This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

# Design of Non-Regenerative MIMO-Relay System with Partial Channel State Information

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***Abstract*—Design strategy for non-regenerative multiple-input multiple-output (MIMO) relay system with partial channel state information (CSI) is developed. We assume that the CSI of the base station-relay link is fully known to the relay, while the CSI of the relay-mobile station link is not known except its channel statistics. Based on this model, weighting matrices to increase ergodic capacity is proposed for downlink and uplink MIMO-relay system, respectively, with the approximation for both the high and low signal-to-noise ratio (SNR) region. We also propose a switching scheme to cover the intermediate SNR region. Numerical results show that the proposed scheme outperforms the conventional one especially when the SNR becomes high.**

## I. Introduction

Wireless relay attracts great attentions because it provides reliable transmission and coverage extension to cellular systems. As the cell size becomes smaller for the newly developed wireless systems because of the higher frequency bands, the role of the wireless relay becomes more critical to reduce the cost of the deployment of expensive base station (BS). The former works on wireless relay may be categorized in two: regenerative and non-regenerative relay. Regenerative relay employs decode-and-forward scheme and regenerates the original information from the source. Non-regenerative relay employs amplify-and-forward scheme, which allows only simple linear processing. Compared to the regenerative relay, it has several advantages: The delay becomes short and fast processing is possible. It allows simple implementation since signal processing for decoding is unnecessary. However, it suffers from the noise enhancement induced by the relay.

Multiple antennas at the wireless transceivers enable high data rate communications over wireless channels. In [1] and [2], the capacity of point-to-point MIMO channel was investigated and extensive studies have been done for a decade [3],[4]. Recently, the study for MIMO systems is extended to the MIMO-relay systems [5]-[8]. In [6], the optimal design of non-regenerative relay for MIMO channel is investigated, and it is shown that the capacity increases significantly employing power allocations at the relay. Assuming full channel state information (CSI) for both source-relay link and relay-destination link, it finds out the optimal weight matrix for relay to maximize the capacity. Without direct link between source and destination, the optimal canonical coordinates are first established, and then, the optimal power allocations along these coordinates are then found. The similar works were done simultaneously in [7]. In [8], the joint optimization for transmitter and the relay is considered with full CSI, and an approximated solution is also proposed.

In practical system, however, the requirements for the use of the relay are critical. Firstly, processing delay at the relay should be minimized so that the relay may be merged into the conventional system without any change of the system configurations. Secondly, the simplicity of the relay is strongly required to lower the cost of the initial deployment and radio resources for feedback channel. Note that wireless relay station (RS) may possibly be located in the fixed position to enable the communication of the mobile stations (MSs) in the shadow area, and the channel of BS-RS link changes very slowly. In contrast, the channel of RS-MS link may change fast because the MS can move in high speed. Therefore the assumption in the previous works that the relay knows the CSI of all links is not feasible to meet the above requirements for the relay system. It causes processing delay for the estimation of the CSI and increases feedback overhead to report the CSI to the relay.

In this paper, design strategy for non-regenerative MIMO-relay with partial CSI is developed. We assume the CSI of the BS-RS link is fully known to the relay while the CSI of the RS-MS link is not known except its channel statistics. We propose weighting matrices for the relay to increase the ergodic capacity of MIMO-relay system based on this assumption, and give approximated solutions for both high and low signal-to-noise ratio (SNR) region in downlink and uplink, respectively. We also propose a switching scheme to cover the intermediate SNR region. These are evaluated in numerical results and compared to the capacity of the conventional scheme and the optimal scheme proposed in [6] which assumes full CSI is given.

The following notations are employed. Boldface capital letters and boldface small letters denote matrices and vectors. We use superscript $^H$ to denote a conjugated transpose operation, $\det(\cdot)$ to denote determinant, $(\cdot)^{-1}$ to denote matrix inversion, $tr(\cdot)$ to denote trace, and $\mathbf{I}$ to denote an identity matrix.

## II. Downlink Systems

### *A. System Model and Problem Formulation*

As depicted in Fig. 1, we assume a MIMO-relay system without direct link between BS and MS. BS, RS, and MS are equipped with $N_B$, $N_R$, and $N_M$ antennas, respectively. As-

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suming a RS with the fixed location, the channel coherence time of the BS-RS link is large and the channel response of this link is assumed to be constant over several time frames, which is represented as constant matrix $\mathbf{H}_1$. In contrast, the coherence time of the RS-MS link may be short, and its channel matrix is represented as a random matrix $\mathbf{H}_2$. The transfer function of the relay that transforms the received signal to the transmit signal is represented as a memoryless weighting matrix $\mathbf{G}$. Then, the signal model of the proposed system is given by

$$\mathbf{y} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2, \tag{1}$$

where $\mathbf{n}_1 \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{n}_2 \in \mathbb{C}^{N_M \times 1}$ are zero-mean complex Gaussian noise vector received at the RS and MS with the covariance matrices $\sigma_1^2\mathbf{I}$ and $\sigma_2^2\mathbf{I}$, respectively. $\mathbf{x} \in \mathbb{C}^{N_B \times 1}$ denotes zero-mean complex Gaussian vector whose covariance matrix is $p_1\mathbf{I}$, which indicates equal power is allocated for each antenna at the BS. BS and RS have individual power constraints $P_1$ and $P_2$, respectively, and $p_1$ is defined as $P_1 / N_B$. $\mathbf{H}_1$ and $\mathbf{H}_2$ are assumed to be complex Gaussian matrices with zero-mean and unit covariance matrix. Then, the ergodic capacity for unknown channel $\mathbf{H}_2$ is given by

$$C = \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{H}_1^H\mathbf{G}^H\mathbf{H}_2^H \cdot \left(\sigma_2^2\mathbf{I} + \sigma_1^2\mathbf{H}_2\mathbf{G}\mathbf{G}^H\mathbf{H}_2^H\right)^{-1}\right)\right\}, \tag{2}$$

where power constraint at the relay is given by

$$tr\left(\mathbf{G}\left(\sigma_1^2\mathbf{I} + p_1\mathbf{H}_1\mathbf{H}_1^H\right)\mathbf{G}^H\right) \le P_2. \tag{3}$$

Note that $\mathbf{H}_1$ is known to the relay, while only the channel statistics of $\mathbf{H}_2$ is known to it. In [6], it is proved that the maximal ergodic capacity is achieved by using a diagonal weighting matrix at the relay when the CSI of both links are unknown, while the capacity is achieved by canonical coordinates utilizing singular matrices of the MIMO channels when the CSI of both links are known. Based on this property, we let $\mathbf{G} = \mathbf{\Sigma}^{1/2}\mathbf{U}_1^H$ for simplicity, where $\mathbf{U}_1$ is the left singular matrix of $\mathbf{H}_1$. Then, (2) and (3) is re-formulated as

$$C = \mathcal{E}\left\{\left(\mathbf{I} + p_1\mathbf{H}_2\mathbf{\Sigma}\mathbf{\Lambda}_1\mathbf{H}_2^H\left(\mathbf{I} + \sigma_1^2\mathbf{H}_2\mathbf{\Sigma}\mathbf{H}_2^H\right)^{-1}\right)\right\}, \tag{4}$$

$$tr\left(\mathbf{\Sigma}\left(\sigma_1^2\mathbf{I}_{N_R} + p_1\mathbf{\Lambda}_1\right)\right) \le P_2. \tag{5}$$

In (4) and (5), $\sigma_2^2$ is assumed to be one for simplicity. It is valid in general because $p_1, p_2$, and $\sigma_1^2$ may be also normalized to $\sigma_2^2$ simultaneously. $\mathbf{\Sigma}$ is a diagonal power allocation matrix, which is defined as

$$\mathbf{\Sigma} = \begin{bmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_{N_m} \end{bmatrix}. \tag{6}$$

To find an appropriate value of $\mathbf{\Sigma}$, we initially investigate extreme cases to gain insight. We propose approximated solutions for both high and low SNR region in this paper. At first, the exact solution without noise at the relay will be found, and it will be extended to the high SNR region in general. Then, the solution at the low SNR region will be derived, and finally a switching scheme for the intermediate SNR region will be proposed.

*B. High SNR at the RS*

By assuming $\mathbf{n}_1$ in (1) is negligible, the exact solution that optimizes the ergodic capacity defined in (4) is obtained. By letting $\mathbf{n}_1 = 0$, (4) and (5) are expressed as

$$C = \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\mathbf{\Sigma}\mathbf{\Lambda}_1\mathbf{H}_2^H\right)\right\}, \tag{7}$$

where

$$tr\left(p_1\mathbf{\Lambda}_1\mathbf{\Sigma}\right) \le P_2. \tag{8}$$

In this scenario, the bottleneck is the RS-MS link because no noise is assumed at the relay so that the capacity of the BS-RS link increases infinitely. Therefore, the transmit covariance matrix at the relay, $p_1\mathbf{\Sigma}\mathbf{\Lambda}$, that maximizes the capacity of the unknown MIMO channel for RS-MS link also maximizes the whole system capacity *C*. In [1], it is proved that the optimal covariance matrix of the open-loop MIMO channel is an identity matrix that is scaled to the given power constraints. The proof is straightforward by utilizing the concavity of log determinant function and permutation matrices [1]. It is interpreted that the optimal power allocation for the unknown MIMO channel is dividing all the power equally to each transmit antennas. Based on this property, $C$ in (7) is maximized when $p_1\mathbf{\Sigma}\mathbf{\Lambda} = p_2\mathbf{I}_{N_R}$, where $p_2$ is defined as $P_2 / N_R$. Therefore, the optimal power allocation at the relay is

$$\mathbf{\Sigma} = \frac{p_2}{p_1}\mathbf{\Lambda}_1^{-1}. \tag{9}$$

The optimal weighting matrix in (9) makes the transmit covariance matrix for the unknown RS-MS link an identity matrix. The transmit power for each transmit antenna at the relay becomes equal with each other.

From the above solution without noise at the relay, the general solution for high SNR region where $p_1$ is assumed to be very large may be inferred. One candidate is making the transmit power of the relay equals for every transmit antennas. As denoted in (5), the covariance matrix of the transmit signal at the relay is represented as $\mathbf{\Sigma}(\sigma_1^2\mathbf{I} + p_1\mathbf{\Lambda})$. To allocate equal power to each antenna, the covariance matrix should be in the form of identity matrix with satisfying the power constraints in (5). The power allocation matrix at the relay that meets this condition is given by

$$\mathbf{\Sigma} = p_2\left(\sigma_1^2\mathbf{I} + p_1\mathbf{\Lambda}\right)^{-1}. \tag{10}$$

Note that it is not the optimal weighting matrix since the noise component is enhanced simultaneously. However, the relative amount of the noise is assumed to be very small in high SNR region so that we can achieve a near-optimal solution, which will be shown in the numerical results. The exact analysis at the high SNR region is a future work.

### C. Low SNR at the RS and BS

At the low SNR region where both $p_1$ and $p_2$ are assumed to be very small, the ergodic capacity in (4) may be approximated as follows.

$$\begin{aligned} C &= \mathcal{E}\Big\{\log\det\left(p_1\mathbf{H}_2\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\mathbf{H}_2^H + \sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}\mathbf{H}_2^H + \mathbf{I}\right) \\ &\qquad - \log\det\left(\sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}\mathbf{H}_2^H + \mathbf{I}\right)\Big\} \\ &\cong \mathcal{E}\Big\{\log\left(1 + tr\left(p_1\mathbf{H}_2\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\mathbf{H}_2^H + \sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}\mathbf{H}_2^H\right)\right) \\ &\qquad - \log\left(1 + tr\left(\sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}\mathbf{H}_2^H\right)\right)\Big\} \\ &\cong \log_2 e \times\left[tr\left(p_1 N_R \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\right)\right]. \end{aligned} \tag{11}$$

The third equality comes from $\log(1+x) \cong x\log e$ for small $x$ [10]. Both the ergodic capacity in (11) and the power constraint in (5) are represented as trace of the linear combination of $\boldsymbol{\Sigma}$. Therefore, to maximize $C$ in (11) satisfying the power constraint in (5) may be solved with linear programming [11], and the solution of this problem is allocating all the power to the stream with the highest singular value. It is given by

$$\boldsymbol{\Sigma} = \frac{P_2}{\sigma_1^2 + p_1}\begin{bmatrix} \delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_{N_R} \end{bmatrix}, \tag{12}$$

where $\delta_i = 1$ if the $i$th stream is selected, and $\delta_i = 0$ if not.

With the proposed weighting matrix, only one data stream is transferred to the MS among multiple data streams, since allocating all the power to the best data stream may be better in terms of ergodic capacity. It is interpreted that avoiding the noise enhancement is desirable at low SNR region where CSI of the RS-MS link is unknown.

### D. Switching for Intermediate SNR

The above solutions are not applicable for the intermediate SNR region. However, both schemes may be still employed by selecting the better one according to $\mathbf{H}_1$ with calculating the ergodic capacity for $\mathbf{H}_2$. Define the candidate solution for high SNR in (10) as $\boldsymbol{\Sigma}_1$, and the candidate solution for low SNR in (12) as $\boldsymbol{\Sigma}_2$. The ergodic capacity for $\boldsymbol{\Sigma}_1$ or $\boldsymbol{\Sigma}_2$ are given by

$$\begin{aligned} \mathcal{E}\{C\} &= \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\boldsymbol{\Sigma}_k\boldsymbol{\Lambda}_1\mathbf{H}_2^H\cdot\left(\sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}_k\mathbf{H}_2^H + \mathbf{I}\right)^{-1}\right)\right\} \\ &= \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\tilde{\boldsymbol{\Sigma}}_k\mathbf{H}_2^H\right) - \log\det\left(\mathbf{I} + \sigma_1^2\mathbf{H}_2\boldsymbol{\Sigma}_k\mathbf{H}_2^H\right)\right\}, \end{aligned} \tag{14}$$

where $\tilde{\boldsymbol{\Sigma}}_k = \boldsymbol{\Sigma}_k(\sigma_1^2/p_1\cdot\mathbf{I} + \boldsymbol{\Lambda}_1)$. Then assuming $\mathbf{H}_2$ as a complex Gaussian, $\mathbf{H}_2\boldsymbol{\Sigma}_k\mathbf{H}_2^H$ and $\mathbf{H}_2\tilde{\boldsymbol{\Sigma}}_k\mathbf{H}_2^H$ may be defined as a positive-definite quadratic forms in complex Gaussian matrix.

$$\mathbf{H}_2\boldsymbol{\Sigma}_k\mathbf{H}_2^H = \mathbf{Y} \sim Q_{N_M,N_R}\left(\mathbf{I},\mathbf{I},\boldsymbol{\Sigma}_k^{1/2}\right), \tag{15}$$

$$\mathbf{H}_2\tilde{\boldsymbol{\Sigma}}_k\mathbf{H}_2^H = \mathbf{Y} \sim Q_{N_M,N_R}\left(\mathbf{I},\mathbf{I},\tilde{\boldsymbol{\Sigma}}_k^{1/2}\right). \tag{16}$$

From [9], we can obtain the ergodic capacities in (14) when the channel and input covariance matrix is represented as positive-definite quadratic forms in complex Gaussian matrix as (15) and (16). Then, comparing the results for a given $\mathbf{H}_1$ we may compare the ergodic capacity with $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$, and select the larger one as final solution. Surprisingly, it is shown in numerical results that the ergodic capacity of MIMO-relay system increases with the proposed switching scheme even though the applied approximations for high and low SNR region are not valid for the intermediate SNR region.

Instead of utilizing the calculation in [9] which requires a little bit complex computations, a heuristic method, namely *sample test method*, may be utilized to select the covariance matrix with the larger ergodic capacity. As sample matrices for $\mathbf{H}_2$, we may randomly choose sample matrices with the same mean and covariance matrix. Then calculate an instantaneous capacity for the randomly chosen $\mathbf{H}_2$ matrices and compare the ergodic capacity for $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$, and we may choose the one with the larger ergodic capacity.

### E. Numerical Results

The ergodic capacities of the proposed schemes are evaluated. It is estimated from (2) and (3) using 1,000 independent realization of the channel matrices $\mathbf{H}_1$ and 1,000 independent realization of $\mathbf{H}_2$ for each $\mathbf{H}_1$, respectively. The proposed scheme is based on the switching scheme between approximated ones for high and low SNR region as described above. The conventional scheme, referred to as identity scheme, employs identity matrix as a weighting matrix of the relay. This indicates that the conventional one only amplifies and forwards the receive signal to each transmit antennas. The ergodic capacity of the optimal scheme proposed in [6] is also evaluated. All the channel state information of the BS-RS link and RS-MS links are utilized to derive the optimal weighting matrix.

Fig. 2 shows the comparisons of the ergodic capacity in downlink MIMO-relay system. SNR1 denotes the signal-to-noise ratio of the BS-RS link $p_1/\sigma_1^2$, while SNR2 denotes the signal-to-noise ratio of the RS-MS link $p_2/\sigma_2^2$. SNR2 is set to 0dB, 10dB, and 20dB, respectively, and the number of antennas of all nodes is set to 2. It is observed that the proposed scheme outperforms the identity scheme especially with high SNR1 with any SNR2 values. Especially when both SNR1 and SNR2 are high, the performance of the proposed scheme approaches the optimal one. For example, with SNR1=35dB, and SNR2=20dB, the proposed scheme shows almost 10% capacity gain over the identity scheme and has similar ergodic capacity with the optimal scheme. It is found that the performance increases a little when both SNR1 and SNR2 is very low. Without the exact information of the channel state, the contribution of the relay is not significant in low SNR region.

## III. Uplink Systems

### A. System Model and Problem Formulation

In uplink systems, we also assume non-regenerative MIMO-relay system without direct link as depicted in Fig. 3.

As discussed in downlink systems, the RS-BS link is static, while the MS-RS link varies fast. Therefore, it is hard to utilize the CSI of the MS-RS link to determine the weighting matrix of the relay. Instead, the proposed weighting matrices increase the ergodic capacity for the unknown MS-RS channel.

The signal model of the proposed non-regenerative MIMO-relay system in uplink is given by

$$\mathbf{y} = \mathbf{H}_1\mathbf{G}\mathbf{H}_2\mathbf{x} + \mathbf{H}_1\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2, \quad (17)$$

where $\mathbf{H}_1 \in \mathbb{C}^{N_B \times N_R}$ denotes MIMO channel matrix of the RS-BS link, which is assumed to be known at the relay. The relay may know $\mathbf{H}_1$ by utilizing the channel reciprocity in time-division duplex system or feedback information in frequency-division duplex system. Because $\mathbf{H}_1$ is assumed to be static, it may be updated with long time interval and feedback overhead required for it is small. $\mathbf{H}_2 \in \mathbb{C}^{N_R \times N_M}$ denotes MIMO channel matrix of the MS-RS link. $\mathbf{n}_1 \in \mathbb{C}^{N_R \times 1}$ and $\mathbf{n}_2 \in \mathbb{C}^{N_B \times 1}$ are zero-mean complex Gaussian noise vector received at the RS and BS whose covariance matrix is $\sigma_1^2\mathbf{I}$ and $\sigma_2^2\mathbf{I}$, respectively. $\mathbf{x} \in \mathbb{C}^{N_M \times 1}$ is assumed to be a zero-mean complex Gaussian vector whose covariance matrix is $p_1\mathbf{I}_{N_M}$, which denotes equal power allocation is assumed at the transmitter. Then the ergodic capacity for $\mathbf{H}_2$ is represented as

$$C = \mathcal{E}\Big\{\log\det\Big(\mathbf{I} + p_1\mathbf{H}_1\mathbf{G}\mathbf{H}_2\mathbf{H}_2^H\mathbf{G}^H\mathbf{H}_1^H \cdot \left(\sigma_2^2\mathbf{I} + \sigma_1^2\mathbf{H}_1\mathbf{G}\mathbf{G}^H\mathbf{H}_1^H\right)^{-1}\Big)\Big\}, \quad (18)$$

where the average power constraint is assumed at the relay.

$$\mathcal{E}\left\{tr\left(\mathbf{G}\left(p_1\mathbf{H}_2\mathbf{H}_2^H + \sigma_1^2\mathbf{I}\right)\mathbf{G}^H\right)\right\} = tr\left(\mathbf{G}\left(p_1N_m + \sigma_1^2\right)\mathbf{G}^H\right) \le p_2. \quad (19)$$

Similar to the downlink system, weighting matrix that achieves the maximum ergodic capacity is represented as $\mathbf{G} = \mathbf{V}_1\boldsymbol{\Sigma}^{1/2}$, where $\mathbf{V}_1$ denotes right singular matrix of $\mathbf{H}_1$ and $\boldsymbol{\Sigma}$ is a power allocation matrix. Then, (18) and (19) is re-formulated as

$$C = \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\mathbf{H}_2^H\boldsymbol{\Sigma}\boldsymbol{\Lambda}\left(\mathbf{I} + \sigma_1^2\boldsymbol{\Sigma}\boldsymbol{\Lambda}\right)^{-1}\right)\right\}, \quad (20)$$

$$tr\left(\boldsymbol{\Sigma}\left(p_1N_m + \sigma_1^2\right)\right) \le P_2. \quad (21)$$

For (20), $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$ is utilized. $p_1$, $p_2$, and $\sigma_1^2$ are normalized by $\sigma_2^2$ without loss of the generality. The definition of $\boldsymbol{\Sigma}$ is the same with that in (6). Based on this, we also propose approximated solutions for both high and low SNR region to initially investigate extreme cases to gain insight.

*B. High SNR*

In the high SNR region where $p_1$ is high, the ergodic capacity is calculated as

$$\begin{aligned} C &\cong \mathcal{E}\left\{\log\det\left(p_1\mathbf{H}_2\mathbf{H}_2^H\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\left(\sigma_1^2\boldsymbol{\Lambda}_1\boldsymbol{\Sigma} + \mathbf{I}\right)^{-1}\right)\right\} \\ &= \mathcal{E}\left\{\log\det\left(p_1\mathbf{H}_2\mathbf{H}_2^H\right)\right\} + \sum_i \log\left(x_i\lambda_i\left(\sigma_1^2x_i\lambda_i + 1\right)^{-1}\right), \end{aligned} \quad (22)$$

where $x_i$ and $\lambda_i$ are the $i$th component of the diagonal matrices $\boldsymbol{\Sigma}$ and $\boldsymbol{\Lambda}_1$. It is easy to show that the last equation of (22) is concave for $x_i$. Introducing Lagrange multipliers,

$$\begin{aligned} L\{x,\upsilon\} &= \mathcal{E}\left\{\log\det\left(p_1\mathbf{H}_2\mathbf{H}_2^H\right)\right\} \\ &+ \sum_i \log\left(x_i\lambda_i\left(\sigma_1^2x_i\lambda_i + 1\right)^{-1}\right) - \upsilon\left(\sum_i x_i\left(p_1N_m + \sigma_1^2\right) - P_2\right), \end{aligned} \quad (23)$$

with the Karush-Kuhn-Tucker (KKT) conditions [12]. Then the optimal $x_i$ becomes

$$x_i = \frac{1}{2\sigma_1^2\lambda_i}\left[\sqrt{1 + 4\mu\sigma_1^2\lambda_i\left(p_1N_R + \sigma_1^2\right)^{-1}} - 1\right], \quad (24)$$

where $\mu = 1/\upsilon$ should be chosen to meet the power constraint. The solution in (24) is similar to the water-filling scheme.

*C. Low SNR*

In the low SNR region where both $p_1$ and $p_2$ are small, the power allocation matrix in (20) may be derived with the following approximation.

$$\begin{aligned} C &= \mathcal{E}\left\{\log\det\left(\mathbf{I} + p_1\mathbf{H}_2\mathbf{H}_2^H\boldsymbol{\Sigma}\boldsymbol{\Lambda}_1\left(\sigma_1^2\boldsymbol{\Lambda}_1\boldsymbol{\Sigma} + \mathbf{I}\right)^{-1}\right)\right\} \\ &\cong \sum_i p_1N_mx_i\lambda_i\left(\sigma_1^2x_i\lambda_i + 1\right)^{-1}\log e. \end{aligned} \quad (25)$$

The third equality comes from $\log(1+x) \cong x\log$ for small $x$. Since the last equation in (25) is concave for $x_i$, the optimal $x_i$ may be found utilizing convex optimization problem like high SNR case. Then, the Lagrange multiplier is given by

$$\begin{aligned} L\{x,\upsilon\} &= \sum_i\left(p_1N_mx_i\lambda_i\left(\sigma_1^2x_i\lambda_i + 1\right)^{-1}\right)\log e \\ &-\upsilon\left(\sum_i x_i\left(p_1N_m + \sigma_1^2\right) - P\right) = 0. \end{aligned} \quad (26)$$

with the following Karush-Kuhn-Tucker (KKT) conditions [12], which is omitted here. Then, letting $\mu = \log e/\upsilon$,

$$x_i = \frac{1}{\sigma_1^2\lambda_i}\left[\sqrt{\mu p_1N_R\lambda_i\left(p_1N_R + \sigma_1^2\right)^{-1}} - 1\right]^+ \quad (27)$$

In (27), $[x]^+$ denotes $x$ when it is equal or larger than zero, while zero when it is smaller when $x$ is smaller than zero. $\mu$ should be chosen to meet the KKT conditions. The solution in (27) is also a kind of the water-filling scheme.

*D. Switching in Intermediate SNR Region.*

Like downlink systems, for the intermediate SNR region, both schemes may be still employed by selecting the better one according to $\mathbf{H}_1$ with calculating the ergodic capacity for $\mathbf{H}_2$. The method to calculate the ergodic capacity is similar to that of the downlink system. We may use either direct calculation utilizing the analytical results in (14)-(16) proposed in [9], or the *sample test method.*

*E. Numerical Results*

The ergodic capacities of the proposed schemes are evaluated. It is estimated from (18) and (19) using 1,000 independent realization of the channel matrices $\mathbf{H}_1$ and 1,000 independent realization of $\mathbf{H}_2$ for each $\mathbf{H}_1$, respectively. The other configurations are the same with those of the downlink system described above. Comparisons of the ergodic capacity of the uplink MIMO-relay system is given in Fig. 4. Different from downlink case, SNR1 denotes the signal-to-noise ratio of the MS-RS link $p_1/\sigma_1^2$, while SNR2 denotes the signal-to-noise ratio of the RS-BS link $p_2/\sigma_2^2$. SNR2 is set to 0dB, 10dB, and 20dB, respectively, and the number of antennas of all nodes is set to 2. The proposed scheme outperforms the conventional one when SNR2=0dB and 10dB, while the performance gain is small when SNR2=20dB. Note that the gain of the water-filling scheme is shown in the low SNR region. The proposed scheme in uplink system is also a kind of water-filling scheme for the known RS-BS link, and the performance gain is expected when SNR2 is low. For example, when SNR1=30dB, and SNR2= 0dB, the capacity gain of the proposed scheme over the identity scheme is almost 25%.

## IV. CONCLUSIONS

In this paper, we have proposed a weighting scheme for the relay of MIMO-relay system with partial channel state information. Reflecting the practical environments, we have assumed that only the channel statistics are known for RS-MS link, while the channel states information of BS-RS link is fully known to the relay. The proposed weighting matrix firstly decomposes the BS-RS channel into parallel streams and performs power allocation for each stream. For both downlink and uplink approximation for high and low SNR region are performed to find out appropriate power allocation matrices. Then, switching scheme to cover the intermediate SNR region is also introduced. Numerical results have showed that the proposed schemes outperform the conventional one and approach the optimal one especially in the high SNR region.

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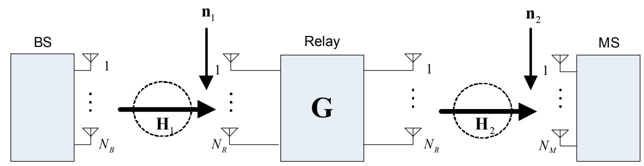


Fig. 1. Non-regenerative MIMO-relay system in downlink.

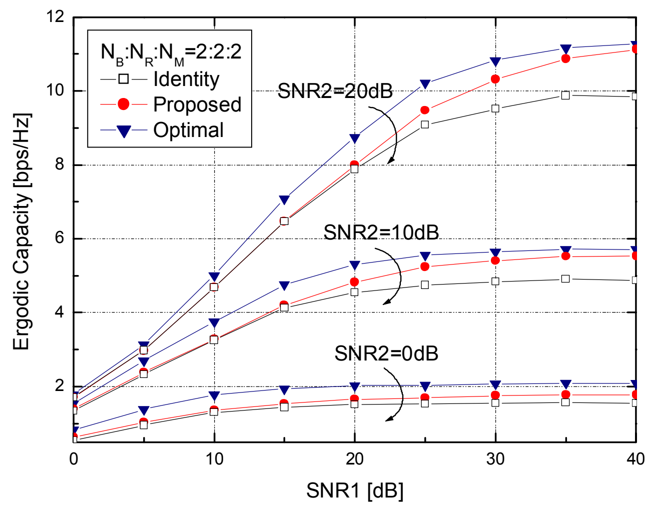


Fig. 2. Comparisons of the ergodic capacity in downlink.

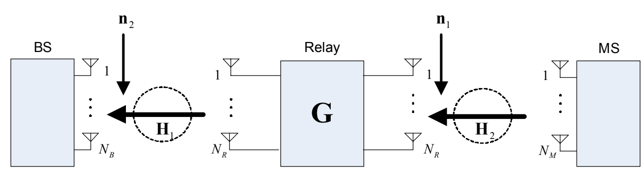


Fig. 3. Non-regenerative MIMO-relay system in uplink.

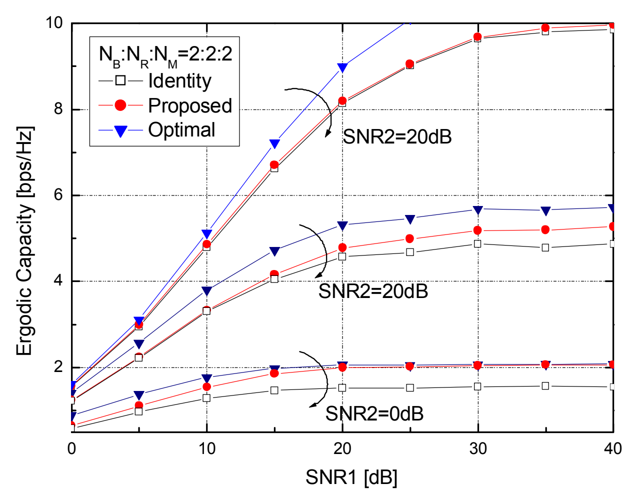


Fig. 4. Comparisons of the ergodic capacity in uplink.