MMSE-Based Precoder Design in Nonregenerative Relay Systems with Direct Link

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***Abstract*—We consider an amplify-and-forward multiple-input multiple-output relay system with a direct source-destination link. We adopt the minimum-mean-square-error (MMSE) criterion at the destination. The problem of interest is to jointly design source and relay precoders so as to minimize mean square error of transmitted symbols under total power constraints at the source and relay nodes. We propose a method which diagonalizes the MSE matrix using singular value decomposition (SVD) and generalized SVD techniques. The proposed approach based on this diagonalized MSE matrix is suboptimal and aims to reduce the design complexity of the precoders. The solution can be obtained via an iterative water-filling technique. Simulations results show the performance advantages of the proposed approach.**

I. Introduction

The cooperative relay system has attracted much attention in recent years since it provides the advantages such as extended cell coverage and improved reliability through the cooperation of the relays [1], [2]. By incorporating the multiple-input multiple-output (MIMO) technology, the system can further realize the spatial diversity and improve the spectral efficiency. In MIMO relay systems, many research works [3]–[6] focused on the amplify-and-forward (AF) strategy, in which received signals at relay nodes are simply amplified without decoding, due to its implementation simplicity and small processing delay.

Most AF MIMO relay systems mentioned above did not consider the direct (source-destination) link in the problem formulation for simple design of precoders. However, the joint consideration of the relay and direct links is able to offer additional performance gain by employing diversity combining and thus should not be neglected. Recently, many works [7]–[12] studied precoder design by considering the direct link. Among them, some works concentrated on linear precoder design only at the relay nodes using the minimum-mean-square-error (MMSE) criterion at the destination [8], [12]. Others addressed the joint design of source-relay precoders for minimizing MSE [7], [10] or maximizing signal-to-interference-plus-noise ratio (SINR) [9], [11].

In this work, we consider the joint source-relay precoder design for the AF MIMO three-node relay system based on the MMSE criterion. Our work is motivated by the work [7] in which the design of precoders aims to diagonalize the MSE matrix based on singular value decomposition (SVD) technique to obtain a tractable MSE upper bound. By minimizing the upper bound under total power constraints at the source and relay nodes, the problem is solvable and a suboptimal closed-form solution can be obtained. However, the performance in [7] decreases as the signal power from the direct link is larger than that from the relay links at the destination node. To overcome this problem, we propose a method that designs precoders based on SVD as well as generalized singular value decomposition (GSVD) techniques to obtain a diagonalized MSE matrix. Simulation results demonstrate that the proposed method indeed improves the performance, particularly when the signal power from the direct link is large.

The rest of this paper is organized as follows. In Section II, we describe the model of the three-node relay system and the problem we address. In Section III, we solve the problem by choosing particular structures of designed matrices so that a suboptimal solution is obtained. Simulation results are given in Section IV. Section V briefly concludes this work.

Notations: Throughout this paper, the following notations are used. A lower case letter denotes a scalar, a boldface lower case letter denotes a vector, and a boldface uppercase letter denotes a matrix. In addition, $\mathbf{A}^T$ and $\mathbf{A}^H$ denote the transpose of $\mathbf{A}$ and the conjugate transpose of $\mathbf{A}$, respectively. The letter $\mathbf{I}$ and $\mathbf{0}$ denote, respectively, an identity matrix and a zero matrix. The operator $\mathrm{diag}(x_1, \cdots, x_M)$ is a diagonal matrix with its mth diagonal element equal to x_m and $\mathrm{tr}(\mathbf{A})$ is the trace of $\mathbf{A}$.

II. System Model and Problem Formulation

We consider a three-node MIMO relay system, as shown in Fig. 1, where the source, relay, and destination nodes are equipped with N, L, and M antennas, respectively. We adopt the AF transmission strategy with a two-phase transmission scheme. In the first phase, the source signal vector $\mathbf{x} \in \mathbb{C}^p$ is transmitted to the relay and destination nodes after multiplying by a precoding matrix $\mathbf{F} \in \mathbb{C}^{N\times p}$. The received signals at the relay and destination are, respectively, $\mathbf{y}_r$ and $\mathbf{y}_1$, which can be written as

$$\mathbf{y}_r = \mathbf{H}_{sr}\mathbf{F}\mathbf{x} + \mathbf{v}_r \quad \text{and} \quad \mathbf{y}_1 = \mathbf{H}_{sd}\mathbf{F}\mathbf{x} + \mathbf{v}_{d,1}, \qquad (1)$$

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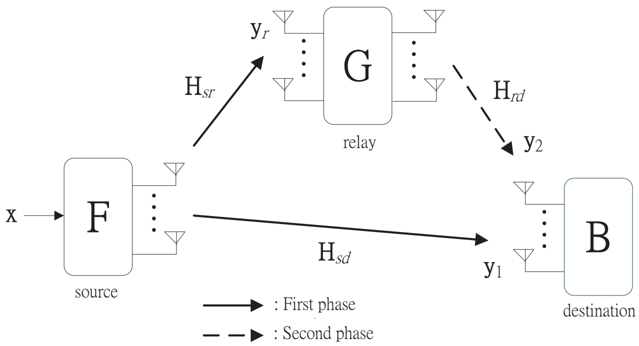

Fig. 1. AF MIMO three-node relay system

where $\mathbf{H}_{sr} \in \mathbb{C}^{L\times N}$ and $\mathbf{H}_{sd} \in \mathbb{C}^{M\times N}$ are the source-relay and source-destination channel matrices; $\mathbf{v}_r \in \mathbb{C}^L$ and $\mathbf{v}_{d,1} \in \mathbb{C}^M$ are the additive noise vectors at the relay and the destination. In the second phase, the received signal $\mathbf{y}_r$ at the relay node is weighted by an amplifying matrix $\mathbf{G} \in \mathbb{C}^{L\times L}$ before sending to the destination. The received signal at the destination in the second phase is

$$\mathbf{y}_2 = \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{F}\mathbf{x} + \mathbf{H}_{rd}\mathbf{G}\mathbf{v}_r + \mathbf{v}_{d,2}, \qquad (2)$$

where $\mathbf{H}_{rd} \in \mathbb{C}^{M\times L}$ is the relay-destination channel matrix and $\mathbf{v}_{d,2} \in \mathbb{C}^M$ is the additive noise vector. By stacking two received vectors at the destination, we have

$$\begin{aligned}\mathbf{y} &= \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{H}_{sd}\mathbf{F} \\ \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{F} \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{v}_{d,1} \\ \mathbf{H}_{rd}\mathbf{G}\mathbf{v}_r + \mathbf{v}_{d,2} \end{bmatrix}. \qquad (3)\end{aligned}$$

The problem formulation of interest is under the settings: (i) $E[\mathbf{x}] = \mathbf{0}$ and $E[\mathbf{x}\mathbf{x}^H] = \sigma_x^2\mathbf{I}$, (ii) $E[\mathbf{v}_r] = \mathbf{0}$ and $E[\mathbf{v}_r\mathbf{v}_r^H] = \sigma_r^2\mathbf{I}$, (iii) $E[\mathbf{v}_{d,i}] = \mathbf{0}$, $E[\mathbf{v}_{d,i}\mathbf{v}_{d,i}^H] = \sigma_d^2\mathbf{I}$, and $E[\mathbf{v}_{d,i}\mathbf{v}_{d,j}^H] = \mathbf{0}$ for $i \neq j$, (iv) $\mathbf{x}$, $\mathbf{v}_r$, and $\mathbf{v}_{d,i}$ are uncorrelated, and (v) $p \le N$ and $N \le \min(M, L)$.

At the destination, we recover the source signal based on $\mathbf{y}$ in (3) using a linear equalizer $\mathbf{B} \in \mathbb{C}^{2M\times p}$. The equalizer is designed to minimize the MSE

$$J = E\left[\|\mathbf{B}^H\mathbf{y} - \mathbf{x}\|^2\right]. \qquad (4)$$

For given $\mathbf{F}$ and $\mathbf{G}$, it is known that the optimal solution is the Wiener filter which is given by [13]

$$\mathbf{B} = \sigma_x^2\left(\sigma_x^2\bar{\mathbf{H}}\bar{\mathbf{H}}^H + \mathbf{R}_v\right)^{-1}\bar{\mathbf{H}}, \qquad (5)$$

where

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{sd}\mathbf{F} \\ \mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{F} \end{bmatrix} \text{ and}$$
$$\mathbf{R}_v = \begin{bmatrix} \sigma_d^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_r^2\mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H + \sigma_d^2\mathbf{I} \end{bmatrix},$$

and the corresponding MSE is

$$J_{mmse} = \mathrm{tr}\{\mathbf{E}\} \qquad (6)$$

with the MSE matrix

$$\begin{aligned}\mathbf{E} = \Big(&\frac{1}{\sigma_x^2}\mathbf{I} + \frac{1}{\sigma_d^2}\mathbf{F}^H\mathbf{H}_{sd}^H\mathbf{H}_{sd}\mathbf{F} + \mathbf{F}^H\mathbf{H}_{sr}^H\mathbf{G}^H\mathbf{H}_{rd}^H \\ &\left(\sigma_r^2\mathbf{H}_{rd}\mathbf{G}\mathbf{G}^H\mathbf{H}_{rd}^H + \sigma_d^2\mathbf{I}\right)^{-1}\mathbf{H}_{rd}\mathbf{G}\mathbf{H}_{sr}\mathbf{F}\Big)^{-1}. \qquad (7)\end{aligned}$$

The problem is to minimize the MSE by the design of the precoding matrix $\mathbf{F}$ and the amplifying matrix $\mathbf{G}$ under total power constraints at the source and relay nodes. The transmitted powers at the source and relay nodes are defined as $E[\|\mathbf{F}\mathbf{x}\|^2] = \sigma_x^2\mathrm{tr}\left\{\mathbf{F}\mathbf{F}^H\right\}$ and $E[\|\mathbf{G}\mathbf{y}_r\|^2] = \mathrm{tr}\left\{\mathbf{G}(\sigma_x^2\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^H\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I})\mathbf{G}^H\right\}$, respectively. If P_s and P_r are the total powers that the source and relay nodes can use, the constraints can be expressed as

$$\begin{array}{ll}\text{source:} & \sigma_x^2\mathrm{tr}\left\{\mathbf{F}\mathbf{F}^H\right\} \le P_s \\ \text{relay:} & \mathrm{tr}\left\{\mathbf{G}(\sigma_x^2\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^H\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I})\mathbf{G}^H\right\} \le P_r.\end{array} \qquad (8)$$

From (6) and (8), the optimization problem can be expressed as

$$\begin{cases} \min\limits_{\mathbf{F},\mathbf{G}} & \mathrm{tr}\{\mathbf{E}\} \\ \text{s.t.} & \sigma_x^2\mathrm{tr}\left\{\mathbf{F}\mathbf{F}^H\right\} \le P_s \\ & \mathrm{tr}\left\{\mathbf{G}(\sigma_x^2\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^H\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I})\mathbf{G}^H\right\} \le P_r. \end{cases} \qquad (9)$$

We note that when $\mathbf{F}$ and $\mathbf{G}$ are obtained, the linear equalizer $\mathbf{B}$ can be evaluated by (5).

III. Proposed Method

Motivated by the capacity achieving linear transceiver design in the three-node relay system, we propose a method that constrains structures of the precoding matrix $\mathbf{F}$ and the amplifying matrix $\mathbf{G}$ so that the MSE matrix in (7) can be diagonalized. Based on the diagonalized MSE matrix, the objective function in (9) has a simple closed form expression and thus the optimization problem can be solved efficiently.

To diagonalize the MSE matrix, we consider to decompose the channel matrices based on singular value decomposition (SVD) as well as generalized singular value decomposition (GSVD) [14] techniques. Specifically, we first express the source-relay channel matrix $\mathbf{H}_{sr}$ as SVD

$$\mathbf{H}_{rd} = \mathbf{U}_{rd}\mathbf{\Lambda}_{rd}\mathbf{V}_{rd}^H, \qquad (10)$$

where $\mathbf{U} \in \mathbb{C}^{M\times M}$ and $\mathbf{V} \in \mathbb{C}^{L\times L}$ are unitary matrices, and $\mathbf{\Lambda}_{rd} \in \mathbb{R}^{M\times L}$ is a nonnegative diagonal matrix with its diagonal elements $\lambda_{rd,i}$, $i = 1, \cdots, k$, and $k = \min(M, L)$. Then, we express the source-destination channel matrix $\mathbf{H}_{sd}$ and the source-relay channel matrix $\mathbf{H}_{sr}$ as GSVD

$$\mathbf{H}_{sr} = \mathbf{U}_{sr}\mathbf{\Lambda}_{sr}\mathbf{X}^H \qquad (11)$$
$$\mathbf{H}_{sd} = \mathbf{U}_{sd}\mathbf{\Lambda}_{sd}\mathbf{X}^H, \qquad (12)$$

where $\mathbf{U}_{sr} \in \mathbb{C}^{L\times L}$ and $\mathbf{U}_{sd} \in \mathbb{C}^{M\times M}$ are unitary matrices, and $\mathbf{X} \in \mathbb{C}^{N\times N}$ is a nonsingular matrix. By assumption (v), since $N \le \min(M, L)$, the diagonal matrices in (11) and (12) have the following forms: $\mathbf{\Lambda}_{sr} = [\tilde{\mathbf{\Lambda}}_{sr}\ \mathbf{0}]^T \in \mathbb{R}^{L\times N}$ and $\mathbf{\Lambda}_{sd} = [\tilde{\mathbf{\Lambda}}_{sd}\ \mathbf{0}]^T \in \mathbb{R}^{M\times N}$. It should be noted that one of the most important properties of GSVD is

$$\tilde{\mathbf{\Lambda}}_{sr}^T\tilde{\mathbf{\Lambda}}_{sr} + \tilde{\mathbf{\Lambda}}_{sd}^T\tilde{\mathbf{\Lambda}}_{sd} = \mathbf{I}, \qquad (13)$$

where $\tilde{\mathbf{\Lambda}}_{sr} = \mathrm{diag}(\lambda_{sr,1}, \cdots, \lambda_{sr,N})$ with $1 \geq \lambda_{sr,1} \geq \cdots \geq \lambda_{sr,N} > 0$ and $\tilde{\mathbf{\Lambda}}_{sd} = \mathrm{diag}(\lambda_{sd,1}, \cdots, \lambda_{sd,N})$ with $0 < \lambda_{sr,1} \leq \cdots \leq \lambda_{sr,N} \leq 1$.

From (7), we see that the MSE matrix $\mathbf{E}$ can be diagonalized when we simultaneously diagonalize the second and third terms in the inverse parentheses on the right hand side. From (12), the second term of the MSE matrix can be rewritten as

$$\mathbf{F}^H \mathbf{H}_{sd}^H \mathbf{H}_{sd} \mathbf{F} = \mathbf{F}^H \mathbf{X} \mathbf{\Lambda}_{sd}^H \mathbf{\Lambda}_{sd} \mathbf{X}^H \mathbf{F}. \tag{14}$$

It is easy to see that (14) is diagonalized if we choose

$$\mathbf{F} = \mathbf{X}^{-H} \mathbf{\Lambda}_s, \tag{15}$$

where $\mathbf{\Lambda}_s \in \mathbb{R}^{N \times p}$ is a diagonal matrix with its diagonal elements $\lambda_{s,i}$, $i = 1, \cdots, p$, which should be determined so that the power constraint at the source node is satisfied. From (10), (11), and (15), the third term of the MSE matrix can be expressed as

$$\begin{aligned} &\mathbf{F}^H \mathbf{H}_{sr}^H \mathbf{G}^H \mathbf{H}_{rd}^H \left(\sigma_r^2 \mathbf{H}_{rd} \mathbf{G} \mathbf{G}^H \mathbf{H}_{rd}^H + \sigma_d^2 \mathbf{I}\right)^{-1} \mathbf{H}_{rd} \mathbf{G} \mathbf{H}_{sr} \mathbf{F} \\ =&\mathbf{\Lambda}_s \mathbf{\Lambda}_{sr}^H \mathbf{U}_{sr}^H \mathbf{G}^H \mathbf{V}_{rd} \mathbf{\Lambda}_{rd}^H \left(\sigma_r^2 \mathbf{\Lambda}_{rd} \mathbf{V}_{rd}^H \mathbf{G} \mathbf{G}^H \mathbf{V}_{rd} \mathbf{\Lambda}_{rd}^H + \sigma_d^2 \mathbf{I}\right)^{-1} \\ &\mathbf{\Lambda}_{rd} \mathbf{V}_{rd}^H \mathbf{G} \mathbf{U}_{sr} \mathbf{\Lambda}_{sr} \mathbf{\Lambda}_s. \end{aligned} \tag{16}$$

It can be shown that (16) is diagonalized if we choose

$$\mathbf{G} = \mathbf{V}_{rd} \mathbf{\Lambda}_r \mathbf{U}_{sr}^H, \tag{17}$$

where $\mathbf{\Lambda}_r \in \mathbb{R}^{L \times L}$ is a diagonal matrix with its diagonal elements $\lambda_{r,i}$, $i = 1, \cdots, L$, which is to be determined. Based on (15) and (17), the MSE matrix is diagonalized and the objective function in (9) can be expressed as

$$\begin{aligned} \mathrm{tr}\{\mathbf{E}\} =&\mathrm{tr}\left\{\left(\left(\frac{1}{\sigma_s^2}\mathbf{I} + \frac{1}{\sigma_d^2}\mathbf{\Lambda}_s^H \mathbf{\Lambda}_{sd}^H \mathbf{\Lambda}_{sd} \mathbf{\Lambda}_s + \mathbf{\Lambda}_s^H \mathbf{\Lambda}_{sr}^H \mathbf{\Lambda}_r^H \mathbf{\Lambda}_{rd}^H \right.\right.\right. \\ &\left.\left.\left. \left(\sigma_r^2 \mathbf{\Lambda}_{rd} \mathbf{\Lambda}_r \mathbf{\Lambda}_r^H \mathbf{\Lambda}_{rd}^H + \sigma_d^2 \mathbf{I}\right)^{-1} \mathbf{\Lambda}_{rd} \mathbf{\Lambda}_r \mathbf{\Lambda}_{sr} \mathbf{\Lambda}_s\right)\right)^{-1}\right\}. \end{aligned} \tag{18}$$

It should be noted that since (18) is obtained by choosing particular structures of $\mathbf{F}$ and $\mathbf{G}$, it can be considered as an upper bound of the true minimum MSE. By minimizing the upper bound under power constraints, the result can be regarded as a suboptimal solution. From (15) and (17), the source power constraint can be rewritten as

$$\sigma_x^2 \mathrm{tr}\left\{\mathbf{F}\mathbf{F}^H\right\} = \sigma_x^2 \mathrm{tr}\left\{\mathbf{\Lambda}_s \mathbf{\Lambda}_s^H \mathbf{S}\right\}, \tag{19}$$

where $\mathbf{S} = (\mathbf{X}\mathbf{X}^H)^{-1}$ with its diagonal elements s_{ii}, $i = 1, \cdots, N$, and the relay power constraint is

$$\begin{aligned} &\mathrm{tr}\left\{\mathbf{G}(\sigma_x^2 \mathbf{H}_{sr} \mathbf{F} \mathbf{F}^H \mathbf{H}_{sr}^H + \sigma_r^2 \mathbf{I}) \mathbf{G}^H\right\} \\ &= \mathrm{tr}\left\{\mathbf{\Lambda}_r^H \left(\sigma_r^2 \mathbf{I} + \sigma_x^2 \mathbf{\Lambda}_{sr} \mathbf{\Lambda}_s \mathbf{\Lambda}_s^H \mathbf{\Lambda}_{sr}^H\right) \mathbf{\Lambda}_r^H\right\}. \end{aligned} \tag{20}$$

Let $\alpha_i = \lambda_{s,i}^2$ and $\beta_i = \lambda_{r,i}^2$. From (18) to (20), the optimization problem in (9) can be rewritten as

$$\begin{cases} \min\limits_{\{\alpha_i,\, \beta_i\}_{i=1}^p} & \displaystyle\sum_{i=1}^p \frac{1}{\sigma_x^{-2} + \sigma_d^{-2}\lambda_{sd,i}^2 \alpha_i + \frac{\lambda_{sr,i}^2 \lambda_{rd,i}^2 \alpha_i \beta_i}{\sigma_r^2 \lambda_{rd,i}^2 \beta_i + \sigma_d^2}} \\ \text{s.t.} & \sigma_x^2 \displaystyle\sum_{i=1}^p s_{ii} \alpha_i \leq P_s \\ & \displaystyle\sum_{i=1}^p (\sigma_r^2 + \sigma_x^2 \lambda_{sr,i}^2 \alpha_i) \beta_i \leq P_r \\ & \alpha_i \geq 0,\ \beta_i \geq 0 \quad i = 1, \cdots, p. \end{cases} \tag{21}$$

The problem (21) can be solved by using the Lagrange technique followed by an iterative water-filling procedure [15]. The resultant solution is

$$\begin{aligned} \beta_i =&\frac{\sigma_d^2(\sigma_x^2 \lambda_{sd,i}^2 \alpha_i + \sigma_d^2)}{c_{r,i} \lambda_{rd,i}^2} \left(\frac{\sigma_x^2}{\sigma_x^2 \lambda_{sd,i}^2 \alpha_i + \sigma_d^2} \times \right. \\ &\left. \sqrt{\frac{\sigma_d^2 \lambda_{sr,i}^2 \lambda_{rd,i}^2 \alpha_i}{\mu_0(\sigma_r^2 + \sigma_x^2 \lambda_{sr,i}^2 \alpha_i)}} - 1\right)^+, \end{aligned} \tag{22}$$

where $(x)^+ = \max(0, x)$, $c_{r,i} = \sigma_d^2 \sigma_r^2 + \sigma_x^2(\sigma_r^2 \lambda_{sd,i}^2 + \sigma_d^2 \lambda_{sr,i}^2)\alpha_i$, and μ_0 is chosen so that the transmitted power at the relay node satisfies the power constraint P_r. Similarly, we can obtain

$$\alpha_i = \frac{\sigma_d^2(\sigma_r^2 \lambda_{rd,i}^2 \beta_i + \sigma_d^2)}{\sigma_x^2 c_{s,i}} \left(\sqrt{\frac{\sigma_x^2 c_{s,i}}{\nu_0 s_{ii} \sigma_d^2(\sigma_r^2 \lambda_{rd,i}^2 \beta_i + \sigma_d^2)}} - 1\right)^+, \tag{23}$$

where $c_{s,i} = \sigma_d^2 \lambda_{sd,i}^2 + \lambda_{rd,i}^2(\sigma_r^2 \lambda_{sd,i}^2 + \sigma_d^2 \lambda_{sr,i}^2)\beta_i$ and ν_0 is chosen so that the transmitted power at the source node satisfies the power constraint P_s.

The proposed method mainly uses SVD of the relay-destination channel matrix in (10), GSVD of the source-relay and source-destination channel matrices in (11) and (12), and the matrix inversion of $\mathbf{X}\mathbf{X}^H$ in (19). Details of the computational complexity for the design of the precoding matrix and the amplifying matrix are given in TABLE I (using the results in [14]).

TABLE I
COMPLEXITY OF THE PROPOSED METHOD

Operation	**Flops**
SVD, (10)	$4M^2L + 8ML^2 + 9L^3$
GSVD, (11), (12)	$12LN^2 + 6MN^2 + 4L^2N + 7N^3$
$\mathbf{S}$, (19)	$1/3N^3 + 3N^2$
α_i and β_i, (22), (23)	$(29pI_r + 24pI_s)I_a$
$\mathbf{F}$ and $\mathbf{G}$, (15), (17)	$2(Lp + L^2p + Np)$

I_r: number of iteration for evaluating β_i
I_s: number of iteration for evaluating α_i
I_a: number of iteration for the water-filling process

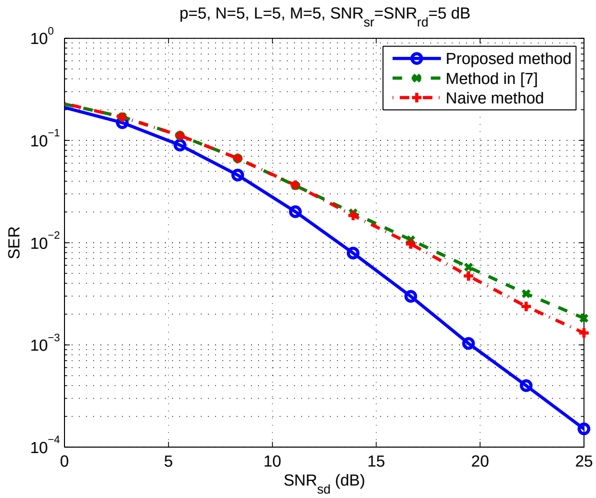

Fig. 2. SER performance comparison with fixed relay links

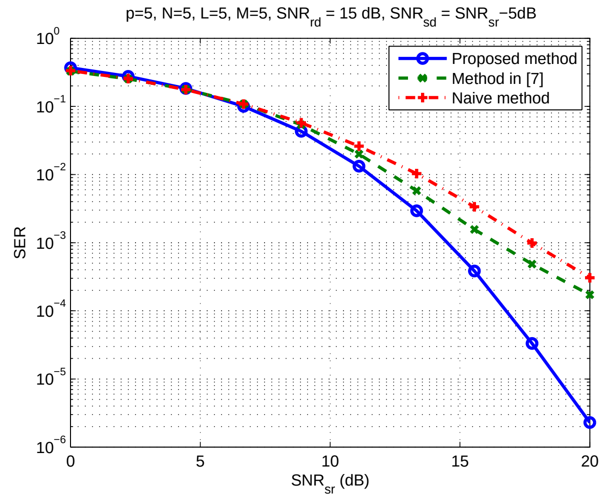

Fig. 3. SER performance comparison with fixed relay-destination link

IV. Simulation Result

In this section, we use a number of numerical simulations to verify the results obtained in Section III. We consider the AF MIMO relay system with $p = N = L = M = 5$. The channel matrices, $\mathbf{H}_{sr}$, $\mathbf{H}_{rd}$, and $\mathbf{H}_{sd}$, have i.i.d complex Gaussian elements with zero mean and unit variance. The transmit symbols are obtained from QPSK constellation. The SNR denotes the signal-to-noise ratio per received antenna and thus the SNR_{sr}, SNR_{sd}, and SNR_{rd} are the SNR of the source-relay, source-destination, and relay-destination links. We compare the proposed method with two methods: the first one is proposed in [7], where the precoding and amplifying matrices are designed based on SVD technique; the second one is the naive amplify-and-forward method, in which the precoding matrix $\mathbf{F} = \sqrt{P_s/N}\ \mathbf{I}$ and the amplifying matrix $\mathbf{G} = \sqrt{P_r/\mathrm{tr}\{\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^H\mathbf{H}_{sr}^H + \sigma_r^2\mathbf{I}\}}\ \mathbf{I}$.

Fig. 2 shows the comparison of the symbol error rates (SERs) between the proposed method and other two methods for fixed relay link conditions $SNR_{sr} = SNR_{rd} = 5$ dB. We can see from the figure that when the direct link SNR varies from 0 dB to 25 dB, the proposed method outperforms the method in [7] and the naive method, especially for high SNR_{sd}. It is also seen that the SERs of the naive method is slightly better than that of the method in [7] when $SNR_{sd} > 15$ dB.

In Fig. 3, we consider the scenario in which the relay-destination link condition is fixed while the source link conditions are varying. Specifically, we set $SNR_{rd} = 15$ dB and $SNR_{sd} = SNR_{sr} - 5$ dB. The figure shows that although the SERs are similar at low SNR for three methods, the performance of the proposed method improves significantly when $SNR_{sr} > 10$ dB.

Fig. 4 shows the performance of three methods in terms of SER versus $SNR = SNR_{sr} = SNR_{rd}$ for a fixed $SNR_{sd} = 10$ dB. It can be seen that the naive method has the worst performance, since it dose not consider channel information for precoder design. Although the proposed method outperforms the other two method, the SER is closed to that of the method in [7] as $SNR > 20$ dB.

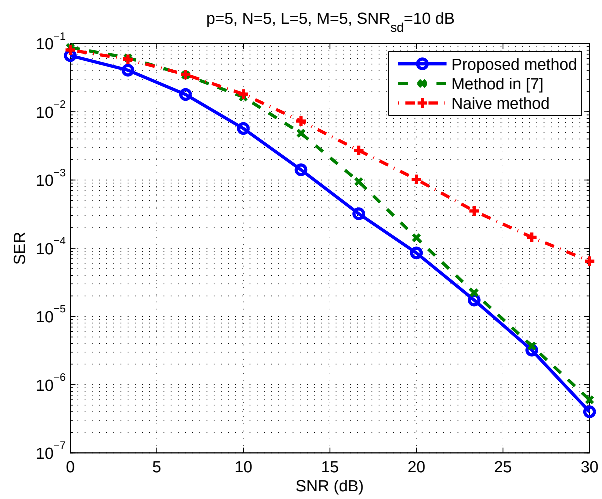

Fig. 4. SER performance comparison with fixed source-destination link

V. Conclusion

We study the AF MIMO three-node relay system. Considering the MMSE criterion at the destination, we design the precoding matrix at the source and the amplifying matrix at the relay to minimize the MSE of transmitted symbols under the source and relay power constraints. Based on SVD of the relay-destination channel matrix and GSVD of the source-relay and source-destination channel matrices, the MSE matrix can be diagonalized by choosing particular structures of the procoding matrix and the amplifying matrix. This suboptimal approach simplifies the design of two matrices and the solution is obtained by using the Lagrange technique followed by an iterative water-filling procedure. From simulation results, we see that the improvement in performance of the proposed method over the method in [7] is more significant when the signal power of the direct link is larger than that of the relay link.

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