Adaptive Robust Output-Feedback Motion/Force Control of Electrically Driven Nonholonomic Mobile Manipulators

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***Abstract*—In this brief, adaptive robust output-feedback force/motion control strategies are presented for mobile manipulators under both holonomic and nonholonomic constraints in the presence of uncertainties and disturbances. The controls are developed on structural knowledge of the dynamics of the robot and actuators and in conjunction with a linear observer. The proposed controls are robust not only to parametric uncertainty such as mass variations but also to external ones such as disturbances. The system stability and the boundedness of tracking and observation errors are proven using Lyapunov stability synthesis. Simulation results validate that the states of the system converge to the desired trajectory, while the constraint force converges to the desired force.**

***Index Terms*—Actuators dynamics, motion/force control, nonholonomic mobile manipulators, output feedback.**

I. INTRODUCTION

THE MOBILE manipulator possesses a complex and strongly coupled dynamics of the mobile platform and the robotic arm. Tracking control of mobile manipulators in practical applications requires both the motion and constraint forces converge to their desired trajectories and constraint forces, respectively, in the presence of parametric uncertainty [3], [6].

With the assumption of known dynamics, much research has been carried out. In [1], nonlinear feedback control for the mobile manipulator was developed to compensate for the dynamic interaction between the mobile platform and the arm to achieve tracking performance. In [2], coordination and control of mobile manipulators were presented with two basic task-oriented controls: end-effector task control and platform self posture control. In [6], force/position control of the end-effector for mobile manipulators was developed using nonlinear feedback linearization and decoupling dynamics.

To solve for the unknown parameters, adaptive schemes were investigated to deal with mobile manipulators with unknown inertia parameters and disturbances. In [3], adaptive trajectory/force tracking controls were investigated for mobile manipulators with unknown inertia parameters and disturbances. With the difficulty in dynamic modeling, adaptive neural network controls, a non-model-based approach [4], were developed for the motion control of mobile manipulators subject to kinematic constraints in [5].

In these schemes, the controls are designed at kinematic level with velocity as input or dynamic level with torque as input, but the actuator dynamics are ignored. As demonstrated in [7], actuator dynamics constitute an important component of the complete robot dynamics, especially in the case of high-velocity movement and highly varying loads. Many control methods have therefore been developed to take into account the effects of actuator dynamics (see, for instance, [7]–[9]). However, most literature assumes that the actuator velocities are measurable [3], [5], which may deteriorate the control performance of these methods, since velocity measurements are often contaminated by a considerable amount of noise. Therefore, it is desired to achieve good control performance by using only joint position measurement. Moreover, in most research conducted on control of mobile manipulators, joint torques are the control inputs, while in reality the joints are driven by actuators (e.g., DC motors). Therefore, using actuator input voltages as control inputs and designing observer-controller structure for mobile manipulators with only the positions and the driving currents of actuators are more realistic. As such, actuator dynamics is combined with the mobile manipulator's dynamics in this brief.

This brief addresses adaptive robust output-feedback control of force/motion for a class of mobile manipulator systems electrically driven by DC motors with both holonomic and nonholonomic constraints in the parameter uncertainties and external disturbances. Simulation results are described in detail that show the effectiveness of the proposed control.

II. SYSTEM DESCRIPTION

Consider an n-degree-of-freedom (DOF) mobile manipulator mounted on a nonholonomic mobile base, the dynamics can be described as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau + f \qquad (1)$$

where $q = [q_v, q_r]^T \in R^n$ with $q_v \in R^v$ denoting the generalized coordinates for the mobile platform and $q_r \in R^r$ denoting the coordinates of the manipulator, and $n = v + r$. The symmetric positive definite inertia matrix $M(q) \in R^{n\times n}$, the Centripetal and Coriolis torques $C(\dot{q}, q) \in R^{n\times n}$, the gravitational torque vector $G(q) \in R^n$, the external disturbances $d(t) \in R^n$, the known full rank input transformation matrix $B(q) \in R^{n\times m}$,

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the control inputs $\tau \in R^m$, and the generalized constraint forces $f \in R^n$ could be represented as, respectively

$$M(q) = \begin{bmatrix} M_v & M_{vr} \\ M_{rv} & M_r \end{bmatrix}$$
$$C(q,\dot{q}) = \begin{bmatrix} C_v & C_{vr} \\ C_{rv} & C_r \end{bmatrix}$$
$$G(q) = \begin{bmatrix} G_v \\ G_r \end{bmatrix}$$
$$d(t) = \begin{bmatrix} d_v \\ d_r \end{bmatrix}$$
$$B(q) = \begin{bmatrix} B_v & 0 \\ 0 & B_r \end{bmatrix}$$
$$\tau = \begin{bmatrix} \tau_v \\ \tau_r \end{bmatrix}$$
$$f = \begin{bmatrix} A & 0 \\ J_v & J_r \end{bmatrix}^T \begin{bmatrix} \lambda_n \\ \lambda_h \end{bmatrix}$$

where M_v and M_r describe the inertia matrices for the mobile platform and the robot manipulator, respectively, M_{vr} and M_{rv} are the coupling inertia matrices of the mobile platform and the robot manipulator, C_v and C_r denote the Centripetal and Coriolis torques for the mobile platform and the robot manipulator, respectively, and C_{vr} and C_{rv} are the coupling Centripetal and Coriolis torques of the mobile platform and the robot manipulator. G_v and G_r are the gravitational torque vectors for the mobile platform and the robot manipulator, respectively, τ_v and τ_r are the control input vectors for the mobile platform and the robot manipulator, $d_v(t)$ and $d_r(t)$ denote the external disturbances on the mobile platform and the robot manipulator, respectively, and λ_n and λ_h are the associated Lagrangian multipliers with the generalized nonholonomic and holonomic constraints, respectively.

Assume that the j non-integrable and independent velocity constraints on the mobile platform

$$A(q_v)\dot{q}_v = 0 \tag{2}$$

which can be viewed as restricting the dynamics on the manifold $\Omega_n = \{(q_v,\dot{q}_v)|A(q_v)\dot{q}_v = 0\}$, where $A = [A_1^T(q_v),\ldots,A_j^T(q_v)]^T : R^v \rightarrow R^{j\times v}$ is the kinematic constraint matrix which is assumed to have full rank j. The effect of the nonholonomic constraints can be viewed as the generalized constraint forces given by $f_v = A^T(q)\lambda_n$. It is always possible to find an $v-j$ rank matrix $H(q_v) \in R^{v\times v-j}$ formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q_v)$, where

$$H^T(q_v)A^T(q_v) = 0. \tag{3}$$

The constraints (2) and (3) imply that

$$\dot{q}_v = H(q_v)\dot{\eta}. \tag{4}$$

Remark 2.1: It should be noted that the $(v-j)$-vector η represent the internal states, so that $(q_v,\dot{\eta})$ is sufficient to describe the constrained motion of the mobile platform [13].

Combining the non-holonomic constraints (2) and the transformation (4) and their derivatives, the dynamics of a mobile manipulator can be expressed as

$$\begin{aligned} &M_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + G_1(q_1) + d_1(t) \\ &= B_1(q_1)\tau + f_1 \end{aligned}$$
$$M_1(q_1) = \begin{bmatrix} H^T M_v H & H^T M_{vr} \\ M_{rv}H & M_r \end{bmatrix}$$
$$q_1 = \begin{bmatrix} \eta \\ q_r \end{bmatrix}$$
$$C_1(q_1,\dot{q}_1) = \begin{bmatrix} H^T C_v \dot{H} + H^T C_v H & H^T C_{vr} \\ M_{rv}\dot{H} + C_{rv}H & C_r \end{bmatrix}$$
$$d_1(t) = \begin{bmatrix} H^T d_v \\ d_r \end{bmatrix}$$
$$G_1(q_1) = \begin{bmatrix} H^T G_v \\ G_r \end{bmatrix}$$
$$B_1(q_1)\tau = \begin{bmatrix} H^T B_v \tau_v \\ B_r \tau_r \end{bmatrix}$$
$$f_1 = \begin{bmatrix} 0 & 0 \\ J_v & J_r \end{bmatrix}^T \begin{bmatrix} 0 \\ \lambda_h \end{bmatrix}. \tag{5}$$

Property 2.1: Matrices $M_1(q_1)$, $G_1(q_1)$ are uniformly bounded and uniformly continuous if q_1 is uniformly bounded and continuous, respectively. Matrix $C_1(q_1,\dot{q}_1)$ is uniformly bounded and uniformly continuous if $\dot{q}_1$ and q_1 are uniformly bounded and continuous, respectively.

When the system (5) is subjected to holonomic constraints, the k independent holonomic constraints can be expressed as

$$h(q_1) = 0 \quad h(q_1) \in R^k \tag{6}$$

where $h(q_1)$ is of full rank, and one has $J(q_1) = \partial h(q_1)/\partial q_1$ and $J(q_1)\dot{q}_1 = 0$. The effect of the holonomic constraints can be viewed as the generalized constraint forces f_1 in the joint space as $f_1 = J^T\lambda_h$. Hence, the holonomic constraint on the robot's end-effector can be viewed as restricting only the dynamics on the constraint manifold Ω_h defined by $\Omega_h = \{(q_1,\dot{q}_1)|h(q_1) = 0, J(q_1)\dot{q}_1 = 0\}$. Assume that the mobile manipulator is a series-chain multilink manipulator with holonomic constraints (i.e., geometric constraints). According to [12], the vector q_r can be properly rearranged and partitioned into the form $q_r = [q_r^{1T}, q_r^{2T}]^T$, $q_r^1 \in R^{r-k}$ describes the constrained motion of the manipulator, $q_r^2 \in R^k$ denotes the remaining joint variable. Then, $J(q_1) = [(\partial h/\partial \eta), (\partial h/\partial q_r^1), (\partial h/\partial q_r^2)]$. Moreover, by the partition of q_r, q_r^2 is uniquely determined by $\zeta = [\eta^T, q_r^{1T}]^T$, there is a unique function such that the q_1 is expressed explicitly as the function of ζ, that is, $q_1 = \vartheta(\zeta)$, and one has $\dot{q}_1 = L(\zeta)\dot{\zeta}$, where $L(\zeta) = \partial q_1/\partial\zeta$, $\ddot{q}_1 = L(\zeta)\ddot{\zeta} + \dot{L}(\zeta)\dot{\zeta}$, and $L(\zeta)$, $J^1(\zeta) = J(\vartheta(\zeta))$ satisfies the relationship $L^T(\zeta)J^{1T}(\zeta) = 0$. The dynamics (5), restricted to the constraint surface, can be transformed into the reduced-order model

$$M_2\ddot{\zeta} + C_2(\zeta,\dot{\zeta})\dot{\zeta} + G_2 + d_2(t) = u + J^{1T}\lambda_h \tag{7}$$

where $M_2 = M_1(q_1)L(\zeta)$, $C_2 = M_1(q_1)\dot{L}(\zeta) + C_1(q_1,\dot{q}_1)L(\zeta)$, $G_2 = G_1$, $d_2 = d_1$, $u = B_1\tau$.

Multiplying $L^T(\zeta)$ on both sides of (7), one has

$$M_L(\zeta)\ddot{\zeta} + C_L(\zeta,\dot{\zeta})\dot{\zeta} + G_L + d_L(t) = L^T u \quad (8)$$

where $M_L(\zeta) = L^T(\zeta)M_2$, $C_L(\zeta,\dot{\zeta}) = L^T(\zeta)C_2$, $G_L = L^T(\zeta)G_2$, $d_L = L^T(\zeta)d_2$.

The force multiplier λ_h can be obtained by (7)

$$\lambda_h = Z\left(C_2(\zeta,\dot{\zeta})\dot{\zeta} + G_2 + d_2(t) - u\right) \quad (9)$$

where $Z = (J^1(M_1)^{-1}J^{1T})^{-1}J^1(M_1)^{-1}$.

Property 2.2: The matrix $M_L(\zeta)$ is symmetric and positive definite.

Property 2.3: The matrix $\dot{M}_L(\zeta) - 2C_L(\zeta,\dot{\zeta})$ is skew-symmetric, and the $C_L(\zeta,\dot{\zeta})$ satisfies $C_L(\zeta,x)y = C_L(\zeta,y)x$ and $C_L(\zeta,z+kx)y = C_L(\zeta,z)y + kC_L(\zeta,x)y\ \forall x,y$, and k is scalar.

Property 2.4: For holonomic systems, matrices $J^1(\zeta)$, $L(\zeta)$ are uniformly bounded and uniformly continuous if ζ is uniformly bounded and continuous, respectively.

Remark 2.2: The matrix Z is uniformly bounded and uniformly continuous since $M_1(q_1)$ and $J^1(\zeta)$ are uniformly bounded and uniformly continuous.

Property 2.5: There exist some finite positive constants $c_i > 0 (1 \le i \le 4)$ and finite non-negative constant $c_i \ge 0 (i = 5)$ such that $\forall \zeta \in R^{n-j-k}\ \forall \dot{\zeta} \in R^{n-j-k}$, $\|M_L(\zeta)\| \le c_1$, $\|C_L(\zeta,\dot{\zeta})\| \le c_2 + c_3\|\dot{\zeta}\|$, $\|G_L(\zeta)\| \le c_4$, and $\sup_{t\ge 0}\|d_L(t)\| \le c_5$.

III. Actuator Dynamics

The joints of the mobile manipulators are assumed to be driven by DC motors. Consider the following notations used to model a DC motor: $\nu \in R^m$ represents the control input voltage vector; I denotes an m-element vector of motor armature current; $K_N \in R^{m\times m}$ is a positive definite diagonal matrix which characterizes the electromechanical conversion between current and torque; $L_a = \mathrm{diag}[L_{a1}, L_{a2}, L_{a3}, \ldots, L_{am}]$, $R_a = \mathrm{diag}[R_{a1}, R_{a2}, R_{a3}, \ldots, R_{am}]$, $K_e = \mathrm{diag}[K_{e1}, K_{e2}, K_{e3}, \ldots, K_{em}]$, $\omega = [\omega_1, \omega_2, \ldots, \omega_m]^T$ represent the equivalent armature inductances, resistances, back EMF constants, angular velocities of the driving motors, respectively; $G_r = \mathrm{diag}(g_{ri}) \in R^{m\times m}$ denotes the gear ratio for m joints; τ_m are the torque exerted by the motor. In order to apply the DC servomotors for actuating an n-DOF mobile manipulator, assume without energy loss, a relationship between the ith joint velocity $\dot{\zeta}_i$ and the motor shaft velocity ω_i can be presented as $g_{ri} = \omega_i/\dot{\zeta}_i = \tau_i/\tau_{mi}$, where g_{ri} is the gear ratio of the ith joint, τ_{mi} is the ith motor shaft torque, and τ_i is the ith joint torque. The motor shaft torque is proportional to the motor current as $\tau_m = K_N I$. The back EMF is proportional to the angular velocity of the motor shaft, then one has

$$L_a\frac{dI}{dt} + R_a I + K_e\omega = \nu. \quad (10)$$

In the actuator dynamics (10), the relationship between ω and $\dot{\zeta}$ is dependent on the type of mechanical system and can be generally expressed as

$$\omega = G_r T\dot{\zeta}. \quad (11)$$

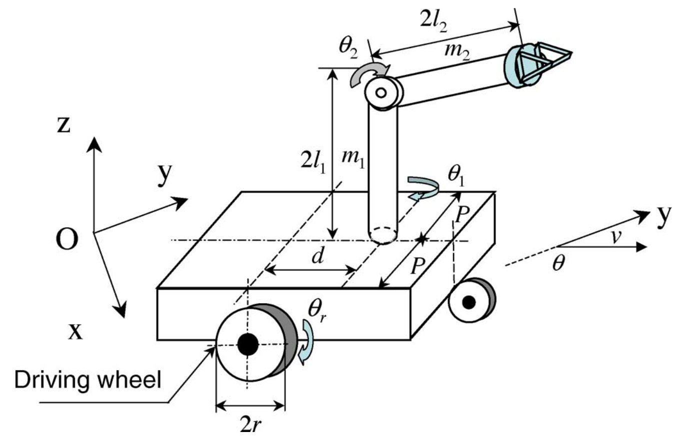

Fig. 1. Two-DOF mobile manipulator.

The structure of T depends on the mechanical systems to be controlled. For instance, in the simulation example, a two-wheel differential drive 2-DOF mobile manipulator is used to illustrate the control design. From [11], one has $v = (r\dot{\theta}_l + r\dot{\theta}_r)/2$, $\dot{\theta} = (r\dot{\theta}_r - r\dot{\theta}_l)/2l$, $\dot{\theta}_1 = \dot{\theta}_1$, $\dot{\theta}_2 = \dot{\theta}_2$, where $\dot{\theta}_l$ and $\dot{\theta}_r$ are the angular velocities of the two wheels, respectively, and v is the linear velocity of the mobile platform, as shown in Fig. 1. Since $\dot{y} = v\cos\theta$, one has $[\dot{\theta}_l\ \dot{\theta}_r\ \dot{\theta}_1\ \dot{\theta}_2]^T = T[\dot{y}\ \dot{\theta}\ \dot{\theta}_1\ \dot{\theta}_2]^T$, $T = [T_1\ T_2\ T_3\ T_4]$, $T_1 = [(1/r\cos\theta)(1/r\cos\theta)\ 0\ 0]^T$, $T_2 = [(l/r) - (l/r)\ 0\ 0]^T$, $T_3 = [0\ 0\ 1\ 0]^T$, $T_4 = [0\ 0\ 0\ 1]^T$, where r and l are shown in Fig. 1.

Eliminating ω from the actuator dynamics (10) by substituting (11) and considering (8), one obtains

$$M_L(\zeta)\ddot{\zeta} + C_L(\zeta,\dot{\zeta})\dot{\zeta} + G_L + d_L(t) = L^T B_1 G_r K_N I \quad (12)$$

$$Z(C_2(\zeta,\dot{\zeta})\dot{\zeta} + G_2 + d_2(t) - B_1 G_r K_N I) = \lambda_h \quad (13)$$

$$L_a\frac{dI}{dt} + R_a I + K_e G_r T\dot{\zeta} = \nu. \quad (14)$$

Assumption 3.1: The unknown actuator parameters L_a, R_a, and K_e are bounded and satisfy $\|L_a\| \le \alpha_1$; $\|R_a\| \le \alpha_2$; $\|K_e\| \le \alpha_3$ where α_ι, $(\iota = 1,\ldots,3)$ are finite positive constants.

Remark 3.1: In reality, these constants $c_i (1 \le i \le 5)$ and $\alpha_\iota (1 \le \iota \le 3)$ cannot be obtained easily. Although any fixed large c_i and α_ι can guarantee good performance, it is not recommended in practice as large c_i and α_ι imply, in general, high noise amplification and high cost of control. Therefore, it is necessary to develop a control law which does not require the knowledge of $c_i (1 \le i \le 5)$ and $\alpha_\iota (1 \le \iota \le 3)$.

IV. Output-Feedback Control Design

Given a desired motion trajectory $\zeta_d(t) = [\eta_d\ q_{rd}^{1T}]^T$ and a desired constraint force $f_d(t)$, or, equivalently, a desired multiplier $\lambda_h(t)$, the trajectory and force tracking control is to determine a control law such that for any $(\zeta(0), \dot{\zeta}(0)) \in \Omega$, ζ, $\dot{\zeta}$, λ_h converge to a manifold $\Omega_d : \Omega_d = \{(\zeta,\dot{\zeta},\lambda_h) | \zeta = \zeta_d, \dot{\zeta} = \dot{\zeta}_d, \lambda_h = \lambda_h^d\}$.

The controller design will consist of two stages: 1) a virtual adaptive control input I_d is designed so that the subsystems (12) and (13) converge to the desired values and 2) the actual control input ν is designed in such a way that $I \to I_d$. In turn, this allows $\zeta - \zeta_d$ and $\lambda_h - \lambda_h^d$ to be stabilized to the origin.

Assumption 4.1: The desired reference trajectory $\zeta_d(t)$ is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivatives up to the second order. The desired Lagrangian multiplier $\lambda_h^d(t)$ is also bounded and uniformly continuous.

Lemma 4.1: For $x > 0$ and $\delta \geq 1$, one has $\ln(\cosh(x)) + \delta \geq x$.

Proof: If $x \geq 0$, one has $\int_0^x (2/(e^{2s}+1))ds < \int_0^x (2/e^{2s})ds = 1 - e^{-2x} < 1$. Therefore, $\ln(\cosh(x)) + \delta \geq \ln(\cosh(x)) + \int_0^x (2/(e^{2s}+1))ds$ with $\delta \geq 1$. Let $f(x) = \ln(\cosh(x)) + \int_0^x (2/(e^{2s}+1))ds - x$, one has $df(x)/dx = \tanh(x) + (2/(e^{2x}+1)) - 1 = ((e^x - e^{-x})/(e^x + e^{-x})) + (2/(e^{2x}+1)) - 1 = 0$. From the mean value theorem [15], one has $f(x) - f(0) = (df(x)/dx)(x-0)$. Since $f(0) = 0$ and $df(x)/dx = 0$, for all x, one has $f(x) = 0$, that is, $\ln(\cosh(x)) + \int_0^x (2/(e^{2s}+1))ds = x$, then, one has $\ln(\cosh(x)) + \delta \geq x$. ∎

Remark 4.1: Lemma 4.1 is used to facilitate the control design.

Definition 4.1: Consider time varying positive functions $\kappa_\iota(t)$ for $\iota = 1, \ldots, 3$ converges to zero as $t \to \infty$ and satisfies $\lim_{t\to\infty} \int_0^t \kappa_\iota(\omega)d\omega = b_\iota < \infty$ with a finite constant b_ι. There are many choices for $\kappa_\iota(t)$ that satisfy the previous condition, for example, $\kappa_\iota = 1/(1+t)^2$.

A. Kinematic and Dynamic Subsystems

Consider the following signals defined as:

$$\dot{\zeta}_r = \dot{\zeta}_d - K_\zeta(\hat{\zeta} - \zeta_d) = \dot{\zeta}_d - K_\zeta e_\zeta + K_\zeta \tilde{\zeta} \quad (15)$$

$$r_1 = \dot{\zeta} - \dot{\zeta}_r = \dot{e}_\zeta + K_\zeta e_\zeta - K_\zeta \tilde{\zeta} \quad (16)$$

$$\dot{\zeta}_o = \dot{\hat{\zeta}} - K_\zeta \tilde{\zeta} \quad (17)$$

$$r_2 = \dot{\zeta} - \dot{\zeta}_o = \dot{\tilde{\zeta}} + K_\zeta \tilde{\zeta} \quad (18)$$

$$r = r_1 + r_2 \quad (19)$$

$$e_\lambda = \lambda_h - \lambda_h^d \quad (20)$$

where $e_\zeta = \zeta - \zeta_d$, $\tilde{\zeta} = \zeta - \hat{\zeta}$ with $\hat{\zeta}$ denoting the estimate of ζ, and K_ζ is diagonal positive.

The linear observer [10] for velocity estimation is introduced to the system

$$\dot{\hat{\zeta}} = z + K_\zeta \tilde{\zeta} + k_d \tilde{\zeta} \quad (21)$$

$$\dot{z} = \ddot{\zeta}_r + k_d K_\zeta \tilde{\zeta} \quad (22)$$

where k_d is a positive constant.

A decoupling control scheme is introduced to control generalized position and constraint force, separatively. Consider the control u in the following form:

$$u = L^{+^T} u_a - J^{1T} u_b \quad (23)$$

where $u_a = B_{1a} G_{ra} K_{Na} I_a$, $u_b = B_{1b} G_{rb} K_{Nb} I_b$ $B_1 = \mathrm{diag}[B_{1a}\ B_{1b}]$, $G_r = \mathrm{diag}[G_{ra}\ G_{rb}]$, $u_a, B_{1a}, G_{ra}, I_a \in R^{n-j-k}$, and $u_b, B_{1b}, G_{rb}, I_b \in R^k$, $L^+ = (L^T L)^{-1} L^T$. Then, (8) and (9) can be rewritten as

$$B_{1a} G_{ra} K_{Na} I_a = M_L(\zeta)\ddot{\zeta} + C_L(\zeta, \dot{\zeta})\dot{\zeta} + G_L + d_L(t) \quad (24)$$

$$\lambda_h = Z\left(C_2(\zeta, \dot{\zeta})\dot{\zeta} + G_2 + d_2(t) - L^{+^T} B_{1a} G_{ra} K_{Na} I_a\right) + B_{1b} G_{rb} K_{Nb} I_b. \quad (25)$$

Consider the following control laws:

$$I_{ra} = (B_{1a} G_{ra} K_{Na})^{-1}\Big[-K_p(r_1 - r_2) - K_i(\tilde{\zeta} + e_\zeta) - \mathrm{sgn}(r)\Big(\ln\Big(\cosh(\hat{\Phi})\Big) + \delta\Big)\Big] \quad (26)$$

$$I_{rb} = (B_{1b} G_{rb} K_{Nb})^{-1}\left[(\ln(\cosh(\hat{\chi})) + \delta)\,\ddot{\zeta}_d + \lambda_h^d - K_f e_\lambda\right] \quad (27)$$

where $\hat{\Phi} = \hat{C}^T \Psi$, $\dot{\hat{C}} = -\Lambda \hat{C} + \|r\| \Gamma \Psi$, $\hat{\chi} = \hat{c}_1 \|Z^* L^{+^T}\|$, $\Psi = [\|\ddot{\zeta}_r\|\ \|\dot{\zeta}_r\|\ \|\dot{\zeta}_r\|^2\ 1\ 1]^T$, $\hat{C} = [\hat{c}_1 \hat{c}_2 \hat{c}_3 \hat{c}_4 \hat{c}_5]^T$; K_p, K_i, K_f are diagonal positive; if $r \geq 0$, $\mathrm{sgn}(r) = 1$, else $\mathrm{sgn}(r) = -1$; $\delta \geq 1$ is constant; $\Gamma = \mathrm{diag}[\gamma_i] > 0$; Λ is a diagonal matrix whose each element Λ_i satisfies Definition 4.1, i.e., $\lim_{t\to\infty} \int_0^t \Lambda_i(\omega)d\omega = a_i < \infty$ with the finite constant a_i, $i = 1, \ldots, 5$; $Z^* = (J^1 (M^*)^{-1} J^{1T})^{-1} J^1 (M^*)^{-1}$ with $\|L^T M^* L\| = \hat{c}_1$, $I_r = [I_{ra}, I_{rb}]^T$ will be defined in the following section.

B. Control Design at the Actuator Level

Till now, ζ tends to ζ_d can be guaranteed, if the actual input control signal of the dynamic system I be of the form I_d which can be realized from the actuator dynamics by the design of the actual control input ν. On the basis of the previous statements, it is concluded that if ν is designed in such a way that I tends to I_d then $(\zeta - \zeta_d) \to 0$ and $(\lambda_h - \lambda_h^d) \to 0$.

Define $e_I = \int_0^t (I - I_d)dt$, and $\dot{e}_I = I - I_d$, $I_r = I_d - K_r e_I$, and $s = \dot{e}_I + K_r e_I$ with $K_r = 0$. Substituting I and $\dot{\zeta}$ of (14), one gets

$$L_a \dot{s} + R_a s + K_e G_r T \dot{e}_\zeta = -\phi + \nu \quad (28)$$

where $\phi = L_a \dot{I}_r + R_a I_r + K_e G_r T \dot{\zeta}_d$.

Remark 4.2: Since I is partitioned into I_a and I_b, one has corresponding partitions $e_I = [e_{Ia}^T, e_{Ib}^T]^T$, $\dot{e}_I = [\dot{e}_{Ia}^T, \dot{e}_{Ib}^T]^T$, $I_r = [I_{ra}^T, I_{rb}^T]^T$, $s = [s_a^T, s_b^T]^T$, $\phi = [\phi_a^T, \phi_b^T]^T$, $T = \mathrm{diag}[T_a, T_b]$, $\nu = [\nu_a^T, \nu_b^T]^T$ with the corresponding $\alpha_a = [\alpha_{a1}, \alpha_{a2}, \alpha_{a3}]^T$ and $\alpha_b = [\alpha_{b1}, \alpha_{b2}, \alpha_{b3}]^T$, $\varphi_a = \hat{\alpha}_a^T \mu_a$ and $\varphi_b = \hat{\alpha}_b^T \mu_b$, $\mu_a = [\|\dot{I}_{ad}\|\ \|I_{ad}\|\ \|G_{ra} T_a \dot{\zeta}_d\|]^T$ and $\mu_b = [\|\dot{I}_{bd}\|\ \|I_{bd}\| 0]^T$, $K_{\nu a}$ and $K_{\nu b}$, $L_a = \mathrm{diag}[L_{aa}, L_{ab}]$, $R_a = \mathrm{diag}[R_{aa}, R_{ab}]$ and $K_e = \mathrm{diag}[K_{ea}, K_{eb}]$.

Consider the adaptive robust control law for I_a and I_b, respectively

$$\nu_a = -\mathrm{sgn}(s_a)\left(\ln\left(\cosh(\hat{\varphi}_a)\right) + \delta\right) - K_{\nu a} s_a \quad (29)$$

$$\nu_b = -\mathrm{sgn}(s_b)\left(\ln\left(\cosh(\hat{\varphi}_b)\right) + \delta\right) - K_{\nu b} s_b \quad (30)$$

where $\hat{\alpha}_a = [\hat{\alpha}_{a1}\ \hat{\alpha}_{a2}\ \hat{\alpha}_{a3}]^T$, $\hat{\alpha}_b = [\hat{\alpha}_{b1}\ \hat{\alpha}_{b2}\ \hat{\alpha}_{b3}]^T$, $\dot{\hat{\alpha}}_a = -\kappa \hat{\alpha}_a + \|s_a\| \beta \mu_a$ and $\dot{\hat{\alpha}}_b = -\kappa \hat{\alpha}_b + \|s_b\| \beta \mu_b$ for I_a and

I_b, respectively, and $K_{\nu a}$ and $K_{\nu b}$ are diagonal positive, $\beta = \text{diag}[\beta_\iota]$, κ is a diagonal matrix whose each element κ_ι satisfies Definition 4.1, i.e., $\lim_{t\to\infty}\int_0^t \kappa_\iota(\omega)d\omega = b_\iota < \infty$ with the finite constant b_ι, $\iota = 1,\ldots,3$.

C. Stability Analysis for the System

Theorem 4.1: Consider the mechanical system described by (1), (2), and (6), using the control law (26), (27), (29), and (30), the following hold for any $(q(0),\dot q(0)) \in \Omega_n \cap \Omega_h$: 1) r and s_a converge to a set containing the origin as $t\to\infty$; 2) e_q and $\dot e_q$ converge to 0 as $t\to\infty$; and 3) s_b converges to a set containing the origin as $t\to\infty$, and e_λ and τ are bounded for all $t\geq 0$.

Proof: 1) By combining (12) with (16) and considering Property 2.3, the closed-loop system dynamics is given by

$$M_L(\zeta)\dot r_1 = B_{1a}G_{ra}K_{Na}I_{ra} + B_{1a}G_{ra}K_{Na}s_a - \gamma - \left(C_L(\zeta,\dot\zeta) + C_L(\zeta,\dot\zeta_r)\right)r_1 \quad (31)$$

where $\gamma = M_L(\zeta)\ddot\zeta_r + C_L(\zeta,\dot\zeta_r)\dot\zeta_r + G_L + d_L$. Differentiating (21) and considering (22), one has $\ddot{\tilde\zeta} = \ddot\zeta_r + k_d\dot{\tilde\zeta} + k_dK_\zeta\tilde\zeta + K_\zeta\dot{\tilde\zeta}$, which leads to

$$\dot r_2 + k_d r_2 = \dot r_1. \quad (32)$$

Substitute (26) and (32) into (31), and consider Property 2.3, the closed-loop dynamic equation is obtained

$$\begin{aligned} M_L(\zeta)\dot r_2 = &- C_L(\zeta,\dot\zeta)r_2 - (k_dM_L(\zeta) - K_p)r_2 - K_pr_1 \\ &+ C_L(\zeta,\dot\zeta_r + r_1)r_2 - C_L(\zeta,r_1)(r_1 + 2\dot\zeta_r) \\ &- K_i(\tilde\zeta + e_\zeta) - \text{sgn}(r)\left(\ln\left(\cosh(\hat\Phi)\right) + \delta\right) \\ &- \gamma + B_{1a}G_{ra}K_{Na}s_a. \end{aligned} \quad (33)$$

Consider the Lyapunov function candidate

$$V = V_1 + V_2 \quad (34)$$

where $V_1 = (1/2)X^T\mathcal{M}X$ and $V_2 = (1/2)s_a^TL_{aa}s_a + (1/2)\tilde\alpha_a^T\beta^{-1}\tilde\alpha_a$, $X = [r_1^T\ e_\zeta^T\ r_2^T\ \tilde\zeta^T\ \tilde C^T]^T$, $\tilde C = C - \hat C$, and $\mathcal{M} = \text{diag}[M_L\ K_i\ M_L\ K_i\ \Gamma^{-1}]$, $\tilde\alpha_a = \alpha_a - \hat\alpha_a$. Differentiating V_1 with respect to time, one has $\dot V_1 = r_1^T(M_L\dot r_1 + (1/2)\dot M_Lr_1) + r_2^T(M_L\dot r_2 + (1/2)\dot M_Lr_2) + \tilde C^T\Gamma^{-1}\dot{\tilde C} + e_\zeta^TK_i\dot e_\zeta + \tilde\zeta^TK_i\dot{\tilde\zeta}$. From Property 2.2 and Property 2.3, the time derivative of V_1 along the trajectory of (33) is

$$\begin{aligned} \dot V_1 = &- r_1^TK_pr_1 + r_1^TK_pr_2 - r_1^TK_i(\tilde\zeta + e_\zeta) \\ &- r_1^T\text{sgn}(r)\left(\ln\left(\cosh(\hat\Phi)\right) + \delta\right) - r_1^T\gamma \\ &- r_1^TC_L(\zeta,\dot\zeta_r)r_1 + r_1^TB_{1a}G_{ra}K_{Na}s_a \\ &- r_2^T(k_dM_L(\zeta) - K_p)r_2 - r_2^TK_pr_1 \\ &+ r_2^TC_L(\zeta,\dot\zeta_r + r_1)r_2 - r_2^TC_L(\zeta,r_1)(r_1 + 2\dot\zeta_r) \\ &- r_2^TK_i(\tilde\zeta + e_\zeta) - r_2^T\text{sgn}(r)\left(\ln\left(\cosh(\hat\Phi)\right) + \delta\right) \\ &- r_2^T\gamma + r_2^TB_{1a}G_{ra}K_{Na}s_a + \tilde C^T\Gamma^{-1}\dot{\tilde C} \\ &+ e_\zeta^TK_i\dot e_\zeta + \tilde\zeta^TK_i\dot{\tilde\zeta}. \end{aligned}$$

Considering Lemma 4.1, one has $\ln(\cosh(\hat\Phi)) + \delta \geq \hat\Phi$, and $\|r\|(\ln(\cosh(\hat\Phi)) + \delta) \geq \|r\|\hat\Phi$, and $\dot e_\zeta = r_1 - K_\zeta e_\zeta + K_\zeta\tilde\zeta$, $\dot{\tilde\zeta} = r_2 - K_\zeta\tilde\zeta$ from (16) and (18), one has

$$\begin{aligned} \dot V_1 \leq &- r_1^TK_pr_1 - r_2^T(k_dM_L(\zeta) - K_p)r_2 - e_\zeta^TK_iK_\zeta e_\zeta \\ &- \tilde\zeta^TK_iK_\zeta\tilde\zeta + \|K_i\|\|e_\zeta\|\|r_2\| + \|K_iK_\zeta\|\|e_\zeta\|\|\tilde\zeta\| \\ &+ \|K_i\|\|\tilde\zeta\|\|r_1\| + \|r\|\|\gamma\| - \|r\|\hat\Phi + \tilde C^T\Gamma^{-1}\Lambda\hat C \\ &- \tilde C^T\Psi\|r\| - r_1^TC_L(\zeta,\dot\zeta_r)r_1 + r_2^TC_L(\zeta,\dot\zeta_r + r_1)r_2 \\ &- r_2^TC_L(\zeta,r_1)(r_1 + 2\dot\zeta_r) + r^TB_{1a}G_{ra}K_{Na}s_a. \end{aligned}$$

Since $\|K_i\|\,\|e_\zeta\|\,\|r_2\| \leq (1/2)\|K_i\|e_\zeta^Te_\zeta + (1/2)\|K_i\|r_2^Tr_2$, $\|K_iK_\zeta\|\,\|e_\zeta\|\,\|\tilde\zeta\| \leq (1/2)\|K_iK_\zeta\|e_\zeta^Te_\zeta + (1/2)\|K_iK_\zeta\|\tilde\zeta^T\tilde\zeta$, $\|K_i\|\,\|\tilde\zeta\|\,\|r_1\| \leq (1/2)\|K_i\|\tilde\zeta^T\tilde\zeta + (1/2)\|K_i\|r_1^Tr_1$, and $\tilde C^T\Gamma^{-1}\Lambda\hat C = \tilde C^T\Gamma^{-1}\Lambda(C - \tilde C) \leq (1/4)C^T\Gamma^{-1}\Lambda C$, and From Property 2.5, one has these valid relationships $\|C_L(\zeta,\dot\zeta_r)\| \leq \mu_1$, $\|C_L(\zeta,\dot\zeta_r + r_1)\| \leq \mu_2$, $\|C_L(\zeta,r_1 + 2\dot\zeta_r)\| \leq \mu_3$, where μ_1, μ_2 and μ_3 are known constants, therefore

$$\begin{aligned} \dot V_1 \leq &- r_1^T\left(K_p - \frac{1}{2}\|K_i\|\mathcal{I} - \mu_1\mathcal{I} - \frac{1}{2}\mu_3\mathcal{I}\right)r_1 \\ &- e_\zeta^T\left(K_iK_\zeta - \frac{1}{2}\|K_iK_\zeta\|\mathcal{I} - \frac{1}{2}\|K_i\|\mathcal{I}\right)e_\zeta \\ &- r_2^T\left(k_dM_L(\zeta) - K_p - \frac{1}{2}\|K_i\|\mathcal{I} - \mu_2\mathcal{I} - \frac{1}{2}\mu_3\mathcal{I}\right)r_2 \\ &- \tilde\zeta^T\left(K_iK_\zeta - \frac{1}{2}\|K_iK_\zeta\|\mathcal{I} - \frac{1}{2}\|K_i\|\mathcal{I}\right)\tilde\zeta \\ &+ \frac{1}{4}C^T\Gamma^{-1}\Lambda C + r^TB_{1a}G_{ra}K_{Na}s_a. \end{aligned} \quad (35)$$

Differentiating $V_2(t)$ with respect to time, considering $n - j - k$ joints, using (28) for ν_a, one has

$$\begin{aligned} \dot V_2 = -s_a^T\big[&-\nu_a + (L_{aa}\dot I_{ra} + R_{aa}I_{ra} + K_{ea}G_{ra}T_a\dot\zeta_d) \\ &+ R_{aa}s_a + K_{ea}G_{ra}T_a\dot e_\zeta\big] + \tilde\alpha_a^T\beta^{-1}\dot{\tilde\alpha}_a. \end{aligned} \quad (36)$$

Substituting ν_a into (36) by the control law (29) for ν_a, one has $\dot V_2 \leq -s_a^T(K_{\nu a} + R_{aa})s_a - s_a^TK_{ea}G_{ra}T(r_1 - K_\zeta e_\zeta + K_\zeta\tilde\zeta) + \|s_a\|\ \|\phi_a\| - s_a^T\text{sgn}(s_a)(\ln\cosh(\hat\varphi_a) + \delta) + \tilde\alpha_a^T\beta^{-1}\dot{\tilde\alpha}_a$. As similar as $\tilde C^T\Gamma^{-1}\Lambda\hat C$, one has $\tilde\alpha_a^T\beta^{-1}\kappa\hat\alpha_a \leq (1/4)\alpha_a^T\beta^{-1}\kappa\alpha_a$. By noting $\|s_a\|(\ln\cosh(\hat\varphi_a) + \delta) \geq \|s_a\|\hat\varphi_a$, therefore, one has

$$\begin{aligned} \dot V_2 \leq -s_a^T(K_{\nu a} + R_{aa})s_a + \frac{1}{4}\alpha_a^T\beta^{-1}\kappa\alpha_a \\ -s_a^TK_{ea}G_{ra}T_a(r_1 - K_\zeta e_\zeta + K_\zeta\tilde\zeta). \end{aligned} \quad (37)$$

Since the last term in (35), $r^TB_{1a}G_{ra}K_{Na}s_a \leq (1/2)\|B_{1a}G_{ra}K_{Na}\|\ \|r_1\|^2 + (1/2)\|B_{1a}G_{ra}K_{Na}\|\|r_2\|^2 + \|B_{1a}G_{ra}K_{Na}\|\ \|s_a\|^2$, and in (37), $-s_a^TK_{ea}G_{ra}T_a(r_1 - K_\zeta e_\zeta + K_\zeta\tilde\zeta) \leq (3/2)\|K_{ea}G_{ra}T_a\|\ \|s_a\|^2 + (1/2)\|K_{ea}G_{ra}T_a\|\ \|r_1\|^2 + (1/2)\|K_{ea}G_{ra}K_\zeta\|\ \|e_\zeta\|^2 + (1/2)\|K_{ea}G_{ra}K_\zeta\|\ \|\tilde\zeta\|^2$, integrating (35) and (37) and considering the previous two inequalities, $\dot{V}$ can be expressed as

$$\dot{V} \leq -r_1^T \mathcal{A} r_1 - r_2^T \mathcal{B} r_2 - s_a^T \mathcal{C} s_a - e_\zeta^T \mathcal{D} e_\zeta - \tilde{\zeta}^T \mathcal{D} \tilde{\zeta} + \frac{1}{4}\left(C^T \Gamma^{-1} \Lambda C + \alpha_a^T \beta^{-1} \kappa \alpha_a\right) \quad (38)$$

where $\mathcal{A} = K_p - (1/2)\|K_i\|\mathcal{I} - \mu_1 \mathcal{I} - (1/2)\mu_3 \mathcal{I} - (1/2)\|K_{ea} G_{ra} T_a\|\mathcal{I} - (1/2)\|B_{1a} G_{ra} K_{Na}\|\mathcal{I}$, $\mathcal{B} = k_d M_L(\zeta) - K_p - (1/2)\|K_i\|\mathcal{I} - \mu_2 \mathcal{I} - (1/2)\mu_3 \mathcal{I} - (1/2)\|B_{1a} G_{ra} K_{Na}\|\mathcal{I}$, $\mathcal{C} = K_{\nu a} + R_{aa} - \|B_{1a} G_{ra} K_{Na}\|\mathcal{I} - (3/2)\|K_{ea} G_{ra} T_a\|\mathcal{I}$, $\mathcal{D} = K_i K_\zeta - (1/2)\|K_i K_\zeta\|\mathcal{I} - (1/2)\|K_i\|\mathcal{I} - (1/2)\|K_{ea} G_{ra} K_\zeta\|\mathcal{I}$. From (38), one can choose K_p, K_i, $K_{\nu a}$, k_d, and K_ζ such that the matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$ are all positive definite. Therefore, one has

$$\begin{aligned}\dot{V} \leq & -\lambda_{\min}(\mathcal{A})\|r_1\|^2 - \lambda_{\min}(\mathcal{B})\|r_2\|^2 - \lambda_{\min}(\mathcal{C})\|s_a\|^2 \\ & - \lambda_{\min}(\mathcal{D})\|e_\zeta\|^2 - \lambda_{\min}(\mathcal{D})\|\tilde{\zeta}\|^2 \\ & + \frac{1}{4}\left(C^T \Gamma^{-1} \Lambda C + \alpha_a^T \beta^{-1} \kappa \alpha_a\right).\end{aligned} \quad (39)$$

Noting $(1/4)(C^T \Gamma^{-1} \Lambda C + \alpha_a^T \beta^{-1} \kappa \alpha_a) \to 0$ as $t \to \infty$ because of Definition 4.1.

Integrating both sides of the previous equation gives

$$\begin{aligned}V(t) - V(0) < & -\int_0^t \left(r_1^T \mathcal{A} r_1 + r_2^T \mathcal{B} r_2 + s_a^T \mathcal{C} s_a \right. \\ & \left. + e_\zeta^T \mathcal{D} e_\zeta + \tilde{\zeta}^T \mathcal{D} \tilde{\zeta}\right) d\varpi + \frac{1}{4} C^T \mathcal{E} C + \frac{1}{4} \alpha_a^T \mathcal{F} \alpha_a\end{aligned} \quad (40)$$

with $\mathcal{E} = \mathrm{diag}[a_i/\gamma_i]$, $i = 1, \ldots, 5$ and $\mathcal{F} = \mathrm{diag}[b_\iota/\beta_\iota]$, $\iota = 1, \ldots, 3$.

2) From (40), one has $V(t) < V(0) + (1/4)C^T \mathcal{E} C + (1/4)\alpha_a^T \mathcal{F} \alpha_a$, therefore, V is bounded, which implies that $r_1, r_2, s_a \in L_\infty^{n-j-k}$. From the definitions of r_1 and r_2 and s_a, it can be obtained that $e_\zeta, \dot{e}_\zeta, e_{Ia}, \dot{e}_{Ia} \in L_\infty^{n-j-k}$. As it has established that $e_\zeta, \dot{e}_\zeta, e_{Ia}, \dot{e}_{Ia} \in L_\infty$, from Assumption 4.1, it concludes that $\zeta(t), \dot{\zeta}(t), \dot{\zeta}_r(t), \ddot{\zeta}_r(t), I_a, \dot{I}_a, I_{ra}, \dot{I}_{ra} \in L_\infty^{n-k}$, and $\dot{q} \in L_\infty^n$.

Therefore, all the signals on the right-hand side of (33) are bounded and it is concluded that $\dot{r}_1$, $\dot{r}_2$, and $\dot{s}_a$ and, therefore, $\ddot{\zeta}$ and $\dot{I}_{ra}$ are bounded. Thus, r, $s_a \to 0$ as $t \to \infty$ can be obtained. Consequently, one has $e_\zeta, e_{Ia} \to 0$, $\dot{e}_\zeta, \dot{e}_{Ia} \to 0$ as $t \to \infty$. It follows that $e_q, \dot{e}_q \to 0$ as $t \to \infty$.

3) Substituting the control (26) and (27) into the reduced-order dynamic system model (25) yields

$$\begin{aligned}(I + K_f)e_\lambda = & -ZL^{+T} M_L(\zeta)\ddot{\zeta} + (\ln(\cosh(\hat{\chi})) + \delta)\ddot{\zeta}_r \\ & + B_{1b} G_{rb} K_{Nb} s_b.\end{aligned} \quad (41)$$

For the k joints in the force space, $V_1 = 0$, $I_b \in R^k$, (14) could be rewritten as $L_{ab}(dI_b/dt) + R_{ab} I_b = \nu_b$, therefore, V_2 could be rewritten as $V_2 = (1/2)s_b^T L_{ab} s_b + (1/2)\tilde{\alpha}_b^T \beta^{-1} \tilde{\alpha}_b$, and the time derivative of V_2 is as similar as (37), one has

$$\begin{aligned}\dot{V}_2 \leq & -s_b^T K_{Nb}(K_{\nu b} + R_{ab})s_b + \frac{1}{4}\alpha_b^T \beta^{-1} \kappa \alpha_b \\ \leq & -\lambda_{min}(K_{\nu b} + R_{ab}) + \frac{1}{4}\alpha_b^T \beta^{-1} \kappa \alpha_b.\end{aligned} \quad (42)$$

Noting $(1/4)\alpha_b^T \beta^{-1} \kappa \alpha_b \to 0$ as $t \to \infty$ because of κ satisfying Definition 4.1.

Integrating both sides of the previous equation gives

$$V_2(t) - V_2(0) < -\int_0^t s_b^T K_{Nb}(K_{\nu b} + R_{ab}) s_b d\varpi + \frac{1}{4}\alpha_b^T \mathcal{F} \alpha_b.$$

Since $K_{Nb}(K_{\nu b} + R_{ab}) > 0$ is bounded, $V_2(t) < V_2(0) + (1/4)\alpha_b^T \mathcal{F} \alpha_b$, therefore, $V_2(t)$ is bounded, one has $V(t)$ is bounded for the joints in the force space and then $s_b \to 0$ as $t \to \infty$. The proof is completed by noticing that since $\ddot{\zeta}$, Z, K_{Nb}, and s_b are bounded. Moreover, $\zeta_r \to \zeta_d$, $\zeta \to \zeta_d$, and the right side of (41) is bounded, the size of e_λ can be adjusted by choosing the proper gain matrices K_λ.

Since r, ζ, $\dot{\zeta}$, ζ_r, $\dot{\zeta}_r$, $\ddot{\zeta}_r$, e_λ, and s are all bounded, it is easy to conclude that τ is bounded from (23).

V. Simulations

Consider the mobile manipulator system shown in Fig. 1 [3] with the non-holonomic constraint $\dot{x}\cos\theta + \dot{y}\sin\theta = 0$, where $q_v = [x, y, \theta]^T$, $q_r = [\theta_1, \theta_2]^T$, $q = [q_v^T, q_r^T]^T$, and $A_v = [\cos\theta, \sin\theta, 0]^T$, $\tau_v = [\tau_{lw}, \tau_{rw}]^T$, $\tau_r = [\tau_1, \tau_2]^T$. The matrix H is chosen as $H = [H_1, H_2, H_3, H_4, H_5]$, $H_1 = [-\tan\theta, 0.0, 0.0, 0.0]^T$, $H_2 = [1.0, 0.0, 0.0, 0.0]^T$, $H_3 = [0.0, 1.0, 0.0, 0.0]^T$, $H_4 = [0.0, 0.0, 1.0, 0.0]^T$, and $H_5 = [0.0, 0.0, 0.0, 1.0]^T$. Given the desired trajectories and the geometric constraint that end-effector is subject to as follows: $y_d = 1.5\sin(t)$, $\theta_d = 1.0\sin(t)$, $\theta_{1d} = \pi/4(1 - \cos(t))$, $\Phi = l_1 + l_2 \sin(\theta_2) = 0$, and $\lambda_d = 10.0N$.

Remark 5.1: The existence of sign-function in the control (26), (29), and (30) may inevitably lead to chattering in control torques. To avoid such a phenomenon, in actual implementation, it can be replaced by the sat-function defined as: if $|\sigma| \geq \varsigma$, $\mathrm{sat}(\sigma) = \mathrm{sgn}(\sigma)$, else, $\mathrm{sat}(\sigma) = \varsigma/\sigma$, where $\sigma = r$ or s [14].

In the simulation, we assume the parameters $m_p = m_1 = m_2 = 1.0$ kg, $I_w = I_p = 1.0\,\mathrm{kg}\cdot\mathrm{m}^2$, $2I_1 = I_2 = 1.0\,\mathrm{kg}\cdot\mathrm{m}^2$, $I = 0.5\,\mathrm{kg}\cdot\mathrm{m}^2$, $d = l = r = 1.0$ m, $2l_1 = 1.0$ m, $2l_2 = 0.6$ m, $q(0) = [0, 2.0, 0.6, 0.5]^T m$, $\dot{q}(0) = [0.0, 0.0, 0.0, 0.0]^T$ m/s, $K_N = \mathrm{diag}[0.01]$ N $\cdot$ m/A, $G_r = \mathrm{diag}[100]$, $L_a = [5.0, 5.0, 5.0, 5.0]^T$ mH, $R_a = [2.5, 2.5, 2.5, 2.5]^T$ Ω, and $K_e = [0.02, 0.02, 0.02, 0.02]^T$ Vs/rad. The disturbances on the mobile base are introduced into the simulation model as $0.1\sin(t)$ and $0.1\cos(t)$. By Theorem 4.1, the observer gain is selected as $k_d = \mathrm{diag}[50]$, the control gains are selected as $K_p = \mathrm{diag}[10.0, 10.0, 10.0]$, $K_\zeta = \mathrm{diag}[1.0, 1.0, 1.0]$, $K_\nu = \mathrm{diag}[10.0, 10.0, 10.0, 10.0]$, $K_i = \mathrm{diag}[0.5]$, and $K_f = 0.5$, $\hat{C}(0) = [1.0, 1.0, 1.0, 1.0, 1.0]^T$, $K_N = \mathrm{diag}[0.1]$, $G_r = \mathrm{diag}[50]$, $\Lambda = \kappa = \mathrm{diag}[1/(1+t)^2]$, $\Gamma = \mathrm{diag}[4.8]$, $\beta = \mathrm{diag}[0.2]$, $\hat{\alpha}(0) = [0.001, 1.0, 0.01]^T$, $\varsigma = 0.01$. The simulation results for motion/force are shown in Figs. 2 and 3. The input voltages on the motors are shown in Fig. 3. In order to validate the better performance of the proposed control, in the same conditions, we compare: 1) the model based control, which assume 40% model uncertainty; 2) robust control by state feedback under the conditions that the constants $c_i (1 \leq i \leq 5)$ and $\alpha_{a\iota}$, $\alpha_{b\iota}$, $(1 \leq \iota \leq 3)$ can be obtained easily, therefore,

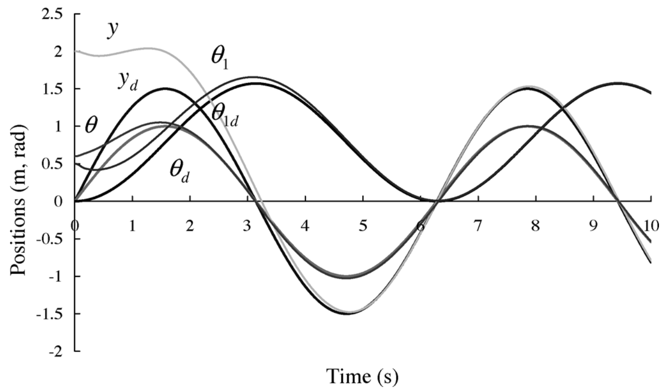

Fig. 2. Positions of the joints.

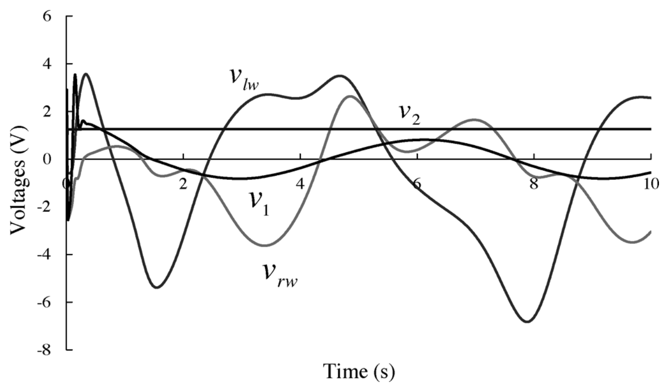

Fig. 3. Input voltages.

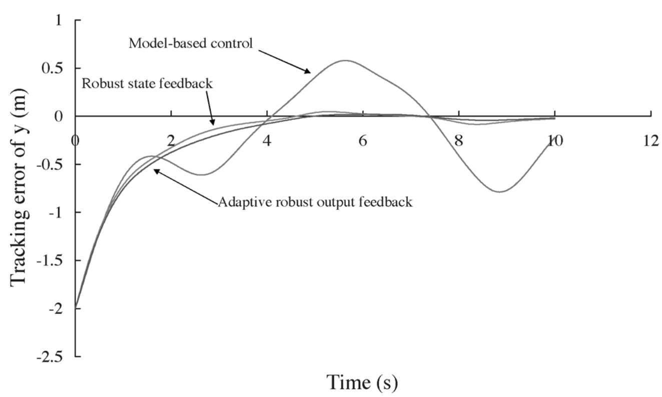

Fig. 4. Comparison of tracking y.

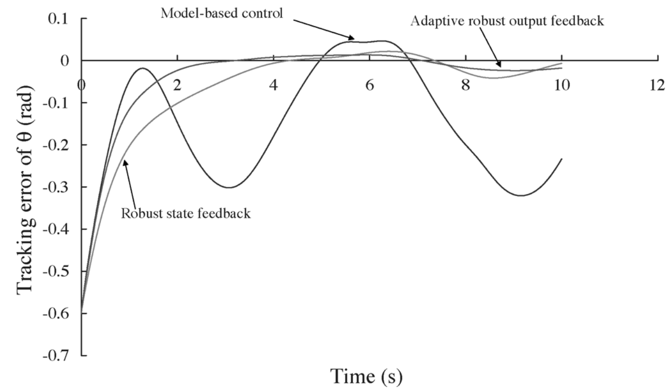

Fig. 5. Comparison of tracking θ.

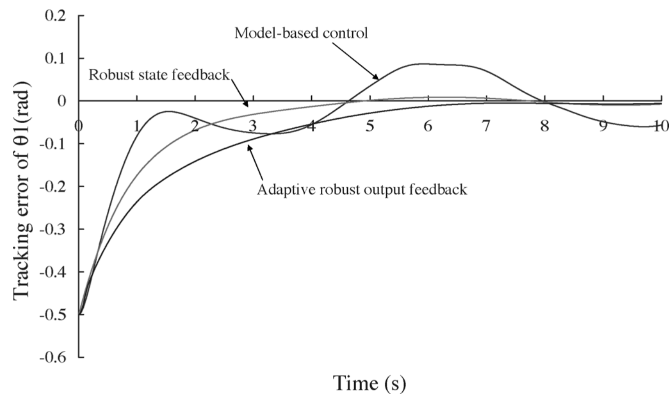

Fig. 6. Comparison of tracking θ_1.

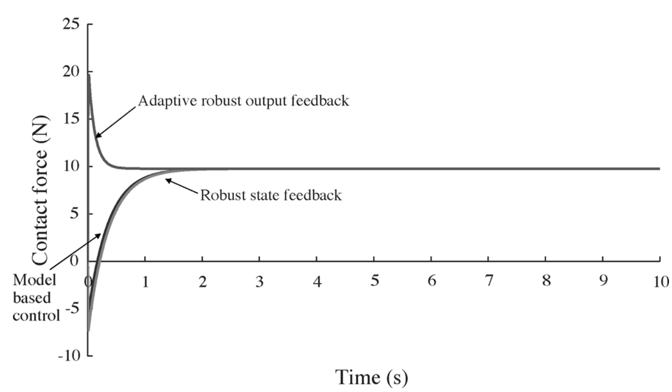

Fig. 7. Constraint force.

the adaptive update laws $\dot{\hat{C}} = -\Lambda\hat{C} + \|r\|\Gamma\Psi$ in (26) and $\dot{\hat{\alpha}}_a = -\kappa\hat{\alpha}_a + \|s_a\|\beta\mu_a$ and $\dot{\hat{\alpha}}_b = -\kappa\hat{\alpha}_b + \|s_b\|\beta\mu_b$ in (29) and (30) are not included in the implemented control, moreover, the velocity of states is assumed to be accurately measured; and 3) the proposed control without the conditions of 2). In the robust control by state feedback, we choose $c_1 = c_2 = c_3 = c_4 = c_5 = 10.0$, and $\alpha_1 = 0.006$, $\alpha_2 = 4.0$, and $\alpha_3 = 0.02$. The comparison of state variables y, θ, θ_1, and the contact force are shown in Figs. 4–7, respectively. The tracking performance of the three controls are illustrated in Figs. 4–6, with the proposed control scheme achieving better tracking performance compared with the model-based control and robust state feedback control. The better tracking performances is largely due to the "adaptive" mechanism. Although the parametric uncertainties and the external disturbances are both introduced into the simulation model, the force/motion control performance of system, under the proposed control, is not degraded. The simulation results demonstrate the effectiveness of the proposed adaptive control in the presence of unknown nonlinear dynamic system and environments. Different motion/force tracking performance can be achieved by adjusting parameter adaptation gains and control gains.

VI. Conclusion

In this brief, adaptive robust controls integrating an observer have been presented to control the holonomic constrained non-holonomic mobile manipulators in the presence of uncertainties and disturbances and actuator dynamics are considered in the controls. The proposed controls are non-regressor-based and require no information on the system dynamics. Simulation studies have verified the effectiveness of the proposed controls.

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