Multihop Analog Network Coding via Amplify-and-Forward: The High SNR Regime

Ivana Marić, *Member, IEEE*, Andrea Goldsmith, *Fellow, IEEE*, and Muriel Médard, *Fellow, IEEE*

***Abstract*—In the simplest relaying strategy, a network node amplifies and forwards a received signal over a wireless channel. Multihop amplify-and-forward allows for a (noisy) linear combination of signals simultaneously sent from multiple sources to be propagated through the network over multiple layers of relays. The performance of multihop amplify-and-forward is limited by noise propagated to the destination over multiple hops, and we expect this strategy to perform well only in high SNR. In this paper, this intuition is formalized and high-SNR conditions under which multihop amplify-and-forward approaches capacity in a layered relay network are determined. By relating the received signal power and the received power of the propagated noise at the nodes, the rate achievable with multihop amplify-and-forward is determined. In particular, when all received powers are lower bounded by $1/\delta$, the noise power propagated to the destination over L layers is of the order $L\delta$. The result demonstrates that multihop amplify-and-forward approaches the cut-set bound as received powers at relays increase. As all powers in the network increase at the same rate, the multihop amplify-and-forward rate and the upper bound are within a gap that is independent of channel gains. This gap grows linearly with the number of nodes.**

***Index Terms*—Amplify-and-forward, capacity, high SNR regime, relaying.**

I. Introduction

FOR noiseless networks on graphs, network coding achieves the *multicast* capacity, i.e., the highest rate at which a source can send information to a set of destination nodes [1]. The multicast capacity can be achieved with *linear* network coding [2]. This result implies that each node only has to send out a linear combination of its incoming packets. Destination nodes effectively obtain source information multiplied by a *transfer* matrix determined by a network graph, and can recover the original data provided that the matrix is invertible [3].

The capacity of wireless networks is still unknown. This is true for the general networks with one or more sources, as well as for simpler, canonical models such as relay channel [4]. The deterministic view of wireless networks [5] led to a new characterization of the network capacity; it has been shown that in a wireless network with a single source-destination pair, compress-and-forward [4] achieves the cut-set bound within a gap that does not depend on channel gains, and increases with the number of network nodes [6]. In [7], it was demonstrated that at high-SNR, decode-and-forward [4] exhibits a good scaling performance where the gap from the cut-set bound increases only logarithmically with the number of nodes. For multiple source networks, an extension of compress-and-forward was more recently developed in [8]. It was demonstrated that the proposed scheme outperforms existing compress-and-forward strategies, without requiring Wyner-Ziv coding [9].

In a wireless channel, signals simultaneously transmitted from multiple sources add up in the air resulting in interference; a receiver obtains a noisy sum of these signals, each scaled by a channel gain. Relays exploit this interference by forwarding it through the network to their destinations. Because relays are not interested in these messages, decoding them can unnecessarily limit the transmission rates. In fact, since the receiver already receive the sum of the signals, a natural strategy, following the idea of network coding, would be to forward the received sum after clearing the noise. A technique that exploits this idea by having relays decode linear functions of sent data, *compute-and-forward*, was recently proposed and demonstrated to perform well in certain regimes [10]. The challenge in this strategy is that the received signal sum has to correspond to an actual codeword.

A simpler strategy that alleviates this need is to amplify and forward the observed noisy signal sum. Unlike amplify-and-forward in the relay channel, in a network, the forwarded signal carries interfering signals sent by multiple sources and possibly over multiple hops, in this way extending the idea of network coding to the physical layer. As such, this scheme is a form of analog network coding [11]. The drawback of amplify-and-forward, especially when the signal is forwarded over multiple hops, is noise propagation. Consequently, at low SNRs amplify-and-forward in relay networks reduces to no relaying, i.e., it reduces to a direct transmission from the source [12], [13].

Gastpar and Vetterli showed that uncoded transmission and two-hop amplify-and-forward achieve a constant gap from the cut-set bound in the limit of a large number of relays [14, Sec.

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VIII]. In the proposed scheme, the source transmits in the first slot and the relays amplify-and-forward the observed noisy signals in the second slot. Therefore, a message reaches the destination in two hops and the noise is propagated only for one hop. This approach avoids noise propagation through the network, but reduces the rate by half. The advantages of two-hop amplify-and-forward have also been demonstrated for multiple antenna networks and fading channels (see [15] and references therein).

In this paper, we consider multihop amplify-and-forward in which data is propagated over many intermediate nodes. The diversity-multiplexing tradeoff (DMT) of multihop amplify-and-forward when relays have multiple antennas was characterized in [16]. By considering a deterministic wireless network, the diversity and degrees-of-freedom were analyzed in [17]. For a special type of networks, DMT of this scheme was also considered in [18], [19]. In this paper, we will derive the rate achievable with multihop amplify-and-forward and show that it achieves capacity in the regime in which the propagated noise is negligible.

Intuition suggests that the noise amplification drawback of multihop amplify-and-forward should diminish at high SNR. In fact, it was shown that, in the high SNR regime, multihop amplify-and-forward achieves full degrees of freedom of the MIMO system [20]. This intuition might seem to contradict results in [6, Sec. III] where it was demonstrated that amplify-and-forward can have an unbounded gap from capacity in the high channel gain regime. As the main contribution in this paper we will validate the intuition that in the high SNR regime multihop amplify-and-forward approaches network capacity. In fact, one of the key insights from our work is that high channel gains do not necessarily lead to the high SNR regime in a multihop network, unlike in a point-to-point channel. Specifically, in a multihop network, even for high channel gains, noise propagation can lead to low SNRs at the nodes. In this paper, we relate the received power and the noise power of the propagated noise at the nodes in multihop amplify-and-forward. We determine high-SNR conditions under which multihop amplify-and-forward approaches the capacity in a layered wireless relay network. We further demonstrate that at high-SNR, the multihop amplify-and-forward rate has a favorable scaling, i.e., as the received powers increase, the multihop amplify-and-forward rate is within a gap from the capacity upper bound that is independent of channel gains. We also demonstrate by an example that multihop amplify-and-forward can perform close to sum-capacity in the multicast case as well.

The remainder of this paper is organized as follows. The considered network model is presented in Section II. The main result on the multihop amplify-and-forward performance and capacity is presented in Section III. A small network illustrating these results is analyzed in Section IV. Two examples demonstrating the capacity-achieving performance of the multihop amplify-and-forward in the high SNR regime are presented in Section V. Section VI extends the analysis to a multicast problem. Section VIII concludes the paper. The proofs are given in the appendix.

II. Network Model

We consider a wireless network with a single source-destination pair and N relays. The channel output at node k is

$$y_k = \sum_{j \in \mathcal{V}(k)} h_{jk} x_j + z_k \tag{1}$$

where x_j is the channel input at node j, h_{jk} is a real number representing the channel gain from node j to node k and z_k is the Gaussian noise with zero mean and variance 1. We assume that the links in the network are directed. Thus, if node k can receive from node j with non-zero channel gain h_{jk}, this does not imply that the opposite is true, i.e, channel gain h_{kj} can be zero. Consequently, $\mathcal{V}(k)$ denotes nodes that can directly transmit to node k, i.e., node $j \in \mathcal{V}(k)$ if $h_{jk} \neq 0$. We assume that there is a power constraint at node k:

$$E[X_k^2] \leq P_k. \tag{2}$$

All nodes are full-duplex. The source S wishes to send a message from a message set $\mathcal{W} = \{1, \ldots, 2^{nR}\}$ to destination node D. The encoding function at the source is given by $X_s^n = f(W)$. An (R, n) code consists of a message set, an encoding function at the source encoder, an encoding function at each node k, that at time i performs $X_{k,i} = f_{k,i}(Y_k^{i-1})$, and a decoding function at the destination node D: $\hat{W} = g(Y_D^n)$. The average error probability of the code is given by $P_e = P\left[\hat{W} \neq W\right]$. A rate R is achievable if, for any $\epsilon > 0$, there exists, for a sufficiently large n, a code (R, n) such that $P_e \leq \epsilon$.

A. Layered Networks

As in [6], we initially consider layered networks in which each path from the source to the destination has the same number of hops. We denote layer l with $\mathcal{L}_l$. We consider the source node to be at layer $\mathcal{L}_0$ and the destination at layer $\mathcal{L}_L$. We denote number of relays at layer $\mathcal{L}_l$ as n_l, hence $\sum_{l=1}^{L-1} n_l = N$. A layered network with 4 layers between the source and the destination and 2 relays at each layer is shown in Fig. 1. In a layered network, the input-output relationship is simplified because all copies of a source input traveling on different paths arrive at the destination at layer $\mathcal{L}_L$ with an $L-1$ time delay. For that reason, from now on we drop the time index in the notation. We denote a transmitted vector at layer $\mathcal{L}_l$ as $\mathbf{x}_l = [x_{l,1} \ldots x_{l,n_l}]^T$ where we use $x_{l,i}$ to denote x_i when $i \in \mathcal{L}_l$. We accordingly define the received signal $\mathbf{y}_l$ and noise $\mathbf{z}_l$ at layer l. We let $\mathbf{H}_l$ denote the channel matrix between layers $\mathcal{L}_l$ and $\mathcal{L}_{l+1}$. An element $\mathbf{H}_l(j,i)$ is the channel gain from node i at layer $\mathcal{L}_l$ to node j at level $\mathcal{L}_{l+1}$. As observed in [20], the received vector at layer $\mathcal{L}_{l+1}$ can then be written as

$$\mathbf{y}_{l+1} = \mathbf{H}_l \mathbf{x}_l + \mathbf{z}_{l+1}. \tag{3}$$

III. Main Result

A. High SNR Regime

We are interested in the regime in which nodes transmit with high enough power so that the noise propagated by multihop

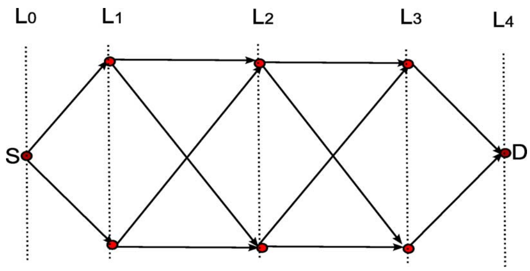

Fig. 1. Layered network with 5 layers and 2 relays at each layer.

amplify-and-forward is low. When each node j transmits with P_j given in (2), we denote the power received at node $k \in \mathcal{L}_l$ as

$$P_{\mathrm{R},k} = \left(\sum_{j \in \mathcal{L}_{l-1}} h_{jk} \sqrt{P_j} \right)^2, \qquad k \in \mathcal{L}_l. \tag{4}$$

Definition: A wireless network is in the high SNR regime if

$$\min_{k \in \mathcal{L}_l} P_{\mathrm{R},k} \geq \frac{1}{\delta}, \qquad l = 1, \ldots, L-1 \tag{5}$$

for some small $\delta \geq 0$.

Remark 1: Condition (5) implies that the received SNR at every relay k is large, i.e.,

$$P_{\mathrm{R},k} \gg 1, \qquad k \in \mathcal{L}_l, \qquad l = 1, \ldots, L-1. \tag{6}$$

From (5) and (6), we observe that the received SNR at the destination does not need to be large. In that case, the bottleneck on the data transfer is on the multi-access (MAC) side of the network. In the MAC cut, the nodes are partitioned such that destination node is in one set, and the rest of the nodes are in the other. The MAC cut-set bound [21, Theorem 14.10.1],[14, Corollary 1], evaluates to

$$C_{\mathrm{MAC}} = \frac{1}{2} \log(1 + P_D) \tag{7}$$

where, for brevity and with a slight abuse of notation, we denote the received power at the destination as P_D:

$$P_D = \left(\sum_{i \in \mathcal{L}_{L-1}} h_{iD} \sqrt{P_i} \right)^2. \tag{8}$$

We will address both cases in the paper: 1) $P_D = \mathrm{const.}$ as $\delta \to 0$ and thus the MAC at the destination is a bottleneck; 2) P_D increases as $\delta \to 0$ such that $\delta P_D = \mathrm{const}$. This is the case when, for example, all transmit powers increase at the same rate.

B. Multihop Amplify-and-Forward

In the considered transmission scheme, the source node encodes using the Gaussian codebook $X_s \sim \mathcal{N}[0, P_s]$ where $\mathcal{N}[0, \sigma^2]$ denotes normal distribution with zero mean and variance σ^2. Each network node j at layer $\mathcal{L}_l, l = 1, \ldots, L-1$ performs multihop amplify-and-forward, i.e., at time i transmits:

$$x_{l,j}(i) = \beta_j y_{l,j}(i-1), \qquad j \in \mathcal{L}_l \tag{9}$$

where the amplification gain β_j is chosen such that the power constraint (2) is satisfied. In a layered network, this corresponds to the transmit vector at layer $\mathcal{L}_l$:

$$\mathbf{x}_l = \mathbf{B}_l \mathbf{y}_l \tag{10}$$

where $\mathbf{B}_l = \mathrm{diag}\{\beta_j\}, j \in \mathcal{L}_l$. From (3) and (10), the received signal at any layer $\mathcal{L}_l$ is given by

$$\begin{aligned} \mathbf{y}_l = {} & \mathbf{H}_{l-1}\mathbf{B}_{l-1} \ldots \mathbf{H}_1 \mathbf{B}_1 \mathbf{H}_0 x_s \\ & + \sum_{i=1}^{l-1} \mathbf{H}_{l-1}\mathbf{B}_{l-1} \ldots \mathbf{H}_i \mathbf{B}_i \mathbf{z}_i + \mathbf{z}_l. \end{aligned} \tag{11}$$

Each term in the sum is the noise that propagated from layer $\mathcal{L}_i$ to layer $\mathcal{L}_l$. We choose the amplification gain at any node $j \in \mathcal{L}_l$ as

$$\beta_j^2 = \frac{P_j}{(1+\delta) P_{\mathrm{R},j}}, \qquad j \in \mathcal{L}_l. \tag{12}$$

Lemma 1: At every node performing multihop amplify-and-forward with the amplification gain (12), the power constraint (2) is satisfied.

Proof: The proof is given in the Appendix. ■

Both Lemma 1 and Theorem 1 rely on the following key lemma.

Lemma 2: At any node $j \in \mathcal{L}_l$, noise propagated from layer $\mathcal{L}_{l-d}, d = 1, \ldots, l-1$ via multihop amplify-and-forward in the high SNR regime has power

$$P_{\mathrm{Z},j}^{(l-d)} \leq \frac{\delta P_{\mathrm{R},j}}{(1+\delta)^d}. \tag{13}$$

Proof: The proof is given in the Appendix. ■

Remark 2: From (13), it follows that the total noise propagated to level $\mathcal{L}_L$ (i.e., the destination) has power

$$P_{Z,D} = \sum_{d=1}^{L-1} P_{\mathrm{Z},D}^{(l-d)} = \delta P_D \sum_{d=1}^{L-1} \frac{1}{(1+\delta)^d} \leq L \delta P_D. \tag{14}$$

The following theorem is the main result of our paper.

Theorem 1: In a layered relay network (1) in the high SNR regime (5), multihop amplify-and-forward achieves the rate

$$R = \frac{1}{2} \log\left(1 + \frac{1}{(1+\delta)^{L-1}} \frac{P_D}{P_{Z,D} + 1}\right) \tag{15}$$

where $P_{Z,D}$ is given by (14).

Proof: The proof is given in the Appendix. ■

Remark 3: For $\delta \to 0$, and $P_D = \mathrm{const.}$, (14) implies that $P_{Z,D} \to 0$; the achievable rate (15) then approaches the MAC cut-set bound (7), and thus the capacity.

Remark 4: From (15), we also obtain the scaling behavior of multihop amplify-and-forward when all the received powers increase at the same rate, i.e., $\delta \to 0$, and $\delta P_D = \mathrm{const}$. By comparing (7) and (15) in this regime, we conclude that the multihop amplify-and-forward rate is within $1/2 \log(L\ \mathrm{const})$ from the MAC cut-set bound.

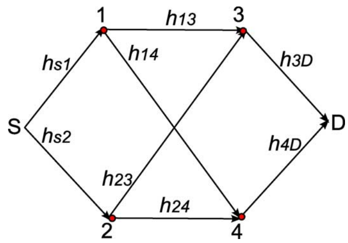

Fig. 2. Network with 4 layers. A source signal propagates over 3 layers to the destination.

Remark 5: For $P_D = \text{const.}$, as $\delta \to 0$ from (14) and (15) we also obtain the first-order approximation as

$$R = \frac{1}{2}\log(1+P_D) - \mathrm{O}\big(L\delta\big) \tag{16}$$

where we use the standard notation to denote $f(x) = \mathrm{O}\big(g(x)\big)$ as x increases, if and only if there exists a positive real number M and a real number x_0 such that

$$|f(x)| \le M|g(x)|, \qquad \text{for all } x > x_0. \tag{17}$$

We next illustrate the result given by Theorem 1 for the network with $L=3$ and $n_l = 2$ for $l = 1,2$ shown in Fig. 2. The proof for a general layered network is given in the Appendix. In Section V, we present numerical examples.

IV. Proof for Networks With $L = 3$

For the network shown in Fig. 2, received signals at nodes at level $\mathcal{L}_1$ are given by

$$\begin{aligned} y_1 &= h_{s1}x_s + z_1 \\ y_2 &= h_{s2}x_s + z_2. \end{aligned} \tag{18}$$

From (9), the power constraint at nodes $j = 1,2,3,4$ is satisfied if

$$\beta_j^2 \le \frac{P_j}{E[Y_j^2]}. \tag{19}$$

From (12), the amplification coefficients are

$$\beta_j^2 = \frac{P_j}{(1+\delta)P_{\mathrm{R},j}}, \qquad j = 1,2,3,4. \tag{20}$$

To prove that the power constraint is satisfied at nodes $j = 1,2$, we observe that

$$\begin{aligned} \beta_j^2 &= \frac{P_j}{(1+\delta)P_{\mathrm{R},j}} \\ &\le^{(a)} \frac{P_j}{h_{sj}^2 P_s + 1} \\ &= \frac{P_j}{E[Y_j^2]} \end{aligned} \tag{21}$$

where (a) follows by (18) and (5). Thus the power constraints (19) are satisfied at level $\mathcal{L}_1$.

Received signals at nodes at level $\mathcal{L}_2$ are

$$y_3 = h_{13}x_1 + h_{23}x_2 + z_3$$

$$y_4 = h_{14}x_1 + h_{24}x_2 + z_4. \tag{22}$$

Substituting (9), (18), and (20) in (22), we obtain

$$\begin{aligned} y_3 = &\frac{x_s}{\sqrt{(1+\delta)P_s}}\big(h_{13}\sqrt{P_1} + h_{23}\sqrt{P_2}\big) \\ &+ h_{13}\beta_1 z_1 + h_{23}\beta_2 z_2 + z_3. \end{aligned} \tag{23}$$

From (23), the power of propagated noise from layer $\mathcal{L}_1$ is

$$\begin{aligned} P_{Z,3}^{(1)} &= h_{13}^2\beta_1^2 + h_{23}^2\beta_2^2 \\ &=^{(a)} \frac{1}{1+\delta}\left(\frac{h_{13}^2 P_1}{P_{\mathrm{R},1}} + \frac{h_{23}^2 P_2}{P_{\mathrm{R},2}}\right) \\ &\le^{(b)} \frac{\delta}{1+\delta}\big(h_{13}^2 P_1 + h_{23}^2 P_2\big) \\ &\le^{(c)} \frac{\delta P_{\mathrm{R},3}}{(1+\delta)} \end{aligned} \tag{24}$$

where (a) follows by (12); (b) follows by (5) and (c) by evaluating the received power $P_{R,3}$ from (22). From (5), (23), and (24), it follows that

$$E[Y_3^2] \le (1+\delta)P_{\mathrm{R},3}. \tag{25}$$

To show that β_3 chosen as in (12) satisfies the power constraint (19), we observe from (20) and (25) that

$$\begin{aligned} \beta_3^2 &= \frac{P_3}{(1+\delta)P_{\mathrm{R},3}} \\ &\le \frac{P_3}{E[Y_3^2]} \end{aligned} \tag{26}$$

and hence (19) is satisfied. The same steps (23)–(25) hold for node 4.

Continuing with the next level, the signal received at the destination, from (9), (20) and (22) evaluates to

$$\begin{aligned} y_D &= h_{3D}x_3 + h_{4D}x_4 + z_D \\ &= \frac{x_s}{(1+\delta)\sqrt{P_s}}\big(h_{3D}\sqrt{P_3} + h_{4D}\sqrt{P_4}\big) \\ &\quad + (h_{13}\beta_1 h_{3D}\beta_3 + h_{14}\beta_1 h_{4D}\beta_4)z_1 \\ &\quad + (h_{23}\beta_2 h_{3D}\beta_3 + h_{24}\beta_2 h_{4D}\beta_4)z_2 \\ &\quad + h_{3D}\beta_3 z_3 + h_{4D}\beta_4 z_4 + z_D \end{aligned} \tag{27}$$

and is also given in (11). From (8) and (27), the received signal power is

$$\hat{P}_D = \frac{1}{(1+\delta)^2}P_D. \tag{28}$$

We next evaluate power of the propagated noise from (27). Following the same steps as in (24), the power of the noise propagated from level $\mathcal{L}_2$, denoted $P_{Z,D}^{(2)}$, from (27) evaluates to

$$\begin{aligned} P_{Z,D}^{(2)} &= h_{3D}^2\beta_3^2 + h_{4D}^2\beta_4^2 \\ &\le^{(a)} \frac{\delta P_D}{(1+\delta)} \end{aligned} \tag{29}$$

where (a) follows from (5), (8) and (12).

We next calculate the noise from level $\mathcal{L}_1$. We denote the coefficient in front of the noise z_j in (27) as k_{jD}, for $j = 1,2$:

$$k_{jD} = h_{j3}\beta_j h_{3D}\beta_3 + h_{j4}\beta_j h_{4D}\beta_4. \tag{30}$$

From (27) and (30), we have

$$P_{Z,D}^{(1)} = k_{1D}^2 + k_{2D}^2 \leq (\sqrt{k_{1D}} + \sqrt{k_{2D}})^2 \leq^{(a)} \frac{\delta P_D}{(1+\delta)^2} \quad (31)$$

as given by Lemma 2. Inequality (a) follows by (5) and by a straightforward calculation. The SNR at the destination is obtained from (27)–(31) as

$$SNR_D > \frac{1}{(1+\delta)^2} \frac{P_D}{\frac{\delta P_D}{(1+\delta)^2} + \frac{\delta P_D}{1+\delta} + 1} \quad (32)$$

in agreement with (15) and Theorem 1.

V. Examples

We next illustrate the above result for several networks.

A. Example 1: Diamond Network

It was observed in [6] that amplify-and-forward in a diamond network (first analyzed in [12] and shown in Fig. 3 for the choice of channel gains as in [6]) cannot achieve the cut-set bound when a is large and transmit powers are set to 1. Rather, the gap between the amplify-and-forward rate and the cut-set bound increases as a increases. However, we next show that, for any value of a, there is a range of power P_s for which amplify-and-forward achieves capacity. To show this we consider the MAC cut-set bound (7) which in a diamond network evaluates to

$$C_{\text{MAC}} = \frac{1}{2}\log(1+h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D}h_{2D}\sqrt{P_1 P_2}) \quad (33)$$

where h_{iD} is the channel gain from relay i to the destination. With amplify-and-forward, the SNR at the destination is given by

$$SNR_D = \frac{h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D}h_{2D}\sqrt{P_1 P_2}}{\frac{h_{1D}^2 P_1}{h_{s1}^2 P_s} + \frac{h_{2D}^2 P_2}{h_{s2}^2 P_s} + 1} \quad (34)$$

where, due to (5), we can approximate $\beta_i^2 = P_i/(h_{si}^2 P_s)$, $i = 1, 2$. Condition (5) implies that

$$SNR_D \geq \frac{h_{1D}^2 P_1 + h_{2D}^2 P_2 + 2h_{1D}h_{2D}\sqrt{P_1 P_2}}{\delta(h_{1D}^2 P_1 + h_{2D}^2 P_2) + 1} \quad (35)$$

and the achievable rate is in the first-order approximation,

$$R = \frac{1}{2}\log\left(1 + (h_{1D}\sqrt{P_1} + h_{2D}\sqrt{P_2})^2\right) - \mathrm{O}(\delta). \quad (36)$$

Hence, for the class of diamond networks in the high-SNR regime, the capacity can be achieved with amplify-and-forward. In terms of powers, from (33) and (34) we conclude that amplify-and-forward approaches capacity in all diamond networks that satisfy

$$\frac{h_{1D}^2 P_1}{h_{s1}^2 P_s} + \frac{h_{2D}^2 P_2}{h_{s2}^2 P_s} \ll 1. \quad (37)$$

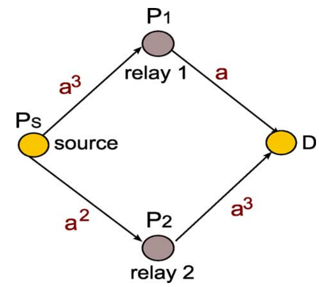

Fig. 3. Diamond network.

For the specific choice of channel gains as in Fig. 3 and for relay powers $P_1 = P_2 = 1$, the MAC bound (33) is

$$C_{\text{MAC}} = \frac{1}{2}\log(1 + a^6 + 2a^4 + a^2) \quad (38)$$

which for large a approximates to

$$C_{\text{MAC}} = 3\log(a). \quad (39)$$

The SNR (34) becomes

$$SNR_D = \frac{a^6 + 2a^4 + a^2}{a^{-4}/P_s + a^2/P_s + 1}. \quad (40)$$

For $P_s \geq a^2$ and large a

$$SNR_D \geq \frac{a^6 + 2a^4 + a^2}{2 + a^{-6}} \to \frac{a^6 + 2a^4 + a^2}{2}. \quad (41)$$

From (34), for large a, the amplify-and-forward rate evaluates to

$$R = 3\log(a) - 1. \quad (42)$$

Furthermore, as P_s increases, the SNR (40) approaches the SNR in the cut-set bound (38).

The comparison of the amplify-and-forward rate with the MAC cut-set bound is shown in Fig. 4. We observe that amplify-and-forward achieves the capacity within a bit for $P_s \geq 35$. For $P_s = 1$, we recover the example from [6], and indeed observe a gap from the capacity.

We also examine the scaling of the achievable rate and the cut-set bound when all the transmit powers in the network increase, while their ratio is kept constant. Equivalently, we can choose $\delta P_D = \text{const}$. The channel gains are fixed as given by the network topology. From (33) and (34), the difference between the two bounds for any value of gains, and for large powers evaluates to a constant

$$C_{MAC} - R = \frac{1}{2}\log c \quad (43)$$

where c denotes the denominator in (34). This behavior is shown in Fig. 5 when all transmit powers are chosen equal (denoted P). All channel gains are constant and equal to 1. The difference between the two bounds (33) and (34) for this choice of parameters evaluates to $0.5\log 3$, for large P.

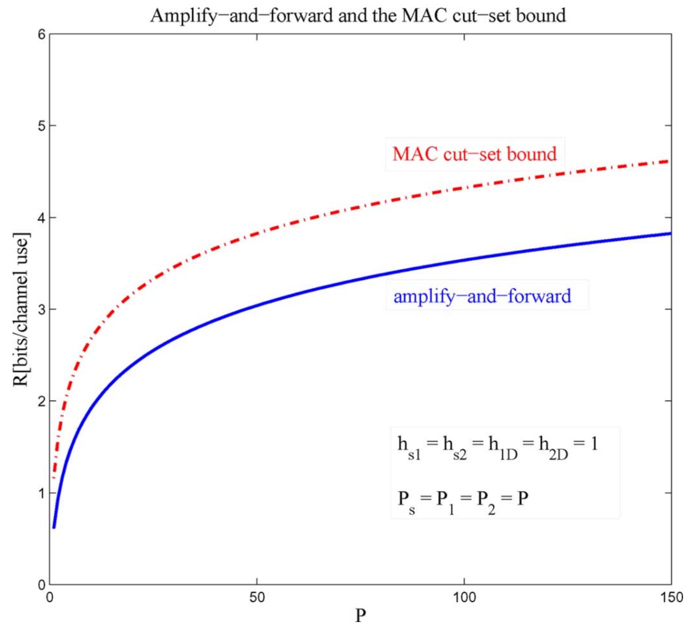

Fig. 5. Amplify-and-forward rate and the MAC cut-set bound in a diamond network as transmit powers increase. We observe the same trend in the two bounds. Amplify-and-forward is within a constant gap from the cut-set bound.

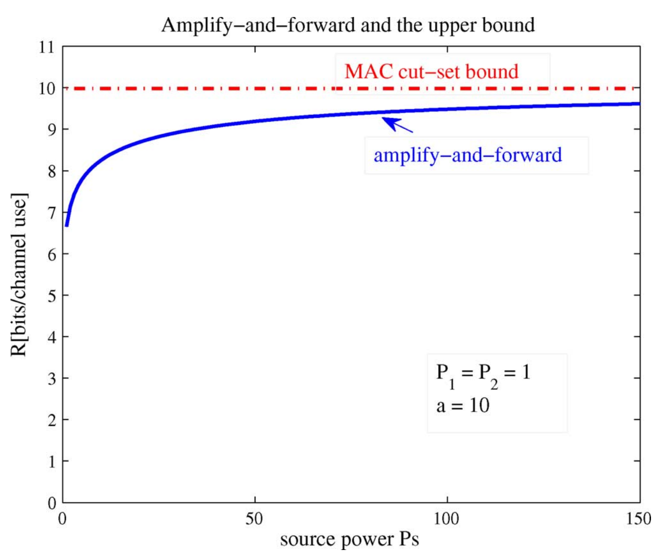

Fig. 4. Amplify-and-forward rate and the MAC cut-set bound in a diamond network for $a = 10$. We observe that the achievable rate approaches the cut-set bound as the source power increases.

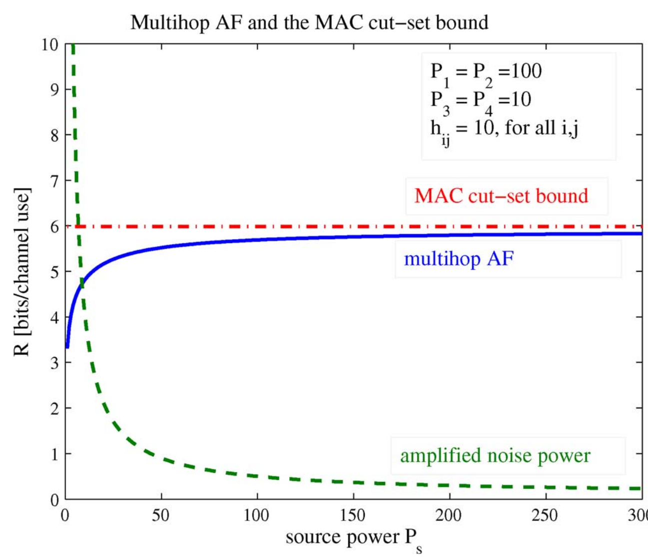

Fig. 6. Multihop amplify-and-forward rate and the MAC cut-set bound in a network with $L = 3$.

B. Example 2: Network With $L = 3$

We next present the performance of multihop amplify-and-forward in a network with 4 layers shown in Fig. 2 and analyzed in Section IV. In (32), we evaluated the SNR at the destination as

$$SNR_D > \frac{1}{(1+\delta)^2} \frac{P_D}{\frac{\delta P_D}{(1+\delta)^2} + \frac{\delta P_D}{1+\delta} + 1}. \quad (44)$$

We observe the following:

1) For P_D constant, SNR_D approaches P_D as $\delta \rightarrow 0$, and multihop amplify-and-forward achieves the MAC cut-set bound (7). This behavior is shown in Fig. 6. We observe that the multihop amplify-and-forward rate approaches the capacity to within one bit as $P_s > 30$, and is within a small fraction of a bit for $P_s > 200$.
2) Fig. 7 shows the achievable rate and the cut-set bound when all the powers in the network increase, and the channel gains are fixed. In this case, $\delta \rightarrow 0$ and $\delta P_D = \text{const}$. As in the previous example, we observe a constant gap between the rate and the cut-set bound.

VI. Multicast

We next illustrate by an example that multihop amplify-and-forward is an efficient transmission scheme also for multicast traffic, when the network is in the high SNR regime.

We consider a 3-layer network (see Fig. 8) with two sources (nodes 1 and 2) and two destinations (nodes 5 and 6). Each

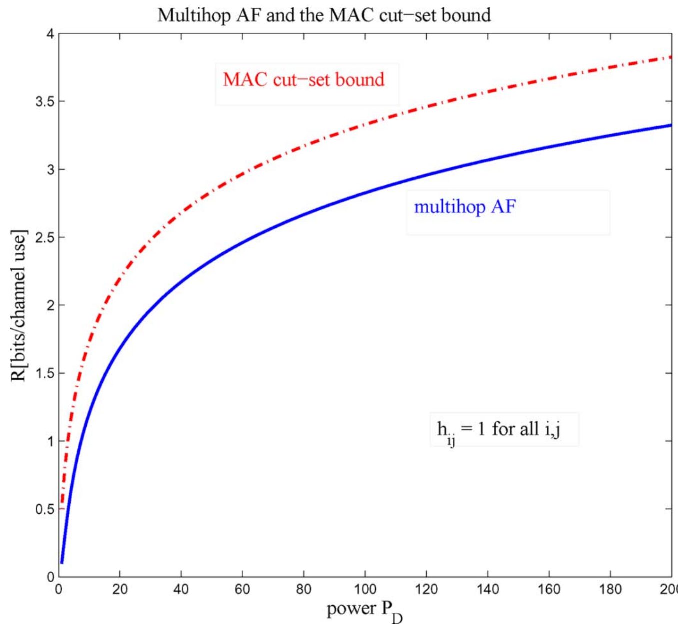

Fig. 7. Multihop amplify-and-forward and the MAC cut-set bound for the network with $L = 3$ as all the powers increase.

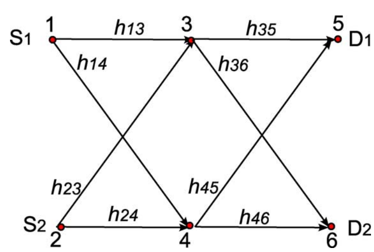

Fig. 8. A two-source network. Sources 1 and 2 wish to multicast independent data to nodes 5 and 6.

source wishes to multicast a message to both destinations. Respective channel inputs at sources 1 and 2 are x_1 and x_2. The received signals at nodes 3 and 4 are

$$y_k = h_{1k}x_1 + h_{2k}x_2 + z_k, \qquad k = 3, 4. \tag{45}$$

As before, nodes perform multihop amplify-and-forward, as given by (9). Note that x_1 and x_2 are independent and hence there is no coherent combining at the receivers. Therefore, amplification gains at nodes 3 and 4 in the high-SNR regime can be approximated as:

$$\beta_k^2 \leq \frac{P_k}{h_{1k}^2 P_1 + h_{2k}^2 P_2}, \qquad k = 3, 4. \tag{46}$$

The received signal at node 5 is:

$$\begin{aligned} y_5 &= h_{35}x_3 + h_{45}x_4 + z_5 \\ &= (h_{35}h_{13}\beta_3 + h_{45}h_{14}\beta_4)x_1 + (h_{35}h_{23}\beta_3 + h_{45}h_{24}\beta_4)x_2 \\ &\quad + h_{35}\beta_3 z_3 + h_{45}\beta_4 z_4 + z_5 \\ &= h_{1,eq}x_1 + h_{2,eq}x_2 + h_{35}\beta_3 z_3 + h_{45}\beta_4 z_4 + z_5 \end{aligned} \tag{47}$$

where we denote

$$h_{j,eq} = h_{35}h_{j3}\beta_3 + h_{45}h_{j4}\beta_4, \qquad j = 1, 2. \tag{48}$$

Equivalent relationship can be obtained at node 6. Equation (48) describes a multiaccess (MAC) channel. The MAC capacity [21] determines the rates achievable at node 5 as

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log(1 + \frac{h_{1,eq}^2 P_1}{1 + P_{Z,eq}}) \\ R_2 &\leq \frac{1}{2}\log(1 + \frac{h_{2,eq}^2 P_2}{1 + P_{Z,eq}}) \\ R_1 + R_2 &\leq \frac{1}{2}\log(1 + \frac{h_{1,eq}^2 P_1 + h_{2,eq}^2 P_2}{1 + P_{Z,eq}}) \end{aligned} \tag{49}$$

where $P_{Z,eq}$ is the power of amplified noise in (47) given by

$$P_{Z,eq} = \frac{h_{35}^2 P_3}{h_{13}^2 P_1 + h_{23}^2 P_2} + \frac{h_{45}^2 P_4}{h_{14}^2 P_1 + h_{24}^2 P_2}. \tag{50}$$

In the high SNR regime, $P_{Z,eq} \rightarrow 0$ and hence the total noise power (and the denominators in (49)) is identity. Therefore, by substituting $\beta_k, k = 3, 4$ and (48) in (49), and by using (50), we obtain that the achievable sum-rate satisfies

$$R_1 + R_2 > 1/2\log(1 + h_{35}^2 P_3 + h_{45}^2 P_4). \tag{51}$$

We next evaluate the MAC cut-set bound at node 5 as

$$\begin{aligned} C_{MAC} &= I(X_3, X_4; Y_5) \\ &= 1/2\log(1 + (h_{35}\sqrt{P_3} + h_{45}\sqrt{P_4})^2). \end{aligned} \tag{52}$$

Following the same steps, we can evaluate the achievable rate and the MAC cut-set bound at node 6. By comparing the sum-rate lower bound (51) and the MAC cut-set bound (52), we observe that the gap is in the coherent combining gain, and hence at most 1/2 bit. Therefore, when the considered network is in the high SNR regime, the sum-rate achievable with multihop amplify-and-forward and the cut-set bound differ due to the coherent combining gain gap by at most 1/2 bit.

VII. Extensions

A. Non-Layered Networks

In non-layered networks, copies of the source signal arrive to the destination with different delays over different routes. The channel is equivalent to the intersymbol interference channel. The channel output at time i is given by

$$y_D(i) = h_0 x_s(i) + \sum_{j=1}^{K_1} q_{j,1} x_s(i-1) + \cdots + \sum_{j=1}^{K_L} q_{j,L} x_s(i-L) + z_e(i) \tag{53}$$

where h_0 is the channel gain on the direct link, K_l is the number of routes of length l, and L is the length of the longest route. Equivalent channel gains $q_{j,i}$ depend on the network topology; each $q_{j,i}$ contains accumulated channel and amplification gains on a source-destination route. $z_e(i)$ denotes the total noise at the destination at time i. The achievable rate with amplify-and-forward can thus be obtained as the capacity of the corresponding channel with intersymbol interference [22], [23].

B. Multiple Antennas

DMT analysis revealed that amplify-and-forward can achieve full degrees of freedom when relays have multiple antennas [20]. It would be interesting to extend the analysis presented in this paper to evaluate achievable rates in multiple input-multiple-output systems where some or all nodes (source, destination and/or relays) have multiple antennas.

C. Undirected Networks

The network model (1) considered in this paper assumes directed links between nodes. This model does not apply to wireless networks in general, e.g., when the nodes have omnidirectional antennas. This directed model is appropriate in some wireless networks such as networks in which sectorized or directional antennas are deployed at the nodes. In networks where nodes have omnidirectional antennas, any two neighboring nodes overhear each other's transmission. The amplify-and-forward scheme then leads to the creation of loops: a relay receives a copy of its own transmitted signal echoed from neighboring relays, along with the noise accumulated along the route. Loops will differ depending on whether relays are full-duplex or half-duplex. The received signal at the destination consists of delayed amplified copies of the source signal propagated over many routes containing such loops. This will result in infinite propagation of signals carrying source messages and accumulation of amplified noise. Loops could be avoided by scheduling source and relay transmissions in a time-sharing fashion according to a desired schedule. Depending on the interference that nodes can cause to each other, a schedule will specify a subset of nodes that transmit at each time instant. An input-output relationship and high-SNR performance could then be evaluated by extending the analysis presented in this paper. In general, we expect that the avoiding loops will reduce achievable rates of multihop amplify-and-forward. Another possibility instead of avoiding loops, may be to allow the loops to occur in the network. Their effect on the transmitted signal will be equivalent to that of the channel with intersymbol interference. It would then be interesting to evaluate the impact of noise propagation and achievable rates in this case, both for full-duplex and half-duplex relays.

VIII. Conclusion

We characterized the behavior of multihop amplify-and-forward in the high SNR regime. In particular, we related the received signal power and the power of the propagated noise at the nodes, to determine the rate achievable with multihop amplify-and-forward. When all received powers are lower bounded by $1/\delta$, the noise power propagated to the destination over L layers is of the order $L\delta$. The result demonstrates that multihop amplify-and-forward approaches the MAC cut-set bound as the received powers at relays increase. As all powers in the network increase, the multihop amplify-and-forward rate and the upper bound are within a gap that is independent of channel gains. The gap depends on number of nodes. Similar behavior was observed for decode-and-forward in a large network [7], and compress-and-forward [6]. As discussed in the previous section, this result assumes directed links between nodes and hence does not consider creation of loops due to multihop amplify-and-forward when nodes are full-duplex. Relaxing this assumption is a topic of our future work. In the high SNR regime, multihop amplify-and-forward seems as a natural choice of the coding strategy for both unicast and multicast traffic, as it allows data that is already mixed in the wireless channel to be jointly forwarded in a simple manner. Furthermore, multihop amplify-and-forward does not require any decoding, which reduces the rate both in decode-and-forward and compute-and-forward schemes; it does not induce a block delay (which is present in the case of decoding); and finally, as demonstrated, the penalty of amplifying noise is small in the high SNR regime characterized by large received powers at the nodes.

Appendix

1) Proof of Lemma 1: From (9), the power constraint at at any node $j \in \mathcal{L}_l$ is satisfied if

$$\beta_j^2 \le \frac{P_j}{E[Y_{l,j}^2]}. \tag{54}$$

We first evaluate the signal power at node $j \in \mathcal{L}_l$. From (11), the received signal at node $j \in \mathcal{L}_l$ can be written as

$$y_{l,j} = \mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\ldots\mathbf{H}_1\mathbf{B}_1\mathbf{H}_0 x_s + \sum_{i=1}^{l-1} \mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\ldots\mathbf{H}_i\mathbf{B}_i\mathbf{z}_i + z_D \tag{55}$$

where $\mathbf{h}_{l-1,j}$ is the j-th row-vector in $\mathbf{H}_l$. We denote the coefficient that multiplies the signal x_s in (55) as

$$\hat{h}_{l,j} = \mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\ldots\mathbf{H}_1\mathbf{B}_1\mathbf{H}_0. \tag{56}$$

Since every $\mathbf{B}_i$ is diagonal, a straight multiplication shows that, for every l

$$\mathbf{B}_{l-1}\dots\mathbf{H}_1\mathbf{B}_1\mathbf{H}_0 = \frac{1}{\sqrt{P_s(1+\delta)^{l-1}}}\mathbf{P}_l[\sqrt{P_1}\dots\sqrt{P_{n_l}}]^T \tag{57}$$

where $\mathbf{P}_l \in R^{n_l\times 1}$ is the vector of transmit powers from all nodes at layer $\mathcal{L}_l$. Note that $\mathbf{H}_0$ is a vector, $\mathbf{H}_0 \in R^{n_1\times 1}$, and since $\mathbf{B}_1$ is diagonal, also $(\mathbf{B}_1\mathbf{H}) \in R^{n_1\times 1}$. We further have that $\mathbf{H}_1 \in R^{n_2\times n_1}$. Therefore, $(\mathbf{B}_2\mathbf{H}_1\mathbf{B}_1\mathbf{H}_0) \in R^{n_2\times 1}$. Continuing in the same fashion, we observe that $(\mathbf{B}_{l-1}\dots\mathbf{H}_1\mathbf{B}_1) \in R^{n_l\times 1}$, and hence the left-hand side and right-hand side of (57) agree.

Since $\mathbf{h}_{l-1,j}$ is a row-vector, multiplying it with the vector given by (57), we obtain $\hat{h}_{l,j}$ as

$$\hat{h}_{l,j} = \frac{\sqrt{P_{\mathrm{R},j}}}{\sqrt{P_s(1+\delta)^{l-1}}}. \tag{58}$$

From (55), (58), and Lemma 2, we obtain

$$\begin{aligned} E[Y_{l,j}^2] &\le \frac{P_{\mathrm{R},j}}{(1+\delta)^{l-1}} + \frac{\delta P_{\mathrm{R},j}}{(1+\delta)^{l-1}}\sum_{i=0}^{l-2}(1+\delta)^i + 1 \\ &\le P_{\mathrm{R},j} + 1 \\ &\le^{(a)} (1+\delta)P_{\mathrm{R},j} \end{aligned} \tag{59}$$

where (a) follows by (5). From (12) and (59), we have that

$$\begin{aligned} \beta_j^2 &= \frac{P_j}{(1+\delta)P_{\mathrm{R},j}} \\ &\le \frac{P_j}{E[Y_{l,j}^2]} \end{aligned} \tag{60}$$

and hence the power constraint (54), and thus (2), is satisfied. □

2) Proof of Lemma 2: We prove the Lemma by induction. We consider the power of the noise propagated to any node j at any level $\mathcal{L}_l$. Consider first the power of noise that propagated to node $j \in \mathcal{L}_l$ from layer $\mathcal{L}_{l-1}$, i.e., $d=1$. From (55)

$$\begin{aligned} P_{Z,j}^{(l-1)} &= E[(\mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\mathbf{z}_{l-1})(\mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\mathbf{z}_{l-1})^T] \\ &=^{(a)} (\mathbf{h}_{l-1,j}\mathbf{B}_{l-1})(\mathbf{h}_{l-1,j}\mathbf{B}_{l-1})^T \\ &=^{(b)} \sum_{i\in\mathcal{L}_{l-1}}(h_{i,j}\beta_i)^2 \\ &\le \left(\sum_{i\in\mathcal{L}_{l-1}} h_{i,j}\beta_i\right)^2 \\ &\le^{(c)} \frac{\delta}{1+\delta}\left(\sum_{i\in\mathcal{L}_{l-1}} h_{i,j}\sqrt{P_i}\right)^2 \\ &\le^{(d)} \frac{\delta P_{\mathrm{R},j}}{1+\delta} \end{aligned} \tag{61}$$

where (a) follows since noise is uncorrelated; (b) by straight multiplication and since $\mathbf{B}_{l-1}$ is diagonal; (c) follows by (12) and (5); (d) follows by (4).

Consider next the power of noise that propagated from layer $\mathcal{L}_{l-2}$ to node $j \in \mathcal{L}_l$, i.e., $d=2$. Similarly to the previous layer, from (55)

$$\begin{aligned} P_{Z,j}^{(l-2)} &= (\mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\mathbf{H}_{l-2}\mathbf{B}_{l-2})(\mathbf{h}_{l-1,j}\mathbf{B}_{l-1}\mathbf{H}_{l-2}\mathbf{B}_{l-2})^T \\ &=^{(a)} \sum_{k\in\mathcal{L}_{l-2}}\left(\sum_{i\in\mathcal{L}_{l-1}} h_{k,i}\beta_k h_{i,j}\beta_i\right)^2 \\ &\le \left(\sum_{k\in\mathcal{L}_{l-2}}\sum_{i\in\mathcal{L}_{l-1}} h_{k,i}\beta_k h_{i,j}\beta_i\right)^2 \\ &\le^{(b)} \frac{\delta P_{\mathrm{R},j}}{(1+\delta)^2} \end{aligned} \tag{62}$$

where (a) follows by multiplication and since $\mathbf{B}_{l-2}$ is diagonal and (b) by (12), (5), and (4).

To prove that the lemma holds for any d, we assume that the Lemma is true for $d-1$. Then

$$P_{Z,j}^{(l-(d-1))} \le \frac{\delta P_{\mathrm{R},j}}{(1+\delta)^{(d-1)}}. \tag{63}$$

The key observation is that the power of the noise propagated from layer $\mathcal{L}_{l-d}$ (i.e., for d layers) to node $j \in \mathcal{L}_l$ can be written as

$$P_{Z,j}^{(l-d)} = \sum_{i\in\mathcal{L}_{l-1}} (h_{ij}\beta_i)^2\, P_{Z,i}^{(l-(d-1))} \tag{64}$$

since this noise propagated for $d-1$ layers to the previous layer. Consequently, we have

$$\begin{aligned} P_{Z,j}^{(l-d)} &= \sum_{i\in\mathcal{L}_{l-1}} (h_{ij}\beta_i)^2\, P_{Z,i}^{(l-(d-1))} \\ &\le \left(\sum_{i\in\mathcal{L}_{l-1}} h_{ij}\beta_i\sqrt{P_{Z,i}^{(l-(d-1))}}\right)^2 \\ &\le^{(a)} \left(\sum_{i\in\mathcal{L}_{l-1}} h_{ij}\sqrt{\frac{P_i}{(1+\delta)P_{\mathrm{R},i}}\frac{\delta P_{\mathrm{R},i}}{(1+\delta)^{(d-1)}}}\right)^2 \\ &= \frac{\delta}{(1+\delta)^d}\left(\sum_{i\in\mathcal{L}_{l-1}} h_{ij}\sqrt{P_i}\right)^2 \\ &=^{(b)} \frac{\delta P_{R,j}}{(1+\delta)^d} \end{aligned} \tag{65}$$

where (a) follows by substituting (12) and (63); and (b) by (4). □

3) Proof of Theorem 1: We denote the signal at the destination node as $y_L = y_D$ and from (11) obtain

$$y_D = \hat{h}_D x_s + \hat{z}_D \tag{66}$$

where

$$\hat{h}_L = \mathbf{H}_{L-1}\mathbf{B}_{L-1}\dots\mathbf{H}_1\mathbf{B}_1\mathbf{H}_0 \tag{67}$$

and $\hat{z}_D$ denotes the total received noise.

As in lemma 1, we can determine $\hat{h}_L$ by using (57) and the fact that $\mathbf{H}_{L-1}$ is a row-vector. From (58) $\hat{h}_L$ evaluates to

$$\hat{h}_L = \frac{\sqrt{P_D}}{\sqrt{P_s(1+\delta)^{L-1}}}. \tag{68}$$

From (66) and (68), the received signal power $\hat{P}_D$ at the destination equals

$$\hat{P}_D = \frac{P_D}{(1+\delta)^{L-1}} \tag{69}$$

where P_D is given by (8).

The total propagated noise is from (14) bounded as

$$P_{Z,D} \leq L\delta P_D \tag{70}$$

and therefore the SNR at the destination evaluates to

$$SNR_D \geq \frac{1}{(1+\delta)^{L-1}} \frac{P_D}{L\delta P_D + 1} \tag{71}$$

as given by (15). □

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