

Efficient Resource Allocation for OFDMA Multicast Systems With Spectrum-Sharing Control

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Abstract—This paper considers the important problem of efficient allocation of available resources (such as radio spectrum and power) in orthogonal frequency-division multiple-access (OFDMA)-based multicast wireless systems. Taking the maximization of system throughput as the design objective, three novel efficient resource-allocation schemes with reduced computational complexity are proposed under constraints on total bandwidth and transmitted power at the base station (BS). Distinct from existing approaches in the literature, our formulation and solution methods also provide an effective and flexible means to share the available radio spectrum among multicast groups by guaranteeing minimum numbers of subcarriers to be assigned to individual groups. The first two proposed schemes are based on the separate optimization of subcarriers and power, where subcarriers are assigned with the assumption of uniform power distribution, followed by water filling of the total available transmitted power over the determined subcarrier allocation. In the third scheme, which is essentially a modified genetic algorithm (GA), each individual of the entire population represents a subcarrier assignment, whose fitness value is the system sum rate computed on the basis of the power water-filling procedure. Numerical results show that with a flexible spectrum-sharing control mechanism, the proposed designs are able to more flexibly and fairly distribute the total available bandwidth among multicast groups and, at the same time, achieve a high system throughput.

Index Terms—Genetic algorithm (GA), orthogonal frequency-division multiple access (OFDMA), resource allocation, spectrum-sharing control, wireless multicast.

I. INTRODUCTION

WITH A SIMPLE receiver structure and the ability to offer high performance in multipath fading environments, orthogonal frequency-division multiplexing (OFDM) is a promising solution for broadband wireless networks [1], [2]. In unicast multiuser OFDM systems, which is commonly

known as orthogonal frequency-division multiple access (OFDMA), the problem of how to optimally allocate the available resources to improve the system performance has recently been the center of intensive research. Two important resources in wireless communications are 1) the available radio spectrum over which signals of all the users may occupy and 2) the available transmitted power. Consequently, how these resources are utilized determines the level of efficiency of a given resource allocation scheme. In the literature, the dynamic allocation of resources in OFDMA-based wireless systems can be categorized into two broad classes: 1) margin class and 2) rate-adaptive class. Specifically, the margin adaptive class aims at minimizing the transmitted power under constraints on the individual user's data rate and/or bit error rate [3], whereas the objective of the rate-adaptive problems is to maximize the data rate of each user subject to a power budget and/or user's achievable data rate [4]–[11].

On the other hand, multicasting, which is an efficient means of transmitting the same information content to multiple receivers while minimizing the use of network resources [12], [13], is certainly an attractive transmission method for resource-constrained wireless networks. For resource allocation in OFDMA multicast wireless networks, the study in [14] proposes a low-complexity algorithm that aims at improving the downlink (information-theoretic) capacity of the systems. First, each of the available subcarriers is assigned to the group with the best channel condition and with the most member users under the assumption of equal transmitted power. Then, the total power is distributed to the already determined subcarrier assignment in a water-filling fashion. In [15], a heuristic solution for allocating resources of OFDM-based multicast systems is suggested to minimize the number of OFDM symbols that each individual user receives, thereby resulting in a reduction of the total power consumed by the users. For the downlink of an OFDM-based multicast wireless network employing multiple description coding, [16] addresses the power control and bit-loading problems for both cases of throughput maximization and proportional fairness. Due to the high complexity of the resulting integer programming, a two-step suboptimal scheme is proposed in [16], wherein subcarriers are assigned by assuming that a constant transmitted power has been distributed to each subcarrier, and then, the bits are loaded to the allocated subcarriers according to the modified Levin–Campello algorithm.

Nevertheless, since the channel quality of every user in a multicast network may be very different, the attainable data rate of each multicast stream is usually restricted by the data rate of the least-capable user. Furthermore, the number of users in a

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multicast group also has a direct impact on the aggregate data rate that can be achieved by that group. These critical factors lead to imbalanced opportunities when gaining access to the available system resources, such as bandwidth and power, of individual multicast groups. When the differences in path loss and/or size among groups are large, it is likely that the typical adaptive resource allocation schemes, which try to maximize the system performance, will distribute most of the available bandwidth (and subsequently power) to groups with high equivalent channel signal-to-noise ratio (CSNR) and/or with larger user sets for a significant portion of time. Consequently, groups with worse channel conditions and/or with fewer member users may not be able to access any of the available resources at all. As the system resources are valuable but scarce, and maximizing the total system throughput is not always the only design priority, the issue of fair resource utilization among multicast groups with diverse CSNR characteristics and with different group sizes becomes particularly important. The fair allocation of available resources in OFDMA-based systems has been discussed in different contexts for both unicast and multicast scenarios, including max–min fairness [4], proportional rate guarantee [6], minimum bandwidth assurance [17], equal bandwidth distribution [7], and proportional fairness [16]. However, none of these solutions accounts for a controllable sharing of the available radio spectrum to flexibly distribute the system resources in wireless multicast settings. Motivated by the works in [4] and [17], this paper addresses the aforementioned shortcoming of the existing solutions.

We first provide a new formulation for the resource allocation problem in OFDMA-based multicast wireless systems that balances the tradeoff between maximizing the total throughput and ensuring a flexible and controllable spectrum sharing among different multicast groups. To this end, by introducing “bandwidth control indexes” that can easily be regulated, we impose constraints on the minimum numbers of subcarriers to be assigned to individual groups. The indexes can be adjusted so that the formulated problem may be cast into the problem of sum rate maximization (SRM). More importantly, if fair bandwidth sharing¹ among different groups with asymmetric links and diverse group sizes is desired, the minimum numbers of subcarriers can always be set to proper values, which are determined from the respective channel conditions and sizes of individual groups. On one hand, this prevents groups with good channels or with large user sets from greedily consuming all the available bandwidth. On the other hand, it guarantees that groups with poorer channel conditions or smaller group sizes still have good opportunities to access the system resources.

We then propose three novel efficient schemes with low computational complexity to solve the formulated NP-hard design problem. In the first and second schemes, the allocation is accomplished via the separate optimization of subcarriers and transmitted power, where, specifically, subcarriers are assigned based on the assumption of uniform power allocation,

¹In this paper, “fair bandwidth sharing” means that a certain multicast group deserves some portion of the total available bandwidth, regardless of its link condition or group size. In addition, the terms “bandwidth” and “radio spectrum” are used interchangeably.

followed by water filling of the total power over the determined subcarrier assignment. In the third scheme, which is based on a modified genetic algorithm (GA) [18]–[20], each individual of the whole population corresponds to a subcarrier allocation whose fitness score is the system throughput computed on the basis of the power water-filling procedure. It is shown that with proper adjustments of the minimum numbers of subcarriers assigned to individual groups, the proposed solutions provide more flexibility in controlling the share of available radio spectrum given to each group and, at the same time, achieve a very high total sum rate. The complexities of the proposed approaches are analyzed, and their potentials are thoroughly verified via simulation with the illustration of numerical examples.

The rest of this paper is organized as follows: Section II formulates the OFDMA-based multicast resource allocation problem with spectrum-sharing constraints. Sections III and IV propose the separate optimization and GA-based schemes, respectively. Section V analyzes the computational complexity and evaluates the performance of the proposed solutions with the support of numerical results. Finally, Section VI concludes this paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a one-cell multicast wireless system employing OFDMA, wherein one base station (BS) transmits G (down-link) traffic flows, each to one distinct multicast group, over M subcarriers. Assume that each user receives one traffic flow at a time; hence, it belongs to only one multicast group. Let K_g and $|K_g|$ ($g = 1, \dots, G$) denote the user set of group g and its cardinality, respectively. Since the g th group is unicast if $|K_g| = 1$, whereas it is multicast if $|K_g| > 1$, the model is valid for both unicast and multicast settings. Clearly, all the users belong to the set $K = \bigcup_{g=1}^G K_g$, and $|K| = \sum_{g=1}^G |K_g|$ is the total number of users in the system. Let B denote the total system bandwidth, and assume that each subcarrier has an equal bandwidth of $B_m = B_0 = B/M$. The system setup is depicted in Fig. 1.

In this paper, resource allocation is accomplished in a centralized manner at the BS, which has perfect channel state information of all the users in the systems via dedicated feedback channels. This is a typical assumption in the literature [3], [5], [6]. The BS is then able to determine the maximum rate at which an individual user can reliably receive data, as well as the corresponding subcarrier over which the data will be transmitted. It is commonly accepted that the maximum attainable rate of user $k \in K_g$ on subcarrier m is

$$r_{k,m} = \frac{B_0}{B} \log_2 \left(1 + \frac{|h_{k,m}|^2 P_m}{B_0 N_0} \right) \quad (1)$$

where $h_{k,m}$ is the channel coefficient from the BS to user k on subcarrier m , P_m is the transmitted power allocated to subcarrier m , and N_0 is the one-sided power spectral density of additive white Gaussian noise. It is further assumed that the channel conditions remain unchanged during the allocation period. This assumption is particularly valid for slowly varying

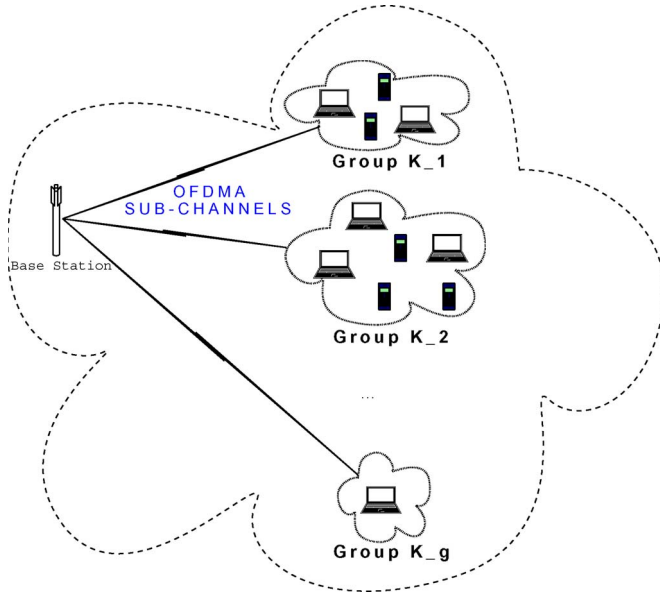


Fig. 1. Downlink of an OFDMA multicast system.

channels, where the channel gains do not vary too significantly over time, for example, in high-data-rate systems and/or environments with reduced degrees of mobility.

An attractive feature of wireless multicast is that multicast data can be transmitted from the BS to multiple mobile users only through a single transmission. However, while all users within a multicast group receive the same rate from the BS, the main issue arises from the mismatch data rates attainable by individual users of that group, whose link conditions are typically asymmetric. If the BS transmits at a rate higher than the maximum rate that a user can handle, then that user cannot decode *any* of the transmitted data at all. Therefore, a conventional approach is to transmit at the lowest rate of all the users within a group, which is determined by the user with the worst channel condition [14]. This assures that the multicast services can be provided to *all* the subscribed users. On one hand, as all the multicast users within a group receive the same data rate from the BS, the total sum rate is scaled by the group size, which is effectively the number of active users of that group. On the other hand, the lowest transmit rate typically decreases as the number of users increases, since it is based on the least capable user. We, however, have established that as the number of users in a multicarrier multicast system tends to infinity, the ergodic system capacity becomes independent of the group size but depends on the total number of subcarriers (see the Appendix for more details). This result confirms that the conventional multicast transmission scheme is indeed both practical and beneficial, particularly with the use of multicarrier transmission, as in OFDM-based wireless networks.

Although it is possible to adopt other techniques (for instance, exploiting the hierarchy in multicast data together with the assumption of multidescription coding [16]) to overcome the issue of data rate mismatch in wireless multicast, such solutions are out of the scope of this paper. Here, the conventional approach is followed by enforcing the BS to transmit at the lowest rate of all the users within a group, which is deter-

mined by the user with the smallest CSNR. On subcarrier m , letting

$$\beta_{g,m} = \min_{k \in K_g} \frac{|h_{k,m}|^2}{B_0 N_0} \quad (2)$$

be the equivalent CSNR of group g , then the maximum rate at which *all* users of group g are able to decode the transmitted data is

$$\check{r}_{g,m} = \frac{B_0}{B} \log_2(1 + \beta_{g,m} P_m). \quad (3)$$

As all the users in a group receive the same rate, the aggregate data rate transmitted to group g on subcarrier m is thus

$$R_{g,m} = \sum_{k \in K_g} \check{r}_{g,m} = |K_g| \check{r}_{g,m}. \quad (4)$$

The goal of this paper is to devise a subcarrier assignment and power-allocation policy that maximizes the system sum rate of all multicast groups while satisfying a constraint on the total transmitted power. Distinct from existing works, here, the important issue of providing a flexible mechanism to effectively govern the share of accessible bandwidth among various multicast groups is also taken into account. One possible way to realize this idea is to guarantee a certain minimum number of subcarriers to be allocated to each group. Specifically, the design problem can be formulated as follows:

$$\max_{\{\rho_{g,m}, P_m\}} \sum_{g=1}^G \sum_{m=1}^M \frac{|K_g|}{M} \rho_{g,m} \log_2(1 + \beta_{g,m} P_m) \quad (5)$$

$$\text{subject to} \quad \sum_{m=1}^M P_m \leq P_{\text{tot}} \quad (6)$$

$$P_m \geq 0, \quad m = 1, \dots, M \quad (7)$$

$$\sum_{g=1}^G \rho_{g,m} = 1, \quad m = 1, \dots, M \quad (8)$$

$$\rho_{g,m} \in \{0, 1\} \quad (9)$$

$$\sum_{m=1}^M \rho_{g,m} \geq \alpha_g, \quad g = 1, \dots, G. \quad (10)$$

In this formulation, the binary variable $\rho_{g,m}$ represents the allocation of subcarrier m to group g . Constraints (6) and (7) express the power limitation at the BS, whereas constraints (8) and (9) ensure a disjoint subcarrier assignment in OFDMA systems, wherein one subcarrier can only be given to at most one group. Constraint (10) reflects the spectrum-sharing control of the design, where the “bandwidth control index” α_g is required to satisfy $\alpha_g \in \mathcal{Z}_+$ and $\sum_{g=1}^G \alpha_g \leq M$. The value α_g manages the priority in terms of spectrum access opportunity provided to each multicast group. It varies from 0 to M and can flexibly be adjusted according to system design specifications. As α_g increases toward M , a higher priority is given to group g . In particular, if all α_g 's approach 0, then the problem in (5)–(10) becomes that of the SRM. Moreover, as all α_g 's approach

$\lfloor M/G \rfloor$, the optimization formulation enforces (almost) a strict bandwidth fairness.

It should be pointed out that the problem in (5)–(10) is NP-hard. Therefore, determining its optimal solution within a given time is very challenging. Performing a direct exhaustive search at the BS would obviously face a prohibitive computational burden, where the optimal solutions must be obtained within a designated time period due to the quick variations of wireless channels. Since such a solution method is too computationally expensive, it is impractical, particularly for systems with a large number of subcarriers (which is often the case in practice). Suboptimal algorithms, which have a low complexity and yet provide good performance, are therefore preferable for cost-effective and delay-sensitive implementations. In the next sections, three efficient solutions to solve the formulated design problem in (5)–(10) are proposed. The first two solutions are based on the separate optimization of subcarriers and power, whereas the last solution is obtained with a modified GA.

III. EFFICIENT RESOURCE ALLOCATION VIA SEPARATE OPTIMIZATION

Ideally, both subcarriers and power should jointly be allocated to achieve the global optimum of (5)–(10). However, this is highly complicated as the total number of variables becomes large. Instead of jointly optimizing $\{\rho_{g,m}\}$ and $\{P_m\}$, a separate optimization over these two sets of variables will be performed. Although suboptimal, this approach enables significantly lower computational complexity since the number of variables in each separate optimization problem is reduced almost by half. Specifically, the subcarrier assignment problem is solved in the first phase by assuming a constant power allocation on subcarriers. In the second phase, the total power is distributed over the available subcarriers in a water-filling fashion.

A. Phase 1—Subcarrier Allocation With Uniform Power Assumption

Under the assumption of equal-power distribution over the subcarriers, the data rate of the downlink traffic flow to multicast group g on subcarrier m in (4) becomes

$$R_{g,m} = \frac{|K_g|}{M} \log_2 \left(1 + \beta_{g,m} \frac{P_{\text{tot}}}{M} \right). \quad (11)$$

The proposed two-step subcarrier allocation is detailed in Algorithm 1. In Step 1, each subcarrier is assigned to the group that has the largest value of $R_{g,m}$ and has not been given its required minimum number of subcarriers. Once a subcarrier is assigned, it will not be considered in all subsequent operations. Further, the group that has already been allocated its minimum number of subcarriers is discarded in all the subsequent iterations. While the largest $R_{g,m}$ corresponds to the group with largest group size and/or the best link condition, the bandwidth constraint actually helps avoid the situation that subcarriers are all granted to the advantageous multicast groups. The procedure is repeated until all groups have been allocated their minimum

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input :  $\bar{R}_{G \times M} = \{R_{g,m}\}$  computed by (11),
          $\mathcal{S} = \{1, 2, \dots, M\}$ , and
          $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_G] \in \mathcal{Z}_+^G$ .
output: Subcarrier allocation  $\{\rho_{g,m}\}$ 

1 begin
   // Initialization
2    $\rho_{g,m} \leftarrow 0, \forall g, m$ 
3    $\Theta \leftarrow [0, 0, \dots, 0] \in \mathcal{Z}_+^G$ 
   // Step 1
4   repeat
5     find pair  $\{g_m^*, m^*\}$  that satisfy both
       conditions:
6       (a)  $\bar{R}(g_m^*, m^*) \geq \bar{R}(g, m), \forall g, m,$ 
7       (b)  $\Theta(g_m^*) \leq \alpha(g_m^*)$ 
8     end
9      $\rho_{g_m^*, m^*} \leftarrow 1$ 
10     $\rho_{g \neq g_m^*, m^*} \leftarrow 0$ 
11     $\Theta(g_m^*) \leftarrow \Theta(g_m^*) + 1$ 
12     $\mathcal{S} \leftarrow \mathcal{S} \setminus \{m^*\}$ 
13    delete  $\bar{R}(:, m^*)$  from  $\bar{R}$ 
14    if  $\Theta(g_m^*) > \alpha(g_m^*)$  then delete  $\bar{R}(g_m^*, :)$  from
        $\bar{R}$ 
15  until  $\Theta_g \geq \alpha_g, \forall g = 1, \dots, G$ 
   // Step 2
16  foreach  $m \in \mathcal{S}$  do
17    find  $g_m^* = \arg_g \max \bar{R}(g, m)$ 
18     $\rho_{g_m^*, m} \leftarrow 1$ 
19     $\rho_{g \neq g_m^*, m} \leftarrow 0$ 
20  end
21 end

```

Algorithm 1. Subcarrier assignment with bandwidth-sharing control.

numbers of subcarriers. In Step 2, the remaining subcarriers left from Step 1 are assigned to the group that has the largest value of $R_{g,m}$ in a sequential manner. Effectively, the allocation controls a certain level of bandwidth sharing as a result of Step 1, whereas the system throughput is further enhanced as a direct consequence of Step 2.

It can easily be seen that the lookup of the subcarrier–group pair (g_m^*, m^*) in lines 5–8 of Algorithm 1 involves a 2-D search, which could highly be intensive for systems with large numbers of subcarriers and multicast groups. To alleviate this drawback, we now propose a reduced-complexity subcarrier assignment based on Algorithm 1. Different from Algorithm 1, the reduced-complexity approach performs the assignment on a per-subcarrier basis in Step 1, where randomization is carried out to pick a subcarrier for which all the eligible groups, that is, the groups that have not reached their minimum numbers of subcarriers, will compete. Since the assignment only requires a 1-D search for each subcarrier, its computational complexity is significantly lower. A full description of this algorithm is provided in Algorithm 2.

B. Phase 2—Water-Filling Power Allocation

Once subcarrier allocation is accomplished, all the values of $\rho_{g,m}$ are known. Hence, power allocation can optimally be


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input :  $\bar{R}_{G \times M} = \{R_{g,m}\}$  computed by (11),
          $\mathcal{S} = \{1, 2, \dots, M\}$ ,
          $\mathcal{G} = \{1, 2, \dots, G\}$ , and
          $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_G] \in \mathcal{Z}_+^G$ .
output:  $\{\rho_{g,m}\}$ 

1 begin
   // Initialization
2    $\rho_{g,m} \leftarrow 0, \forall g, m$ 
3    $\Theta \leftarrow [0, 0, \dots, 0] \in \mathcal{Z}_+^G$ 
   // Step 1
4   repeat
5     randomly pick  $m^*$  from  $\mathcal{S}$ 
6     find  $g_m^* = \arg \max_{g \in \mathcal{G}} \bar{R}(g, m^*)$ 
7      $\rho_{g_m^*, m^*} \leftarrow 1$ 
8      $\rho_{g \neq g_m^*, m^*} \leftarrow 0$ 
9      $\Theta(g_m^*) \leftarrow \Theta(g_m^*) + 1$ 
10     $\mathcal{S} \leftarrow \mathcal{S} \setminus \{m^*\}$ 
11    if  $\Theta(g_m^*) > \alpha(g_m^*)$  then  $\mathcal{G} = \mathcal{G} \setminus \{g_m^*\}$ 
12  until  $\Theta_g \geq \alpha_g, \forall g = 1, \dots, G$ 
   // Step 2
13  foreach  $m \in \mathcal{S}$  do
14    find  $g_m^* = \arg_g \max \bar{R}(g, m)$ 
15     $\rho_{g_m^*, m} \leftarrow 1$ 
16     $\rho_{g \neq g_m^*, m} \leftarrow 0$ 
17  end
18 end

```

Algorithm 2. Reduced-complexity subcarrier assignment.

completed on a per-subcarrier basis. The optimization problem in (5)–(10) now becomes

$$\begin{aligned}
 & \max_{P_m \geq 0, m=1, \dots, M} \sum_{m=1}^M \frac{|K_{g_m^*}|}{M} \log_2 (1 + \beta_{g_m^*, m} P_m) \\
 & \text{subject to} \quad \sum_{m=1}^M P_m \leq P_{\text{tot}} \quad (12)
 \end{aligned}$$

where each subcarrier m has been assigned to group g_m^* .

Clearly, (12) involves the maximization of a concave function over a linear set; thus, it is a convex optimization problem. The closed-form solution can then be obtained by employing the Lagrange multiplier method. The Lagrangian of (12) can be expressed as

$$\begin{aligned}
 \mathcal{L}(P_m, \mu) = & \sum_{m=1}^M \frac{|K_{g_m^*}|}{M} \log_2 (1 + \beta_{g_m^*, m} P_m) \\
 & - \mu \left(\sum_{m=1}^M P_m - P_{\text{tot}} \right) \quad (13)
 \end{aligned}$$

where $\mu > 0$ is a Lagrange multiplier. The optimal power allocation can be derived from the Karush–Kuhn–Tucker conditions to be [21]

$$P_m = \max \left(\frac{|K_{g_m^*}|}{\mu M \log 2} - \frac{1}{\beta_{g_m^*, m}}, 0 \right). \quad (14)$$

It can be observed that the solution in (14) has the form of water filling, where μ can easily be found from the total power constraint $\sum_{m=1}^M P_m \leq P_{\text{tot}}$.

Combining Phases 1 and 2 results in two complete efficient resource-allocation schemes, which will be referred to as bandwidth control-separate optimization (BC-SO) and reduced-complexity BC-SO (RCBC-SO), respectively. Although being simple, the allocation schemes devised in this section are suboptimal due to the separation of optimization variables in each allocation phase. In the next section, we propose another efficient scheme that utilizes the GA to provide a global search for a jointly optimal subcarrier and power allocation.

IV. EFFICIENT RESOURCE ALLOCATION VIA MODIFIED GENETIC ALGORITHM

A. Overview of GA

GA [18]–[20], which is categorized as global search heuristics, is a search technique used to find exact or approximate solutions to both constrained and unconstrained optimization problems. It is based on natural selection, i.e., the process driving biological evolution. In brief, the GA is implemented as a computer simulation wherein a population of abstract representations (called *chromosomes* or the genotype of the genome) of candidate solutions (called *individuals* or creatures) to an optimization problem evolves toward better solutions. Specifically, the evolution usually starts from a population of randomly generated individuals and happens in generations. The GA repeatedly modifies a population of individuals through iterations, and at each iteration, the algorithm randomly picks individuals from the current population to be parents, which are then used to produce the *children* (or *offsprings*) for the next generation. Since the population evolves toward an optimum over successive generations, a sufficiently good solution to the optimization problem can finally be found. The attractiveness of GA comes from its simplicity and elegance as a robust search algorithm, as well as from its power to rapidly discover good solutions for difficult high-dimensional problems. As such, it has been employed in numerous applications in different fields, such as machine learning, bioinformatics, economics, chemistry, manufacturing, mathematics, physics, and so on.

A typical GA is presented in Table I [18], [20]. As can be seen, a GA by its nature does not begin its optimization process from a single point in the search space but rather from an entire set of individuals, which form the initial population. Hence, the GA may be invoked in robust global search and optimization procedures that do not require knowledge of the objective function's derivatives or any gradient-related information concerning the search space. It is, therefore, particularly suitable for optimization problems that are not well suited for standard optimization algorithms, including problems whose objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear.² Regarding the NP-hard design problem in (5)–(10), the objective function involves both continuous

²See, for instance, the adaptive resource-allocation and call-admission control problem in [22] or the multiple-antenna OFDM multiuser-detection problem in [23].

TABLE I
OUTLINE OF A BASIC GA

1. **[Start]** Generate a random population consisting N_p chromosomes (the suitable solutions for the problem)
2. **[Fitness]** Evaluate the fitness $f(x)$ of each individual x in the population
3. **[New Population]** Create a new population by repeating the following steps until the new population is complete
 - a. **[Elite]** Select some of the individuals from the current population who have high fitness values, called the *elite*, and pass them to the next population without any modification
 - b. **[Selection]** Select two parent individuals from the current population according to their fitness (the better fitness, the bigger chance to be chosen)
 - c. **[Crossover]** With a certain crossover probability, cross over the parents to form a new offspring. If no crossover was performed, offspring is an exact copy of parents.
 - d. **[Mutation]** With a certain mutation probability, mutate new offspring at a position in chromosome
 - e. **[Accepting]** Place new offspring in a new population
4. **[Replace]** Use newly generated population for a further run of the algorithm
5. **[Test]** If an end condition is met, stop, and return the best solution in current population
6. **[Loop]** Go to step 2

and discrete variables and, thus, represents a class of problems for which GA can efficiently be applied.

B. Proposed GA for Multicasting Resource Allocation

The proposed efficient scheme, which will be referred to as bandwidth control-GA (BC-GA), follows the general procedure of a GA together with the following features to specifically solve the design problem under investigation [see (5)–(10)].

1) *Coding of Individuals*: Each individual of the population corresponds to a subcarrier allocation. It is coded as a vector of length M , whose indexes represent the subcarriers, and the value of each vector entry is an integer in the range $[1, G]$, representing the group that has been assigned the subcarrier corresponding to that entry. For instance, the m th entry of an individual that has the value of g implies that subcarrier m is designated to multicast group g . Fig. 2 depicts the coding of individuals and the entire population in one generation.

2) *Initial Population*: The initial population of size N_p can randomly be generated with high-quality individuals possibly being fed into the population. A fine individual could be either a good subcarrier allocation generated by appropriate randomization or the suboptimal solutions derived via the proposed BC-SO and RCBC-SO schemes in Section III. With a well-

chosen starting population, the time required for BC-GA to reach an optimum solution would substantially be reduced.

3) *Fitness Function*: For each individual, its fitness value is the corresponding total sum rate. To compute this value, first, the bandwidth control constraint in (10) is checked against each individual (that is, each subcarrier allocation). If the constraint is unsatisfied, then the individual will be given the fitness value of $-\infty$. Otherwise, by performing the water filling of power over the known subcarrier assignment as described in Section III, the fitness score of this subcarrier power allocation can be computed. Since the objective is to maximize the system throughput, individuals with higher fitness values (that is, higher sum rates) are preferable in the proposed solution.

4) *Producing Next Generation*: To produce the next generation, the following rules apply, and their operations are also illustrated in Fig. 3:

- 1) **Elite Children Rule**: Elite children are individuals in the current generation with the best fitness values. These individuals automatically survive to the next generation. We propose that the number of elite children N_e in our GA to be fewer than 5, as setting this number to a high value causes the fittest individuals to dominate the population, which in turn may lead to a less-effective search.
- 2) **Crossover Rule**: Crossover enables the algorithm to extract the best genes from different individuals and recombine them into potentially superior children. In our proposed scheme, we apply a two-point crossover rule that selects two unequal points M_A and M_B at random ($1 \leq M_A, M_B \leq M$). The child has the vector entries (genes) of the first parent at the locations before M_A and after M_B and the vector entries (genes) of the second parent after M_A and before M_B .
- 3) **Mutation Rule**: The mutation process adds to the diversity of a population and, hence, increases the likelihood that the algorithm will generate individuals whose fitness values are better. Here, we propose a swapping of two randomly selected entries in a single parent to produce a new child.
- 5) *Stopping Criteria*: The proposed GA is terminated when at least one of the following conditions is met:
 - 1) A maximum number of generations L_{\max} is exceeded.
 - 2) The number of generations, over which a cumulative change in fitness function value is less than a tolerance value ϵ , exceeds L_{\lim} .

V. COMPLEXITY ANALYSIS AND PERFORMANCE EVALUATION

A. Complexity Analysis

Regarding the resource allocation problem in (5)–(10), an optimal search can be accomplished via exhaustive comparison of all G^M possible subcarrier assignments, each of which requires a total of M runs of power water filling to compute the achieved throughput. As a result, the direct search has an exponential complexity of $\mathcal{O}(G^M M)$. On the other hand, after obtaining the matrix \bar{R} via GM operations, both BC-SO and RCBC-SO

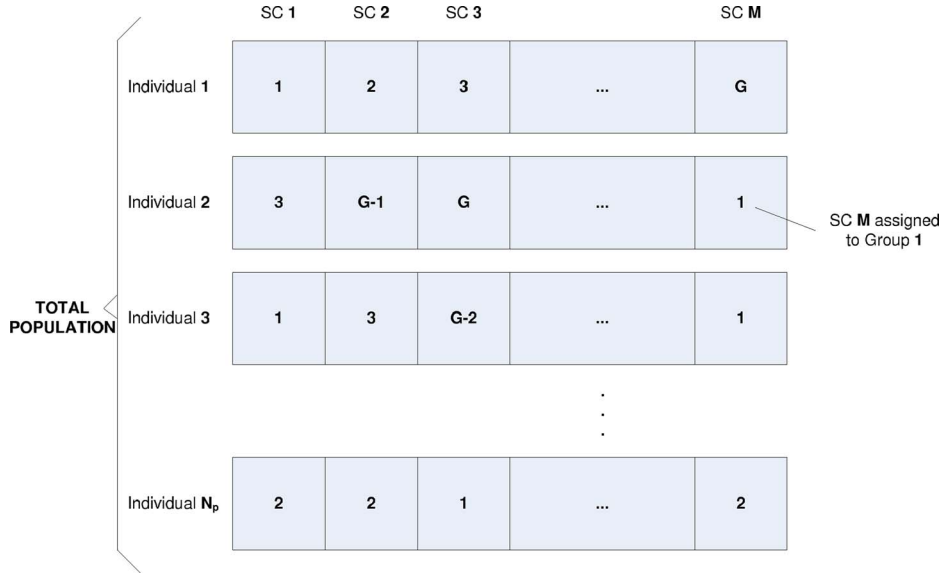


Fig. 2. Coding of individuals and total population in one generation.

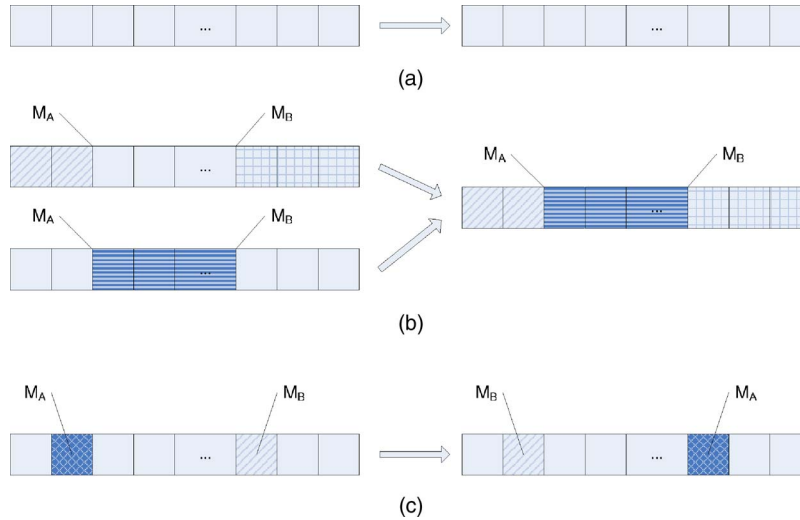


Fig. 3. Operations to create new generations. (a) ELITE. (b) CROSSOVER. (c) MUTATION.

schemes only need to perform either 1-D or 2-D search to find the eligible group corresponding to individual subcarriers. Once an optimal solution of subcarrier assignment has been found, the actual number of water-filling executions in these cases is merely M . Assuming that a search through a 1-D (unsorted) list is of quadratic order, then the total number of operations required by the BC-SO approach is indeed $GM + G^2M^3 + M$, whereas that by the RCBC-SO design is only $GM + G^2M + M$. It is worth commenting that the considerably low complexity of these two algorithms mainly stems from the separation of $\{\rho_{g,m}\}$ and $\{P_m\}$ variable sets with the assumption of uniform power distribution, as previously discussed in Section III-A. In contrast, the complexity of the BC-GA scheme depends on the maximum number of generations L_{\max} required to be produced before the algorithm terminates, as well as on the size N_p of each generated population. Within a population, the water filling of power is completed for each individual in the computation of fitness scores, followed by a 1-D search

TABLE II
COMPLEXITY ANALYSIS

Algorithm	Number of Operations	Order of Complexity
Optimal Search	$G^M M$	$\mathcal{O}(G^M M)$
BC-SO	$GM + G^2M^3 + M$	$\mathcal{O}(G^2M^3)$
RCBC-SO	$GM + G^2M + M$	$\mathcal{O}(G^2M)$
BC-GA	$L_{\max}(N_p M + N_p^2)$	$\mathcal{O}(L_{\max}(N_p M + N_p^2))$

to select the most fit individuals. Notice that the efficiency of a GA-based approach also depends on other factors, which can be difficult to explicitly quantify, such as the choice of initial population, the rules to produce new generations, and the tolerance allowable for cumulative changes in fitness scores. Excluding these parameters, the total complexity of the BC-GA scheme can be shown to be $\mathcal{O}(L_{\max}(N_p M + N_p^2))$.

All of the aforementioned analyses are summarized in Table II. Compared with the optimal exhaustive search, the

TABLE III
PARAMETERS FOR THE BC-GA SCHEME

Parameter	N_p	N_e	L_{\max}	L_{\lim}	ϵ
Value	32	2	60	20	10^{-6}

three proposed methods apparently demand far less computational effort. However, this benefit comes at the cost of sacrificing the attained system throughput as the devised schemes, by their nature, are suboptimal. In selecting suitable algorithms for different applications, it is therefore critical to balance the contradicting requirements of reducing the computational burden and achieving the highest possible sum rates. In what follows, we will provide some numerical examples to evaluate the performance of the proposed designs in various scenarios.

B. Numerical Examples

Considered is an OFDMA system with $M = 9$ subcarriers, wherein the BS communicates with $G = 3$ multicast groups, each has equal $|K_1| = |K_2| = |K_3| = 4$ users. Assuming that K_1 is located closer to the BS thus causes a path loss advantage of 1.5 dB to K_2 and of 3 dB to K_3 . To have a meaningful interpretation of the results, 100 sets of independent channel coefficients $\{h_{k,m}\}$ are randomly generated according to the Rayleigh distribution in each simulation study. The equivalent CSNR of group K_g on subcarrier m is computed as $\beta_{g,m} = \min_{k \in K_g} |h_{k,m}|^2$. The final results are then averaged for plotting. For simplicity, the average channel gain, the noise power in each subcarrier, and the individual subcarrier bandwidth are all normalized to 1. We will now demonstrate three illustrative examples wherein the values of bandwidth control indexes α_g ($g = 1, 2, 3$) are properly adjusted to either provide throughput maximization or offer a fair spectrum sharing by guaranteeing certain portions of the total available bandwidth to be designated to individual multicast groups. The performance of the proposed solutions (namely, BC-SO, RCBC-SO, and BC-GA) are to be compared against one another, as well as with that of optimal exhaustive search. In all the examples presented here, the parameters used for the BC-GA scheme are listed in Table III.

First of all, notice that by not guaranteeing any minimum numbers of subcarriers to be allotted to individual multicast groups, that is, by setting $\alpha_1 = \alpha_2 = \alpha_3 = 0$, the formulation in (5)–(10) actually becomes the problem of throughput maximization. In this case, there is a free competition among K_1 , K_2 , and K_3 , and the group that contributes the most to the total sum rate will finally secure the available system resources for its own usage. We will refer to this example as SRM, where it is apparent from Fig. 4(a) that all the proposed algorithms approach optimality. This numerical result, in particular, verifies that an equal transmit power allocation hardly decreases the data throughput of OFDMA-based systems since each subchannel is only given to a user whose channel gain is good for it. Further, because Step 1 of both Algorithms 1 and 2 is omitted in the SRM example, the BC-SO and RCBC-SO schemes reduce to a throughput maximization algorithm and, hence, perform identically. In terms of bandwidth sharing, the

proposed algorithms allocate more subcarriers to the group with better link conditions in this case, as can clearly be seen in Fig. 4(b). It should also be pointed out that although optimal search assigns more subcarriers to the advantageous groups, these subcarriers might have been distributed zero power by the water-filling procedure, resulting in no improvement in the attained throughput at all.

Since groups K_2 and K_3 are located farther away from the BS, their effective equivalent channel gains (including long-term path loss and short-term fading) are potentially smaller than that of group K_1 . The former groups are therefore likely to be in disadvantageous position, having fewer chances to gain access into the available radio spectrum. As such, we, in this second example, impose $\alpha_1 = 1$, $\alpha_2 = 2$, and $\alpha_3 = 3$ to ensure a fairer allocation in terms of bandwidth to the disadvantaged (inferior) groups K_2 and K_3 . The remaining three subcarriers are then open for competition among the three groups. We will refer to this example as inferior-fair bandwidth allocation (IBA). From Fig. 5(a), it is clear that the sum rates achieved by the BC-SO and BC-GA schemes are only 5% away from optimality, whereas the simple RCBC-SO design attains even more than 82% of the optimal throughput. In addition, the total bandwidth has been shared out more fairly among the multicast groups, as can be seen in Fig. 5(b). Note that the values of α_g in this example are chosen for illustrative purposes only, and they are completely adjustable at the discretion of the system designer. If the channel condition of the worst user in group K_2 or K_3 remains unfavorable for a relatively long period of time, it becomes necessary to readjust the value of α_g to avoid an unacceptable sacrifice in system throughput (one, for instance, may opt to increase α_1 and decrease α_2 and α_3).

Even more strictly, a totally fair bandwidth allocation for all three multicast groups can be enforced by setting $\alpha_1 = \alpha_2 = \alpha_3 = 9/3 = 3$, in which case, each group will exactly be given a third of the accessible bandwidth regardless of its respective channel state. This example will be referred to as equal bandwidth allocation (EBA). Fig. 6(a) illustrates that the sum rates obtained by the proposed solutions are very close to that offered by optimal search, with both the BC-SO and the BC-GA algorithms achieving more than 97% of the optimal throughput and that for the RCBC-SO solution being above 91%. Regarding the distribution of available bandwidth, Fig. 6(b) verifies that subcarriers have been shared equally among individual multicast groups by all the schemes under investigation in this third example.

The preceding numerical results have clearly confirmed that by properly adjusting the minimum numbers of subcarriers allotted to individual multicast groups, the design formulation and the proposed schemes offer more flexibility in controlling the share of available radio spectrum given to each group and, at the same time, still achieve a high system throughput. In particular, the BC-GA algorithm, with an appropriate choice of parameters, always offers the highest attainable data rate among the three proposals. This is expected since the BC-GA scheme performs a robust global search for the jointly optimal solution of subcarriers and power, as opposed to the separate optimization of those two variable sets in the BC-SO and RCBC-SO solutions. Moreover, the RCBC-SO

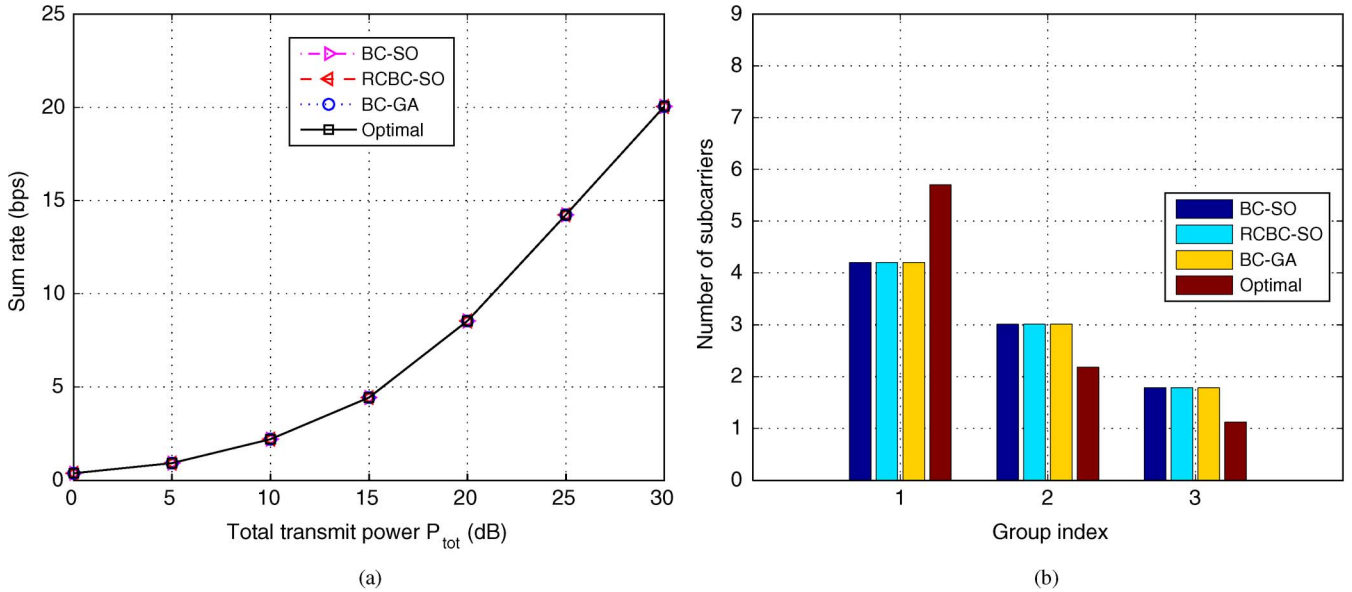


Fig. 4. Performance in the SRM example. (a) Achieved throughput. (b) Distribution of total bandwidth (averaged).

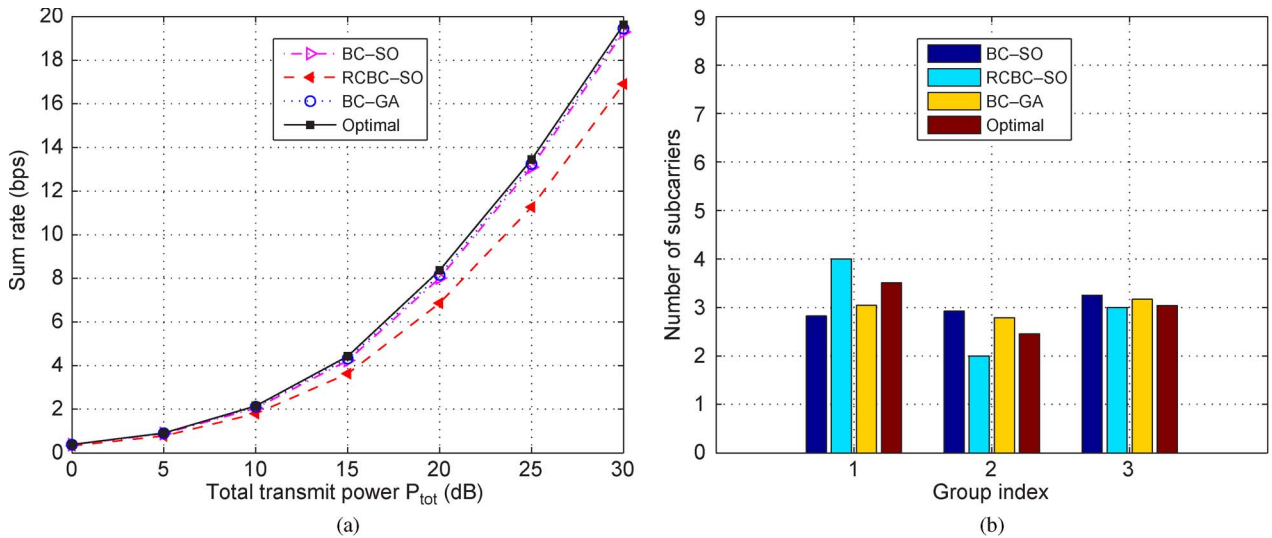


Fig. 5. Performance in the IBA example. (a) Achieved throughput. (b) Distribution of total bandwidth (averaged).

design yields the lowest throughput among the three and have the benefit of significantly lower computational complexity.

VI. CONCLUSION

This paper has proposed three efficient low-complexity resource-allocation schemes for OFDMA-based multicast wireless systems. The novelty in the proposed schemes is that the issue of controllable and flexible distribution of the available radio spectrum among multicast groups was explicitly taken into account. In separate optimization schemes, the subcarrier allocation ensures minimum numbers of subcarriers to be assigned to individual groups according to their respective channel gains and group sizes, whereas power is allocated in a water-filling fashion. With the scheme based on the modified GA, the jointly optimal subcarrier power allocation is iteratively evolved through a global search while satisfying the imposed

bandwidth constraints among different multicast groups. Numerical examples showed that the proposed designs can be utilized to attain a high total sum rate and, at the same time, more flexibly and fair distribution of the available bandwidth among multicast groups. The computational complexity of our proposed approaches has been analyzed, and their benefits have also been confirmed by numerical examples.

APPENDIX

ERGODIC CAPACITY OF MULTICARRIER SYSTEMS EMPLOYING CONVENTIONAL MULTICAST TRANSMISSION

In this Appendix, we show that for multicarrier systems employing the conventional multicast scheme (that is, transmit at the lowest rate of all users within a group), the ergodic system capacity becomes independent of the group size but depends on the total number of subcarriers, as the number

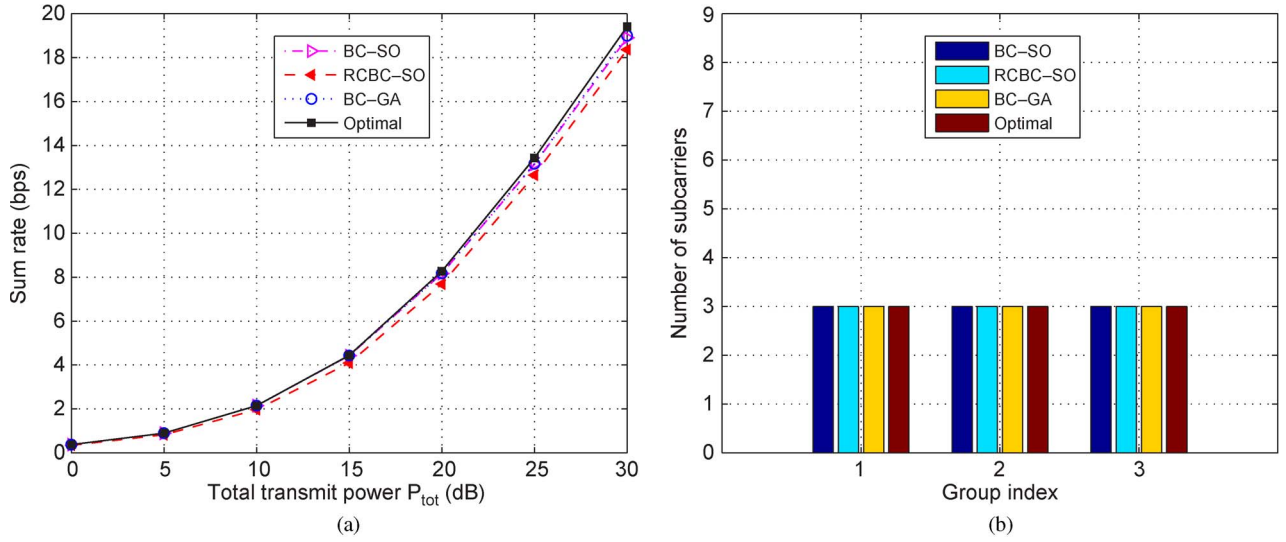


Fig. 6. Performance in the EBA example. (a) Achieved throughput. (b) Distribution of total bandwidth (averaged).

of active users tends to infinity. Considered is a conventional multicast transmission from BS to a group of K active users over M OFDM subcarriers. Upon defining $X_k^{(m)}$ ($m = 1, \dots, M; k = 1, \dots, K$) as the random variable representing the SNR of user k on subcarrier m and denoting $X_{(1)}^{(m)} = \min\{X_1^{(m)}, X_2^{(m)}, \dots, X_K^{(m)}\}$ as the group equivalent SNR on that same subcarrier, the multicast transmission rate at which BS transmits to all the K users on subcarrier m can be written as

$$R_{\text{MC}}^{(m)} = \log_2 \left(1 + X_{(1)}^{(m)} \right). \quad (15)$$

The system multicast capacity over all M subcarriers is then

$$C_{\text{MC}} = \sum_{m=1}^M K \cdot R_{\text{MC}}^{(m)}. \quad (16)$$

The ergodic capacity for multicast service now becomes

$$E[C_{\text{MC}}] = \sum_{m=1}^M E \left[K \cdot \log_2 \left(1 + X_{(1)}^{(m)} \right) \right]. \quad (17)$$

For Rayleigh fading channels, we have the following result.

Proposition 1: Assume that $X_k^{(m)}$, $k = 1, \dots, K$, are independent identically distributed exponential random variables with parameter $\beta^{(m)}$, and the ergodic capacity defined in (17) only depends on $\beta^{(m)}$ in the limit as $K \rightarrow \infty$. If we further assume that $\beta^{(1)} = \beta^{(2)} = \dots = \beta^{(M)}$, then the ergodic capacity linearly increases with M in the limit as $K \rightarrow \infty$.

Proof: From the probability density function (pdf) of $X_k^{(m)}$, which is

$$f(x) = \frac{1}{\beta^{(m)}} e^{-\frac{x}{\beta^{(m)}}} \quad (18)$$

the pdf of $X_{(1)}^{(m)}$ can be derived via order statistics as

$$f_{(1)}(x) = \frac{K}{\beta^{(m)}} e^{-\frac{Kx}{\beta^{(m)}}}. \quad (19)$$

The ergodic capacity multicarrier multicast system can be expressed as

$$\begin{aligned} E[C_{\text{MC}}] &= E \left[\sum_{m=1}^M C_{\text{MC}}^{(m)} \right] \\ &= \sum_{m=1}^M E \left[K \cdot \log_2 \left(1 + X_{(1)}^{(m)} \right) \right] \\ &= \sum_{m=1}^M K \cdot \int_0^{\infty} \log_2(1+x) f_{(1)}(x) dx \\ &= \sum_{m=1}^M \frac{K^2}{\beta^{(m)}} \cdot \int_0^{\infty} \log_2(1+x) e^{-\frac{Kx}{\beta^{(m)}}} dx. \quad (20) \end{aligned}$$

Applying the result in [16] and by the change of variable $u = t - (K/\beta^{(m)})$, (20) simplifies to

$$E[C_{\text{MC}}] = \log_2 e \cdot \sum_{m=1}^M \beta^{(m)} \int_0^{\infty} \frac{e^{-u}}{1 + \frac{\beta^{(m)}u}{K}} du. \quad (21)$$

As $K \rightarrow \infty$, (21) becomes

$$\lim_{K \rightarrow \infty} E[C_{\text{MC}}] = \log_2 e \cdot \sum_{m=1}^M \beta^{(m)}. \quad (22)$$

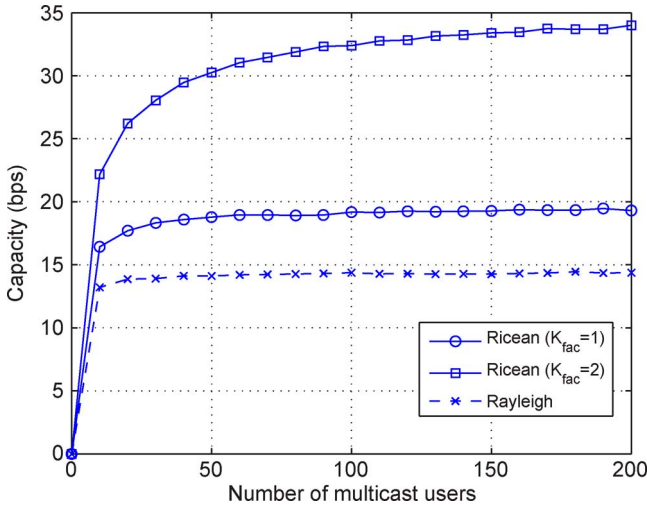


Fig. 7. Ergodic multicast capacity as a function of group size (all users are assumed to have an identical K_{fac}).

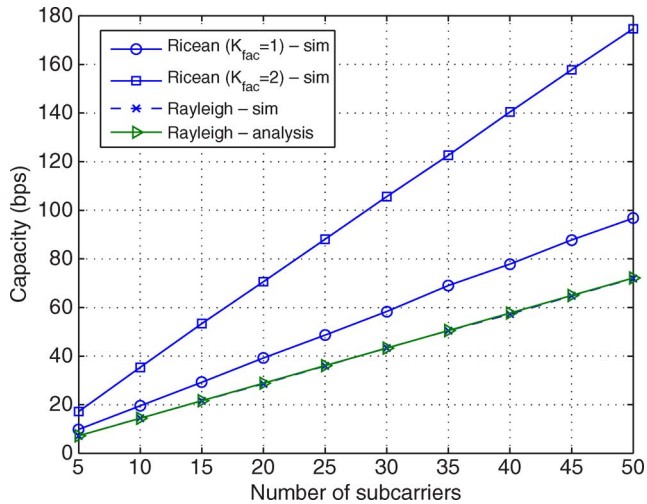


Fig. 8. Ergodic multicast capacity as a function of the number of subcarriers (all users are assumed to have an identical K_{fac}).

If we further assume that $\beta^{(m)} = \bar{\beta}, \forall m$, then (22) evaluates to

$$\lim_{K \rightarrow \infty} E[C_{MC}] = \log_2 e \cdot M \cdot \bar{\beta}. \quad (23)$$

This completes the proof. ■

For Ricean fading channels, analyzing the ergodic multicast capacity is challenging since the Ricean distribution involves the modified Bessel function. Instead, we claim that a similar result applies for the case of Ricean fading and verify it with simulation results in the following.

Assuming that the average SNR is normalized to 1, Figs. 7 and 8 demonstrate the dependence of multicast ergodic capacity on the group sizes and on the number of subcarriers, respectively. As the group size K increases, the capacity becomes saturated and independent of K for multicast systems employing $M = 10$ subcarriers. However, the capacity of a multicast system with $K = 100$ users employing conventional transmission increases linearly with the number of subcarriers.

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