On the Diversity-Multiplexing Tradeoff of Multiuser Amplify & Forward Multihop Networks

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***Abstract*— We consider an amplify & forward multihop network with n single-antenna nodes in the source and destination stage each, as well as $n_{\mathcal{R}}^{(l)}$, $l \in \{1, \dots, L\}$, single-antenna relay nodes in the l-th relay stage. The relay gain allocation scheme proposed in [1] decouples this network into parallel single-input single-output (SISO) links between n source-destination pairs under certain conditions on the network topology. Hence, all degrees of freedom are achieved in a distributed fashion. We extend this as follows: for i.i.d. $\mathcal{CN}(0, 1)$ fading coefficients and SISO coding/decoding at source and destination nodes, we derive an upper-bound on the diversity-multiplexing tradeoff curve of the network. Moreover, we devise a scheme that we conjecture to achieve the bound, if the network can be decoupled and $L \geq 2$.**

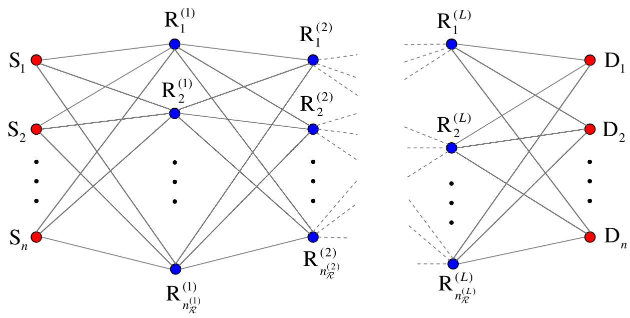

Fig. 1. Network Graph.

I. Introduction

The understanding of fundamental performance limits in wireless networks is one of the main foci of current wireless research. An insightful and popular performance measure in this context are degrees of freedom. This measure is particularly meaningful in the regime of high signal-to-noise ratio, where it is the key indicator for spectral efficiency.

As a prominent example for results of this kind, it has recently been shown in [2] that $n/2$ degrees of freedom are achievable in an n-user interference network with single-antenna terminals in sufficiently frequency selective or time variant environments. This insight has triggered a lot of research activity in the field, and in the meanwhile significant progress in the characterization of the ergodic capacity of such networks [3], [4] has been made.

The schemes that yield the above results require time variance and/or frequency selectivity in general. In contrast, our work is concerned with channels that are frequency flat and constant over time. In this case, the best known upper-bound on degrees of freedom of n-user interference networks is $n/2$ as for time varying/frequency selective channels [5], to the best of our knowledge. However, there is the conjecture that degrees of freedom are limited to one [5].

For channels that are constant both with respect to time and frequency, it is known that single-antenna relay nodes are a valuable means to establish n/p degrees of freedom in multihop interference networks with n nodes in the source and destination stage each [1][1], see Fig. 1. Here, $1/p$ is the pre-log due to the use of multiple time or frequency slots required for half-duplex relays or for orthogonalization of transmissions in adjacent hops. More specifically, these degrees of freedom are achieved for a certain class of network topologies through a coherent amplify & forward scheme. This scheme decouples the network into n parallel single-input single-output (SISO) links between the source-destination (S-D) pairs. We conjecture the subsequent conditions to be necessary (each) and sufficient (both conditions together) for the feasibility of this decoupling in a network with L relay stages and $n_{\mathcal{R}}^{(l)}$ nodes in the l-th relay stage [1]:

$$\sum_l n_{\mathcal{R}}^{(l)} \geq n^2 - n + L, \tag{1}$$

$$n_{\mathcal{R}}^{(l)} \geq n, \qquad \forall l \in \{1, \dots, L\}. \tag{2}$$

If the conjecture of [5] on degrees of freedom in n-user interference networks was true, our conjecture would imply that multihop interference networks can exhibit more degrees of freedom than single-hop networks do.

Since the capacity of slowly fading and frequency flat channels is formally zero for most common fading distributions, outage capacity/probability becomes the most striking performance measure in this case. In order to characterize performance at high signal-to-noise ratio, it is therefore essential not only to consider achievable degrees of freedom, but also achievable diversity, which determines outage behavior at high signal-to-noise ratio. Accordingly, our contribution takes the prior work [1] as a starting point and broadens the focus to not only studying degrees of freedom, but the full diversity-multiplexing tradeoff (DMT) curve [7].

Specifically, our contributions are summarized as follows:

- Under the assumption of i.i.d. $\mathcal{CN}(0, 1)$ fading coefficients, we develop an upper-bound on the achievable

[1]For the time varying and/or frequency selective case degrees of freedom of multiuser multihop networks have also been studied in [6].

DMT curve. The bound is found through a specific isolation of each hop in the network.

- We extend the network diagonalization scheme proposed in [1] by incorporating relay set selection and selection among relay gain allocations (from a specific finite set). We conjecture this scheme to achieve the upper-bound on the DMT, if the network is both diagonalizable and has more than two hops. This conjecture is supported by numerical evidence.

Notation: We use boldface lowercase and capital letters to indicate vectors and matrices, respectively. Alike, we use the notation $(a_{ij})_{i=1,\ldots m,j=1,\ldots n}$ for an $m \times n$ matrix $\mathbf{A}$. A diagonal matrix with vector $\mathbf{x}$ on its diagonal is denoted by $\mathrm{diag}(\mathbf{x})$. The probability/conditional probability of the event A is denoted by $\mathsf{P}[A]$ and $\mathsf{P}[A|\cdot]$, respectively.

II. Network Topology & System Model

We consider a coherent multiuser amplify & forward multihop network. The network consists of a source stage $\mathcal{S}$, a destination stage $\mathcal{D}$ and the relay stages $\mathcal{R}^{(l)}$, $l \in \{1, \ldots, L\}$. Relay stage $\mathcal{R}^{(l)}$ comprises the single-antenna relay nodes $\mathrm{R}_k^{(l)}$, $k \in \{1, \ldots, n_{\mathcal{R}}^{(l)}\}$. The n single-antenna nodes in $\mathcal{S}$ and $\mathcal{D}$ are matched to S-D pairs $(\mathrm{S}_i, \mathrm{D}_i)$, $i \in \{1, \ldots, n\}$ that wish to communicate over the same physical channel and make use of standard SISO codes. Signals are fed into the network by all source nodes simultaneously in every p-th time slot, traverse all L relay stages, and arrive at the destination nodes after $L + 1$ time slots. A sketch of the network graph is provided in Fig. 1. Channel coefficients between nodes are assumed to be quasi-static and i.i.d. $\mathcal{CN}(0, 1)$, if nodes are located in adjacent stages, and zero otherwise[2]. We denote by h_{IJ} the complex multiplicative fading coefficient that corresponds to the transmission from node J to node I, and by g_{R} the complex relay gain coefficient of relay R. We define the *effective* multiplicative fading coefficient $d_{\mathrm{D}_i\mathrm{S}_j}$ as the superposition of all paths between S_j and D_i in the network graph:

$$d_{\mathrm{D}_i\mathrm{S}_j} = \sum_{\substack{(\mathrm{R}_{k_1}^{(1)}, \ldots, \mathrm{R}_{k_L}^{(L)}) \\ \in \mathcal{R}^{(1)} \times \ldots \times \mathcal{R}^{(L)}}} h_{\mathrm{R}_{k_1}^{(1)}\mathrm{S}_j} h_{\mathrm{R}_{k_2}^{(2)}\mathrm{R}_{k_1}^{(1)}} \cdots h_{\mathrm{D}_i\mathrm{R}_{k_L}^{(L)}} g_{\mathrm{R}_{k_1}^{(1)}} \cdots g_{\mathrm{R}_{k_L}^{(L)}}.$$

For notational convenience we define the following matrices:

$$\mathbf{D} = \left(d_{\mathrm{D}_i\mathrm{S}_j}\right)_{i=1,\ldots,n;j=1,\ldots,n},$$

$$\mathbf{G}_l = \mathrm{diag}\left(\left(g_{\mathrm{R}_k^{(l)}}\right)_{k=1,\ldots,n_{\mathcal{R}}^{(l)}}\right),$$

$$\mathbf{H}_l = \begin{cases} \left(h_{\mathrm{R}_i^{(1)}\mathrm{S}_j}\right)_{i=1,\ldots,n_{\mathcal{R}}^{(1)};j=1,\ldots,n} & \text{if } l = 1, \\ \left(h_{\mathrm{R}_i^{(l)}\mathrm{R}_j^{(l-1)}}\right)_{i=1,\ldots,n_{\mathcal{R}}^{(l)};j=1,\ldots,n_{\mathcal{R}}^{(l-1)}} & \text{if } 2 \leq l \leq L, \\ \left(h_{\mathrm{D}_i\mathrm{R}_j^{(L)}}\right)_{i=1,\ldots,n,j=1,\ldots,n_{\mathcal{R}}^{(L)}} & \text{if } l = L + 1. \end{cases}$$

With this notation, the vector of received signals at the destination antennas (without noise) is obtained from the vector of source transmit signals through the linear map determined by the matrix $\mathbf{D} = \mathbf{H}_{L+1}\mathbf{G}_L\mathbf{H}_L \cdots \mathbf{G}_1\mathbf{H}_1$.

The receive signal of each relay and destination node is assumed to be distorted by additive white Gaussian noise of variance σ^2. We allocate a common transmit power $P_{\mathcal{S}}/n$ to all source nodes, where $P_{\mathcal{S}}$ is the sum-transmit power of $\mathcal{S}$. Relay nodes within a stage $\mathcal{R}^{(l)}$ are subject to a sum-power constraint $\sum_k P_{\mathcal{R}_k^{(l)}} \leq P_{\mathcal{R}^{(l)}}$ each, where $P_{\mathcal{R}_k^{(l)}}$ denotes the transmit power of relay $\mathrm{R}_k^{(l)}$.

[2]The minimum p that does not cause interference between signals that are injected into the network in different time slots is thus given by 3 for $L \geq 2$.

III. Reprise of Basic Concepts

A. Network Diagonalization

The concept of distributed network diagonalization as introduced in [1] is briefly reviewed. Here, the term “distributed” refers to the fact that cooperation among relay nodes in the same relay stage is disabled (in the sense of exchanging transmit symbols). The considered network is said to be diagonalized, if the effective multiple-input multiple-output (MIMO) channel from source to destination nodes decouples into n parallel SISO channels between the pairs $(\mathrm{S}_i, \mathrm{D}_i)$, i.e. if $\mathbf{D}$ is diagonal with nonzero entries on its diagonal. This corresponds to fulfilling the following conditions:

$$d_{\mathrm{D}_i\mathrm{S}_j} = 0 \text{ for all } (\mathrm{D}_i, \mathrm{S}_j) \in \mathcal{D} \times \mathcal{S} \text{ s.t. } i \neq j, \quad (3)$$

$$d_{\mathrm{D}_i\mathrm{S}_i} \neq 0 \text{ for all } i \in \{1, \ldots, n\}. \quad (4)$$

We conjecture that (3) and (4) can be fulfilled simultaneously, iff both conditions (1) and (2) are fulfilled. This conjecture is proved for the special cases (i) $n = 2$ and (ii) $L = 1$, and numerically confirmed for several topologies. Since the system of equations (3) is polynomial with respect to the unknown relay gain coefficients, there are, in general, multiple solutions to this system of equations even for the minimum feasible number of relays.

In the following we consider only networks that are diagonalizable and study such networks in terms of the achievable DMT [7].

B. Diversity-Multiplexing Tradeoff

The definition of the DMT according to [7] tailored to our network is as follows. We assume that each S-D pair makes use of the same set of SISO codes. For a specific value of the average receive signal-to-noise ratio SNR (averaged over channel realizations and S-D pairs) each source node chooses the same SISO code in a way such that the code rate R as a function of SNR fulfills

$$\lim_{\mathsf{SNR}\to\infty} \frac{R}{\log \mathsf{SNR}} \triangleq \frac{r}{n}.$$

The quantity r is referred to as multiplexing gain. The diversity achieved by the code set at multiplexing gain r is given by

$$-\lim_{\mathsf{SNR}\to\infty} \frac{\log \mathsf{P}\left[\cup_{i=1}^n E_i \;\middle|\; R, \mathsf{SNR}\right]}{\log \mathsf{SNR}} \triangleq d(r),$$

where E_i denotes the event of a maximum likelihood decoding error at destination node D_i. We refer to $d(r)$ as DMT curve. The quantity SNR is taken to infinity by assuming $P_{\mathcal{S}} = P_{\mathcal{R}^{(1)}} = \ldots = P_{\mathcal{R}^{(L)}} \triangleq P$ and taking P to infinity.

IV. Upper-Bound on DMT Curve

We develop $L+1$ upper-bounds $\bar{d}_l(r)$, $l \in \{1, \ldots, L+1\}$, on the achievable DMT curve $d(r)$. Each bound $\bar{d}_l(r)$ is obtained through a specific isolation of the l-th hop in the network. Eventually, we combine all these bounds into the bound

$$d(r) \leq \min_l \bar{d}_l(r) \triangleq \bar{d}(r).$$

To obtain the $\bar{d}_l(r)$, we apply the following relaxations, which for each value of the multiplexing gain r can only increase the DMT curve of the network $d(r)$:

- We neglect all noise in the network except for that introduced in the respective receive stage of hop l, i.e. in $\mathcal{R}^{(l)}$ if $l \leq L$, and in $\mathcal{D}$ if $l = L+1$.
- If $l > 1$, we replace the subnetwork from source stage $\mathcal{S}$ to relay stage $\mathcal{R}^{(l-1)}$ by an arbitrary linear map defined by the matrix $\mathbf{G}_t^{(l-1)} \in \mathbb{C}^{n_{\mathcal{R}}^{(l-1)} \times n}$ that fulfills the sum-power constraint on $\mathcal{R}^{(l-1)}$. Likewise, if $l \leq L$, we replace the subnetwork from relay stage $\mathcal{R}^{(l)}$ to the destination stage $\mathcal{D}$ by an arbitrary linear map defined by the matrix $\mathbf{G}_r^{(l)} \in \mathbb{C}^{n \times n_{\mathcal{R}}^{(l)}}$.

The second relaxation yields an upper-bound on the DMT, since we allow for an arbitrary linear processing on the transmit and/or receive side of the hop. That is, neither $\mathbf{G}_r^{(l)}$ needs to follow the structure $\mathbf{H}_{L+1}\mathbf{G}_L \cdots \mathbf{H}_2\mathbf{G}_l$ nor $\mathbf{G}_t^{(l)}$ needs to follow the structure $\mathbf{G}_l\mathbf{H}_l \cdots \mathbf{G}_1\mathbf{H}_1$. Note that in the physical network only the diagonal elements of the $\mathbf{G}_l$ can be varied in order to control the multiuser interference.

For the evaluation of the resulting upper-bounds, it turns out that three cases have to be distinguished. These are (i) $l = 1$, (ii) $l = L+1$ and (iii) $2 \leq l \leq L$.

Case $l = 1$: This case corresponds to the bound obtained through the isolation of the hop between $\mathcal{S}$ and $\mathcal{R}^{(1)}$. The subnetwork from $\mathcal{R}^{(1)}$ to $\mathcal{D}$ is replaced by an arbitrary linear map defined by the matrix $\mathbf{G}_r^{(1)} \in \mathbb{C}^{n \times n_{\mathcal{R}}^{(1)}}$. Thus, a single-hop network is obtained that corresponds to a MIMO multiple access scenario with a linear receive structure. A sketch of this network is depicted in Fig. 2a. The DMT curve of this network is derived in [8]. It is achieved through receive zero-forcing and given in terms of the function $x^+ \triangleq \max(0, x)$ by (taking again into account the pre-log $1/p$)

$$\bar{d}_1(r) = \left(n_{\mathcal{R}}^{(1)} - n + 1\right) \cdot \left(1 - \frac{p \cdot r}{n}\right)^+ .$$

Case $2 \leq l \leq L$: This case corresponds to the single-hop channels between any two adjacent relay stages $\mathcal{R}^{(l-1)}$ and $\mathcal{R}^{(l)}$. The subnetwork from $\mathcal{S}$ to $\mathcal{R}^{(l-1)}$ is replaced by a linear map determined by the matrix $\mathbf{G}_t^{(l-1)} \in \mathbb{C}^{n_{\mathcal{R}}^{(l-1)} \times n}$ that fulfills the power constraint on $\mathcal{R}^{(l-1)}$. Likewise, the subnetwork from $\mathcal{R}^{(l)}$ to $\mathcal{D}$ is replaced by a linear map determined by the matrix $\mathbf{G}_r^{(l)} \in \mathbb{C}^{n \times n_{\mathcal{R}}^{(l)}}$. Thus, a single-hop channel is obtained that corresponds to a point-to-point MIMO link with joint linear transmit and receive beamforming. A sketch of this channel is depicted in Fig. 2b. To obtain a suitable upper-bound on the DMT curve, we must insist on n spatial streams supporting a rate $r/n \log \mathsf{SNR}$ each. The

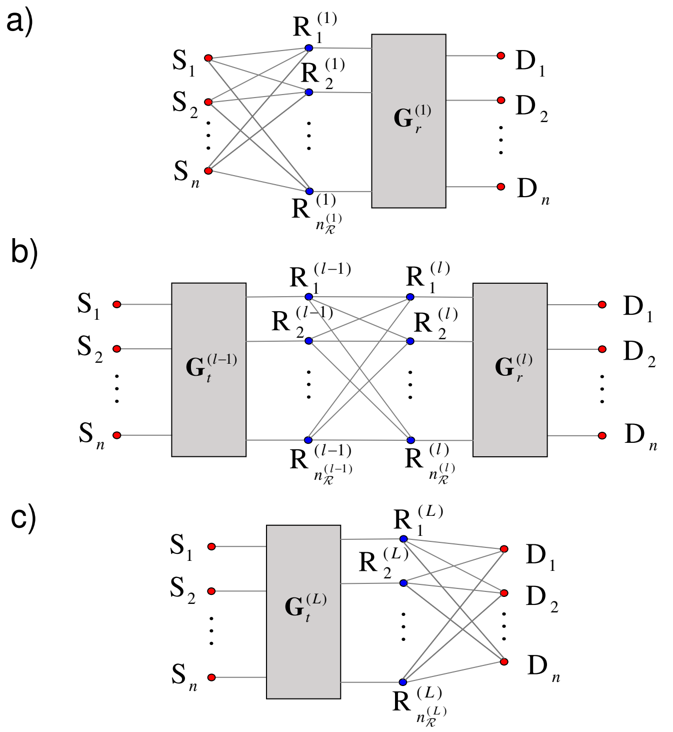

Fig. 2. Receive (a), joint transmit/receive (b), transmit (c) beamforming.

DMT curve of this channel is achieved by diagonalization via singular value decomposition and equalization of the signal-to-noise ratios through power loading. It is given by (proof omitted due to space constraints)

$$\bar{d}_l(r) = \left(n_{\mathcal{R}}^{(l)} - n + 1\right) \cdot \left(n_{\mathcal{R}}^{(l-1)} - n + 1\right) \cdot \left(1 - \frac{p \cdot r}{n}\right)^+ .$$

Case $l = L+1$: This case corresponds to the bound obtained through the isolation of the hop between $\mathcal{R}^{(L)}$ and $\mathcal{D}$. The subnetwork from $\mathcal{S}$ to $\mathcal{R}^{(L)}$ is replaced by an arbitrary linear map defined by the matrix $\mathbf{G}_t^{(L)} \in \mathbb{C}^{n_{\mathcal{R}}^{(L)} \times n}$ that fulfills the power constraint on $\mathcal{R}^{(L)}$. Thus, the actual network is transformed into a single-hop network that corresponds to a MIMO broadcast scenario with a linear transmitter whose n spatial streams are constrained to sum-power P. A sketch of this network is depicted in Fig. 2c. We can argue based on the uplink-downlink duality [9] that the sets of achievable signal-to-interference-plus-noise ratios coincide with those of the MIMO multiple access scenario from the previous case, when the per node power constraints of the source nodes are relaxed to a sum-power constraint. Accordingly, we can consider an equivalent multiple-access problem and (analogously to the derivation of $\bar{d}_1$) infer from [8][3] that the corresponding DMT curve is achieved through transmit zero-forcing and given by

$$\bar{d}_{L+1}(r) = \left(n_{\mathcal{R}}^{(L)} - n + 1\right) \cdot \left(1 - \frac{p \cdot r}{n}\right)^+ .$$

[3] In [8] it is considered to allocate equal transmit power to each of the n spatial streams due to the absence of transmit channel state information. However, it can be shown that relaxing these per node power constraints to a sum-power constraint does not affect the achievable DMT.

V. Achievability: Two Approaches

Assuming that the obtained upper-bound on the DMT curve is achievable appears to be overly optimistic at first glance. The derivation not only ignores most of the noise in the network, it also assumes full cooperation among relay nodes within the same stage. Nevertheless, we are confident of this being feasible, if both of the following conditions are fulfilled:

- the network is diagonalizable, i.e. (1) and (2) are fulfilled,
- the network extends over at least three hops, i.e. $L \geq 2$.

Subsequently, we distinguish three different cases. We conjecture that the upper-bound can be achieved in the first two cases and devise corresponding schemes.

1) Case $L \geq n$: In this case n relay nodes in each stage suffice for diagonalizing the network according to (1) and (2). We devise a relay set selection algorithm that tests $\bar{d}(0)$ (an integer that corresponds to the diversity order for constant code rates) different relay sets. Each relay stage contributes exactly n out of its $n_{\mathcal{R}}^{(l)}$ relays to each test set. In every test cycle we obtain a network diagonalizing gain allocation by fixing $\mathbf{G}_l \propto \mathbf{I}_n$ in each of the stages $\mathcal{R}^{(1)}, \dots, \mathcal{R}^{(L-n)}$ such that the per stage sum-power constraints are fulfilled and using the gain coefficients of the n relay nodes in each of the stages $\mathcal{R}^{(L-n+1)}, \dots, \mathcal{R}^{(L)}$ to solve the resulting system (3). We randomly pick a single out of the finitely many solutions (that fulfill the power constraint) as potential relay gain allocation and test it with respect to the signal-to-noise ratio of the weakest S-D pair. Eventually, the best among the tested relay sets is scheduled for data transmission. The assembling of the test sets is designed according to the following criterion.

Design Criterion: In test cycle m, the corresponding relay test set must fulfill the following property for each of the n S-D pairs: there exists a path between source and destination node whose $L+1$ fading coefficients h_{IJ} form a set that is disjoint to the set of fading coefficients involved in any of the prior test cycles $m' < m$.

We propose a relay set selection algorithm that fulfills this criterion (proof omitted due to space constraints). It is formulated in pseudo-code in Algorithm 1. A corresponding illustration is provided in Fig. 3. We distinguish relay stages with odd and even index l. For the stages with odd indices the algorithm uses the initial test set $\{\mathrm{R}_1^{(l)}, \dots, \mathrm{R}_n^{(l)}\}$ in cycle one. In the following, we remove the relay with smallest index from the current test set and add the relay whose index is the next larger compared to the largest index in the current set, to obtain the next test set each. The algorithm keeps on proceeding this way until it either arrives in test cycle $\bar{d}(0)$ or reaches the set $\{\mathrm{R}_{n_{\mathcal{R}}^{(l)}-n+1}, \dots, \mathrm{R}_{n_{\mathcal{R}}^{(l)}}^{(l)}\}$. If test cycle $\bar{d}(0)$ is not yet reached at this point, the algorithm starts over with the first test set.

Also for the stages with even indices the initial test set is given by $\{\mathrm{R}_1^{(l)}, \dots, \mathrm{R}_n^{(l)}\}$. However, this test set is kept in the next test cycle, unless a test set in an adjacent stage (either a relay stage with odd index or the destination stage) is starting over with its first test set. Thereby, the destination stage is treated like a relay stage that starts over in every test cycle. The algorithm introduces the notation

Algorithm 1 Relay Set Selection

$m_1 = n_{\mathcal{R}}^{(1)} - n + 1$, $m_{L+1} = n_{\mathcal{R}}^{(L)} - n + 1$
for $l = 2, \dots, L$ **do**
  $m_l = (n_{\mathcal{R}}^{(1)} - n + 1)(n_{\mathcal{R}}^{(2)} - n + 1)$
end for
$M = \min_l m_l$
$n_{\mathcal{R}}^{(L+1)} \triangleq n$
$\mathsf{SNR}_{\mathrm{th}} = 0$
$k(l) = 0 \;\forall\; l \in \{1, \dots, L\}$
for $i = 1$ to M **do**
  for $l = 1, 3, 5, \dots$ **do**
    $k(l) = \left((i-1) \bmod (n_{\mathcal{R}}^{(l)} - n + 1)\right) + 1$
  end for
  for $l = 2, 4, 6, \dots$ **do**
    $k(l) = \left\lceil i / \min(n_{\mathcal{R}}^{(l-1)} - n + 1, n_{\mathcal{R}}^{(l+1)} - n + 1) \right\rceil$
  end for
  $\mathcal{R}_{\mathrm{tmp}} = \left\{ \left\{ \mathrm{R}_{k(1)}^{(1)}, \dots, \mathrm{R}_{k(1)+n-1}^{(1)} \right\}, \left\{ \mathrm{R}_{k(2)}^{(2)}, \dots \mathrm{R}_{k(2)+n-1}^{(2)} \right\}, \dots, \left\{ \mathrm{R}_{k(L)}^{(L)}, \dots, \mathrm{R}_{k(L)+n-1}^{(L)} \right\} \right\}$
  compute gain coefficients for $\mathcal{R}_{\mathrm{tmp}}$
  for $j = 1$ to n **do**
    SNR_j = signal-to-noise ratio at D_j
  end for
  if $\min_j \mathsf{SNR}_j \geq \mathsf{SNR}_{\mathrm{th}}$ **then**
    $\mathsf{SNR}_{\mathrm{th}} = \min_j \mathsf{SNR}_j$
    $\mathcal{R} = \mathcal{R}_{\mathrm{tmp}}$
  end if
end for
return $\mathcal{R}$

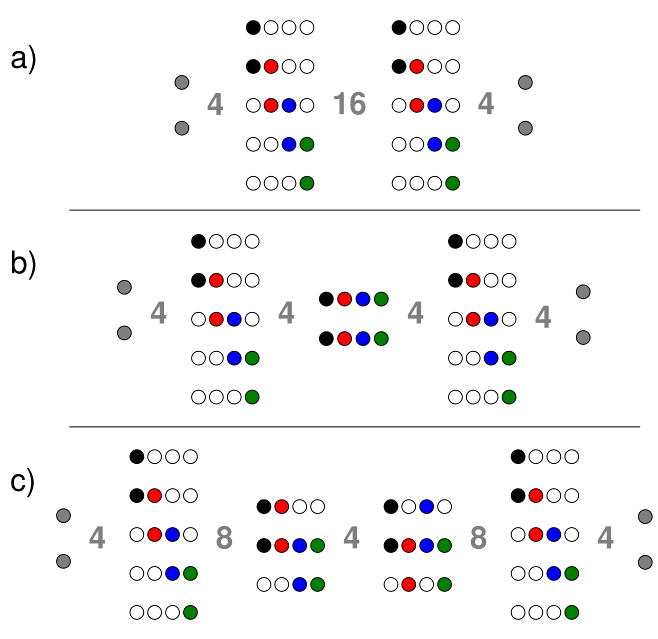

Fig. 3. Relay set selection algorithm applied to different topologies with $\bar{d}(0) = 4$: $2 \times 5 \times 5 \times 2$ (a), $2 \times 5 \times 2 \times 5 \times 2$ (b), $2 \times 5 \times 3 \times 3 \times 5 \times 2$ (c). Selected relays: black (cycle 1), red (cycle 2), blue (cycle 3), green (cycle 4). Bounds $\bar{d}_l(0)$ are indicated in grey.

$n_{\mathcal{R}}^{(L+1)} = n$ for the number of nodes in the destination stage in this context. Relay stages with even indices update their test set according to the same procedure as stages with even index. Since for relay stages with even indices $\bar{d}(0)/\min\{n_{\mathcal{R}}^{(l-1)} - n + 1, n_{\mathcal{R}}^{(l+1)} - n + 1\} \leq n_{\mathcal{R}}^{(l)} - n + 1$, the algorithm changes the test set $n_{\mathcal{R}}^{(l)} - n + 1$ times at most in these stages. Thus, they never start over with the first set.

We have applied the algorithm to several topologies with $n = 2$ and $L \in \{2, 3, 4\}$ and numerically evaluated the corresponding diversity performance at multiplexing gain $r = 0$. We observed that the upper-bound $\bar{d}(0)$ appears to be achieved in all examined cases. Due to space constraints we restrict us to providing outage performance graphs for a network with configuration $2 \times n_{\mathcal{R}} \times 2 \times n_{\mathcal{R}} \times 2$, $n_{\mathcal{R}} \in \{2, 3, 4, 5\}$. The upper-bound on diversity at $r = 0$ is given by $\bar{d}(0) = n_{\mathcal{R}} - 1$. For $n_{\mathcal{R}} = 5$ the tested relay sets are depicted in Fig. 3a. In the simulation we fix $R = 1$ bit/channel use ($r = 0$). The outage probability versus SNR curves are shown in Fig. 4. The asymptotic slope of the curves in log-log scale appears to tend to $1 - n_{\mathcal{R}}$, which corresponds to the bound.

2) Case $1 < L < n$: In the previous case we have assumed that the network extends over $n + 1$ hops at least. This was the key for the identification of $\bar{d}(0)$ relay sets that fulfill the design criterion. If $L < n$, a relay set selection algorithm with $\bar{d}(0)$ different test sets that fulfills the design criterion does not exist (proof omitted). We therefore introduce selection among network diagonalizing relay gain allocations in minimum-configurations, which fulfill (1) with equality. As a complement to relay set selection this scheme might close the persisting diversity gap. Multiple such relay gain allocations exist, whenever $L \geq 2$, i.e the system of equations (3) is nonlinear. Generally, the number of solutions seems to increase rapidly as the network dimensions grow (either in L or in n) [1] and is not a scarce resource even for small networks. Let us consider a $3 \times 4 \times 4 \times 3$-network for instance. The DMT upper-bound for this network evaluates to $d(r) \leq 2 \cdot (1 - p \cdot r/n)^+$ and $\bar{d}(0) = 2$. Since all relay nodes in both relay stages are indispensable for diagonalizing the network in this case, relay set selection is not an option for meeting the upper-bound. Our approach is to test two (randomly chosen out of the conjectured twelve) different solutions with respect to the achievable rate of the weakest of the three S-D pairs, and to choose the better of these solutions for data transmission. A respective numerical experiment shows that the outage probability versus SNR curve at $r = 0$ in log-log scale (not shown due to space constraints) approaches slope -2 for large values of SNR and thus confirms the effectiveness of the approach. We could imagine that the combination of gain allocation selection and relay set selection, i.e. adding a gain allocation selection step to each relay set selection step, suffices for achieving the upper-bound, whenever $1 < L < n$.

3) Case $L = 1$: The two approaches above do not suffice in order to attain the upper-bound. Since a minimum of $n(n-1) + 1$ relays is required for diagonalizing the network, a relay set selection algorithm that fulfills the design criterion

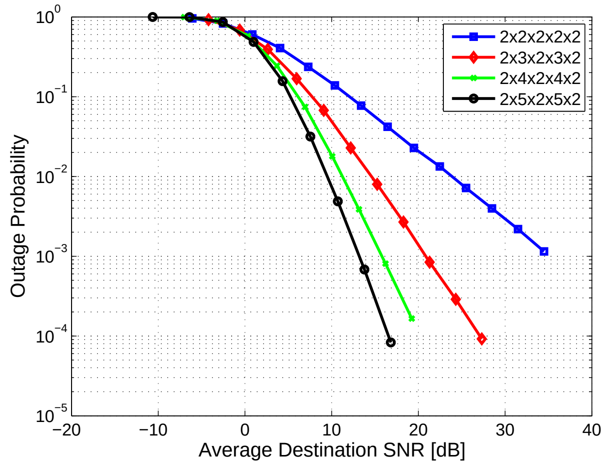

Fig. 4. Outage probability versus SNR for various four-hop networks. The code rate is fixed to $R = 1$ bit/channel use which corresponds to $r = 0$.

cannot comprise more than $n_{\mathcal{R}}^{(1)} - n(n-1)$ test sets. Moreover, the network diagonalizing gain allocation in a minimum-configuration is unique, since the system (3) is linear. Thus, selection among different solutions is not feasible. The number $n_{\mathcal{R}}^{(1)} - n(n-1) < \bar{d}(0)$ coincides with the dimension of the subspace of network diagonalizing gain vectors, and we conjecture that the upper-bound is not achievable.

VI. Conclusion

This paper provides evidence that in terms of the DMT $n_{\mathcal{R}}^{(l)}$ distributed single-antenna relay nodes in a stage achieve the performance of a relay node with $n_{\mathcal{R}}^{(l)}$ collocated antennas in multiuser amplify & forward multihop networks. Proving achievability of the provided upper-bound strikes us as being a difficult task. The key challenge comes along with the dependence of the relay gain coefficients on the channel state.

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