Direction Finding and Positioning Algorithm with COLD-ULA Based on Quaternion Theory

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Abstract —Electromagnetic vector sensor arrays have been widely applied in the communication, radio, navigation, and so on. The application of electromagnetic vector sensor antenna array will greatly improve the overall performance of the communication system. In this paper, a novel quaternion-ESPRIT (estimation of signal parameters via rotational invariance techniques) algorithm for direction finding and positioning based on COLD (cocentered orthogonal loop and dipole) uniform linear array (COLD-ULA) is proposed. First, quaternion data model of COLD-ULA is deduced and constructed. Second, the array steering vector and the angle of arrival (DOA) are estimated using a quaternion eigenvalue decomposition of the data covariance matrix. Finally, the estimation of polarization parameters are required using the relationships between the dipoles and the loops. The proposed technique not only decouples the DOA estimation information from the polarization estimation information, but also improves the ability of signal detection. Moreover, the proposed technique has the advantage of small amount of calculation and parameter automatic matching. Simulation results show that the performance of quaternion method is obviously better than that of the long-vector method.

Index Terms—Array signal processing, direction of arrival, quaternion-ESPRIT, polarization, antenna array, subspace

I. Introduction

Electromagnetic vector sensor (EMVS) array is a new type of array to obtain the electromagnetic signal spatial and polarization domain information, which has the broad application prospect in the communication technique, signal detection technique (SDT), mobile communication and so on [1]-[23]. Direction of arrival (DOA) estimation based on the EMVS array has become a hot topic in the research field of signal processing, and it has made many valuable research results [1], [4]-[6], [8]-[12], [15]-[22]. The authors in literatures [2]-[12] have studied the direction finding parameter estimation of complete EMVS array, the parameter estimation of incomplete EMVS has been discussed in literatures [13]-[18], the direction finding and positioning parameter estimation of spatially collocated vector-sensors composed of orthogonally oriented dipole(s) and /or loop(s) have also been investigated by Wong & Yuan in [19], [20]. Various algorithms have been developed to estimate the DOA and polarization parameters of multiple electromagnetic signals. The first direction-finding algorithms, explicitly exploiting all six electromagnetic components, have been developed by Nehorai and Paldi [2] and Li [3], respectively. The cross-product-based DOA estimation algorithm was first adapted to ESPRIT (estimation of signal parameters via rotational invariance techniques) by Wong and Zoltowski [9]-[12]. A uni-vector-sensor ESPRIT algorithm was put forward in [6]. The maximum likelihood approach was presented in [1]; the trilinear decomposition algorithm was proposed in [4] and [5]; two distinct versions of ESPRIT estimators were reported in [3] and [8] respectively; the multiple signal classification (MUSIC) technique was investigated in [9] and [10]. However, the above-mentioned estimation methods are based on the long vector data model of EMVS. Under this condition, complex-valued vectors are used to represent the output of each EMVS in the array, and the collection of an EMVS array is arranged via concatenation of these vectors into a long vector. Consequently, the corresponding algorithms somehow destroy the vector nature of incident signals carrying multidimensional information in space, time, and polarization. In recent years a few research have been made on estimating the DOA of EMVS within the algebraic system theory for quaternion and its extension [21]-[28]. Quaternion MUSIC technique was proposed based on the quaternion formalism of the two component vector sensor array in [25]. The three component vector sensor was expressed as a biquaternion number, then biquaternion-based MUSIC was proposed in [26]. The six component electromagnetic (EM) vector sensor array was represented by a quad-quaternion model in [27].

The advantages of using quaternion for EMVS is that the local vector nature of a EMVS array is preserved in multiple imaginary parts, and it could result in a more compact representation. The use of quaternion allows us to skip the parametrization step used in long-vector techniques as it intrinsically includes the vector dimension in the process. Quaternionic matrix operations can provide a better subspace approximation than the long-vector approach.

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Compared with the COLD (cocentered orthogonal loop and dipole) pairs oriented along *x*-axis, *y*-axis and cocentered dipole and dipole pairs, cocentered loop and loop pairs oriented along *x*- and *y*-axis respectively, we can see that COLD pairs along the *z* axis is more easier to realize the decoupling of polarization and angle of arrival parameters because of its simple structure [13]-[18]. A novel quaternion-ESPRIT algorithm for estimating signal DOA and polarization using the COLD uniform linear array (COLD-ULA) is proposed in this paper. In the algorithm, the DOA information and polarization information of incident signals are decoupled, errors of DOA and polarization herein do not cumulate. Hence, this proposed algorithm: 1) is computationally less intensive than many open-form search methods; 2) produces closed-form solution with no extra computation needed for signal parameter estimates; 3) has the advantage of parameter automatic matching and without spectral peak searching. The simulation results show that the performance of quaternion method is better than that of long-vector method.

The paper is organized as follows. In Section II, a mathematical theory of quaternion and its relevant properties are presented. Section III introduces the quaternion signal model for six-component and two-component vector sensor. In Section IV, the proposed quaternion-ESPRIT algorithm is introduced. Simulation results are presented in Section V. In Section VI, conclusions are given.

II. Quaternion Theory

Quaternion, which extends imaginary numbers into a four-dimensional space, began to be developed by William Hamilton in 1843. A complex number has two components: the real and the imaginary part. The quaternion has four components, i.e., one real part and three imaginary parts, and can be represented in Cartesian form as:

$$q = a + \mathrm{i}b + \mathrm{j}c + \mathrm{k}d \qquad a,b,c,d \in \mathrm{R} \tag{1}$$

where i, j and k are complex operators which obey the following rules.

$$\begin{aligned} &\mathrm{ij} = -\mathrm{ji} = \mathrm{k} \qquad \mathrm{jk} = -\mathrm{kj} = \mathrm{i} \\ &\mathrm{ki} = -\mathrm{ik} = \mathrm{j} \qquad \mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = -1 \end{aligned} \tag{2}$$

Quaternion can also be expressed as the following form:

$$q = a + \mathrm{i}b + \left(c + \mathrm{i}d\right)\mathrm{j} = \alpha + \beta\mathrm{j} \tag{3}$$

which is known as the "Cayley-Dickson representation".

The quaternion conjugate and the quaternion modulus are respectively given by

$$q^* = a - \mathrm{i}b - \mathrm{j}c - \mathrm{k}d \tag{4}$$

$$\|q\| = \sqrt{a^2 + b^2 + c^2 + d^2} \tag{5}$$

Let q_i be $q_i = a_i + \mathrm{i}b_i + \mathrm{j}c_i + \mathrm{k}d_i$, where $a_i, b_i, c_i, d_i \in \mathrm{R}$, $i=1,2$, then the addition of two quaternion expressed in terms of their real and imaginary parts is given by

$$q_1 + q_2 = \left(a_1 + a_2\right) + \mathrm{i}\left(b_1 + b_2\right) + \mathrm{j}\left(c_1 + c_2\right) + \mathrm{k}\left(d_1 + d_2\right) \tag{6}$$

From the rules in equation (2), it is clear that multiplication is not commutative, namely $q_1 \cdot q_2 \neq q_2 \cdot q_1$. The inner product of quaternion can be represented as:

$$\left\langle p, q\right\rangle_{\mathrm{H}} = p^* q \tag{7}$$

If the inner product of quaternion meets with $\left\langle p, q\right\rangle_{\mathrm{H}} = 0$, it is called that the quaternion p and q is orthogonal.

For some reason, it is sometimes useful to consider the quaternion as composed of a vector part and a scalar part, thus q can also be expressed as

$$q = S\left(q\right) + V\left(q\right) \tag{8}$$

where the scalar part, $S\left(q\right)$ is the real part i.e. , $S\left(q\right) = a$ and the vector part is a composite of three imaginary components, i.e., $V\left(q\right) = a\mathrm{i} + b\mathrm{j} + c\mathrm{k}$.

The product of two quaternions expressed in terms of their scalar and vector parts is given by

$$\begin{aligned} qp = &S\left(q\right)S\left(p\right) - V\left(q\right)V\left(p\right) + S\left(q\right)V\left(p\right) \\ &+ S\left(p\right)V\left(q\right) + V\left(q\right) \times V\left(p\right) \end{aligned} \tag{9}$$

where $\cdot$ and $\times$ denotes the vector dot cross products, respectively. It follows from this that the dot and cross products of two pure quaternions m and n are given by

$$m \cdot n = -\frac{1}{2}\left(mn + nm\right), \qquad m \times n = \frac{1}{2}\left(mn - nm\right) \tag{10}$$

From the point of view of algebra, quaternion is an extension of the complex field. Quaternion has more compact data expression. A quaternion contains more than two components than that of complex number, so that it can contain more information in one operation.

III. Signal and Data Models

A six-component electromagnetic vector sensor consists of three spatially co-located identical but orthogonally oriented, electrically short dipoles and magnetically small loops separately measuring all three electric-field components and three magnetic-field components of incident signals. Such an electromagnetic vector sensor can exploit any polarization diversity among the impinging sources, as shown in Fig. 1.

An ideal electromagnetic vector sensor was called auxiliary element. The auxiliary element is idealized, by overlooking all mutual coupling among its six collocated constituent antennas. Such an idealized vector sensor's array manifold (also called unit-power electromagnetic field vector) would be a concatenation of the 3×1 electric-field vector $\mathbf{e}$ with the 3×1 magnetic-field vector $\mathbf{h}$, to be [3, 8]:

$$\mathbf{a}(\theta,\phi,\gamma,\eta)=\begin{bmatrix}e_x\\e_y\\e_z\\h_x\\h_y\\h_z\end{bmatrix}=\begin{bmatrix}\cos\theta\cos\phi & -\sin\phi\\ \cos\theta\sin\phi & \cos\phi\\ -\sin\theta & 0\\ -\sin\phi & -\cos\theta\cos\phi\\ \cos\phi & -\cos\theta\sin\phi\\ 0 & \sin\theta\end{bmatrix}\begin{bmatrix}\sin\gamma e^{j\eta}\\ \cos\gamma\end{bmatrix} \quad (11)$$

where $\theta\in[0,\pi]$ is the signal's elevation angle measured from the positive z-axis, $\phi\in[0,2\pi]$ denotes the azimuth angle measured form the positive x-axis, $\gamma\in[0,\pi/2]$ represents the auxiliary polarization angle, and $\eta\in[-\pi,\pi]$ symbolizes the polarization phase difference.

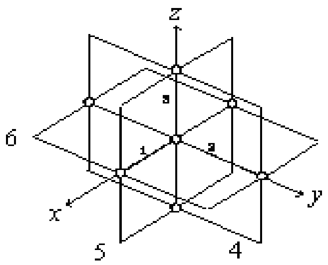

Fig. 1. A six-component electromagnetic vector sensor.

In this paper, we make the following assumptions. K far-field narrowband completely polarized electromagnetic plane wave source signals in the *x*-*z* plane impinge upon a uniform linear array (ULA), which is composed of M $(M>K)$ identical COLD pairs, as shown in Fig. 2. For the COLD pairs, the dipoles parallel to the *z*-axis are referred to as the *z*-axis dipoles and the loops parallel to the *x*-*y* plane as the *x*-*y* plane loops, respectively measuring the *z*-axis electric field components and the *z*-axis magnetic field components. The COLD pairs' steering vector of the *k*th $(1\le k\le \mathrm{K})$ unit-power electromagnetic source signal is the following 2×1 vector [2]:

$$\mathbf{a}(\theta_k,\gamma_k,\eta_k)=\begin{bmatrix}e_{kz}\\h_{kz}\end{bmatrix}=\begin{bmatrix}-\sin\theta_k\sin\gamma_k e^{j\eta_k}\\ \sin\theta_k\cos\gamma_k\end{bmatrix} \quad (12)$$

where $\theta_k\in[0,\pi/2]$ is the signal's elevation angle measured from the positive z-axis, $\gamma_k\in[0,\pi/2]$ represents the auxiliary polarization angle, and $\eta_k\in[-\pi,\pi]$ symbolizes the polarization phase difference.

The *z*-axis electric field e_{kz} and the *z*-axis magnetic field h_{kz} both involve the same factor $\sin\theta_k$, so polarization estimation based on COLD pairs is independent of the source's direction of arrival and it requires no prior information of azimuth and elevation angles. Without loss of generality, we assume that the inter-element spacing of the ULA is d, where $d<0.5\lambda$ and $\lambda=c/f$, with f be the frequency of the signals.

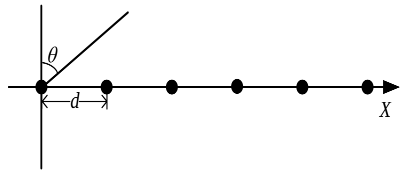

Fig. 2. Uniform COLD linear array geometry.

The phase differences between the M array elements and the origin constitute the spatial steering vector, i.e.:

$$\mathbf{q}(\theta_k)=\begin{bmatrix}1\\ e^{j\frac{2\pi d}{\lambda}\sin(\theta_k)}\\ \vdots\\ e^{j\frac{2\pi(M-1)d}{\lambda}\sin(\theta_k)}\end{bmatrix} \quad (13)$$

The joint expression of e_{kz} and h_{kz} are indicated as with a quaternion c_k.

$$c_k=e_{kz}+ih_{kz}=-\sin\theta_k\sin\gamma_k e^{j\eta_k}+i\sin\theta_k\cos\gamma_k \quad (14)$$

The output of array response for the *k*th incident signal can be expressed as follows:

$$x_k(t)=\underbrace{c_k\mathbf{q}(\theta_k)}_{\mathbf{a}(\theta_k,\gamma_k,\eta_k)}s_k(t) \quad (15)$$

where $s_k(t)$ is the *k*th incident signal and $\mathbf{a}(\theta_k,\gamma_k,\eta_k)$ is defined as $c_k\mathbf{q}(\theta_k)$.

The received data collected by the COLD-ULA at time t can be represented as

$$\mathbf{X}(t)=\mathbf{AS}(t)+\mathbf{N}(t) \quad (16)$$

where $\mathbf{X}(t)$, $\mathbf{S}(t)$, $\mathbf{N}(t)$ and $\mathbf{A}=[\mathbf{a}_1,\cdots,\mathbf{a}_K]$ are the received data, the uncorrelated incident signals, the zero-mean additive complex Gaussian noise and the steering vector matrix of incident signals, respectively, i.e.,

$$\mathbf{X}_i(t)=\begin{bmatrix}\mathbf{x}_{i,0}(t)\\ \mathbf{x}_{i,1}(t)\\ \vdots\\ \mathbf{x}_{i,M-1}(t)\end{bmatrix},\quad \mathbf{S}(t)=\begin{bmatrix}s_1(t)\\ s_2(t)\\ \vdots\\ s_K(t)\end{bmatrix}$$

$$\mathbf{N}_i(t)=\begin{bmatrix}\mathbf{n}_{i,0}(t)\\ \mathbf{n}_{i,1}(t)\\ \vdots\\ \mathbf{n}_{i,M-1}(t)\end{bmatrix},\quad \mathbf{A}_i^T=\begin{bmatrix}\mathbf{a}_i^T(\theta_1,\gamma_1,\eta_1)\\ \mathbf{a}_i^T(\theta_2,\gamma_2,\eta_2)\\ \vdots\\ \mathbf{a}_i^T(\theta_K,\gamma_K,\eta_K)\end{bmatrix},$$

with $\mathbf{a}_k=\mathbf{a}(\theta_k,\gamma_k,\eta_k)$, $k=1,\cdots,K$.

According to formulas (14), (15) and (16), the matrix $\mathbf{A}$ can be rewritten as:

$$\mathbf{A}=\mathbf{A}_e+\mathbf{A}_h\mathrm{i} \tag{17}$$

The relationship between $\mathbf{A}_e$ and $\mathbf{A}_h$ can expressed as :

$$\mathbf{A}_e=\mathbf{A}_h\mathbf{\Omega} \tag{18}$$

where

$$\mathbf{\Omega}=\begin{bmatrix} -\tan\gamma_1 e^{j\eta_1} & & \\ & \ddots & \\ & & -\tan\gamma_K e^{j\eta_K} \end{bmatrix}$$

The first and last $M-1$ rows of matrix $\mathbf{A}$ constitute two new matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ respectively, the relationship between $\mathbf{A}_1$ and $\mathbf{A}_2$ can expressed as

$$\mathbf{A}_2=\mathbf{A}_1\mathbf{\Phi} \tag{19}$$

where

$$\mathbf{\Phi}=\begin{bmatrix} e^{j\frac{2\pi d}{\lambda}\sin\theta_1} & & \\ & \ddots & \\ & & e^{j\frac{2\pi d}{\lambda}\sin\theta_K} \end{bmatrix} \tag{20}$$

IV. Quaternion-ESPRIT Algorithm

The correlation matrix of received data $\mathbf{X}(t)$ is

$$\mathbf{R}_x=\mathrm{E}\left[\mathbf{X}\mathbf{X}^{\mathrm{H}}\right]=\mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}+\sigma^2\mathbf{I} \tag{21}$$

with $\mathrm{E}[\cdot]$ symbolizing the statistical mean, $(\cdot)^{\mathrm{H}}$ is the complex conjugate transpose, σ^2 indicating the white noise power and $\mathbf{R}_s=\mathrm{E}\left[\mathbf{S}(t)\mathbf{S}^{\mathrm{H}}(t)\right]$ representing the source covariance matrix. Let $\mathbf{E}_s$ be the $\mathrm{N}\times\mathrm{K}$ matrix composed of the K eigenvectors corresponding to the K largest eigenvalues of $\mathbf{R}_x$ and let $\mathbf{E}_n$ denote the $\mathrm{N}(\mathrm{N\text{-}K})$ matrix composed of the remaining N-K eigenvectors of $\mathbf{R}_x$. Based on the subspace theory, there exists $\mathrm{K}\times\mathrm{K}$ nonsingular matrix $\mathbf{T}$, and the signal subspace can be expressed explicitly as

$$\mathbf{E}_s=\mathbf{A}\mathbf{T} \tag{22}$$

The first M-1 rows and last M-1 rows of matrix $\mathbf{E}_s$ constitute two new matrices $\mathbf{E}_1$ and $\mathbf{E}_2$ respectively. According to the subspace theory, the signal subspace can be expressed explicitly as:

$$\mathbf{E}_1=\mathbf{A}_1\mathbf{T} \quad \mathbf{E}_2=\mathbf{A}_2\mathbf{T}=\mathbf{A}_1\mathbf{\Phi}\mathbf{T} \tag{23}$$

From equation (23), it can be obtained that:

$$\mathbf{E}_1^{\#}\mathbf{E}_2\mathbf{T}^{-1}=\mathbf{T}^{-1}\mathbf{\Phi} \tag{24}$$

where $\mathbf{E}_1^{\#}=\left(\mathbf{E}_1^{\mathrm{H}}\mathbf{E}_1\right)^{-1}\mathbf{E}_1^{\mathrm{H}}$.

Let $\boldsymbol{\psi}=\mathbf{E}_1^{\#}\mathbf{E}_{s2}=\left(\mathbf{E}_{s1}^{\mathrm{H}}\mathbf{E}_{s1}\right)^{-1}\mathbf{E}_{s1}^{\mathrm{H}}\mathbf{E}_{s2}$, then formula (24) can be rewritten as

$$\boldsymbol{\psi}\mathbf{T}^{-1}=\mathbf{T}^{-1}\mathbf{\Phi} \tag{25}$$

Equation (25) implies that the estimate of $\hat{\mathbf{\Phi}}$ is a matrix whose diagonal elements are composed of the K largest eigenvalues of matrix $\boldsymbol{\psi}$ and the full-rank matrix $\mathbf{T}^{-1}$ is composed of the K eigenvectors of corresponding to the K largest eigenvalues of matrix $\boldsymbol{\psi}$. The estimations of $\hat{\mathbf{A}}_1$, $\hat{\mathbf{A}}_2$ and $\hat{\mathbf{A}}$ can be obtained:

$$\hat{\mathbf{A}}_1=\mathbf{E}_1\mathbf{T}^{-1},\ \hat{\mathbf{A}}_2=\mathbf{E}_2\mathbf{T}^{-1},\ \hat{\mathbf{A}}=\mathbf{E}_s\mathbf{T}^{-1} \tag{26}$$

From the equation (20), the estimation of direction of arrival is given as:

$$\hat{\theta}_k=\sin^{-1}\left[\frac{\lambda}{2\pi d}\arg\left(\hat{\Phi}_{kk}\right)\right] \tag{27}$$

From the formulas (17) and (18) and using the theory of quaternion, estimations of matrix $\hat{\mathbf{A}}_e$ and $\hat{\mathbf{A}}_h$ can be obtained from $\hat{\mathbf{A}}$. Their relationship is follow:

$$\hat{\mathbf{A}}_e=\hat{\mathbf{A}}_h\hat{\mathbf{\Omega}} \tag{28}$$

$$\hat{\mathbf{\Omega}}=\left(\hat{\mathbf{A}}_h^{\mathrm{H}}\hat{\mathbf{A}}_h\right)^{-1}\hat{\mathbf{A}}_h^{\mathrm{H}}\hat{\mathbf{A}}_e \tag{29}$$

where

$$\hat{\mathbf{\Omega}}=\begin{bmatrix} -\tan\hat{\gamma}_1 e^{j\hat{\eta}_1} & & \\ & \ddots & \\ & & -\tan\hat{\gamma}_K e^{j\hat{\eta}_K} \end{bmatrix} \tag{30}$$

According to equation (30), the polarization parameters estimation are presented as

$$\hat{\gamma}_k=\tan^{-1}\left(\left|\hat{\Omega}_{kk}\right|\right) \qquad \hat{\eta}_k=\arg\left(-\hat{\Omega}_{kk}\right) \tag{31}$$

V. Simulation Results

In this section, some simulations are conducted to evaluate the performances on DOA and polarization estimation by the proposed method. Three uncorrelated equal-powered signals with parameters $(\theta_1,\gamma_1,\eta_1)=(45^\circ,44^\circ,60^\circ)$, $(\theta_2,\gamma_2,\eta_2)=(30^\circ,33^\circ,45^\circ)$ and $(\theta_3,\gamma_3,\eta_3)=(10^\circ,22^\circ,30^\circ)$ impinging upon a COLD-ULA with M=5 sensors. The inter-element spacing of ULA is 0.5λ. 1024 snapshots are used in simulation experiments.

In the first experiment, we consider a scenario with 500 independent Monte Carlo trials running on the corresponding COLD ULA. We choose the first signal for this experiment and the SNR is set to 15 dB. The sets of values of the DOA and polarization variables have been represented in scatter diagrams (Fig. 3 to Fig. 6).

From Fig. 4, it is shown that almost all estimated values are located in the vicinity of actual value $\theta_1=45^\circ$

by using quaternion method, the estimated value of $\hat{\theta}_1$ range from the numerical range $(44.75^\circ, 45.25^\circ)$. On the contrary, from Fig. 3, the estimated values $\hat{\theta}_1$ using long vector method are distributed in the range of $\left(44.05^\circ, 45.95^\circ\right)$. The estimated error using long vector method is much larger than that of quaternion method.

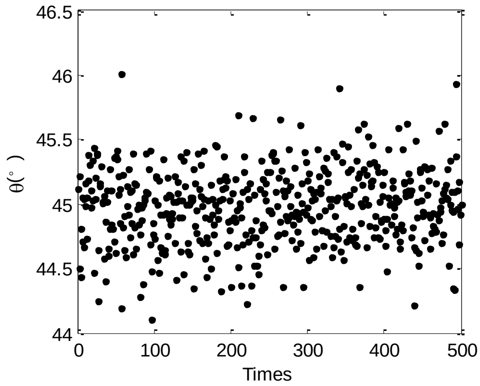

Fig. 3. DOA scatter diagram using long vector method.

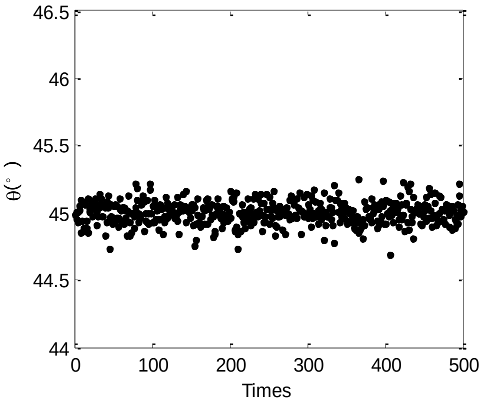

Fig. 4. DOA scatter diagram using quaternion method.

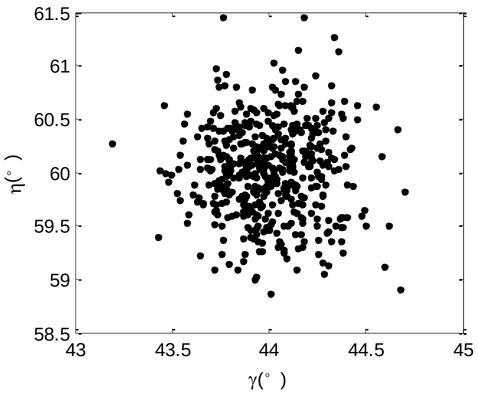

Fig. 5. Polarization scatter diagram using long vector method.

From Fig. 6, it is shown that almost all estimated values are located in the vicinity of actual value $(\gamma_1, \eta_1) = \left(44^\circ, 60^\circ\right)$ by using quaternion method, the estimated value of $\hat{\gamma}_1$ and $\hat{\eta}_1$ range from the numerical range $\left(43.6^\circ, 44.4^\circ\right)$ and $\left(59.25^\circ, 60.05^\circ\right)$, respectively. On the contrary, from Fig. 5, the estimated values are distributed in the range of $\hat{\gamma}_1$ $\left(43.2^\circ, 44.75^\circ\right)$ and $\hat{\eta}_1$ $\left(58.8^\circ, 61.5^\circ\right)$. The estimated error using long vector method is much larger than that of quaternion method.

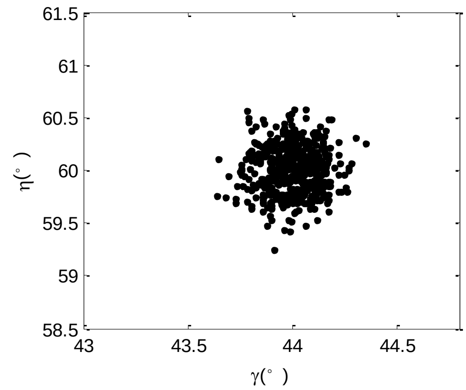

Fig. 6. Polarization scatter diagram using quaternion method.

In the second experiment, we first compare the performance of quaternion method with long vector method with respect to SNR. In the simulations, the signal-to-noise ratio (SNR) is from 0 to 45dB, 1024 snapshots are used in each of the 500 independent Monte Carlo simulation experiments. The performance of standard deviation and probability of success is illustrated, and the corresponding results are shown in Fig. 7- Fig. 9.

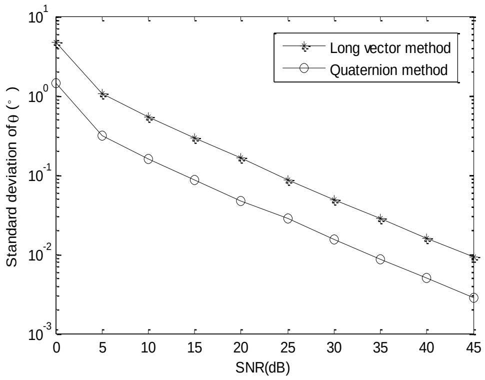

Fig. 7. Standard deviation of DOA versus SNR.

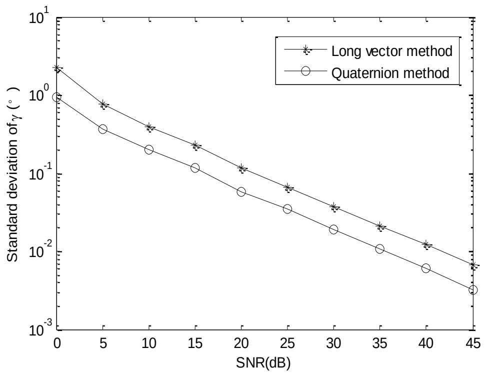

Fig. 8. Standard deviation of APA versus SNR.

The curves with star and circular data points in Fig. 7 to Fig. 9 respectively plot the standard deviation of $\hat{\theta}$, $\hat{\gamma}$

and $\hat{\eta}$, respectively estimated by long vector and the proposed quaternion method, at various signal-to-noise ratio (SNR) levels. The proposed quaternion procedure is better than long vector. The estimation precision at 0 dB based on the quaternion model has improved larger than 4.89° for $\hat{\theta}$, 1.12° for $\hat{\gamma}$, 1.15° for $\hat{\eta}$, compared with that of the long vector method. The enhanced performance is rooted in the special data model of quaternion.

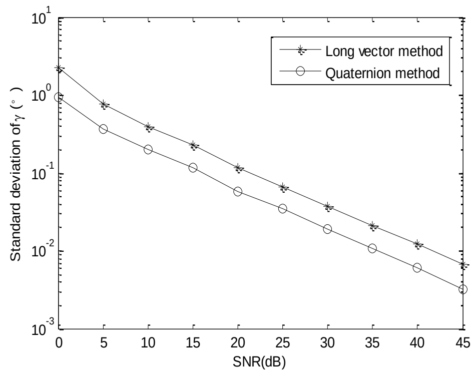

Fig. 9. Standard deviation of APA versus SNR.

Then, we consider the probability of success of DOA and polarization estimations. The result is shown in Fig. 10 and Fig. 11.

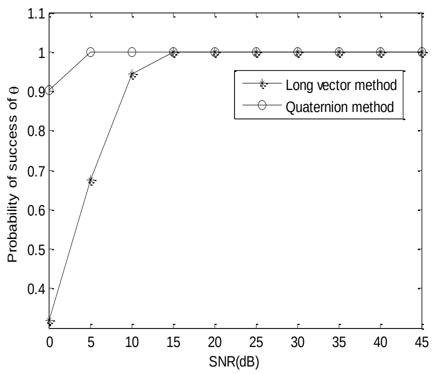

Fig. 10. Probability of success of DOA versus SNR.

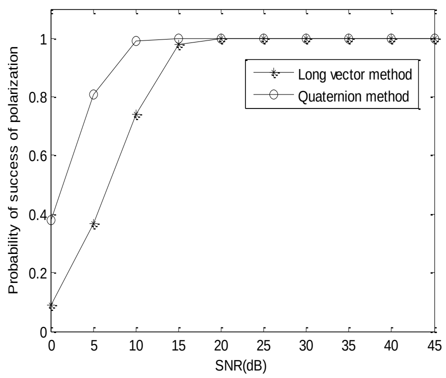

Fig. 11. Probability of success of polarization versus SNR.

The curves with star and circular data points in Fig. 10 and Fig. 11 respectively plot the probability of success of DOA and polarization, respectively estimated by long vector and the proposed quaternion method, at various signal-to-noise ratio (SNR) levels. The proposed quaternion procedure is better and more robust than long vector procedure.

VI. Conclusions

A closed form estimation of DOA and polarization, based on quaternion-ESPRIT algorithm is studied. The proposed algorithm can decouple DOA estimation from the polarization estimation. Because in this algorithm quaternion is used to retain the vector nature of each vector sensor, there exists better estimated accuracy of parameters than that of the traditional long vector data model method.

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Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this article.

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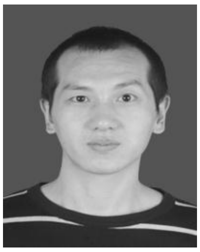

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