A constant modulus blind adaptive receiver for multiuser interference suppression

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Abstract

In this paper we investigate the ability of the Constant Modulus (CM) criterion to adaptively suppress Multiple Access Interference (MAI) in Code Division Multiple Access (CDMA) channels without the need of training sequences. In the absence of noise, we show that the resulting receiver is able to completely remove the MAI. We also show that, for high SNR values, CM receivers perform the same as Minimum Mean Square Error (MMSE) receivers therefore exhibiting a desirable balance between MAI suppression and noise enhancement. The main limitation of CM receivers is that they can be captured by an interferent user. However, we present an initialization strategy that practically overcomes this problem. © 1998 Published by Elsevier Science B.V. All rights reserved.

Zusammenfassung

In diesem Artikel untersuchen wir die Fähigkeit des Constant Modulus (CM) Kriteriums, Störungen zu unterdrücken, die durch vielfachen Benutzerzugriff (*Multiple Access Interference*, MAI) innerhalb eines *Code Division Multiple Access* (CDMA) Kanals anfallen, wobei keine Trainingsfolgen benötigt werden. Wir zeigen, daß der sich ergebende Empfänger unter der Voraussetzung, daß kein Rauschen vorliegt, in der Lage ist, MAI vollständig zu entfernen. Wir zeigen außerdem, daß CM Empfänger ebenso leistungsfähig sind wie *Minimum Mean Square Error* (MMSE) Empfänger, vorausgesetzt, es liegt ein großes SNR vor. Daher weisen die Empfänger den wünschenswerten Kompromiß zwischen MAI-Unterdrückung und Rauschverstärkung auf. Die wesentliche Einschränkung von CM Empfängern besteht darin, daß sie durch einen störenden Benutzer "eingefangen" werden können. Wir legen jedoch eine Initialisierungsstrategie dar, die dieses Problem praktisch löst. © 1998 Published by Elsevier Science B.V. All rights reserved.

Résumé

Dans cet article, nous étudions la capacité du critère de module constant (MC) à supprimer de façon adaptative les interférences d'accès multiples dans des canaux d'accès multiple par division de codes (CDMA), sans avoir recours à des séquences d'entraînement. En l'absence de bruit, nous montrons que le récepteur résultant est capable d'éliminer complètement les interférences d'accès multiples. Nous montrons aussi que, pour de hauts rapport signal-à-bruit, des récepteurs MC ont les mêmes performances que des récepteurs à erreur quadratique moyenne minimale, montrant ainsi un équilibre souhaitable entre suppression des interférences et surcroît de bruit. La limitation principale des récepteurs

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CM est qu'ils peuvent être capturés par un utilisateur interférent. Cependant, nous présentons une stratégie d'initialisation qui, pratiquement, permet de dépasser ce probléme. © 1998 Published by Elsevier Science B.V. All rights reserved.

Notation

A	amplitude matrix
b	transmitted symbols vector
C	code matrix
D	$(CA)^{-1}(AC^{T})^{-1}$
g	output amplitudes vector
H	Hessian matrix
n	noise vector
R	code correlation matrix
w	weight vector
x	observations vector
k	normalized fourth-order moment of the transmitted symbols
m	second-order moment of the transmitted symbols
q	fourth-order moment of the transmitted symbols
y	transmitted symbol estimate
$J(w)$	constant modulus cost function
$\tilde{J}(w)$	constant modulus cost function (with noise)
$\phi(g)$	constant modulus cost function in terms of the output amplitudes
$\tilde{\phi}(g)$	constant modulus cost function in terms of the output amplitudes (with noise)
γ	inverse of the SNR
μ	adaptive algorithm step size parameter
ρ_{ij}	correlation coefficient of the ith and jth codes

1. Introduction

Code Division Multiple Access (CDMA) is likely to be the multiple access technique chosen by the next generation of mobile communication systems. In CDMA, different users simultaneously transmit over the same bandwidth. Each user signal is modulated by a unique code or signature waveform that is orthogonal to the other codes. This property allows the extraction of the desired user from the received signal by means of a filter matched to the desired signature waveform. In practical situations, however, perfect orthogonality is not achieved due to diverse phenomena such as asynchronous transmission, multipath propagation or limited bandwith. The output of the matched filter receiver, therefore, contains components of Multiple Access Interference (MAI). This interference is particularly harmful when differences between the received power of the desired user and the interfering ones are large. This is known as the near–far problem [9] and is currently solved by imposing a stringent power control on the system.

Different techniques have been proposed to adaptively suppress MAI [8,6]. Minimum Mean Square Error (MMSE) approaches [8] suffer from the need of training sequences and, therefore, blind adaptive receivers are preferred. However, the existing Linearly Constrained Minimum Variance (LCMV) [6] receivers require an extremely precise knowledge of the desired user code and do not perform adequately. In this paper we investigate the use of the Constant Modulus (CM) criterion for the blind suppression of MAI in CDMA systems. This criterion was originally proposed for channel equalization and exploits the statistical independence between transmitted symbols to remove the Inter-Symbol Interference (ISI). See [12] and references therein for an overview of the CM criterion. As will be shown in subsequent sections, when applying this criterion to CDMA receivers, interferences can be optimally cancelled relying only on its statistical independence property. No a priori knowledge of the desired user code is initially required. The main disadvantage of the proposed CM receiver is that it may capture an interfering signal instead of the desired one [11]. However, we will show that with an adequate initialization of the adaptive algorithm using a rough estimate of the desired user code provided by a code acquisition circuit, this problem is considerably reduced.

This paper is organized as follows. Section 2 introduces the signal model. In Section 3 we demonstrate that the CM receiver performs the same as the MMSE receiver for high values of the Signal to

Noise Ratio (SNR). In Section 4 we discuss the capture problem. In Section 5 we present some computer simulation results that illustrate the performance of the proposed receiver and, finally, Section 6 is devoted to the conclusions.

2. Signal model

Let us consider a synchronous baseband Direct-Sequence Spread-Spectrum (DSSS) CDMA system with N users. Each user i is assigned a unique code sequence defined by L chip coefficients $c_i[j]$, $j = 0, \ldots, (L-1)$. The received signal is

$$r(t) = \sum_{i=1}^{N} A_i b_i \sum_{j=0}^{L-1} c_i[j] p(t - jT_c) + n(t),$$
$$0 \leqslant t \leqslant T_b, \tag{1}$$

where b_i and A_i are the transmitted symbols and the received amplitude of the ith user, respectively, $p(t)$ is the chip pulse waveform, T_c is the chip period, $T_b = LT_c$ is the bit period and $n(t)$ is the Additive White Gaussian Noise (AWGN) in the channel.

Fig. 1 represents the block diagram of a linear receiver for single user extraction in CDMA systems. It consists of a chip-matched filter and a T_c-tapped delay line FIR filter with coefficients $w = [w_0, \ldots, w_{L-1}]^T$ followed by a bit-rate sampler. Assuming perfect estimation of the chip timing, the output of the chip-matched filter is the sequence $x = [x_0, \ldots, x_j, \ldots, x_{L-1}]^T$ defined as

$$x_j = \int_{jT_c}^{(j+1)T_c} s(t)p(t - jT_c)\,\mathrm{d}t = \sum_{i=1}^{N} A_i b_i c_i[j] + n_j,$$
$$\tag{2}$$

where $n_j = \int_{jT_c}^{(j+1)T_c} n(t)p(t)\,\mathrm{d}t$. Rewrite x in a more compact form,

$$x = \sum_{i=1}^{N} A_i b_i c_i + n = CAb + n, \tag{3}$$

where $c_i = [c_i[0], \ldots, c_i[L-1]]^T$, $n = [n_0, \ldots, n_{L-1}]^T$, $b = [b_1, \ldots, b_N]^T$, $C = [c_1, \ldots, c_N]$ and A is the input amplitude matrix

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_N \end{bmatrix}. \tag{4}$$

Finally, the receiver output can be written as

$$y = w^T x = w^T CAb + w^T n, \tag{5}$$

and is intended to be an estimate of the bits transmitted by the desired user when the filter weights w are adequately chosen.

3. The constant modulus receiver

In the CM receiver, the filter coefficients w are selected to minimize the following cost function:

$$J(w) = E[(y^2 - 1)^2]. \tag{6}$$

Optimum weights can be computed using the stochastic gradient algorithm

$$w(n+1) = w(n) - \mu(y^2(n) - 1)y(n)x(n), \tag{7}$$

where μ is the step size parameter. The constant modulus cost function (6) is not a quadratic form of w because it involves higher-order statistics. Therefore, it contains multiple stationary points that may impair the convergence of the adaptive algorithm (7).

Let us analyze the stationary points in $J(w)$ for a noiseless environment containing $N \leqslant L$ statistically independent users. In the absence of noise, the receiver output can be written as $y = g^T b$, where $g = AC^T w = [g_1, \ldots, g_N]^T$ is the vector containing the amplitudes of the different user symbols at the receiver output. The cost funtion (6) can be written in terms of g as follows:

$$J(w) = \phi(g) = E[((g^T b)^2 - 1)^2]$$
$$= E[(g^T b)^4] + 2E[(g^T b)^2] + 1. \tag{8}$$

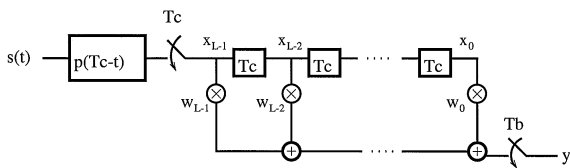Fig. 1. Block diagram of the linear single user receiver structure.

Taking into account the statistical independence between the components in $\boldsymbol{b}$, it can be shown that

$$E[(\boldsymbol{g}^{\mathrm{T}}\boldsymbol{b})^2] = E[b_i^2](\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g}), \qquad (9)$$

$$E[(\boldsymbol{g}^{\mathrm{T}}\boldsymbol{b})^4]$$
$$= (E[b_i^4] - 3E^2[b_i^2])\sum_{i=1}^{N} g_i^4 + 3E^2[b_i^2](\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g})^2, \quad (10)$$

and substituting Eqs. (9) and (10) into Eq. (8) yields

$$\phi(\boldsymbol{g}) = (k-3)\sum_{i=1}^{N} g_i^4 + 3(\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g})^2 - 2(\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g}) + 1, \quad (11)$$

where $k = E[b_i^4]/E^2[b_i^2]$ is the normalized fourth-order moment of the transmitted symbols and it is assumed that the transmitted symbols, b_i, have unit power ($E[b_i^2] = 1 \; \forall i$). The value $k - 3$ is referred in the sequel as the kurtosis.

As shown in Appendix A, computing the points where the gradient $\nabla_g \phi$ vanishes and analyzing the Hessian matrix at these points, three groups of stationary points can be identified:

- $\boldsymbol{g} = [0, 0, \ldots, 0]^{\mathrm{T}}$. This undesired stationary point corresponds to the complete suppression of the signals from all users. The Hessian matrix is negative definite at this point and it is, therefore, a maximum.
- $\boldsymbol{g} = [0, 0, \ldots, g_l^0, \ldots, 0]^{\mathrm{T}}$. This is the zero-forcing solution that corresponds to the extraction of the lth user and the perfect cancellation of the other $N - 1$ users. Analyzing the Hessian, it can be shown that these N points are minima when the kurtosis of the modulation format is negative. This is always the case for DSSS.
- $\boldsymbol{g} = [g_1, g_2, \ldots, g_R, 0, \ldots, 0]^{\mathrm{T}}$. These stationary points correspond to the extraction of a linear combination of the signals from R different users. These points are not minima because the Hessian is not positive definite when the kurtosis of the transmitted symbols is negative.

The above analysis shows that the adaptation rule (7) will only converge to solutions that enable the extraction of a single user. It also shows that the CM receiver may suffer from a capture problem because it can extract an interference instead of the desired signal. However, if a code acquisition circuit is available at the receiver, the capture problem

can be considerably reduced using an adequate initialization strategy as will be shown in Section 4. Assuming that we are able to control the adaptation rule (7) so that it converges to the extraction of the desired user, it is apparent that the CM receiver is near–far resistant since it performs the same as the zero-forcing or decorrelation receiver [7].

Let us consider now the effect of the AWGN in the channel. Assuming the SNR is sufficiently high, it can be shown (see Appendix B) that the noisy optimum amplitude vector is a perturbed version of the noiseless optimum,

$$\boldsymbol{g}_{\mathrm{CM}} = [\gamma\Delta g_1, \gamma\Delta g_2, \ldots, g_l^0 + \gamma\Delta g_l, \ldots, \gamma\Delta g_N]^{\mathrm{T}}, \quad (12)$$

where $\gamma = \mathrm{SNR}^{-1}$. Note that at this point interferences are not completely suppressed. This is because the receiver achieves a desirable balance betweeen interference suppresion and noise enhancement. In fact, the analysis presented in Appendix B shows that the MSE value corresponding to Eq. (12) is roughly the same achieved with an MMSE receiver for a high SNR in a full load CDMA channel and can be written as

$$\mathrm{MSE} \approx \gamma(\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}})^{\mathrm{T}}\boldsymbol{D}\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}}, \qquad (13)$$

where $\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}} = [0, \ldots, g_{l,\mathrm{opt}}^{\mathrm{MMSE}} = 1, \ldots, 0]^{\mathrm{T}}$ and $\boldsymbol{D} = (\boldsymbol{CA})^{-1}(\boldsymbol{CA})^{-\mathrm{T}}$. Fig. 2 illustrates this idea and shows the validity of our approximation. It plots three curves of MSE with respect to SNR. Two curves correspond to values obtained through simulation of the MSE achieved by the CM and MMSE receivers, respectively, in a CDMA channel with 16 users (i.e., 15 interfering ones) employing 31 length Gold codes as spreading sequences. The third one plots the theoretical curve given by Eq. (13). It is apparent that the three curves are the same for SNR values above 10 dB.

4. Capture analysis

The main limitation of the CM receiver is the capture problem described in the previous section. In this section we present an analysis of the transient behavior of the adaptation rule (7) in order to determine the relevant factors in the prediction of
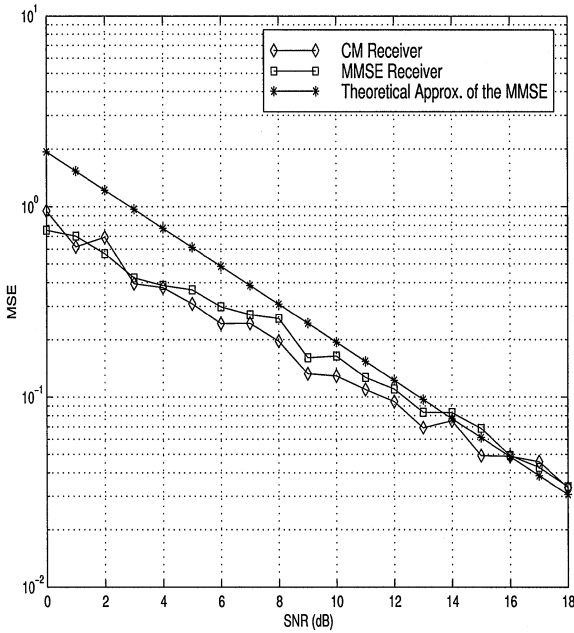
Fig. 2. Theoretical MMSE given by Eq. (13) and MSE obtained for the CM and MMSE receivers in a CDMA channel with AWGN and 16 users using 31 length Gold sequences as spreading codes. All users are received with the same power.

the extracted user and test the performance of a proposed initialization strategy.

Assuming the step size parameter μ is small enough, the expectation of the trajectories described by the stochastic algorithm (7) is given by the Ordinary Difference Equation (ODE)

$$w(n + 1) = w(n) - \mu E[(y^2(n) - 1)y(n)x(n)]$$
$$= w(n) - \mu E[(y^2(n) - 1)x(n)x^T(n)]w(n). \quad (14)$$

In a noise free environment $y = g^T b$ and taking into account the statistical independence between the components in b, the expectation in Eq. (14) can be reduced to

$$E[(y^2(n) - 1)x(n)x^T(n)]$$
$$= CA(E[bb^T g(n)g^T(n)bb^T - bb^T])AC^T$$
$$= CAQ_g(n)AC^T, \quad (15)$$

where

$$Q_g(n) = \begin{bmatrix} v_1 & \alpha_{12} & \dots & \alpha_{1N} \\ \alpha_{21} & v_2 & \dots & \alpha_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1} & \alpha_{N2} & \dots & v_N \end{bmatrix}_{N \times N}. \quad (16)$$

The terms v_i and α_{ij} are defined as

$$v_j = q_j g_j^2(n) + m_j \sum_{\substack{i=1 \\ i \ne j}}^{K} m_i g_i^2(n) - m_j, \quad (17)$$

$$\alpha_{ij} = \alpha_{ji} = 2m_i m_j g_i(n)g_j(n), \quad (18)$$

where $m_i = E[b_i^2]$, $q_i = E[b_i^4]$ and $g_i(n) = w^T(n)c_i$ is the ith user amplitude at the receiver output. Left-multiplying both sides of Eq. (14) by AC^T it is straightforward to obtain the mean amplitude vector recursion

$$g(n + 1) = g(n) - \mu ARAQ_g(n)g(n), \quad (19)$$

where $R = C^T C$ is the $N \times N$ code correlation matrix. Recursion (19) describes the transient behavior of the CM receiver in terms of the amplitude vector g.

For simplicity reasons, let us assume a scenario containing only two users with perfectly orthogonal codes (i.e., $c_1^T c_2 = 0$). In this case, Eq. (19) reduces to two scalar recursions on g_1 and g_2,

$$g_1(n + 1) = g_1(n)(1 - \mu f_1(g_1(n), g_2(n))), \quad (20)$$

$$g_2(n + 1) = g_2(n)(1 - \mu f_2(g_1(n), g_2(n))), \quad (21)$$

where

$$f_1(g_1(n), g_2(n))$$
$$= A_1^2 \rho_{11}(q_1 g_1^2(n) + 3m_1 m_2 g_2^2(n) - m_1), \quad (22)$$

$$f_2(g_1(n), g_2(n)) = A_2^2 \rho_{22}(q_2 g_2^2(n) + 3m_1 m_2 g_1^2(n) - m_2) \quad (23)$$

and $\rho_{ii} = c_i^T c_i$ is the autocorrelation coefficient of the ith code. It is extremely difficult to solve the system of difference equations (20) and (21). Nevertheless, analyzing the sign of $f_i(g_1(n), g_2(n))$, $i = 1, 2$, the algorithm convergence can be predicted in terms of the filter amplitudes magnitude. Note that if

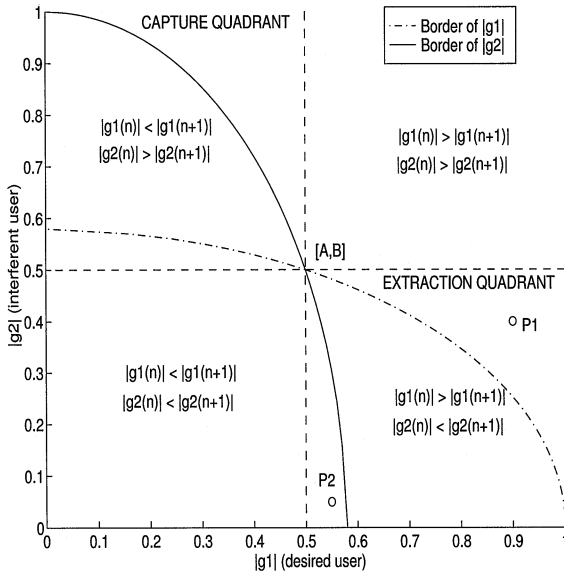$$f_i(g_1(n), g_2(n)) < 0 \quad (f_i(g_1(n), g_2(n)) > 0), \quad (24)$$

Fig. 3. Boundaries of the growth/decay regions of the receiver amplitudes $|g_1|$ and $|g_2|$. Initial capture and extraction quadrants are also depicted. The plot has been obtained with parameters $m_i = 1$, $q_i = 1$.
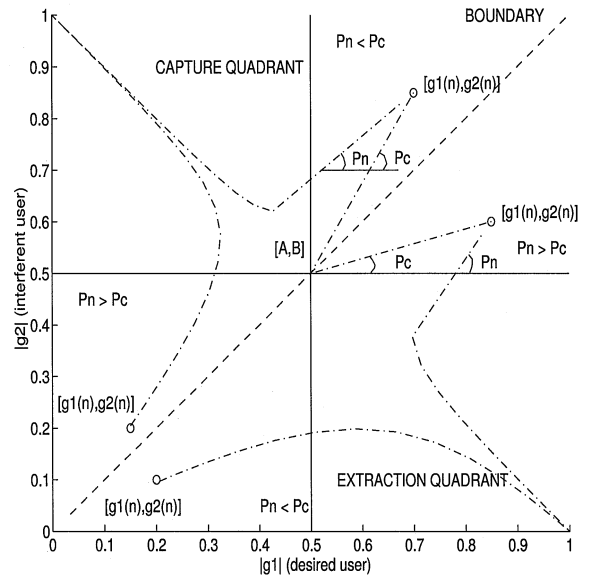

Fig. 4. Global boundary between the extraction and capture regions. The trajectories (i.e., the convergence of the CM algorithm) with origin at points on both sides of the boundary depend on the sign of the difference $p_n - p_c$. The plot has been obtained with parameters $m_i = 1$, $q_i = 1$.

then

$$|g_i(n + 1)| > |g_i(n)| \quad (|g_i(n + 1)| < |g_i(n)|). \tag{25}$$

As a result, the curves $f_i(g_1(n),g_2(n)) = 0$, $i = 1,2$, represent the boundaries of the growth/decay regions of the user amplitudes. Fig. 3 depicts these boundaries for $m_i = 1$ and $q_i = 1$. It is clearly seen that both curves cross in a point $[A,B]$. Let us divide the $[|g_1|,|g_2|]$ plane into four quadrants around this point. It is straightforward to show that if the initial conditions $[|g_1(0)|,|g_2(0)|]$ lie in the lower-right quadrant, the receiver extracts the desired user. Indeed, if P_1 in Fig. 3 is the starting point of the algorithm, $|g_1|$ and $|g_2|$ will diminish until the trajectory crosses the curve $f_1(g_1(n),g_2(n)) = 0$. Then, $|g_1|$ will begin to grow while $|g_2|$ still declines. Eventually, the algorithm will converge to $|g_1| = 1$ and $|g_2| = 0$. Similarly, if the algorithm starts in P_2, it will eventually extract the desired user. An equivalent argument shows why the receiver will be captured by the interferent user when the initial conditions lie in the upper left quadrant.

In order to explain the algorithm's behavior in the upper-right and lower-left quadrants, let us

define the slope of the line between $[|g_1(n)|,|g_2(n)|]$ and $[|g_1(n + 1)|,|g_2(n + 1)|]$ by

$$
\begin{aligned}
p_n &= \frac{|g_2(n + 1)| - |g_2(n)|}{|g_1(n + 1)| - |g_1(n)|} \\
&= \frac{A_2^2 \rho_{22} |g_2(n)| (q_2 g_2^2(n) + 3m_1 m_2 g_1^2(n) - m_2)}{A_1^2 \rho_{11} |g_1(n)| (q_1 g_1^2(n) + 3m_1 m_2 g_2^2(n) - m_1)},
\end{aligned}
\tag{26}
$$

and the slope of the line between $[|g_1(n)|,|g_2(n)|]$ and $[A,B]$ as

$$p_c = \frac{|g_2(n)| - B}{|g_1(n)| - A}. \tag{27}$$

As can be graphically deduced from Fig. 4, when $p_n > p_c$ in the upper-right quadrant (alternatively, $p_n < p_c$ in the lower-left quadrant) the trajectory described by algorithm (19) converges to the point $[1, 0]$, which is the desired extraction solution. On the other hand, when $p_n < p_c$ in the upper-right quadrant (alternatively, $p_n > p_c$ in the lower-left quadrant) algorithm (19) converges to the point

[0, 1], which is the undesired capture solution. The boundary between the two regions is $p_n = p_c$, i.e., the curve given by the equation

$$\frac{A_2^2\rho_{22}|g_2(n)|(q_2 g_2^2(n) + 3m_1 m_2 g_1^2(n) - m_2)}{A_1^2\rho_{11}|g_1(n)|(q_1 g_1^2(n) + 3m_1 m_2 g_2^2(n) - m_1)}$$

$$- \frac{|g_2(n)| - B}{|g_1(n)| - A} = 0. \tag{28}$$

Given a set of initial conditions $[g_1(0), g_2(0)]$, the above analysis allows us to predict which user the algorithm is going to extract. If $[|g_1(0)|, |g_2(0)|]$ lies at the right of curve (28) the desired user will be extracted and if it lies at the left, the algorithm will extract the interfering user.

The capture problem is avoided if we can assume that the algorithm is initialized in a point $[g_1(0), g_2(0)]$ lying always in the extraction region. This can be accomplished if we select the initial conditions $\boldsymbol{w}(0) = \boldsymbol{c}_1/(\boldsymbol{c}_1^{\mathrm{T}}\boldsymbol{c}_1)$, where $\boldsymbol{c}_1$ is the desired user code. In this case $[g_1(0), g_2(0)] = [1,0]$ and the algorithm always starts in the extraction region. At a first glance, it may seem that there is a contradiction between stating that an advantage of CM receivers with respect to LCMV receivers is that the desired user code is not necessary and eventually assuming that this code must be known in order to avoid the capture problem. However, note that the performance of LCMV receivers severely degrades when the channel distorts $\boldsymbol{c}_1$, whereas CM receivers perform optimally as long as the channel distortion does not move the initial conditions into the capture region. In applications where short period spreading sequences are used, it is quite reasonable to assume that an approximate knowledge of $\boldsymbol{c}_1$ is available at the receiver. Note that existing synchronization techniques for single user CDMA receivers are based on the acquisition of the desired user spreading code [4] and, as a consequence, a code acquisition circuit is always present in such receivers. These circuits, however, only provide an estimate $\hat{\boldsymbol{c}}_1$ of the received code $\boldsymbol{c}_1$. In order to determine the robustness of our initialization strategy, we carried out computer simulations assuming that the estimate error $\triangle \boldsymbol{c}_1 = \hat{\boldsymbol{c}}_1 - \boldsymbol{c}_1$ is a vector of Gaussian random variables with zero mean and autocorrelation matrix $\sigma_c^2 \boldsymbol{I}_L$. Fig. 5 plots the capture probability with respect to σ_c^2. It is clearly seen that the
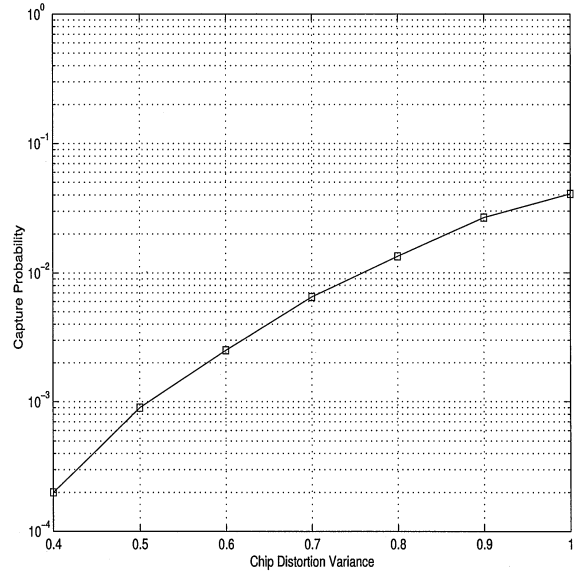

Fig. 5. Capture probability of the CM receiver versus variance of chip distortion (σ_c^2). The channel contains 16 users employing Gold sequences of length 31 as signatures. All the interfering users are 6 dB stronger than the desired one.

distortion variance σ_c^2 must be very large ($\sigma_c^2 > 0.4$) in order to obtain capture probabilities above 10^{-4}. Note, then, that an approximate knowledge of $\boldsymbol{c}_1$ overcomes the capture problem in CM receivers whereas is not enough to ensure an adequate performance of LCMV receivers, as shown in the following section.

5. Computer simulations

In this section we present computer simulation results that illustrate the performance of the proposed CM receiver. We also compare the obtained results with those corresponding to the MMSE receiver with training sequences [8] and the blind LCMV receiver [6].

Fig. 6 shows the Bit Error Rate (BER) of the CM, MMSE and LCMV receivers in a CDMA channel with 16 users, AWGN and an ideal channel impulse response $h(t) = \delta(t)$, where $\delta(t)$ is Dirac's delta function. The received power of each one of the 15 interfering users is 6 dB stronger than the desired one to simulate an environment with
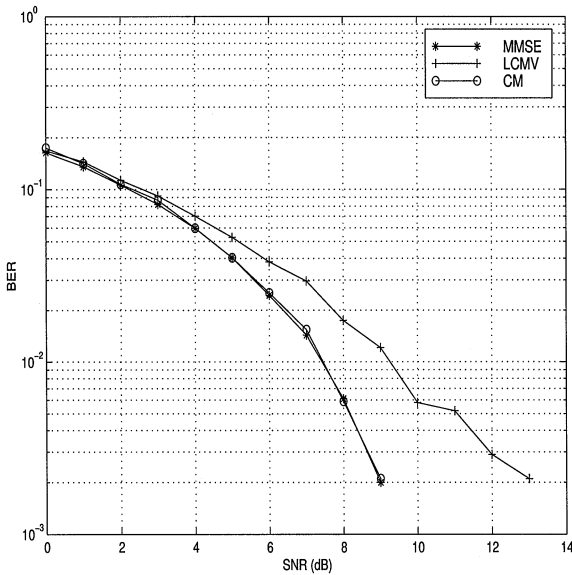
Fig. 6. BER versus SNR for the CM, MMSE and LCMV receivers in a CDMA system with Gold codes of length 31, 16 active users and AWGN. Each interfering user is 6 dB stronger in received power than the desired one.
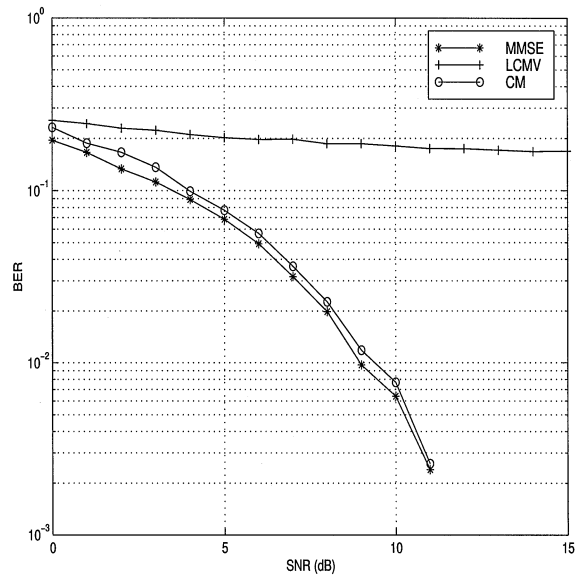

Fig. 7. BER versus SNR for the CM, MMSE and LCMV receivers in a CDMA system with Gold codes of length 31 and 16 users. The curves correspond to a channel with AWGN and impulse response $h(t) = \delta(t) + 0.5\delta(t - T_c) + 0.1\delta(t - 2T_c)$. Each interfering user is 6 dB stronger in received power than the desired one.

near–far. We also assume perfect synchronization of the receiver and, thus, an exact knowledge of the received code c_1. The initialization strategy for the CM receiver is the one presented in Section 4, i.e.,

$$w_0 = c_1/(c_1^T c_1). \tag{29}$$

The MMSE receiver is implemented with a Least Mean Squares (LMS) algorithm and the same initial conditions. The linear constraint in the LCMV receiver is implemented using the desired user transmitted code, c_1, and the optimization approach is a stochastic gradient algorithm that also starts at the value of w_0 given by Eq. (29). It can be seen that the CM and MMSE receivers perform exactly the same, whereas the performance of the LCMV receiver becomes comparatively worse as the SNR grows. This is because the target of the linear constraint in the LCMV receiver is to protect the desired user signal from cancellation when minimizing the output variance [6]. However, even if there is no mismatch in the constraint, the error in the implicit estimation of the desired signal subspace by the stochastic gradient algorithm causes partial cancellation of the desired user signal [3].

The BER of LCMV receivers degrades even more severely when the version of the desired user code available at the receiver to implement the constraint is just an estimate of the actually received code. This is the case in multipath propagation channels. Fig. 7 plots the BER of the CM, MMSE and LCMV receivers when the channel impulse response in the simulation is

$$h(t) = \delta(t) + 0.5\delta(t - T_c) + 0.1\delta(t - 2T_c), \tag{30}$$

where T_c is the chip period. We can see how the curves corresponding to the CM and MMSE receivers are practically the same and both clearly outperform the LCMV receiver. This occurs due to the great sensitivity of LCMV receivers to errors in the linear constraint. As said before, we have implemented this constraint with the transmitted version of the desired user code, that is just an estimate of the actual received code. Some schemes have been recently proposed that partially mitigate this problem [6,10,1]. However, these modifications of the LCMV receiver try to improve performance imposing additional constraints on the LCMV cost
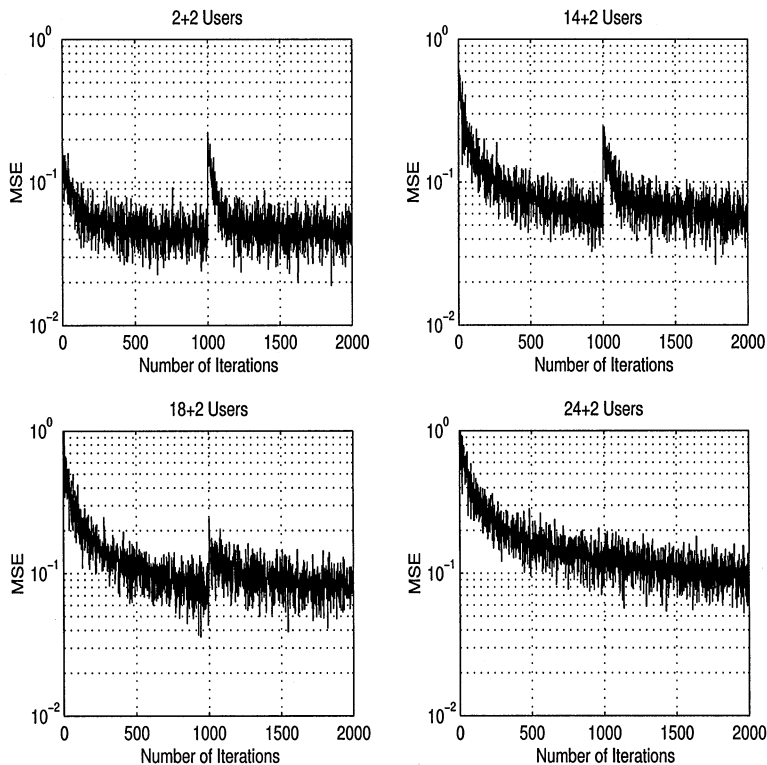
Fig. 8. MSE at the output of a CM receiver when 2 new users start transmitting after 1000 symbols. The spreading sequences are Gold codes of length 31. The SNR is 15 dB and each interfering user (including those that start transmitting after 1000 symbols) is received with a power 6 dB higher than the desired one. The channel impulse response is $h(t) = \delta(t) + 0.5\delta(t - T_c) + 0.1\delta(t - 2T_c)$.

function. Since the new constraints also intend to protect the desired user signal from cancellation, the resulting receivers have essentially the same problems as the original LCMV.

Finally, we study the ability of the proposed CM receiver to track the sudden changes in the CDMA channel caused by those users that start or stop transmitting. For the experiment, we have considered an SNR of 15 dB and the same spreading sequences, multipath propagation channel, near–far environment and initialization strategy described before. Fig. 8 plots the evolution of the Mean Square Error (MSE) at the CM receiver output for several system loads when two new users start transmitting after 1000 symbols. It can be seen that, for average loads (up to 20 users for a maximum of 31), the channel variation due to the increment in the number of users is rapidly compensated by the CM receiver. Notice the advantage of a blind

receiver in such situations: a conventional MMSE receiver would require the transmission of a training sequence each time the BER becomes too high, with the corresponding loss in efficiency, whereas a blind receiver does not have to stop data transmission.

It can also be observed from Fig. 8 that convergence of the CM receiver depends on the number of users in the channel. It varies between a few hundred iterations for a small number of users and a few thousand for heavy system loads. This may be too slow for certain applications and new optimization approaches should be investigated. Some efforts have already been done in this direction [5].

6. Conclusions

We have investigated the application of the CM criterion to CDMA receivers. We have shown that the minimization of the CM cost function leads to

the extraction of a single user with the same MSE as the MMSE receiver for high SNR values (above 5 dB). The main limitation of the proposed CM receiver is that it can be captured by an interfering user. However, further analysis of this capture problem reveals that a correct initialization of the adaptive algorithm based on a rough approximation of the desired user code is sufficient to succesfully reduce the capture probability to acceptable levels in practical situations.

Acknowledgements

Appendix A. Analysis of the stationary points

In this appendix we present an analysis of the stationary points of the CM cost function, $\phi(g)$, given by Eq. (11). Namely, it is shown that the only existing minima correspond to the perfect extraction of a single user.

The stationary points of $\phi(g)$ are the points where the gradient

$$\nabla_g \phi = 12(g^T g)g - 4g + 4(k-3)\begin{bmatrix} g_1^3 \\ \vdots \\ g_N^3 \end{bmatrix} \qquad (A.1)$$

vanishes. In order to determine the nature of these stationary points (maxima, minima or saddle points) we study the definite positiveness of the Hessian matrix given by the general expression

$$H(g) = \begin{bmatrix} \theta_1 & 24g_1g_2 & \cdots & 24g_1g_N \\ 24g_2g_1 & \theta_2 & \cdots & 24g_2g_N \\ \vdots & \vdots & \ddots & \vdots \\ 24g_Ng_1 & 24g_Ng_2 & \cdots & \theta_N \end{bmatrix}, \quad (A.2)$$

where

$$\theta_i = 12(k-3)g_i^2 + 12(g^T g) + 24g_i^2 - 4. \qquad (A.3)$$

When solving equation $\nabla \phi(g) = 0$, three groups of solutions can be identified.

- In the first group, we have a single solution, $g = 0$, that corresponds to the cancellation of all users. The Hessian computed at this point takes the form

$$H(0) = -4I_N, \qquad (A.4)$$

where I_N is the $N \times N$ identity matrix. This is clearly a negative definite matrix and, therefore, the point $g = 0$ is a maximum.

- In the second group of solutions, we have the N stationary points that correspond to the extraction of a single user. Vector g is given by $g^0 = [0, \ldots, g_l^0 = \sqrt{1/k}, \ldots, 0]^T$ and it can be easily shown that computing the Hessian at these points yields

$$H(g^0) = \begin{bmatrix} \theta_1 & 0 & \cdots & 0 \\ 0 & \theta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \theta_N \end{bmatrix}, \qquad (A.5)$$

where $\theta_i = -4(k-3)/k$ for $i \neq l$ and $\theta_l = 8$. Therefore, $H(g^0)$ is clearly positive definite if we assume a negative kurtosis modulation format[1] and, as a consequence, the considered N stationary points are all of them minima.

- In the third group of solutions, we have multiple stationary points that correspond to the extraction of a linear combination of several user signals. Vector g takes the form $g_R = [g_1, \ldots, g_R, 0, \ldots, 0]^T$, where the R nonzero components satisfy

$$g_i^2 = \begin{cases} \dfrac{1 - 3_{g^T g}}{k - 3}, & i = 1, \ldots, R, \\ 0, & i = R+1, \ldots, N. \end{cases} \qquad (A.6)$$

Note that there is no loss of generality in considering that the nonzero amplitudes are the first $R \leqslant N$ ones since no a priori order is imposed on the users. We will show that the Rth upper left

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[1] This assumption is true for most modulation schemes and in particular for DSSS.

square submatrix of the Hessian evaluated at g_R

$$H_R(g_R) = \begin{bmatrix} \theta_1 & 24g_1g_2 & \cdots & 24g_1g_R \\ 24g_2g_1 & \theta_2 & \cdots & 24g_2g_R \\ \vdots & \vdots & \ddots & \vdots \\ 24g_Rg_1 & 24g_Rg_2 & \cdots & \theta_R \end{bmatrix}$$

(A.7)

is neither positive definite nor positive semidefinite. Therefore, there exists at least one upper left square submatrix of $H_R(g_R)$ that has a negative determinant. Since this submatrix is also an upper left submatrix of $H(g_R)$ it follows that the Hessian is not definite positive.

To demostrate that $H_R(g_R)$ is not positive definite, it is enough to find an $N \times 1$ vector z such that

$$z^T H_R(g_R)z < 0.$$

(A.8)

Indeed, substituting Eq. (A.6) into Eq. (A.8),

$$z^T H_R(g_R)z = 24(z^T g)^2 + 8(1 - 3(g^T g))z^T z,$$

(A.9)

which is negative when z is orthogonal to g since $1 - 3(g^T g) < 0$ when $k - 3 < 0$.

Appendix B. Analysis of the noisy channel

The analysis presented below follows the work in [2] and its objective is to show that the proposed CM receiver performs the same as the MMSE receiver in the presence of AWGN for high SNR values.

First, we will obtain an adequate expression of the CM cost function in terms of g. Recall from Eq. (5) that the receiver output is

$$y = w^T CAb + w^T n = g^T b + w^T n,$$

(B.1)

where n is a vector of Gaussian noise components with zero mean and variance σ_n^2. Since b and n are statistically independent, the CM cost function can be expressed as

$$\tilde{J}(w) = E[(g^T b)^4] + E[(w^T n)^4]$$
$$+ 6E[(g^T b)^2]E[(w^T n)^2]$$
$$- 2E[(g^T b)^2] - 2E[(w^T n)^2] + 1.$$

(B.2)

A complete analysis of all the stationary points in Eq. (B.2) is rather involved. However, since the SNR is high, it is reasonable to assume that the number and nature of the stationary points in $\tilde{J}(w)$ are the same as those in $J(w)$. The noise only introduces a perturbation into the stationary points of $J(w)$. Therefore, we will conjecture that the minima of $\tilde{J}(w)$ correspond to the output amplitudes

$$\tilde{g}_{opt} \approx g_{opt} + \gamma \Delta g$$
$$= \left[\frac{1}{\sqrt{k}} + \gamma \Delta g_1, \gamma \Delta g_2, \ldots, \gamma \Delta g_N \right]^T,$$

(B.3)

where $g_{opt} = [1/\sqrt{k}, 0, \ldots, 0]^T$ are the optimum output amplitudes for the noiseless case obtained in Appendix A.

In the sequel, we will prove the above conjecture. Assuming a full rank square code matrix C we can express w in terms of g as

$$w^T CA = g^T \Rightarrow w^T = g^T (CA)^{-1}.$$

(B.4)

Substituting Eq. (B.4) into Eq. (B.2) and assuming unit transmitted power ($E[b_i^2] = 1$) the cost function can be rewritten as

$$\tilde{\phi}(g) \triangleq \tilde{J}(w) \approx (k - 3) \sum_{i=1}^{N} g_i^4 + 3(g^T g)^2$$
$$+ 6\gamma(g^T g)(g^T Dg) - 2(g^T g) - 2\gamma(g^T Dg),$$

(B.5)

where $D = (CA)^{-1}(CA)^{-T}$ and $\gamma = \sigma_n^2/E[b_i^2]$ is the inverse of the SNR. To obtain Eq. (B.5) we have neglected the terms multiplied by γ^2 and γ^3. Computing the gradient of $\tilde{\phi}(g)$ it is obtained

$$\nabla_g \tilde{\phi} = 4(k - 3) \begin{bmatrix} g_1^3 \\ \vdots \\ g_N^3 \end{bmatrix} + 12(g^T g)g + 12\gamma(g^T Dg)g$$
$$+ 12\gamma(g^T g)Dg - 4g - 4\gamma Dg.$$

(B.6)

Substituting Eq. (B.3) into Eq. (B.6) and neglecting again the terms multiplied by γ^2 and γ^3,

$$\nabla_g \tilde{\phi}|_{g=\tilde{g}_{\mathrm{opt}}} \approx 4(k-3) \begin{bmatrix} 1/k^{3/2} + 3/k\gamma\Delta g_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$+ 12 \begin{bmatrix} 1/k^{3/2} + 3/k\gamma\Delta g_1 \\ \gamma\Delta g_2/k \\ \vdots \\ \gamma\Delta g_N/k \end{bmatrix} + \begin{bmatrix} 12y\Delta d_{11}/k^{3/2} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$+ \frac{12\gamma}{k^{3/2}} \begin{bmatrix} d_{11} \\ d_{12} \\ \vdots \\ d_{1N} \end{bmatrix} - 4 \begin{bmatrix} 1/\sqrt{k} + \gamma\Delta g_1 \\ \gamma\Delta g_2 \\ \vdots \\ \gamma\Delta g_N \end{bmatrix} - \frac{4\gamma}{\sqrt{k}} \begin{bmatrix} d_{11} \\ d_{12} \\ \vdots \\ d_{1N} \end{bmatrix},$$

$$\text{(B.7)}$$

where d_{ij} is the element in column i and row j of matrix $\boldsymbol{D}$. If we set Eq. (B.7) to zero and solve for Δg_i we find that $\nabla_g \tilde{\phi}$ vanishes when

$$\Delta g_1 = d_{11}(k-6)/(2k\sqrt{k}), \qquad \text{(B.8)}$$

$$\Delta g_i = -d_{1i}/\sqrt{k}, \quad i > 1. \qquad \text{(B.9)}$$

Therefore, the minima of $\tilde{\phi}(g)$ can be approximated as Eq. (B.3). Under the same assumption of high SNR, it can be easily shown that the optimum output amplitude vector $\tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}}$ obtained when minimizing the MMSE cost function

$$\tilde{J}_{\mathrm{MMSE}}(\boldsymbol{w}) = E[(b_1 - y)^2] \qquad \text{(B.10)}$$

is also a perturbation of the zero-forcing solution, i.e.,

$$\tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}} \approx g_{\mathrm{opt}}^{\mathrm{MMSE}} + \gamma\Delta g$$

$$= [1 + \gamma\Delta g_1, \gamma\Delta g_2, \ldots, \gamma\Delta g_N]^{\mathrm{T}}, \qquad \text{(B.11)}$$

where $g_{\mathrm{opt}}^{\mathrm{MMSE}} = [1,0,\ldots,0]^{\mathrm{T}}$. The MMSE is the MSE value achieved when $g = \tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}}$,

$$\mathrm{MMSE} \approx [E[(b_1 - y)^2]]_{g=\tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}}}$$

$$= [1 - 2g_1 + \boldsymbol{g}^{\mathrm{T}}\boldsymbol{g} + \gamma\boldsymbol{g}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{g}]_{g=\tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}}}$$

$$= [(\boldsymbol{g} - \boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}})^{\mathrm{T}}(\boldsymbol{g} - \boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}})$$

$$\quad + \gamma(\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}})^{\mathrm{T}}\boldsymbol{D}\boldsymbol{g}]|_{g=\tilde{g}_{\mathrm{opt}}^{\mathrm{MMSE}}}$$

$$\approx \gamma(\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}})^{\mathrm{T}}\boldsymbol{D}\boldsymbol{g}_{\mathrm{opt}}^{\mathrm{MMSE}}, \qquad \text{(B.12)}$$

where unit transmitted power is assumed. On the other hand, it is straightforward to show that the MSE value achieved at the minima of the CM cost function is given by

$$[\mathrm{MSE}_{\mathrm{CM}}]_{g=\tilde{g}\mathrm{opt}}$$

$$= [E[(b_1 - \sqrt{k}y)^2]]_{g=\tilde{g}_{\mathrm{opt}}}$$

$$= [1 + kg^{\mathrm{T}}g - 2\sqrt{k}g_1 + \gamma\sqrt{k}g^{\mathrm{T}}Dg]_{g=\tilde{g}_{\mathrm{opt}}}$$

$$\approx \gamma k g_{\mathrm{opt}}^{\mathrm{T}} \boldsymbol{D} g_{\mathrm{opt}}$$

$$= \gamma(g_{\mathrm{opt}}^{\mathrm{MMSE}})^{\mathrm{T}} \boldsymbol{D} g_{\mathrm{opt}}^{\mathrm{MMSE}}$$

$$\approx \mathrm{MMSE}, \qquad \text{(B.13)}$$

where the output has been multiplied by $\sqrt{k}$ to account for the difference between g_{opt} and $g_{\mathrm{opt}}^{\mathrm{MMSE}}$. Thus, we show that the CM receiver performs the same as the MMSE receiver for high SNR values.

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