A Subspace-Tracking Approach to Interference Nulling for Phased Array-Based Radio Telescopes

Steven W. Ellingson, *Member, IEEE,* and Grant A. Hampson

***Abstract*—Several next-generation radio telescopes, now in the planning stages, are based on phased-array technology. One reason for this is to make use of adaptive nulling techniques to combat radio frequency interference, which is a growing problem for radio astronomy. This paper presents a low-complexity approach to interference nulling which is suitable for use in such systems. The approach uses subspace tracking to identify interference, followed by spatial projections to place deep nulls in the directions of interferers. This technique overcomes two limitations of power-minimization algorithms (e.g., "minimum variance"), namely *power inversion* and *pattern rumble*, which create serious problems for radio astronomy. Furthermore, this technique imposes a lower computational burden and provides side information which is useful in later stages of data processing. Performance results from a phased array demonstrator system and a simulation are presented.**

***Index Terms*—Beamforming, phased arrays, radio astronomy, spatial nulling, subspace tracking.**

I. Introduction

SEVERAL next-generation radio telescopes, now in the planning stages, are based on phased-array technology. In fact, the astronomical community is now making detailed plans for a radio telescope with 1 km^2 effective aperture, known as the square kilometer array (SKA) [1]. In one design concept for SKA, the basic unit is a "one square meter array" (OSMA). Fig. 1 shows a OSMA technology demonstrator that is currently in operation [2]. In this concept, SKA would consist of perhaps one million such systems working in unison. Other SKA concepts are also based on arrays, the main differences being the number and directivity of the elements. For example, arrays consisting of small paraboloids or Luneburg lenses are also being considered [3], [4]. In all cases, however, a very large number of elements is needed.

Strong motivation for this architecture comes from the desire to use adaptive nulling techniques to combat radio frequency interference (RFI), which is a growing problem for radio astronomy. Perhaps the best-known approach to adaptive nulling for phased arrays is constrained power minimization, using algorithms such as minimum variance (MV) [5]. For an N-element array with a beamformer of the form

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{1}$$

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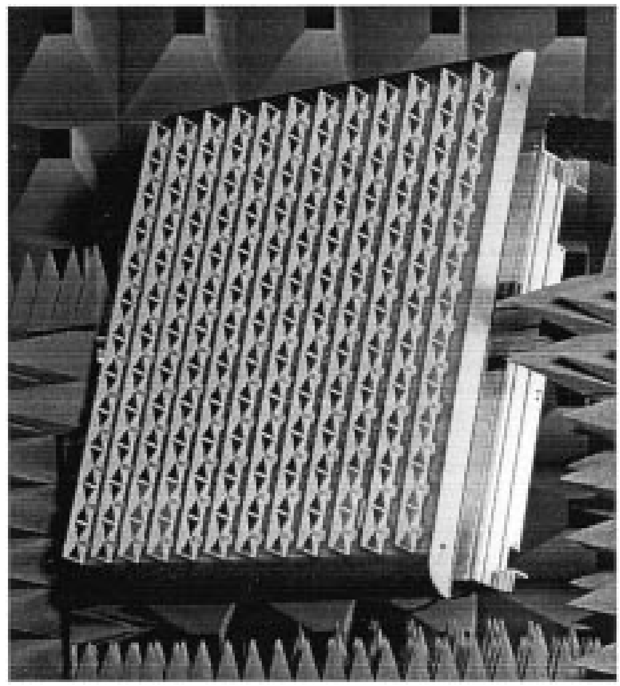

Fig. 1. The OSMA experimental phased array.

the MV weights are given by

$$\mathbf{w} = \frac{R^{-1}\mathbf{a}_0}{\mathbf{a}_0^H R^{-1} \mathbf{a}_0} \tag{2}$$

where $\mathbf{x}(t)$ is the $N \times 1$ vector of element outputs at time t, y is the beamformer output, $\mathbf{a}_0$ is the $N \times 1$ steering vector which defines the desired beam pointing direction and R is the array covariance matrix, defined as the expected value of the outer product $\mathbf{x}(t)\mathbf{x}^H(t)$. The superscript H denotes the conjugate transpose.

MV is not suitable for astronomy applications, for several reasons. First, the *power inversion* property of MV limits null depth to be proportional to the interference-to-noise ratio (INR) [6]. This is acceptable in many communications and radar applications, since traditional demodulation/detection algorithms perform well even as the INR approaches unity. In contrast, radio astronomy involves no processing that is analogous to demodulation, so it is important to eliminate any interference which can be detected. For example, a common observation consists of identifying a spectral line with power spectral density that is many orders of magnitude weaker than the passband noise power spectral density. Such an observation may require seconds, minutes, or even hours of integration to achieve a positive detection. Thus, intermittent interference may have INR much less than 1 and yet be orders of magnitude stronger than the desired signal, with the potential to wreck the results for that integration period. MV is ineffective against such interference.

Second, MV is subject to a phenomenon known as "weight jitter" or "pattern rumble." MV relies on inversion of R for estimation of the interference. However, R also includes information about the noise, which is represented by the high-order eigenvalues of R. Inversion causes the noise eigenvalues to become large, allowing the noise eigenvectors—which are effectively random—to make a significant contribution to the calculated weights [7]. The resulting patterns exhibit considerable variability between updates; even in the main beam. The use of additional constraints as in the linearly-constrained minimum variance technique [8] is not an effective solution since the randomness of the noise subspace is not affected.

Sources of RFI in radio astronomy include nongeosynchronous satellites and land-mobile radio. To deal effectively with these dynamic signals, it is expected that the weight update period for nullforming algorithms will need to be on the order of 10 ms. At the same time, there are numerous other environmental and instrumental errors which must be corrected in order to generate high-quality astronomical images. These errors are mitigated using a process known as selfcalibration [9]. This process is computationally intensive and is typically updated on the order of once per minute. Current selfcalibration methods require that the adapted patterns be approximately stationary between selfcalibration updates; i.e., only the desired nulls should be allowed to move. Therefore, MV-induced pattern rumble greatly complicates the process of making astronomical images.

To further clarify the reasons for the undesirable behavior of MV and introduce alternative approaches, consider the following; it is well known that R can be decomposed as follows:

$$R = U\Sigma U^H \tag{3}$$

where U is an $N \times N$ matrix whose columns are the eigenvectors of R and Σ is an $N \times N$ diagonal matrix whose elements are the corresponding eigenvalues of R. R can be further decomposed as follows:

$$R = U_S \Sigma_S U_S^H + U_N \Sigma_N U_N^H \tag{4}$$

where Σ_S is a $k \times k$ diagonal matrix whose elements are the k largest eigenvalues, U_S is an $N \times k$ matrix whose columns are the associated eigenvectors, Σ_N is a $(N-k) \times (N-k)$ diagonal matrix whose elements are the remaining $N-k$ eigenvalues, and U_N is an $N \times (N-k)$ matrix whose columns are the associated eigenvectors. If k is selected appropriately, then the term $U_S \Sigma_S U_S^H$ completely describes interference, whereas $U_N \Sigma_N U_N^H$ describes only noise. In this case, we refer to the column span of U_S as the "interference subspace" and to the column span of U_N as the "noise subspace." Since the eigenvectors of R are orthogonal, k is the rank of the interference subspace. For readers who are unfamiliar with radio astronomy, we wish to emphasize that there is only interference and noise present in the observation; i.e., the "signals" are not apparent over the timeframe of the weight update. Thus, it is the noise subspace that we wish to preserve.

For MV, the properties of power inversion and pattern rumble can be traced to the use of the entire covariance matrix, as opposed to simply the interference subspace, in the calculation of the adaptive weights. An alternate approach, which is not subject to this liability, is to explicitly estimate the interference subspace and then to calculate weights corresponding to beams in the orthogonal complement of this subspace. This method of forming nulls is known as "spatial projection" or "orthogonal projection" [7], [10]. A straightforward implementation of this approach is to compute an estimate of R as in MV and then identify the interference subspace using an eigendecomposition, such as a singular value decomposition (SVD). This concept has also recently been suggested for existing radio telescope arrays consisting of small numbers of dish-type antennas [11]. For large arrays, however, both the computation of R and the eigendecomposition impose very large computational burdens.

An attractive alternative to this approach is *subspace tracking* [12]. Using subspace tracking, one can develop rank-ordered[1] estimates of the eigenvectors and eigenvalues of R on a sample-by-sample (iterative) basis, with no need to explicitly estimate R. The purpose of this paper is to demonstrate the use of subspace tracking followed by spatial projections in OSMA-type systems designed for radio astronomy. For illustration purposes, we use the "projection approximation subspace tracking with deflation" (PASTd) method of Yang [13] in this paper, which has relatively low complexity, is simple to implement and yields the eigenvectors directly. However, other subspace trackers can be used in place of PASTd.

The remainder of this paper is arranged as follows. Section II describes a candidate implementation of this approach, based on the PASTd method. In Section III, we describe an experiment demonstrating the feasibility of subspace tracking using OSMA. We also demonstrate in simulation the feasibility of accurate interference subspace identification for $INR = 1$. Section IV considers the relative computational burden associated with the proposed approach and MV. We conclude that this approach is able to generate the desired nulls without pattern rumble and with computational cost comparable to or better than MV.

II. Subspace-Tracking Spatial Projections

In this section, we describe a specific implementation of the proposed subspace-tracking spatial projections (STSP) approach for a single-beam system of the form given in (1). This implementation is shown in Fig. 2. The steps in this implementation are as follows:

PASTd: Digitized complex data from the array is processed using PASTd. For each new snapshot from the array, the PASTd algorithm proceeds as follows:

For $n = 1$ to $k_{\max}$ do

$$\begin{aligned}
\gamma_n &\leftarrow \mathbf{u}_n^H \mathbf{x}_i \\
d_n &\leftarrow \beta d_n + |\gamma_n|^2 \\
\mathbf{u}_n &\leftarrow \mathbf{u}_n + \frac{(\mathbf{x}_i - \mathbf{u}_n \gamma_n)\gamma_n^*}{d_n} \\
\mathbf{x}_i &\leftarrow \mathbf{x}_i - \mathbf{u}_n \gamma_n.
\end{aligned}$$

[1]By this we mean that eigenvalue-eigenvector pairs are obtained in sequence, in descending order by eigenvalue.

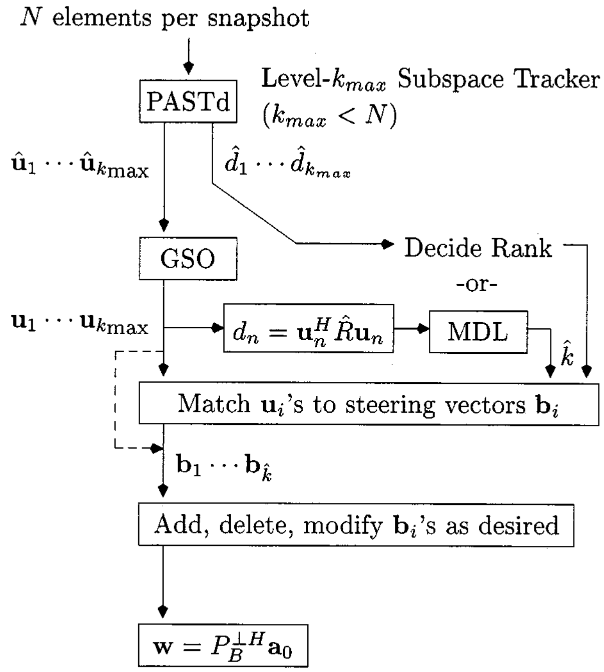

Fig. 2. One possible implementation of the STSP approach.

Above, $\mathbf{u}_n$ is an estimate of the nth eigenvector and d_n is a weighted estimate of the nth eigenvalue ($\beta = 0.99$ is recommended). The number $k_{\max}$ of eigenvectors and eigenvalues to track is an *a priori* decision; thus, $k_{\max}$ is also the maximum rank of the interference subspace that can be correctly estimated. The initial values of the $\mathbf{u}_n$'s and d_n's are not critical.

GSO: The eigenvectors produced by PASTd are not guaranteed to be orthonormal, whereas those of the true R are orthonormal. Therefore, orthonormalization tends to improve the accuracy of the eigenvector estimates. The Graham–Schmidt orthonormalization (GSO) procedure [14] is suitable for this task. It need be performed only once, after PASTd has finished.

Rank Estimation: There are a variety of methods by which the rank of the interference subspace can be estimated from the output of PASTd and GSO. A simple method, used in the simulation described in the next section, is as follows: First, improved eigenvalue estimates d_n are computed using the GSO-processed eigenvector estimates $\mathbf{u}_n$ and a covariance matrix estimate $\hat{R}$ computed from a small number (say, $L/10$) of samples. The minimum description length (MDL) approach [15] can then be used to estimate the rank $\hat{k}$ of the interference subspace, using the d_n. For this, we propose the algorithm of Wax and Kailath [16]. Alternatively, various other criteria can be used to determine the rank of the signal subspace; for example, threshold testing for the eigenvalues, or arbitrary limits on the rank; e.g., "choose $\hat{k}$ to be at least 2." Choice of an optimal rank estimator for radio astronomy applications is an important problem, but outside the scope of this paper.

Steering Vector Matching: In principle, there is no difficulty in using the estimated eigenvectors $\mathbf{u}_n$ directly to compute a suitable nullspace projection. However, it may be of interest to know the directions of arrival (DOA) associated with the various interferers. Such information is useful in radio astronomy for the purposes of array calibration, RFI characterization, and anticipating problems with pattern distortion, i.e., when interferers approach the main beam. In the absence of strongly-correlated multipath, the MUSIC technique can be used to estimate the DOAs [17]. Otherwise, a maximum likelihood method, such as that of Ziskind and Wax [18], can be used to estimate the k DOAs, ψ_i, as follows:

$$\arg\max_{\{\psi_i\}} \operatorname{Tr}\left[P_{A(\{\psi_i\})}\hat{R}\right] \tag{5}$$

where $\hat{R}$ is the estimate of R constructed from eigenvectors and eigenvalues obtained from the previous steps. $A(\{\psi_i\})$ is a matrix whose columns are the steering vectors associated with the set of k "trial" DOA's, which depend on the array geometry. P_A refers to the projection operator $A(A^H A)^{-1}A^H$ and "Tr" denotes the trace; that is, the sum of the diagonal elements. For the case of $k = 1$, this simplifies to a procedure in which one attempts to maximize the inner product $\mathbf{a}^H(\psi)\mathbf{u}_1$, in which $\mathbf{a}(\psi)$ is the steering vector associated with look direction ψ.

Furthermore, it may be desired to replace one or all of the interference eigenvectors with steering vectors obtained using the above procedure. In this way, one may generate nulls which are guaranteed to be as deep as possible in the appropriate directions. Thus (referring to Fig. 2), the selected vectors $\mathbf{b}_i$ may be either the estimated interference eigenvectors, the associated steering vectors, or some combination thereof.

Also, it should be noted that this has implications for array calibration. If the array is calibrated, then the interferer steering vectors computed above infer the DOAs. If the array is not calibrated, but the interferer DOAs are known, then the computed steering vectors can be used to aid in calibration of the array. However, in no case is calibration required to null interference using STSP.

Nulling Options: In addition to the possibility of using steering vectors in lieu of the eigenvectors to represent the interference subspace, there are other options to tailor the "desired" interference subspace before nulling. If $N \gg k$, it may be practical to create additional nulls. This is useful, for example, when a dangerous interferer is known to be present, but has INR < 1 and therefore cannot be reliably detected or estimated in the short period between adaptive weight updates. This process involves identifying the appropriate steering vector and adding it to the list of vectors to include in the nullspace projection. Alternatively, it may be desired to delete vectors from the estimated interference subspace. In radio astronomy, this might occur if the damage caused by an unsuppressed interferer is preferable to the pattern distortion resulting from a particular null. A simple method is to compute the inner product of each interference steering vector with the desired steering vector for beam pointing. If both vectors have unit norm, than a result close to 1 indicates that the associated null is too close to the main beam and so that vector should be excluded. In practice, a threshold would be set based on the user's ability to tolerate distortion. More elegant algorithms are of course possible, but beyond the scope of this paper.

Finally, we note that it may be desired to modify the vectors defining the interference subspace to shape the pattern in some way other than to form a deep null. For example, if the interference is fast-moving, it may be desirable to create a pattern depression or "flat null" as opposed to a zero in the pattern.

Beamforming: Referring again to Fig. 2, we now form the matrix B, representing the interference subspace, by concatenating the selected vectors $\mathbf{b}_i$ obtained from the previous steps. The nullspace projection is then defined as the matrix $P_B^{\perp}$ which, when applied to an input snapshot $\mathbf{x}$, yields another snapshot which lies in the nullspace of B. Therefore, $P_B^{\perp} = I - P_B$. If the desired steering vector for beam-pointing is $\mathbf{a}_0$, then the desired adaptive weights are $\mathbf{w} = \left(P_B^{\perp}\right)^H \mathbf{a}_0$.

III. Measured and Simulated Results

An important test for the feasibility of STSP for OSMA-like systems is to verify the operation of subspace tracking in actual hardware. For this purpose, we used the OSMA technology demonstrator shown in Fig. 1. This system consists of 64 bow-tie elements connected to a dual hierarchical beamforming system consisting of RF and digital beamforming. Four column antennas are combined into one RF signal (frequency range 1.5–3.5 GHz) which is frequency down converted and digitized using a sample rate of 8 MHz. The resulting array configuration for the digital beamformer is eight columns by two rows. The effective column and row spacing is then half and double wavelength, respectively, at the test frequency of 2 GHz.

The 16 complex signals obtained from the array are then processed using PASTd with $k_{\max} = 5$. In this experiment, the array is illuminated by an interferer with INR $= +32$ dB incident from 70° with respect to broadside in the H-plane. Due to the limited size of the test chamber, the array operates in the near field of the source (source distance is 4 meters, whereas the physical aperture of the array is 0.525 meters). Fig. 3 shows the eigenvalue estimates from PASTd. Note that the largest eigenvalue converges to the true INR of $+32$ dB within about 200 samples, whereas the remaining eigenvalues assume various values around 0 dB. In this case, these eigenvalues represent not only noise but also the combined effects of wavefront curvature and spurious signals inside the test chamber. In Fig. 4, we show convergence of the eigenvectors in the same experiment. In this case, we are plotting the magnitude of the inner product of each eigenvector computed by PASTd $+$ GSO with the associated eigenvector computed from the SVD of the array covariance matrix R, obtained from the same snapshots. Referring back to the figure, it is clear that the primary eigenvector has converged to the appropriate value with high accuracy within the first 32 samples. Interestingly, subsequent eigenvectors also converge, one after another, separated by about 350 samples. In terms of interference suppression performance, however, it should be noted that the comparison is meaningful only for the primary eigenvector; i.e., the one associated with the interference.

As mentioned previously, the convergence of the PASTd algorithm used here is controlled by the parameter β. We found that the value of $\beta = 0.99$ used here consistently yielded reliable performance as demonstrated in the scenarios above. However, other subspace trackers are known to outperform PASTd; see [12] for some general insights and [19] for more recent findings, including comparisons between PASTd and other subspace trackers.

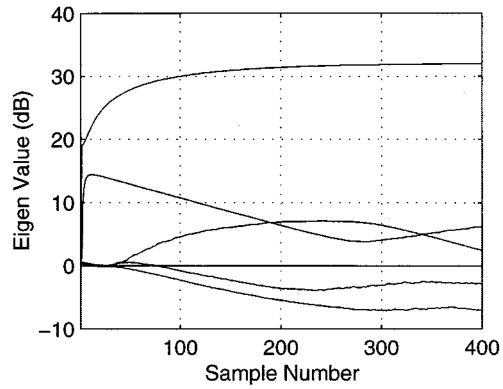

Fig. 3. Eigenvalue convergence in the OSMA experiment.

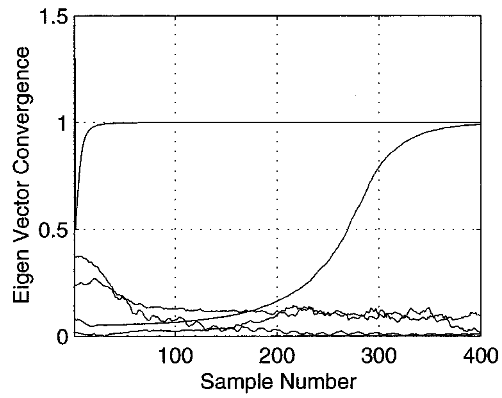

Fig. 4. Eigenvector convergence in the OSMA experiment.

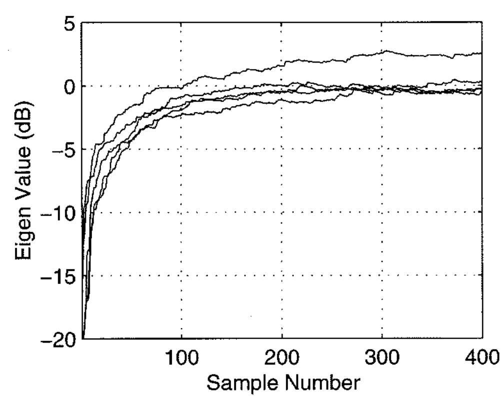

Fig. 5. Eigenvalue convergence from the INR $= 0$ dB simulation.

Due to instrumental limitations of the OSMA test facility, it is not possible to test at low INR, or to accurately measure array patterns. For this, we conducted a simulation with planar wavefronts and $INR = 0$ dB. The simulation is conducted in two dimensions with an $N = 8$ uniform linear array with elements having identical omnidirectional patterns. The element spacing is identical to the column spacing in the above study.

Figs. 5 and 6 are analogous to Figs. 3 and 4, respectively. Note that the primary eigenvalue converges to the correct value of $+3$ dB within about 300 samples, whereas the remaining values are clustered close to 0 dB, as expected. Similarly, the primary eigenvector has converged with high accuracy within

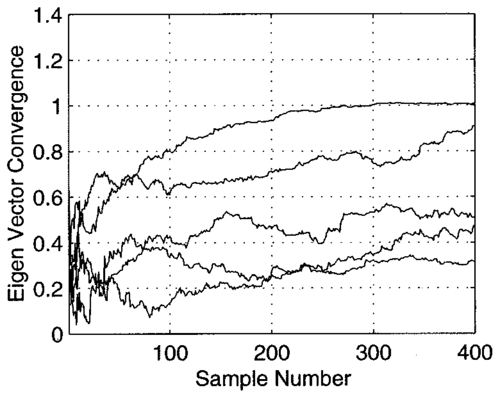

Fig. 6. Eigenvector convergence from the INR $= 0$ dB simulation.

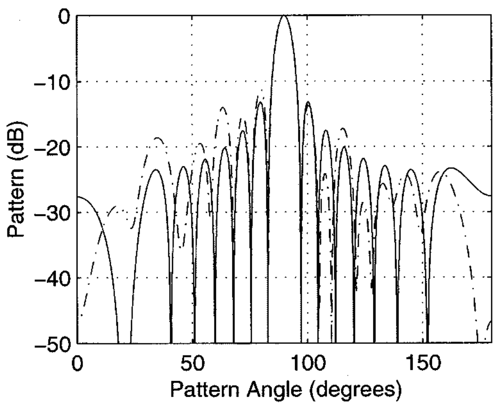

Fig. 7. Adapted patterns for MV (dashed) and STSP (solid) from the INR $=$ 0 dB simulation.

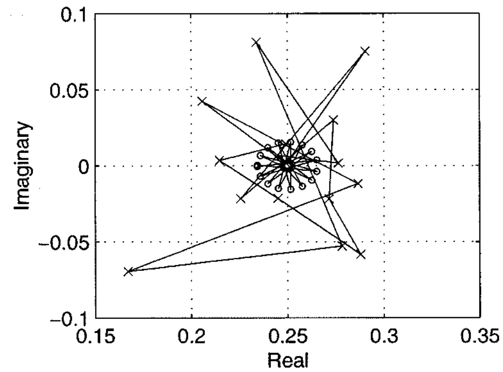

Fig. 8. Values of the adapted weights for MV (+) and STSP (°) from the INR $= 0$ dB simulation.

300 samples. Fig. 7 shows the pattern resulting from the STSP procedure suggested in the previous section, using the proposed MDL-based procedure to estimate the rank of the interference subspace. In this example, the desired beam is pointed at broadside, which corresponds to 90° in the figure. We found that MDL consistently estimates the correct value $k = 1$ and that the resulting pattern has a zero at the correct location (20°, as shown on the plot). For comparison, we also show one trial of the MV algorithm for the same scenario, using a covariance matrix estimate using 1000 samples. Note the weak suppression of the interference (characteristic of power inversion) and also the perturbed sidelobe structure, due to the extraneous noise subspace information included in the MV weights. Fig. 8 illustrates the values of the weights for this same experiment, for both the STSP and MV algorithms. For reference, note that the ideal normalized quiescent weights for broadside pointing would all have the value $0.25 + j0$. Note that STSP makes the minimum modification necessary to form the null, which consequently results in negligible perturbation of the pattern elsewhere. MV, on the other hand, yields weights which deviate excessively from the quiescent values, leading to the observed pattern rumble.

IV. Computational Burden

In this section we compare the computational burden of the STSP approach (using PASTd) to that of MV. The basis for the comparison is an asymptotic estimate of the number of floating point operations (FLOPs) required to compute an update of the beamforming weights. If a FLOP is defined as a single real-valued addition or multiplication, then a single complex addition requires 2 FLOPs and a single complex multiplication requires 6 FLOPs. Then, the inner product of two length-N complex-valued vectors requires $8N - 2$ FLOPs. Using these rules, one quickly finds that a single iteration of the PASTd procedure requires $22k_{\max}N + 3k_{\max}$ FLOPs. In STSP, this is repeated L times per weight update. If $L \gg N$, then PASTd dominates the number of FLOPs required for a single STSP update, so that the approximate result for an STSP update is $22k_{\max}LN + 3k_{\max}L$.

To compute the number of FLOPs required for an MV update, we first note that a single outer product of two length N complex vectors requires $6N^2$ FLOPs. Exploiting the Hermetian property of $\hat{R}$, this can be reduced to $3LN^2 - 3LN$ FLOPs to compute compute $\hat{R}$ from L snapshots. If $L \gg N$, then this cost dominates the number of FLOPs required for a single MV weight update, so that the approximate cost is $3LN^2 - 3LN$.

For arrays with $N > 4$ or so, the ratio of the FLOPs required for an STSP update to FLOPs required for an MV update, based on the above analysis, is approximately $7.3k_{\max}N^{-1}$. Thus, the computational burden of the two methods is the about the same for $N = 8$ and $k_{\max} = 1$. In general, the computational burden of STSP will be favorable to MV whenever $Nk_{\max}^{-1} > 7.3$. In the case of large arrays with small numbers of interferers, it is clear that STSP (using PASTd) will have an enormous advantage in terms of computational burden.

V. Conclusion

In the context of radio astronomy, use of constrained minimization interference suppression algorithms, such as MV, presents several problems. Among these are power inversion and pattern rumble. We have introduced an alternative approach to interference suppression using subspace tracking to identify interference and spatial projections to form nulls. This approach is not subject to power-inversion limitations. Despite the lack of a pointing constraint in null forming, this approach results in much less main beam distortion and pattern rumble than MV. The STSP implementation described above is computationally more efficient than MV, especially for large arrays. Because this approach divides the problems of interference estimation and interference suppression into separate steps, useful side information is generated and greater user control is possible over the behavior of the algorithm. Measurements from a prototype demonstrator system were used to illustrate the viability of subspace tracking for a proposed array configuration. Simulations indicate excellent performance even at low INR. Further research is required to develop suitable interference subspace rank estimators for radio astronomy applications.

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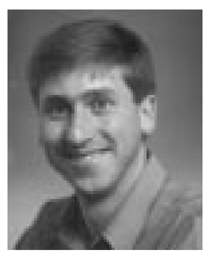

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