

Setup Adjustment for Discrete-Part Manufacturing Processes with Asymmetric Cost Functions

B.M. Colosimo¹, R. Pan², E. Del Castillo^{3*}

¹ Dipartimento di Meccanica - Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133 Milano, Italy

² Department of Mechanical and Industrial Engineering
University of Texas at El Paso, El Paso TX 79968

³Department of Industrial & Manufacturing Engineering
The Pennsylvania State University, University Park, PA 16802

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Abstract

This paper presents a feedback adjustment rule for discrete-part manufacturing processes that experience errors at the setup operation which are not directly observable due to part-to-part variability and measurement error. In contrast to previous work on setup adjustment, the off-target cost function of the process is not symmetric around its target. Two asymmetric cost functions – constant and quadratic functions – are considered in this paper. By introducing a bias term in the feedback adjustment rule, the process quality characteristic converges to the optimal steady-state target from the lower cost side of the cost function. This minimizes the off-target loss incurred during the transient phase of adjustment. A machining application is used to illustrate the proposed adjustment procedure and to demonstrate the savings generated by the proposed feedback adjustment rule compared to an adjustment rule due to Grubbs and to an integral controller. It is shown that the advantage of the proposed rule is significant when the cost of the items is high, items are produced in small lot sizes and the asymmetry of the cost function is large.

Keywords: Process adjustment, Asymmetric cost function, Short-run manufacturing

1 Introduction

We consider adjusting a discrete-part manufacturing process that is *setup dominant*. Such processes “have high stability for the entire length of the batch to be made”, hence, the control system “emphasizes verification of the setup before production proceeds” (Gyrna and Juran, 2000: 452). If an imperfect setup operation takes place and no verification and adjustment is done, the machine

*corresponding author

will produce a systematic error or offset which will show on the quality characteristic of all the produced items in the batch.

The literature contains two different members in this class of problems. In the first one, knowledge about process parameters that can be manipulated to improve start-up operations is available because either Design Of Experiments (DOE) techniques have been used before production takes place or data from “good” setups has been collected and modelled using regression techniques. Optimal parameters settings obtained by DOE or regression models are however only a guide since other exogenous factors (e.g., raw materials variability, improper set-up, environmental conditions, etc.) require on-line tuning of the parameter settings. Thus, the problem consists in building the models and tuning their parameters on-line. Examples of this type of approach are Nembhard and Birge (1998) who optimize regression models for the start-up of a continuous process, and more recently Xu and Albin (2002) who developed an interesting approach with reference to a filament extrusion process. In their approach, process settings recommended for a new batch are obtained by solving a mixed-integer quadratic program that uses the multivariate model developed by Wurl *et al.* (2001) to model the relationship between process and product variables under good production conditions. Their approach is applicable for processes characterized by complex interdependencies among product characteristics and multiple process parameters (temperatures, pressures, rotation speed) that can be frequently collected.

In contrast to this first type of “start-up” problems, our approach belongs to what in the literature has been referred to as the *Setup Adjustment Problem* (Grubbs, 1954, Trietsh, 1998). This problem refers to a different class of production processes in which a process parameter is usually *not* available on-line. This situation characterizes most of the machine-tool cutting processes (e.g., turning, milling, grinding) and a good number of semiconductor manufacturing process where “in situ” measurements are not possible due to the harsh production environment that makes sensing a very difficult, if not impossible, task. This has created a family of on-line process adjustment policies, which include setup adjustment, under the name “run to run” control, popular in the semiconductor industry (Moyne et al., 2001).

Our emphasis, however, is on setup adjustment of more classical discrete-part machining process, as it will be seen in the example of Section 5. In this type of processes, the position of the tool directly affects dimensions obtained on the machined components. This position is usually set off-line by operators in the tool-room but the actual position of the tool once mounted on the spindle is usually computed indirectly, by measuring dimensions obtained on the machined part. In these processes an improper position of the tool translates into a systematic error or offset on the quality characteristic of all the machined items in the batch. Unfortunately, dimensions obtained on the machined part are affected by inherent randomness due to both the machining and the measurement processes. Therefore, a sequence of adjustments that utilizes the process information

obtained by measuring machined parts is useful for eventually removing the setup error.

Adjustment procedures required to solve the Setup Adjustment Problem as studied herein were first studied by Grubbs (1954, 1983), who proposed an adjustment rule which has been more recently discussed by Trietsch (1998), Guo, Chen and Chen (2001), Pan and Del Castillo (2003), and by Del Castillo, Pan and Colosimo (2003a, b). The latter two references show the relations between Grubbs' rule and stochastic approximation techniques where the loss function is symmetric and quadratic. Former research on the Setup Adjustment Problem has only dealt with the case of symmetric cost functions. It is well-known that in many industrial applications asymmetric cost functions can be more appropriate, since the costs of oversized and undersized quality characteristics are often different.

The impact of asymmetric cost functions has been studied from several different perspectives other than for the setup adjustment problem. Wu and Tang (1998) and Maghsoodloo and Li (2000) have considered tolerance design with asymmetric cost functions, while Moorhead and Wu (1998) have analyzed the effect of this type of cost function on parameter design. Ladany (1995) presented a solution to the problem of setting the optimal target of a production process *prior* to starting the process under a constant asymmetric cost function. Harris (1992) discussed the design of optimal feedback controllers with asymmetric cost functions for a process characterized by a linear dynamic model and ARIMA (AutoRegressive Integrated Moving Average) noise. In Harris' model, however, correcting for a process setup error was not taken into consideration.

The idea of having the quality characteristic converge to the optimal setting from the lower cost side is related to certain stochastic approximation techniques in which a bias term is added to allow for one-side convergence, as discussed by Anbar (1977) and Krasulina (1998). These efforts were oriented to finding asymptotic or long-term one-sided convergence conditions of stochastic approximation methods instead of process adjustment applications. Furthermore, the convergence conditions they imposed on the parameters of their estimation schemes are too complicated for any engineering application and are asymptotic. Since short-run production processes have become more common with the advent of modern flexible manufacturing environments, small sample properties of sequential adjustment procedures need to be studied.

In this paper, we propose a solution to the Setup Adjustment Problem for asymmetric cost functions and focus on its small sample performance on machining-type processes. First, two asymmetric cost functions widely used in manufacturing are presented. In Section 3, we describe how the bias terms can be included in a general linear feedback control rule. The optimal value of these bias terms, in the sense of minimizing the expected manufacturing cost at each time step, is then derived in Section 4. In Section 5 a hole-finishing process is used to demonstrate the advantages of the proposed adjustment procedure, where it is compared with other adjustment methods used in industrial practice by evaluating and comparing their short-run off-target costs

over a broad set of conditions.

2 Process and cost models

Suppose the quality characteristic Y_n of the n^{th} machined part represents deviations from a nominal value, which is assumed, without loss of generality, to be equal to zero (the Appendix summarizes notation for ease of reference). After the machine setup operation, the process is supposed to be off-target by d units, i.e., $Y_1 = d + \varepsilon_1$, where ε_1 models both the inherent production variability and the error of measurement. Once the first quality characteristic is measured the value of the control parameter U_1 , which is assumed to have an immediate effect on the process output, is set, thus inducing a change in the next quality characteristic: $Y_2 = d + U_1 + \varepsilon_2$. The procedure is thus iterated and the general expression for the quality characteristic at the n^{th} step, Y_n , is given by:

$$Y_n = d + U_{n-1} + \varepsilon_n \quad (1)$$

where:

- $n = 1, \dots, N$ denotes a discrete time index or part number;
- U_{n-1} is the value of the controllable variable at the $n - 1^{th}$ step of the adjustment procedure, with $U_0 = 0$;
- d is the initial unknown offset (a constant);
- $\{\varepsilon_n\}_{n=1}^N$ represents normally distributed white noise: $\varepsilon_n \sim N(0, \sigma_\varepsilon^2)$, thus the errors are i.i.d. random variables.

Model (1), although a simple transfer function with no “process dynamics” (other than the delay) and no “noise dynamics” underlies most of the literature on SPC methods for non-correlated data because it is appropriate to model a wide variety of discrete-part manufacturing processes. It is also the model assumed by Grubbs (1954, 1983), Trietsch (1998), Sullo and Vandeven (1996, 1999), and Del Castillo et al. (2003a, b) who studied different aspects of the setup adjustment problem. Guo et al. (2001) mainly consider this model as well.

To evaluate the costs associated with the control procedure, we utilize two asymmetric quality loss functions introduced by Taguchi *et al.* (1989). In the first case, costs are assumed to arise only when the part processed is non-conforming, i.e., when the quality characteristic is out of the specification limits. In particular, it will be assumed that the violation of the lower or upper specification limit could lead to different costs. c_1^c is the constant cost when the product quality characteristic is lower than LSL and c_2^c is the constant cost when the product quality characteristic is greater than USL. The superscript c indicates the *constant* cost model.

$$C_n^c = \begin{cases} c_1^c & \text{if } Y_n < LSL \\ 0 & \text{if } LSL \leq Y_n \leq USL \\ c_2^c & \text{if } Y_n > USL \end{cases} \quad (2)$$

(see Figure 1).

insert Figure 1 about here

Another asymmetric cost model considered is based on a piecewise quadratic cost function. In this case, the cost function can be more properly considered as a penalty function, in which the loss is assumed to be proportional to the square of the distance of the quality characteristic from its nominal value.

$$C_n^q = \begin{cases} c_1^q Y_n^2 & \text{if } Y_n < 0 \\ c_2^q Y_n^2 & \text{if } Y_n \geq 0. \end{cases} \quad (3)$$

Here, c_1^q is the quadratic cost coefficient for products with quality deviations less than the target and c_2^q is the quadratic cost coefficient for products with quality deviations greater than the target. The values of c_1^q and c_2^q can be computed with reference to the specification limits as suggested by Taguchi et al (1989), and more recently, by Wu and Tang (1998). The distance between the nominal value and the LSL or USL is denoted by Δ , and the cost corresponding to a quality characteristic equal to LSL or USL is L_1 or L_2 , respectively. The constants, c_1^q and c_2^q , are given by:

$$c_1^q = \frac{L_1}{\Delta^2} \quad \text{and} \quad c_2^q = \frac{L_2}{\Delta^2} \quad (4)$$

(see Figure 2).

insert Figure 2 about here

Modern sensors for on-line inspection and measurement can transmit the data acquired to the machine controller, allowing for automatic feedback control. In this scenario, the cost of adjustment can be neglected and therefore has not been considered in the following analysis.

The asymmetry in the cost function implies two issues that have to be considered in designing the adjustment rule. The first is related to the steady-state optimal process setting T . The problem of determining the value of T for asymmetric cost functions, which is often referred as the *optimum target point*, has been addressed by Ladany (1995) and Wu and Tang (1998). The second issue is related to the way in which, starting from an initial offset, the quality characteristic converges to the optimal target as determined by an adjustment procedure. This paper will mainly deal with the second issue.

Overview of rationale for proposed solution

The proposed solution to the asymmetric Setup Adjustment Problem is presented in Sections 3 and 4. The steps followed for obtaining the solution we propose can be summarized as follows:

- i)* a linear feedback adjustment rule based on ideas taken from stochastic approximation is proposed and has the general form $U_n = U_{n-1} + k_n(Y_n - T + b_n)$ where U_n is the value of the controllable factor, Y_n is the quality characteristic for part n , and where the weights $\{k_n\}_{n=1}^N$, the “bias terms” $\{b_n\}_{n=1}^N$ and the steady-state optimal target T need to be determined. The intuition behind this form of a controller is that for an asymmetric cost function it makes sense to approach the target T from the lower cost side of the cost function;
- ii)* the optimal process means $\{m_n\}_{n=1}^N$ are obtained by minimizing the sum of expected off-target costs over a finite number of parts n ;
- iii)* the steady-state target T is set to be the long-run limit that the optimal process mean should follow;
- iv)* the weights $\{k_n\}_{n=1}^N$ are chosen to assure convergence of the quality characteristic to T ; and finally
- v)* the sequence of bias terms $\{b_n\}_{n=1}^N$ is found such that the actual process mean after adjustment equals to the optimal process mean at each step.

While the derivations and proofs are involved, the proposed linear adjustment rule is very simple to implement in practice, as will be shown in Section 5. We first turn to the details behind the steps above. Readers interested in the use of the solution rather than its rationale can consult the summaries found at the end of Section 4.

3 Biased feedback adjustment rule

Since a control variable is available for removing a potential start-up error of a process, it is possible to design a feedback adjustment rule to manipulate this variable. A common feedback linear adjustment rule is one of the form:

$$U_n = U_{n-1} - k_n(Y_n - T) . \quad (5)$$

That is, the adjustments $U_n - U_{n-1}$ are proportional to the latest measured deviation of the quality characteristic Y_n from the steady-state target T . Del Castillo et al. (2003a) showed that, depending on the selection of the sequence $\{k_n\}_{n=1}^N$, this form of feedback adjustment results in: 1) Grubbs’ rule (Grubbs, 1954, 1983) which in turn is a direct application of stochastic approximation techniques (Robbins and Monro, 1951), 2) the EWMA or integral controller (Box and Luceño, 1995, 1997), 3) the Kalman filter (Kalman, 1960), and 4) an approach based on Recursive Least Squares.

Since the asymmetry in the cost model induces different losses depending on the side from which the quality characteristic approaches the steady-state target, the performance of the linear

adjustment rule could be enhanced by introducing a bias term in (5) to allow convergence from the lower cost side. Anbar (1977) proposed a biased stochastic approximation procedure, further studied by Krasulina (1998), for the problem of one-side convergence. Both authors treated an abstract estimation problem from the point of view of stochastic approximation and did not consider process adjustment scenarios. We apply Anbar's stochastic approximation equation for process adjustment purposes, in which case a bias term b_n is introduced into the adjustment rule (5), and the rule becomes

$$U_n = U_{n-1} - k_n(Y_n - T + b_n) . \quad (6)$$

The effect of the first few bias terms is to “push” the quality characteristic to the lower cost side of T , but eventually the bias term will diminish to zero as n increases. Using the law of the repeated logarithm, Anbar (1977) demonstrated that the convergence of Y_n as $n \rightarrow \infty$ when b_n converges to zero is in $n^{\frac{1}{2}}(\log(\log n))^{-\frac{1}{2}}$.

However, practical insights on the selection of the sequence $\{b_n\}_{n=1}^N$ had not been provided in previous literature. The adjustment procedure we propose is instead based on finding a sequence of bias coefficients $\{b_n\}_{n=1}^N$ that minimizes the costs incurred during the *transient* phase of convergence of the quality characteristic to its steady-state target. In order to preserve its ease-to-use, the adjustment rule should allow to compute the bias sequence $\{b_n\}_{n=1}^N$ as soon as the process offset is found. This property will assure that the control rule is applicable to any discrete-part manufacturing process regardless of the time between observations.

By recursively substituting (6) in (1), the general expression of the quality characteristic at the n^{th} step of the procedure is given by:

$$Y_n = \prod_{i=1}^{n-1} (1 - k_i) d - \sum_{i=1}^{n-1} \left[k_i (\varepsilon_i + b_i) \prod_{j=i+1}^{n-1} (1 - k_j) \right] + T \sum_{i=1}^{n-1} \left[k_i \prod_{j=i+1}^{n-1} (1 - k_j) \right] + \varepsilon_n \quad (7)$$

where we define the product $\prod_{j=n}^{n-1} (1 - k_j)$ to be equal to one.

Since process errors are normally distributed, the quality characteristic Y_n at each step of the procedure is also normally distributed, i.e., $Y_n \sim N(\mu_n, \sigma_n^2)$, with mean and variance equal to:

$$\mu_n = \prod_{i=1}^{n-1} (1 - k_i) d - \sum_{i=1}^{n-1} \left[k_i b_i \prod_{j=i+1}^{n-1} (1 - k_j) \right] + T \sum_{i=1}^{n-1} \left[k_i \prod_{j=i+1}^{n-1} (1 - k_j) \right] \quad (8)$$

$$\sigma_n^2 = \sigma_\varepsilon^2 \left[1 + \sum_{i=1}^{n-1} k_i^2 \prod_{j=i+1}^{n-1} (1 - k_j)^2 \right] . \quad (9)$$

The sequence of bias terms $\{b_n\}_{n=1}^N$ and the offset d affect the mean value μ_n of the quality characteristic but not its variance σ_n^2 .

The adjustment rule is determined by minimizing all the costs associated with the adjustment transient period, i.e., the Average Integrated Expected Cost (AIEC):

$$\text{AIEC} = \frac{1}{N} \sum_{n=1}^N E(C_n) \quad (10)$$

where the cost function C_n can be either constant asymmetric cost function or quadratic cost function as given by (2) and (3), respectively. This represents the average off target cost that we can expect to incur in a lot of N parts (a scaled average expected cost used is introduced in Section 5).

Define $\mathbf{m} = \{m_n\}_{n=1}^N$ to be the collection of the optimal process means. \mathbf{m} describes an optimal average “trajectory” over time that the quality characteristic should follow in order to minimize (10). The goal of the adjustment rule (6) is to keep the process close to this trajectory. The selection of the m_n ’s will be addressed in the next section. For a given selection of the k_n ’s, the bias terms $\{b_n\}_{n=1}^N$ can be determined by equating the right hand side of expression (8) to m_n .

Although the approaches in Anbar (1977) and Krasulina (1998) utilize the harmonic sequence for $\{k_i\}_{n=1}^N$, i.e., $k_i = 1, \frac{1}{2}, \frac{1}{3}, \dots$, it is in principle possible to consider a different sequence, while maintaining the form of the controller given by (6). For example, besides considering the harmonic sequence (Grubbs’ approach), a constant sequence (the EWMA or integral control approach) can be considered instead.

In the case when k_i is a harmonic series, the value of the mean and the variance of the quality characteristic at each step are given by:

$$\mu_n = T - \frac{1}{n-1} \sum_{i=1}^{n-1} b_i \quad (11)$$

$$\sigma_n^2 = \sigma_\varepsilon^2 \left(\frac{n}{n-1} \right). \quad (12)$$

If k_i is instead set equal to a constant λ , as in the EWMA approach, the resulting mean and variance are:

$$\mu_n = T + (1-\lambda)^{n-1}(d-T) - \lambda(1-\lambda)^{n-1} \sum_{i=1}^{n-1} \frac{b_i}{(1-\lambda)^i} \quad (13)$$

$$\sigma_n^2 = \sigma_\varepsilon^2 \left[\frac{2 - \lambda(1-\lambda)^{2(n-1)}}{2-\lambda} \right]. \quad (14)$$

We notice that in Equation (11) the value of μ_n does not depend on the initial unknown offset d , thus an off-line computation of b_n is possible in the proposed biased scheme. In contrast, for an analogous “biased EWMA” approach, μ_n (Eq. 13) is a function of the unknown offset d , so the sequence of biased coefficients $\{b_n\}_{n=1}^N$ can not be computed off line and such a rule is not possible to implement. If the simplest integral or EWMA controller with no bias terms $U_n = U_{n-1} - \lambda(Y_n - T)$

were to be applied instead, the optimal value of λ would again depend on the unknown offset d , as it can be seen from Equation (13). Therefore, in our later comparison, the EWMA scheme will assume varying magnitudes of the standardized offset size $A = d/\sigma_\varepsilon$.

For the proposed biased adjustment scheme, using equation (11), the general expression for b_n can be obtained by equating the mean of the response to the optimal mean at the n^{th} and the $n + 1^{th}$ steps, i.e.,

$$-\frac{1}{n-1} \sum_{i=1}^{n-1} b_i + T = m_n$$

and

$$-\frac{1}{n} \left(\sum_{i=1}^{n-1} b_i + b_n \right) + T = m_{n+1} ,$$

and therefore, the general expression for the bias term b_n is given by

$$b_n = n(T - m_{n+1}) - (n-1)(T - m_n) . \quad (15)$$

4 The optimal target and the sequence of bias terms

To complete the adjustment rule, the optimal steady-state target T and the sequence of bias terms $\{b_n\}_{n=1}^N$ need to be determined. T can be seen as the optimal mean m_n as $n \rightarrow \infty$, while $\{b_n\}_{n=1}^N$ can be computed using (15), once the optimal means \mathbf{m} is known. To compute the means, the following minimization problem has to be solved:

$$\min_{\boldsymbol{\mu}} \text{AIEC} \quad (16)$$

(so, $\mathbf{m} = \arg \min_{\boldsymbol{\mu}} \text{AIEC}$), where $\boldsymbol{\mu} = \{\mu_n\}_{n=1}^N$ is the collection of means of the response at each step of the procedure, and AIEC is the performance index given by equation (10). As shown by Pan (2002), when the linear control rule (6) is in use, the optimization in (16) is equivalent to the following series of minimization problems:

$$\min_{\mu_n} E(C_n) , \quad n = 1, 2, \dots, N . \quad (17)$$

The optimization problems in (17) can be solved for the two types of cost functions under study.

4.1 Constant asymmetric cost function

Consider first the constant asymmetric cost function. The expected cost at time n is given by:

$$\begin{aligned} E(C_n^c) &= c_1^c \int_{-\infty}^{LSL} f_N(y_n; \mu_n, \sigma_n^2) dy_n + c_2^c \int_{USL}^{\infty} f_N(y_n; \mu_n, \sigma_n^2) dy_n \\ &= c_1^c \Phi \left(\frac{LSL - \mu_n}{\sigma_n} \right) + c_2^c \left[1 - \Phi \left(\frac{USL - \mu_n}{\sigma_n} \right) \right] \end{aligned} \quad (18)$$

where $f_N(\cdot)$ is the normal density function, $\Phi(\cdot)$ is the standard normal distribution function, and c_1^c and c_2^c are the constant cost coefficients as defined in Section 2. The minimum of this function with respect to μ_n can be derived by computing the first and second order derivatives of $E(C_n^c)$. The optimal mean m_n^c , obtained by equating the first derivative of $E(C_n^c)$ to zero, is given by

$$m_n^c = \frac{\sigma_n^2 \ln\left(\frac{c_1^c}{c_2^c}\right)}{(USL - LSL)} + \frac{1}{2}(USL + LSL). \quad (19)$$

The second derivative with respect to μ_n is always greater than zero when $LSL < \mu_n < USL$. However, this is not enough to show that m_n^c is a minimum, since m_n^c as in (19) can be outside the interval (LSL, USL) . To prove the optimality of (19), it can be shown that when $\mu_n > m_n^c$ then $\partial E(C_n^c)/\partial \mu_n > 0$, and when $\mu_n < m_n^c$, then $\partial E(C_n^c)/\partial \mu_n < 0$. Thus m_n^c is indeed the minimizer of the expected cost $E(C_n^c)$. A particular case of (19) is when the cost function is symmetric, i.e. $c_1^c = c_2^c$, then the result obtained is $m_n^c = \frac{1}{2}(USL + LSL)$.

The steady-state target T^c can be derived from (19) and (12) by considering the limit as $n \rightarrow \infty$, that is

$$T^c = \frac{\sigma_\varepsilon^2 \ln\left(\frac{c_1^c}{c_2^c}\right)}{(USL - LSL)} + \frac{1}{2}(USL + LSL). \quad (20)$$

Substituting (20) and (19) into the expression of the bias term, given by (15), the values of the bias terms b_n for the asymmetric constant cost function can be directly computed. In this case, all b_n 's except the first one equal to zero, i.e.,

$$b_n = \begin{cases} -\frac{\ln\left(\frac{c_1^c}{c_2^c}\right)\sigma_\varepsilon^2}{(USL - LSL)} & \text{if } n=1 \\ 0 & \text{if } n=2, \dots, N. \end{cases} \quad (21)$$

Although the feedback adjustment procedure has a non-zero bias b_n only at the first step, b_1 affects the subsequent adjustments through the U_{n-1} term in the expression of the controller (6).

4.2 Quadratic asymmetric cost function

Consider now the quadratic asymmetric cost function. The expected cost at the n^{th} step of the procedure is given by

$$E(C_n^q) = c_1^q \int_{-\infty}^0 y_n^2 f_N(y_n; \mu_n, \sigma_n^2) dy + c_2^q \int_0^{\infty} y_n^2 f_N(y_n; \mu_n, \sigma_n^2) dy,$$

where c_1^q and c_2^q are the quadratic cost coefficients as defined in Section 2. By solving the two integrals (as reported in Pan, 2002), the following expression for the expected value of the cost is obtained:

$$E(C_n^q) = c_2^q(\mu_n^2 + \sigma_n^2) + (c_2^q - c_1^q) \left[\sigma_n \mu_n \phi\left(\frac{\mu_n}{\sigma_n}\right) - (\mu_n^2 + \sigma_n^2) \Phi\left(-\frac{\mu_n}{\sigma_n}\right) \right]. \quad (22)$$

Computing the first derivative with respect to μ_n and equating it to zero, the optimal mean m_n^q is determined by the following equation:

$$m_n^q + \left(1 - \frac{c_1^q}{c_2^q}\right) \left[\sigma_n \phi\left(\frac{m_n^q}{\sigma_n}\right) - m_n^q \Phi\left(-\frac{m_n^q}{\sigma_n}\right) \right] = 0 \quad (23)$$

where $\phi(\cdot)$ is the standard normal density function and $\Phi(\cdot)$ is the standard normal distribution function. Although there is no closed form expression for m_n^q , it can be computed numerically off line, since none of the quantities in expression (23) depend on the observations of the quality characteristic. Similarly as in the case of a constant cost function, if the quadratic cost function is symmetric, i.e., $c_1^q = c_2^q$, the optimal mean m_n^q is zero for $n = 1, 2, \dots, N$.

The second derivative of $E(C_n^q)$ with respect to μ_n is always positive, so m_n^q given by equation (23) determines a minimum of the expected cost. Again, the steady-state target T^q can be computed by considering $\lim_{n \rightarrow \infty} \sigma_n = \sigma_\varepsilon$, in equation (23), so T^q is the solution of

$$T^q + \left(1 - \frac{c_1^q}{c_2^q}\right) \left[\sigma_\varepsilon \phi\left(\frac{T^q}{\sigma_\varepsilon}\right) - T^q \Phi\left(-\frac{T^q}{\sigma_\varepsilon}\right) \right] = 0. \quad (24)$$

Therefore, in the case of the quadratic cost model, the feedback adjustment rule can be obtained by first evaluating numerically the optimal means m_n^q that satisfy equation (23) for $n = 1, 2, \dots, N$, and the optimal steady-state target T^q obtained from equation (24), then substituting these values in equation (15) to obtain the sequence of bias coefficients $\{b_n\}_{n=1}^N$.

In summary, the biased linear adjustment procedure for constant and quadratic cost functions is as follows:

Solution to the Asymmetric Constant Cost Model

Given: $r = c_1^c/c_2^c$, USL , LSL , σ_ε , N .

1. Compute the steady-state target T^c using (20);
2. Compute the bias coefficient b_1 using (21);
3. Adjust the control variable on line according to the following equation:

$$U_n = \begin{cases} -[Y_1 - T^c + b_1] & \text{if } n = 1 \\ U_{n-1} - \frac{1}{n}[Y_n - T^c] & \text{if } n = 2, \dots, N. \end{cases} \quad (25)$$

Solution to the Asymmetric Quadratic Cost Model

Given: $r = c_1^q/c_2^q$, σ_ε , N .

1. Compute the steady-state target T^q by solving numerically equation (24);
2. Find the sequence of bias terms $\{b_n\}_{n=1}^N$:
 - Compute the optimal mean m_n^q by solving numerically equation (23) where $\sigma_n = \sigma_\varepsilon \sqrt{\frac{n}{n-1}}$;
 - Substitute T_n^q and m_n^q into (15) to obtain b_n ;
3. Adopt the biased linear adjustment rule for on-line process adjustment:

$$U_n = U_{n-1} - \frac{1}{n}(Y_n - T^q + b_n) .$$

5 An application to a real machining process

5.1 The machining process

This example comes from a major Italian auto part manufacturing company. We consider a hole-finishing operation that is performed on a pre-existing hole in a raw aluminum part made by pressure casting. The design specification limits of the final hole diameter are at LSL=57.000 USL=57.074 mm, while the nominal value set by manufacturing personnel to produce the part is set at the midpoint of the tolerance interval, i.e., at 57.037 mm. After the execution of the operation, the diameter of the hole is measured in an automatic inspection station constituted by a probe that acquires the diameter while the workpiece rotates 360 degrees around the axis of the hole. The mean diameter is computed and recorded. Due to the materials machined and the tools used (polycrystalline inserts), the tool wear can be neglected and no trend is present in the data collected. Therefore no noise dynamics needs to be considered, a typical characteristic of discrete-part manufacturing processes.

insert Figure 3 about here

The costs related to non-conforming parts are different depending on whether the diameter obtained is below the lower or above the upper specification limit (LSL and USL, respectively). Indeed, when the hole diameter is less than LSL, an additional machining operation can correct the defect by adequately selecting the depth of cut. On the other hand, when the diameter obtained is greater than USL, the part has to be scrapped, since it is impossible to recover the nonconforming workpiece. The cost of an undersized hole is determined by considering the additional repairing operation while the cost of an oversized hole is equal to the margin lost minus the salvage value of the scrap.

The problem is to find a sequence of adjustments in the radial position of the tool (the controllable factor which modifies the hole diameter of the parts, see Figure 3) in an economic way under a

given asymmetric cost function. Due to the nature of the machine tools and measurement systems utilized, the cost of performing the adjustments themselves is negligible and the only significant cost component is given by an asymmetric off-target cost function. The problem and solutions demonstrated by this example are applicable to a wide range of discrete-part manufacturing processes with asymmetric cost structures, especially for the high value-added operations performed in the short-run manufacturing.

5.2 Solution applied to the machining process

A Pareto analysis of the principal causes of stops in production and scraps was conducted by the manufacturer. This analysis shows that the execution of the hole diameter represents the first cause for scraps and the second cause for stops in production.

Table 1 reports the summary statistics of the diameter data (deviations from nominal, in microns) from 30 days production. The process capability can be estimated by $PCR = \frac{USL-LSL}{6\hat{\sigma}}$, which equals to 1.10. We note that the process is almost centered. In addition, an ANOVA one-way analysis conducted by the company based on historical data shows that most of the process variability comes from initial offsets due to inaccurate setup operations. After eliminating initial offsets, the actual standard deviation of the process reduces to $\hat{\sigma}_\varepsilon = 7.26$ microns, which means the process capability could reach a potential value of $PCR = \frac{USL-LSL}{6\hat{\sigma}_\varepsilon} = 1.70$. The historical data also shows that the mean sample size between two setup operations is around 15 parts and the average initial offset d is equal to 19.16 microns. Therefore, the constant A ($A = \frac{d}{\sigma_\varepsilon}$, i.e., the offset value in standard deviations) is equal to 2.64.

insert Table 1 about here

The cost of an undersized hole, c_1^c , is determined by considering the additional repairing operation while the cost of an oversized hole, c_2^c , is equal to the margin lost minus the value of the scrap. In this case, the asymmetry ratio r ($r = c_2^c/c_1^c$) is 6.5. If a quadratic cost model is assumed, the same ratio between c_2^q and c_1^q can be obtained. The resulting steady-state targets are $T^c = -1$ and $T^q = -5$.

As shown in Figure 3, the controllable variable U_n is the radial position of the tool. By adequately selecting this variable, the depth of cut can be changed. Furthermore, the adoption of a parametric program (as available in CNC machines) can in principle allow for an automatic adjustment procedure: once a diameter is measured, the value of the controllable variable can be determined and transmitted to the control unit of the machining center that will process the next part accordingly. In practice, the resolution of the machine in setting the tool position should be considered in order to derive the approximation of the adjustment size. In this case the precision

in the order of microns determines that we should round the adjustment to zero decimal place.

Assuming the asymmetric constant cost function model, the expected value of the cost can be rewritten as a function of r by scaling the expected costs at each step as follows:

$$\frac{E(C_n^c)}{c_1^c} = \Phi\left(\frac{LSL - \mu_n}{\sigma_n}\right) + r \left[1 - \Phi\left(\frac{USL - \mu_n}{\sigma_n}\right)\right] \quad (26)$$

Therefore, the performance comparisons among the different control rules will be evaluated using as performance index the Scaled Average Integrated Expected Cost (SAIEC), defined as:

$$SAIEC^c = \frac{1}{N-1} \sum_{n=2}^N \frac{E(C_n^c)}{c_1^c} \quad (27)$$

where the index in the summation starts from 2, since the quality characteristic of the first part machined does not depend on the adjustment procedure.

For the quadratic cost function model, an analogous comparison was performed. In this case, the expected cost reported in Equation (22) can be rewritten in scaled form by manipulating the expression as follows:

$$E(C_n^q) = c_1^q \sigma_n^2 \left\{ \frac{c_2^q}{c_1^q} \left(\frac{\mu_n^2}{\sigma_n^2} + 1 \right) + \left(\frac{c_2^q}{c_1^q} - 1 \right) \left[\frac{\mu_n}{\sigma_n} \phi\left(\frac{\mu_n}{\sigma_n}\right) - \left(\frac{\mu_n^2}{\sigma_n^2} + 1 \right) \Phi\left(-\frac{\mu_n}{\sigma_n}\right) \right] \right\} .$$

Since the variance at each step of the adjustment procedure (12) is proportional to the variance of the error σ_ε^2 , the expected cost at the n^{th} step of the procedure is given by:

$$\frac{E(C_n^q)}{c_1^q} = s_n \sigma_\varepsilon^2 \left\{ r(\delta_n^2 + 1) + (r-1) [\delta_n \phi(\delta_n) - (\delta_n^2 + 1) \Phi(-\delta_n)] \right\} ,$$

where $s_n = \left[1 + \sum_{i=1}^{n-1} k_i^2 \prod_{j=i+1}^{n-1} (1 - k_j)^2 \right]$, r denotes the ratio between c_2^q and c_1^q and δ_n the ratio between μ_n and σ_n .

Similar to the constant cost function case, the performance index in the quadratic case is the Scaled Average Integrated Expected Cost defined as:

$$SAIEC^q = \frac{1}{N-1} \sum_{n=2}^N \frac{E(C_n^q)}{c_1^q} \quad (28)$$

Figure 4 shows plots of the mean of the quality characteristic obtained with Grubbs' rule (for which the mean is constant and equal to the steady-state target value), and the Biased rule (for which the mean converges to the target value from the lowest cost side) under the constant and the quadratic cost models. The piecewise behavior of the biased mean converging to the target value is due to the approximation (rounding) adopted to consider the precision of the machine in setting of the tool position.

insert Figure 4 about here

The expected savings in cost obtained by the Biased rule are shown in Figure 5, where the percentage difference in $SAIEC^c$ and $SAIEC^g$ determined by the Biased and Grubbs' procedures is reported as a function of the items processed. For the constant cost model, the savings are almost constant as the number of parts increases. This is because after the first few adjustments the process mean is in the zero cost region (far from the specification limits) and the $E(C_n^c)$ will be close to zero for large n . For the quadratic cost case, the savings decrease with n , with significantly savings occurring in the first few parts only.

In contrast, comparisons with an EWMA or integral control rule (Box and Luceño, 1997) indicates that the savings obtained by using the proposed biased rule are significant regardless of the value of λ chosen (see Figure 6). This is partly due to the long-run behavior of the process mean under the EWMA control actions. As n tends to infinity, the mean of the Y_n regulated by the EWMA controller approaches zero, but the variance approaches the value $\frac{2\sigma_\varepsilon^2}{2-\lambda}$, which is greater than σ_ε^2 . This inflation in variance has been discussed by Box and Luceño (1997) and Del Castillo (2001).

insert Figure 5 about here

insert Figure 6 about here

5.3 Sensitivity analysis

A comprehensive sensitivity analysis of the application above in terms of varying the initial offset value, asymmetric ratio, process capability ratio, and batch size was conducted. For reasons of space, we only summarize the findings here, referring readers to Pan (2002) for details. It was found that, comparing to Grubbs' rule or the EWMA controller, the Biased adjustment procedure can reduce the process quality loss significantly on the first few parts produced, which implies that this procedure is suitable for short-run manufacturing. When the process capability ratio is large (i.e., the inherent process variation is relatively small), the advantage of the Biased rule is more clear, even if batch sizes are large. When the asymmetry ratio increases or the initial offset value increases, the advantage of the biased adjustment rule over other adjustment rules increases as well.

One problem with the EWMA controller used for setup adjustment is that it is not clear how to choose λ a priori, since the best choice of λ depends on the unknown offset d . As it is well-known in the area of PID control (Box and Luceño, 1997, Del Castillo, 2001), an integral controller with large λ (integral constant) will bring the mean of the process back to target rapidly after a sudden

shift or offset, but this will inflate the variance of the adjusted process. This is the reason why in most of our comparisons large λ seems to perform relatively better, although the proposed biased rule still dominates overall. This is in contrast to the usual recommendations for PI controllers, based on non-stationary disturbances (e.g., IMA(1,1) noise, very unlikely in CNC machining), since no offsets are included in such models, and a smaller value of λ in an EWMA will predict such process relatively well.

6 Conclusions

The problem of designing an adjustment rule to correct a process start-up error has recently received a renewed attention in the literature. This is related to two trends that have been shown in modern manufacturing – shortened production runs, which leads to an increase in the number of setups required on the machine, and growing frequency in changing product specifications, which increases the chance of systematic errors at the start-up of a manufacturing process. Therefore, applying feedback adjustments for process start-up errors becomes an effective way to reduce the number of non-conforming parts. Up to now, previous approaches to setup adjustment problems have only considered symmetric cost functions. This paper presented a feedback adjustment rule that can be adopted when an asymmetric cost model better represents the process quality losses entailed. In particular, two asymmetric cost functions that are often encountered in manufacturing have been considered. In the first case, the cost of a non-conforming item is assumed constant but changes depending whether the quality characteristic is below the lower or above the upper specification limit. In the second case, costs are supposed to be proportional to the square of the distance of the quality characteristic from the nominal value and the proportional value is allowed to change.

Based on a general form of a linear controller, a biased feedback adjustment rule was derived by minimizing all the off-target costs incurred during the transient phase in which the quality characteristic converges to its steady-state target from the lowest cost side. A numerical comparison of the cost incurred by the adjustment rule proposed and other rules used in practice showed that the proposed procedure is effective, especially when the asymmetry in the cost function or the initial process offset are large. The proposed Biased adjustment rule is recommended for manufacturing expensive parts produced in small lots (e.g., as in the aerospace industry).

Besides the two specific cost functions studied herein, the proposed adjustment approach can be easily extended to deal with other production situations in which the cost function is also asymmetric, such as a piece-wise linear function used in filling processes (Misiorek and Barnett, 2000). The present work could also be extended to account for a quadratic adjustment cost. Del Castillo *et al.* (2003a) show how this can be done in the symmetric off-target cost case.

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Appendix: notation

LSL	lower specification limit
USL	upper specification limit
r	asymmetric ratio
Y_n	deviation from the nominal value of a quality characteristic of the n^{th} part
U_n	value of the controllable variable
c_1^c & c_2^c	cost coefficients of the constant cost model
c_1^q & c_2^q	cost coefficients of the quadratic cost model
T^c	steady-state target for the constant cost case
T^q	steady-state target for the quadratic cost case
$\{b_n\}_{n=1}^N$	a sequence of bias terms used in the adjustment rule
$\{m_n\}_{n=1}^N$	a sequence of optimal process means of the adjusted process
μ_n	mean of the adjusted process
σ_n	standard deviation of the adjusted process
$AIEC$	average integrated expected cost
$SAIEC$	scaled average integrated expected cost
$\Phi()$	cumulative normal distribution function

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Tables

Variable	N	Mean	Median	StDev
hole diameter deviations	344	2.732	1.523	11.203

Table 1: Descriptive statistics for hole diameter deviations from nominal, Y_n , in microns. The data were obtained from 30 days of production under no on-line adjustments.

List of Figure Captions

1. The asymmetric constant cost function with different costs when the quality characteristic is below LSL or above USL.
2. The asymmetric quadratic cost function with different costs when the quality characteristic is below LSL or above USL.
3. The hole finishing operation.
4. Trajectory of the optimal mean of the quality characteristic under the constant (m_n^c , left) and the quadratic (m_n^q , right) cost models using Grubbs' rule and the Biased procedure (considering 0 and 2 decimal places), hole finishing example. Here $r = 6.5$, $N = 15$, and $PCR = 1.7$.
5. The percentage savings in SAIEC $\left(\frac{\text{SAIEC}_G - \text{SAIEC}_B}{\text{SAIEC}_G} \times 100\right)$ obtained by using the Biased procedure compared to Grubbs' rule under the constant (SAIEC^c) and the quadratic (SAIEC^q) cost function models ($r = 6.5$, $N = 15$ and $PCR = 1.7$), hole finishing example.
6. The percentage savings in SAIEC $\left(\frac{\text{SAIEC}_{EWMA} - \text{SAIEC}_B}{\text{SAIEC}_{EWMA}} \times 100\right)$ obtained by using the Biased procedure compared to the EWMA rule (with $\lambda = 0.2, 0.4, 0.6$ and 0.8) under the constant (SAIEC^c) and the quadratic (SAIEC^q) cost function model ($r = 6.5$, $N = 15$, $A = 2.64$ and $PCR = 1.7$), hole finishing example.

Figures

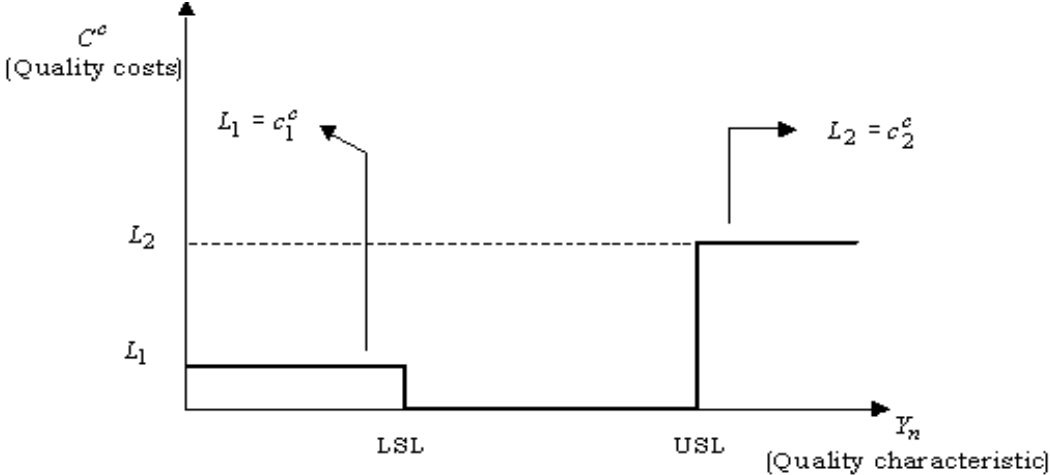


Figure 1: The asymmetric constant cost function with different costs when the quality characteristic is below LSL or above USL.

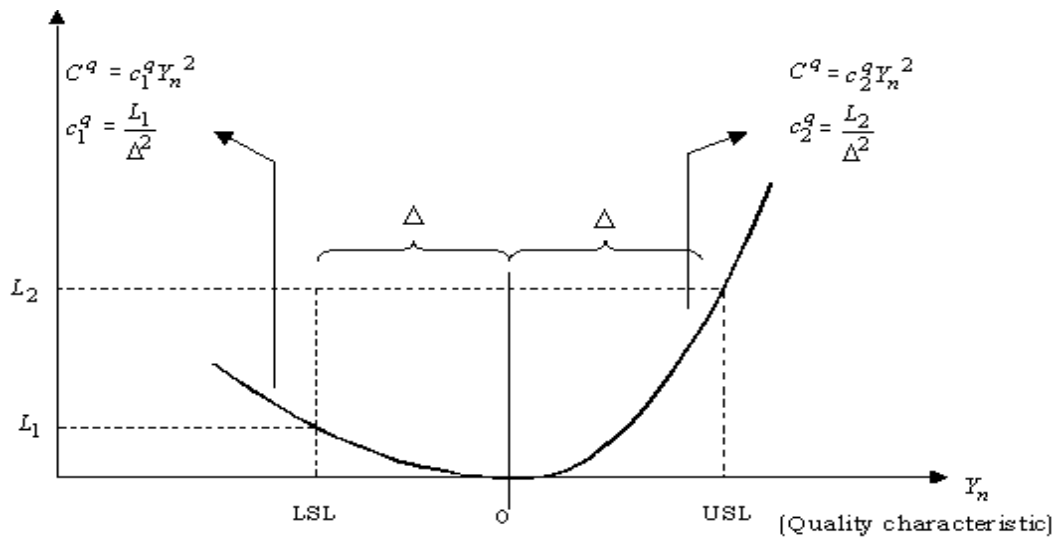


Figure 2: The asymmetric quadratic cost function with different costs when the quality characteristic is below LSL or above USL.

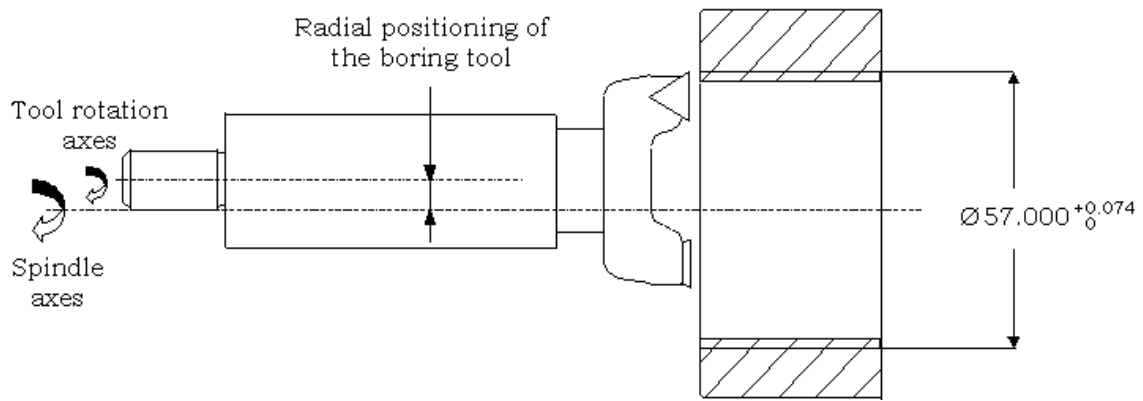


Figure 3: The hole finishing operation.

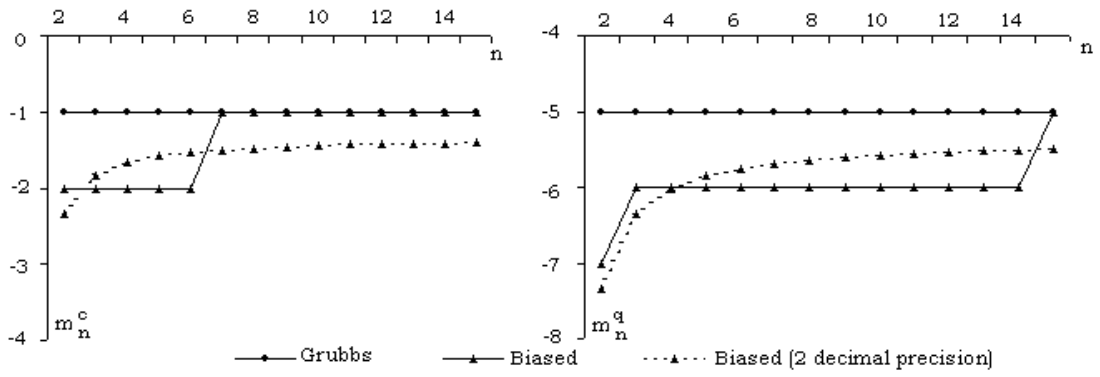


Figure 4: Trajectory of the optimal mean of the quality characteristic under the constant (m_n^c , left) and the quadratic (m_n^q , right) cost models using Grubbs' rule and the Biased procedure (considering 0 and 2 decimal places), hole finishing example. Here $r = 6.5$, $N = 15$, and $PCR = 1.7$.

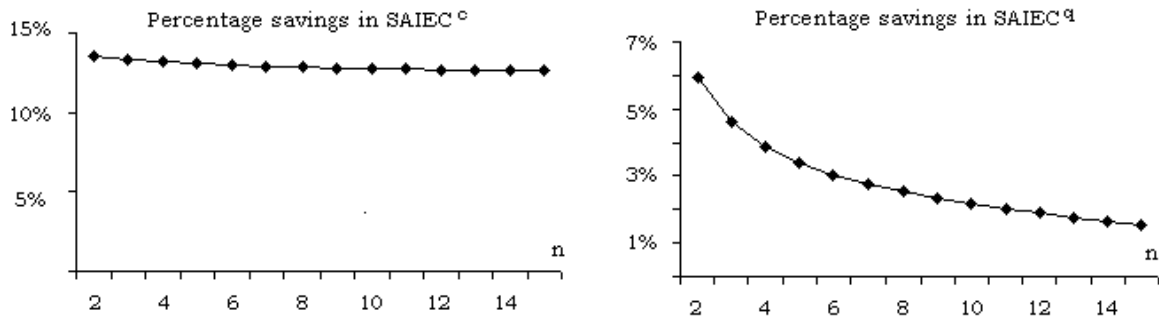


Figure 5: The percentage savings in SAIEC $\left(\frac{\text{SAIEC}_G - \text{SAIEC}_B}{\text{SAIEC}_G} \times 100\right)$ obtained by using the Biased procedure compared to Grubbs' rule under the constant (SAIEC^c) and the quadratic (SAIEC^q) cost function models ($r = 6.5$, $N = 15$ and $PCR = 1.7$), hole finishing example.

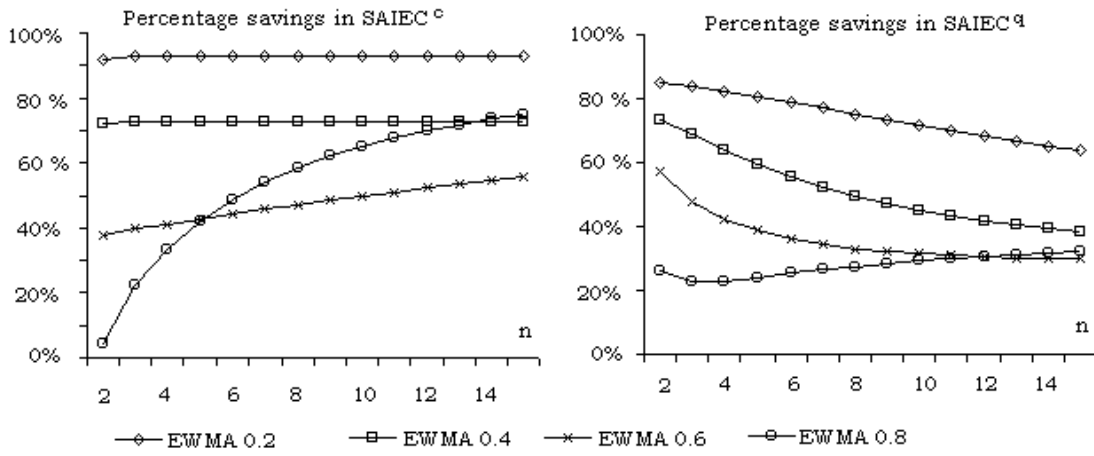


Figure 6: The percentage savings in SAIEC $\left(\frac{\text{SAIEC}_{EWMA} - \text{SAIEC}_B}{\text{SAIEC}_{EWMA}} \times 100 \right)$ obtained by using the Biased procedure compared to the EWMA rule (with $\lambda = 0.2, 0.4, 0.6$ and 0.8) under the constant (SAIEC^c) and the quadratic (SAIEC^q) cost function model ($r = 6.5, N = 15, A = 2.64$ and $PCR = 1.7$), hole finishing example.