r_2 r_3\right) \end{aligned} \tag{3}$$

with $I_\nu(\cdot)$ being the first kind of νth order modified Bessel function. Without loss of generality and for simplification purposes of the matrix $\mathbf{\Sigma}$, it was assumed in (3) (as in [8]) that $\Omega_i = (\overline{r_i^2})/(m) = 2{\sigma_i}^2$ and the variance ${\sigma_i}^2 = 1$.

The corresponding trivariate cdf is derived in [5] as

$$\begin{aligned} F_{R_1,R_2,R_3}(R_1,R_2,R_3) = {} & \frac{|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} \\ & \times \frac{1}{i_1! i_2! \Gamma(i_1+m)\Gamma(i_2+m)} \\ & \times \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}}{p_{1,1}^{i_1+m} p_{2,2}^{i_1+i_2+m} p_{3,3}^{i_2+m}} \\ & \times \gamma\left(i_1+m, \frac{1}{2}p_{1,1}R_1^2\right) \\ & \times \gamma\left(i_1+i_2+m, \frac{1}{2}p_{2,2}R_2^2\right) \\ & \times \gamma\left(i_2+m, \frac{1}{2}p_{3,3}R_3^2\right) \end{aligned} \tag{4}$$

with $\gamma(\cdot)$ being the "upper" incomplete Gamma function [9, eq. (6.5.2)]. Note that in the special theoretical case of exponential correlation [8], the normalized correlation matrix can be written as $\mathbf{\Sigma}_{i,j} \equiv \rho^{|i-j|}$. The exponential correlation model corresponds to the important practical case of linear arrays consisted from equispaced diversity antennas [8]. In this case, $\mathbf{\Sigma}^{-1}$ has a tridiagonal form, the first step can be neglected, and $\mathbf{W}$ can be written as

$$\mathbf{W} = \mathbf{\Sigma}^{-1} = \frac{1}{\rho^2 - 1}\begin{bmatrix} -1 & \rho & 0 \\ \rho & -(\rho^2+1) & \rho \\ 0 & \rho & -1 \end{bmatrix}. \tag{5}$$

Equations (3) and (4) can be used to evaluate directly the performance of a triple SC receiver, finding the one-dimensional pdf and cdf at the output. The SC output cdf can be used to evaluate the outage probability, while the SC output pdf for the evaluation of the average SNR and the average BER [1].

Let us define now the instantaneous SNR per symbol and per channel $\zeta_i = r_i^2 E_s/N_0$, $i = 1, 2, 3$, with E_s/N_0 being the symbol energy-to-Gaussian noise spectral density ratio and the average SNR per symbol $\overline{\zeta} = \overline{\zeta_i} = \overline{r_i^2} E_s/N_0 = 2m\, E_s/N_0$. The assumption of identical power in all three branches is reasonable if the diversity channels are closely spaced and the gain of each channel is such that all the noise powers are equal [8]. The joint trivariate cdf of ζ_1, ζ_2, and ζ_3 can be found directly from (4) as

$$\begin{aligned} F_{\zeta_1,\zeta_2,\zeta_3}(\zeta_1,\zeta_2,\zeta_3) = {} & \frac{|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} \\ & \times \frac{1}{i_1! i_2! \Gamma(i_1+m)\Gamma(i_2+m)} \\ & \times \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}}{p_{1,1}^{i_1+m} p_{2,2}^{i_1+i_2+m} p_{3,3}^{i_2+m}} \\ & \times \gamma\left(i_1+m, p_{1,1}\frac{m\zeta_1}{\overline{\zeta}}\right) \\ & \times \gamma\left(i_1+i_2+m, p_{2,2}\frac{m\zeta_2}{\overline{\zeta}}\right) \\ & \times \gamma\left(i_2+m, p_{3,3}\frac{m\zeta_3}{\overline{\zeta}}\right). \end{aligned} \tag{6}$$

The cdf of the triple SC output SNR can be derived from (6) equating the three arguments $\zeta_1 = \zeta_2 = \zeta_3 = \zeta$ as

$$F_\zeta(\zeta) = F_{\zeta_1,\zeta_2,\zeta_3}(\zeta,\zeta,\zeta). \tag{7}$$

The corresponding pdf of the triple SC output SNR can be extracted—after manipulations—differentiating (7) with respect to ζ as

$$\begin{aligned} f_\zeta(\zeta) &= \frac{dF_\zeta(\zeta)}{d\zeta} \\ &= \frac{|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}}{p_{1,1}^{i_1+m} p_{2,2}^{i_1+i_2+m} p_{3,3}^{i_2+m}} \\ &\quad \times \frac{[G_1+G_2+G_3]}{i_1! i_2! \Gamma(m+i_1)\Gamma(m+i_2)} \end{aligned} \tag{8}$$

where

$$\begin{aligned} G_1 &= \left(\frac{p_{1,1}m}{\overline{\zeta}}\right)^{i_1+m} \zeta^{i_1+m-1} e^{-\frac{p_{1,1}m}{\overline{\zeta}}\zeta} \\ &\quad \times \gamma\left(i_1+i_2+m, \frac{p_{2,2}m}{\overline{\zeta}}\zeta\right) \gamma\left(i_2+m, \frac{p_{3,3}m}{\overline{\zeta}}\zeta\right) \\ G_2 &= \left(\frac{p_{2,2}m}{\overline{\zeta}}\right)^{i_1+i_2+m} \zeta^{i_1+i_2+m-1} e^{-\frac{p_{2,2}m}{\overline{\zeta}}\zeta} \\ &\quad \times \gamma\left(i_1+m, \frac{p_{1,1}m}{\overline{\zeta}}\zeta\right) \gamma\left(i_2+m, \frac{p_{3,3}m}{\overline{\zeta}}\zeta\right) \\ G_3 &= \left(\frac{p_{3,3}m}{\overline{\zeta}}\right)^{i_2+m} \zeta^{i_2+m-1} e^{-\frac{p_{3,3}m}{\overline{\zeta}}\zeta} \\ &\quad \times \gamma\left(i_1+m, \frac{p_{1,1}m}{\overline{\zeta}}\zeta\right) \gamma\left(i_1+i_2+m, \frac{p_{2,2}m}{\overline{\zeta}}\zeta\right). \end{aligned}$$

The average output SNR $\overline{\zeta}_{\mathrm{OUT}}$ can be derived by averaging the instantaneous SNR ζ, over the pdf of ζ. Using (8), the average SNR at the output of a triple SC can be written as

$$\begin{aligned}\overline{\zeta}_{\mathrm{OUT}} &= \int_0^\infty \zeta f_\zeta(\zeta) d\zeta \\ &= \int_0^\infty \zeta \frac{|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^\infty \sum_{i_2=0}^\infty \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}}{p_{1,1}^{i_1+m} p_{2,2}^{i_1+i_2+m} p_{3,3}^{i_2+m}} \\ &\quad \times \frac{[G_1+G_2+G_3]}{i_1! i_2!\, \Gamma(m+i_1)\Gamma(m+i_2)} d\zeta. \end{aligned} \tag{9}$$

As the quantity in the double sum in (9) is Riemann integrable and converges uniformly in the range $[0, \infty)$, the order of integration and summation is interchangeable and (9) can be written as

$$\begin{aligned}\overline{\zeta}_{\mathrm{OUT}} = \frac{|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^\infty \sum_{i_2=0}^\infty \frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}}{p_{1,1}^{i_1+m} p_{2,2}^{i_1+i_2+m} p_{3,3}^{i_2+m}} \\ \times \frac{\left[\int_0^\infty \zeta G_1 d\zeta + \int_0^\infty \zeta G_2 d\zeta + \int_0^\infty \zeta G_3 d\zeta\right]}{i_1! i_2!\, \Gamma(m+i_1)\Gamma(m+i_2)}. \end{aligned} \tag{10}$$

The three integrals in (10) have the following form:

$$\int_0^\infty e^{-ax} x^b \gamma(d_1, c_1 x)\gamma(d_2, c_2 x) dx \tag{11}$$

which can be written as

$$\begin{aligned}&\int_0^\infty e^{-ax} x^b \gamma(d_1, c_1 x)\gamma(d_2, c_2 x) dx \\ &= \frac{c_1^{d_1} c_2^{d_2}}{d_1 d_2} \int_0^\infty x^{d_1+d_2+b} e^{-(a+c_1+c_2)} \\ &\quad \times {}_1F_1(1, d_1+1; c_1 x)\, {}_1F_1(1, d_2+1; c_2 x) dx \end{aligned} \tag{12}$$

using [9, (6.5.12)] and applying Kummer transformation [10, (9.20)]. In (12), ${}_1F_1(u_1, u_2, x)$ is the well-known confluent hyper-geometric function. Now, the integral in the second part of (12) can be written in a closed form using [11, Appendix C] resulting in

$$\begin{aligned}&\int_0^\infty e^{-ax} x^b \gamma(d_1, c_1 x)\gamma(d_2, c_2 x) dx \\ &= \frac{c_1^{d_1} c_2^{d_2} \Gamma(b+d_1+d_2+1)}{d_1 d_2 (a+c_1+c_2)^{(b+d_1+d_2+1)}} \\ &\quad \times F_2\left(b+d_1+d_2+1; 1, 1; d_1+1, d_2+1; \frac{c_1}{a+c_1+c_2}, \frac{c_2}{a+c_1+c_2}\right) \end{aligned} \tag{13}$$

with F_2 being the hyper-geometric function of two variables defined in [10, (9.1.80/2)]. Using (13), and after some algebraic manipulations, the average output SNR of a triple SC can be finally written as in (14), shown at the bottom of the page.

It must be noted here that the double infinite sum in (14) converges rapidly and a mean number of 14 terms for each sum is adequate for accuracy at the fifth significant digit (Table I).

TABLE I
TERMS NEED TO BE SUMMED IN THE DOUBLE-INFINITE SERIES OF (14), ASSUMING ACCURACY AT THE FIFTH SIGNIFICANT DIGIT

	m=1	*m*=3
ρ	Terms	Terms
0.3	6	8
0.5	12	16
0.7	21	27

III. Numerical Results and Discussion

In this section, numerical results are presented to illustrate the formulation presented in Section II. More specifically, the improvement of the triple SC compared to the dual diversity case is studied, for several correlation models, such as exponential [8], linearly arbitrary [12], and constant [8].

Fig. 1 plots the first branch normalized average output SNR versus the correlation coefficient, for dual and triple SC with exponential correlation. It is evident that the effect of the correlation to the output SNR performance is higher for the triple diversity case, compared with the dual one. The normalized average

$$\overline{\zeta}_{\mathrm{OUT}} = \frac{\overline{\zeta}|\mathbf{W}|^m}{\Gamma(m)} \sum_{i_1=0}^\infty \sum_{i_2=0}^\infty \left[\frac{|p_{1,2}|^{2i_1}|p_{2,3}|^{2i_2}\Gamma(3m+2i_1+2i_2+1)(H_1+H_2+H_3)}{m\Gamma(m+i_1)\Gamma(m+i_2) i_1! i_2! (p_{1,1}+p_{2,2}+p_{3,3})^{(3m+2i_1+2i_2+1)}}\right] \tag{14}$$

with

$$H_1 = \frac{F_2\left(3m+2i_1+2i_2+1; 1, 1; m+i_1+i_2+1, m+i_2+1; \frac{p_{2,2}}{p_{1,1}+p_{2,2}+p_{3,3}}, \frac{p_{3,3}}{p_{1,1}+p_{2,2}+p_{3,3}}\right)}{(m+i_1+i_2)(m+i_2)}$$

$$H_2 = \frac{F_2\left(3m+2i_1+2i_2+1; 1, 1; m+i_1+1, m+i_2+1; \frac{p_{1,1}}{p_{1,1}+p_{2,2}+p_{3,3}}, \frac{p_{3,3}}{p_{1,1}+p_{2,2}+p_{3,3}}\right)}{(m+i_1)(m+i_2)}$$

$$H_3 = \frac{F_2\left(3m+2i_1+2i_2+1; 1, 1; m+i_1+1, m+i_1+i_2+1; \frac{p_{1,1}}{p_{1,1}+p_{2,2}+p_{3,3}}, \frac{p_{2,2}}{p_{1,1}+p_{2,2}+p_{3,3}}\right)}{(m+i_1)(m+i_1+i_2)}$$

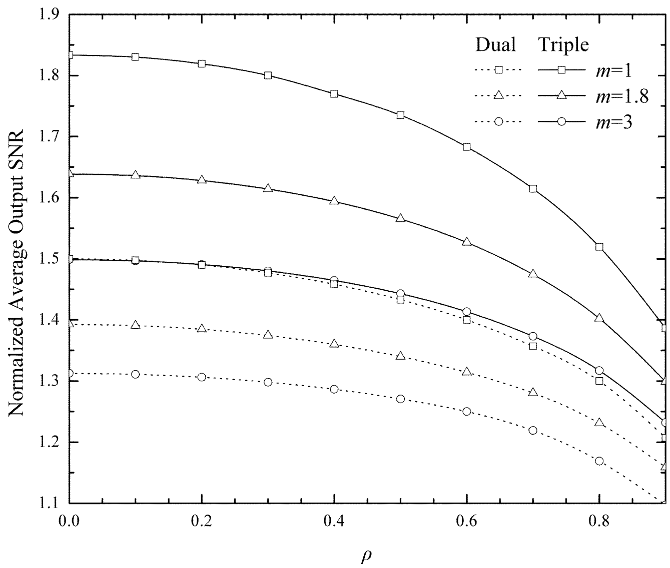

Fig. 1. Normalized average output SNR versus the correlation coefficient ρ for dual and triple SC with exponential correlation.

output SNR degrades as the correlation coefficient increases, since the SNR gain increases when the fading severity parameter m decreases. These results were also observed in [3]. Moreover, as m decreases, the improvement offered by the triple SC compared to the dual one is more significant.

When the branches of the triple SC are linearly arbitrarily correlated, the correlation decreases as the distance of the elements increases. An example of such a correlation matrix is given in [5] and [12] as

$$\mathbf{\Sigma_1} = \begin{bmatrix} 1.000 & 0.795 & 0.605 \\ 0.795 & 1.000 & 0.795 \\ 0.605 & 0.795 & 1.000 \end{bmatrix}.$$

and its Green's matrix approximation can be found as

$$\mathbf{C_1} = \begin{bmatrix} 1.000 & 0.786 & 0.617 \\ 0.786 & 1.000 & 0.786 \\ 0.617 & 0.786 & 1.000 \end{bmatrix}.$$

In the case of equidistant antennas (e.g., placed at the edges of a triangular), the branches of the triple SC are constant correlated and an example of such a correlation matrix is given in [5]

$$\mathbf{\Sigma_2} = \begin{bmatrix} 1.000 & 0.600 & 0.600 \\ 0.600 & 1.000 & 0.600 \\ 0.600 & 0.600 & 1.000 \end{bmatrix}$$

which has a Green's matrix approximation

$$\mathbf{C_2} = \begin{bmatrix} 1.000 & 0.699 & 0.473 \\ 0.699 & 1.000 & 0.699 \\ 0.473 & 0.699 & 1.000 \end{bmatrix}.$$

In Table II, a comparison between triple and dual diversity is made, using the approximated matrices $\mathbf{C}_1$ and $\mathbf{C}_2$ and (14) for several values of the parameter m. The correlation coefficients for the dual diversity case are 0.699 and 0.786, respectively. The curves and the results for the dual selection diversity case were obtained using the expression for the bivariate Nakagami-m cdf proposed in [13] and following the same procedure as in Section II.

TABLE II
COMPARISON OF THE NORMALIZED OUTPUT SNR BETWEEN THE TRIPLE AND THE DUAL SC FOR THE APPROXIMATED MATRICES $\mathbf{C}_1$ AND $\mathbf{C}_2$

Model	***m***	$\overline{\zeta}_{OUT}/\overline{\zeta}$	
		Triple SC	**Dual SC**
Linearly Arbitrary (Matrix $\mathbf{C}_1$)	1	1.5438	1.3091
	1.5	1.4584	1.2623
	2	1.4026	1.2316
	2.5	1.3636	1.2054
	3	1.3340	1.1826
	3.5	1.3106	1.1678
	4	1.2913	1.1512
Constant (Matrix $\mathbf{C}_2$)	1	1.6986	1.3576
	1.5	1.5972	1.3035
	2	1.5306	1.2682
	2.5	1.4843	1.2428
	3	1.4494	1.2235
	3.5	1.4219	1.2081
	4	1.3994	1.1955

IV. CONCLUSION

In this paper, capitalizing on a useful expression for the trivariate Nakagami-m pdf, recently presented by the authors, an analytical expression for the average output SNR of a triple SC over correlated Nakagami-m fading channels is derived. Numerical results show the usefulness of the proposed formula, pointing out the improvement obtained using three-branch SC compared to the dual diversity case.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers and the editor, Dr. A. Swami, for their valuable comments that considerably improved the quality of this letter.

REFERENCES

[1] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels*, First ed. New York: Wiley, 2000.
[2] T. S. Rappaport, *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ: PTR Prentice-Hall, 1996.
[3] Y.-K. Ko, M.-S. Alouini, and M. K. Simon, "Average SNR of dual selection combining over correlated Nakagami-m fading channels," *IEEE Commun. Lett.*, vol. 4, pp. 12–14, Jan. 2000.
[4] N. Kong and L. B. Milstein, "Average SNR of a generalized diversity selection combining scheme," *IEEE Commun. Lett.*, vol. 3, pp. 57–59, Mar. 1999.
[5] G. K. Karagiannidis, D. A. Zogas, and S. A. Kotsopoulos, "An efficient approach to multivariate Nakagami-m distribution using Green's matrix approximation," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 883–889, Sept. 2003.
[6] M. Nakagami, "The m-distribution—A general formula if intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, W. G. Hoffman, Ed. Oxford, U.K.: Pergamon, 1960.
[7] R. Nabben, "On Green's matrices of trees," *SIAM J. Matrix Anal. Applicat.*, vol. 4, pp. 1014–1026, 2000.
[8] V. A. Aalo, "Performance of maximal—ratio diversity systems in a correlated Nakagami-fading environment," *IEEE Trans. Commun.*, vol. 43, pp. 2360–2369, Aug. 1995.
[9] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1972.
[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, Fifth ed. New York: Academic, 1994.
[11] A. Annamalai, C. Tellambura, and V. K. Bhargava, "Equal-gain diversity receiver performance in wireless channels," *IEEE Trans. Commun.*, vol. 48, pp. 1732–1745, Oct. 2000.
[12] Q. T. Zhang, "Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1141–1150, July 1999.
[13] C. C. Tan and N. C. Beaulieu, "Infinite series representation of the bivariate Rayleigh and Nakagami-m distributions," *IEEE Trans. Commun.*, vol. 45, pp. 1159–1161, Oct. 1997.