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Multi-agent gradient climbing via extremum seeking control ⋆

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Abstract: A unified framework based on discrete-time gradient-based extremum seeking control is proposed to localise an extremum of an unknown scalar field distribution using a group of equipped with sensors. The controller utilises estimates of gradients of the field from local dithering sensor measurements collected by the mobile agents. It is assumed that distributed coordination which ensures uniform asymptotic stability with respect to a prescribed formation of the agents is employed. The framework is useful in that a broad range of nonlinear programming algorithms can be combined with a wide class of cooperative control laws to perform extreme source seeking. Semi-global practical asymptotically stable convergence to local extrema is established in the presence of bounded field sampling noise.

Keywords: Extremum seeking, gradient climbing, multi-agent systems, cooperative control

1. INTRODUCTION

The problem of localising the source of an unknown or uncertain scalar field environment may arise from the need to identify a source flow (e.g. chemical pollution) in the ocean with a fleet Autonomous Underwater Vehicles (AUVs) (Fiorelli et al., 2006; Leonard et al., 2010), for example. The signal or field distribution can be the concentration of a chemical or electromagnetic entity with its strength decaying away from the source. As such, an extreme source seeking which relies on gradient information is also called gradient climbing/descending in the literature (Ögren et al., 2004). It is common that knowledge about the scalar signal field to be optimised is not readily available, suggesting that extremum seeking (Ariyur and Krstić, 2003) is a suitable tool for tackling the problem.

In Zhang et al. (2007), the case of a single autonomous vehicle without position measurements is investigated. An extremum-seeking input with periodic dithers based on the continuous-time ideas of Ariyur and Krstić (2003) is proposed to estimate the gradient of the potential field on the fly and steer the vehicle to the source with this information. Extensions to a non-holonomic vehicle are considered in Cochran and Krstic (2009), and in Liu and Krstić (2010) using stochastic methods. The deployment of multiple agents is examined in Ghods and Krstić (2012) with heat-diffusion coordination rules, which results in a formation that has higher density near the source.

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There are potential advantages to using multiple agents for extremum seeking instead of a single one such as robustness to vehicle failure, scalability, increased reliability and search speed etc. In Ögren et al. (2004), a network of sensor-enabled agents is employed to seek out local extrema in a distributed environment. Collectively the mobile sensors form an intelligently interacting sensor array and they are coordinated using virtual bodies and artificial potentials. Continuous-time gradient descent updates are applied to selected virtual leaders for the network to cooperatively perform the gradient climbing task. In Biyik and Arcak (2008), a control algorithm is developed for a vehicle to lead a group of others in a prescribed formation to the source via a passivity-based distributed coordination framework (Arcak, 2007) and discrete-time Newton method. There, the sensor-enabled leader is driven by a reference velocity returned by the extremum seeker while the sensor-disabled followers reconstruct this information adaptively (Bai et al., 2011).

This paper considers a fleet of autonomous point-mass vehicles endowed with uniformly asymptotically stabilising cooperative control laws. It is assumed that only samples of measurements of the field are available as in Mayhew et al. (2008), i.e. a vehicle cannot measure a continuum of the signal field. Such an assumption is justified, for example, when data collection is costly and/or time-consuming. Extremum seeking is performed in the discrete time within the general frameworks of Khong et al. (2013). Using gradient-based extremum seeking, local dither measurements are taken by a group of mobile sensors and then used to estimate the gradients of the objective field distribution. This information is utilised by the discrete-time extremum seeking controller to determine the subsequent fleet manoeuvre. The fleet formation is designed in such a way that the measurements required to approximate the gradients can be taken simultaneously, i.e. the dithering motions of the leader in Biyik and Arcak (2008) are avoided altogether. A broad class of optimisation algorithms fit within the framework, allowing the user to base the selection on the complexity of implementation, speed of convergence, robustness, etc. at the control design stage. Furthermore, the fleet formation can be maintained by applying various consensus algorithms known in the literature (Ren and Beard, 2008; Bai et al., 2011). Semi-global practical asymptotic stability of local extrema is established in the presence of norm-bounded noise to the sampled measurements on the objective field distribution. The main contribution of the paper is that of identifying generic conditions about an optimisation algorithm and a cooperative control law such that combining the two yields a convergent extremum seeking scheme which manoeuvres a group of agents towards the extreme source.

The paper has the following structure. First, a unifying discrete-time gradient-based extremum seeking control framework from Khong et al. (2013) is reviewed in the forthcoming section. In Section 3. it is shown how the framework can be adapted for extreme source seeking with a single autonomous vehicle. Source seeking with multiple sensor-enabled agents is considered in Section 4. Simulation examples are provided in Section 5 and conclusions in Section 6.

2. DISCRETE-TIME EXTREMUM SEEKING

The real and natural numbers are denoted $\mathbb{R}$ and $\mathbb{N}$ respectively. Given a vector x in $\mathbb{R}^n$, its components are denoted by x^i for $i = 1, 2, \ldots, n$. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class-$\mathcal{K}$ (denoted $\gamma \in \mathcal{K}$) if it is continuous, strictly increasing, and $\gamma(0) = 0$. If γ is also unbounded, then $\gamma \in \mathcal{K}_\infty$. A continuous function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class-$\mathcal{KL}$ if for each fixed t, $\beta(\cdot, t) \in \mathcal{K}$ and for each fixed s, $\beta(s, \cdot)$ is decreasing to zero (Khalil, 2002). The Euclidean norm is denoted $\|\cdot\|_2$. Let $\mathcal{X}$ be a Banach space with norm $\|\cdot\|$. Given any subset $\mathcal{Y}$ of $\mathcal{X}$ and a point $x \in \mathcal{X}$, define the distance of x from $\mathcal{Y}$ as $\|x\|_\mathcal{Y} := \inf_{a \in \mathcal{Y}} \|x - a\|$.

Consider the optimisation problem:

$$y^* := \max_{x \in \Omega} Q(x), \tag{1}$$

where $Q : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is an unknown, stationary, and Lipschitz continuous function or scalar field distribution which takes its maximum value on $\mathcal{C} \subset \Omega$, i.e. $Q(x) = y^*$ for all $x \in \mathcal{C}$. It is assumed that Q can only be sampled discretely in its domain Ω. Let Σ be a discrete-time extremum seeking algorithm for (1).

In the presence of bounded additive perturbations on the measurements as illustrated in Figure 1, i.e. $y_k = Q(x_k) + w_k$ with $|w_k| \leq \nu$ for $k = 0, 1, \ldots$ and some $\nu > 0$, the following assumption is important to establish convergence of the extremum seeking schemes in subsequent sections.

Assumption 1. The extremum seeking controller Σ satisfies the following conditions:

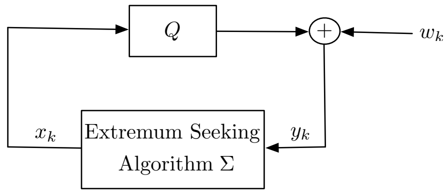

Fig. 1. Extremum seeking algorithm with noisy output measurement.

(i) Σ is time-invariant. Denote by $\{\hat{x}_k\}_{k=0}^\infty \subset \Omega$ the output sequence Σ generates based on input to Σ, $\{\hat{y}_k\}_{k=0}^\infty$, where $\hat{y}_k := Q(\hat{x}_k)$. Σ is causal in the sense that the output at any time $N \in \mathbb{N}$, i.e. $\hat{x}_N$, is determined based only on $\hat{x}_k$ and $\hat{y}_k$ for $k = 0, 1, \ldots, N-1$, that is the past probe values to Q and the corresponding measurements.

(ii) Denote by $\mathcal{S}(\hat{x}_0)$ the set of all admissible output sequences of Σ with respect to the initial point $\hat{x}_0$. There exists a class-$\mathcal{KL}$ function β such that for any initial point $\hat{x}_0 \in \Omega$, all outputs $\hat{x} \in \mathcal{S}(\hat{x}_0)$ satisfy for some $\delta \geq 0$,

$$\|\hat{x}_k\|_\mathcal{C} \leq \beta(\|\hat{x}_0\|_\mathcal{C}, k) + \delta \quad \forall k \geq 0. \tag{2}$$

(iii) Let $y_k := Q(x_k) + w_k$, where $w_k \in \mathbb{R}$. Denote by $\{x_k\}_{k=0}^\infty$ the output sequence Σ generates based on input $\{y_k\}_{k=0}^\infty$. The pair (x, y) is multi-step consistent/close (Nešić et al., 1999) with $(\hat{x}, \hat{y})$, in the sense that for any positive (Δ, η) and $N \in \mathbb{N}$, there exists a $\nu > 0$ such that if $\|x_0\|_\mathcal{C} \leq \Delta$ and $|w_k| \leq \nu$ for $k = 0, \ldots, N$, then there exists a $\hat{x} \in \mathcal{S}(x_0)$ satisfying

$$\|x_k - \hat{x}_k\|_2 \leq \eta \quad \text{for } k = 0, 1, \ldots, N.$$

Assumption 1 covers a wide class of optimisation algorithms. The next subsection provides some examples.

2.1 Examples of extremum seeking algorithms

It is often the case that gradient-based extremum seeking algorithms can be serially decomposed into a derivative estimator and a nonlinear programming method as shown in Figure 2. This paradigm is analogous to its *continuous-time* counterpart in Nešić et al. (2010), where the singular perturbation technique and time-scale separation are used to establish convergence of the extremum seeking scheme therein.

Let the initial output of the extremum seeking controller be x_0. As determined by the derivative estimator, the following length-p sequence of points can be used to probe the field distribution Q along the directions given by the basis vectors $z_1, \ldots, z_m$:

$$(x_0 + d_1(x_0), \ldots, x_0 + d_p(x_0)), \tag{3}$$

where $d_i : \Omega \to \mathbb{R}^n$ denote the dither signals. The corresponding outputs of Q are then sampled and collected by the derivative estimator to numerically approximate the first N-order partial derivatives of Q at x_0 using the Euler methods, trapezoidal rule, or the more sophisticated Runge-Kutta method, as needed by the optimisation algorithm. Exploiting this information, the optimisation algorithm can then update its output to x_1, and the series of steps described above repeats. The well-known gradient descent and Newton methods operating with approximated derivatives are two examples that satisfy Assumption 1 (Khong et al., 2013) for twice continuously differentiable Q:

$$x_{k+1} = x_k - \lambda_k \nabla Q(x_k);$$
$$x_{k+1} = x_k - \nabla^2 Q(x_k)^{-1} \nabla Q(x_k),$$

where λ_i denotes the step size and $\nabla Q(\cdot)$ and $\nabla^2 Q(\cdot)$ denote, respectively, the Jacobian and Hessian of Q. For instance, using the simplest first-order gradient approximation method, the Euler's finite-difference, the gradient of a field distribution $Q : \mathbb{R}^2 \to \mathbb{R}$ at $x \in \mathbb{R}^2$ can be approximated with three samples x, $x + hz^1$, and $x + hz^2$, where z^1 and z^2 denote the canonical basis vectors for $\mathbb{R}^2$ and h is a small step size. In particular, the approximation is given by

$$\nabla Q(x) \approx \begin{bmatrix} \frac{Q(x+hz^1)-Q(x)}{h} \\ \frac{Q(x+hz^2)-Q(x)}{h} \end{bmatrix}. \quad (4)$$

More examples of extremum seeking algorithms can be found in Teel and Popović (2001); see Khong et al. (2013).

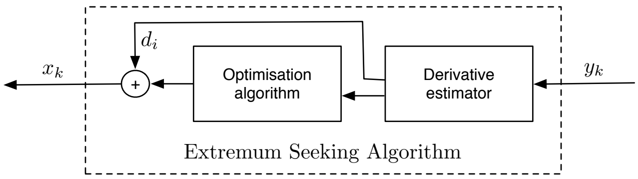

Fig. 2. A gradient-based extremum seeking controller.

3. SINGLE-AGENT EXTREME SOURCE SEEKING

This section demonstrates how extreme source seeking of a $Q : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ described in the previous section can be accomplished with a single controllable point-mass vehicle. It is assumed that measurements are corrupted by norm-bounded noise. Here, $n = 1$, 2, or 3. The class of controllable vehicles include, for example, autonomous vehicles considered in Ögren et al. (2004); Biyik and Arcak (2008). Semi-global practical asymptotic stability of the extrema of field distributions is established.

Assumption 2. The vehicle dynamics are

$$\dot{x} = f(\xi, u, v); \qquad \dot{\xi} = g(\xi, u), \quad (5)$$

where $x(t) \subset \mathbb{R}^n$ and $\xi(t) \in \mathbb{R}^p$ denote, respectively, the position with respect to an inertial frame and internal dynamics of the vehicle, $u(t) \in \mathbb{R}^m$ the (cooperative) control input, and $v(t) \in \mathbb{R}^n$ the reference velocity. Both f and g are locally Lipschitz in each argument. The vehicle is controllable in the following sense. Suppose $x(0) = x_0$ for some $x_0 \in \mathbb{R}^n$. Given any sequence $\{x_k\}_{k=0}^{\infty}$ in $\mathbb{R}^n$, there exist an increasing sequence $\{\tau_k\}_{k=1}^{\infty}$ in $\mathbb{R}$ and a piecewise continuous reference velocity v such that when applied to (5) with $u = 0$ results in $x(\tau_k) = x_k$ for $k = 1, 2, \ldots$. Suppose also that given any $\epsilon > 0$, there exists a $T > 0$ such that $\|v(t)\|_2 \leq \epsilon$ for $t \in [\tau_k, \tau_{k+1}]$ if $\tau_{k+1} - \tau_k \geq T$. In other words, a vehicle travelling at a lower speed takes longer time to get from one point to another.

Remark 3. The vehicle dynamics in Assumption 2 may also take the form

$$\ddot{x} = f(\xi, u, v); \qquad \dot{\xi} = g(\xi, u),$$

where $v(t) \in \mathbb{R}^n$ now denotes the reference applied force, which serves the same controllability function as above.

Various types of vehicular dynamics satisfy Assumption 2. For instance, the single and double integrator dynamics that are well-studied in the multi-agent literature (Ren and Beard, 2008; Olfati-Saber and Murray, 2002; Bai et al., 2011).

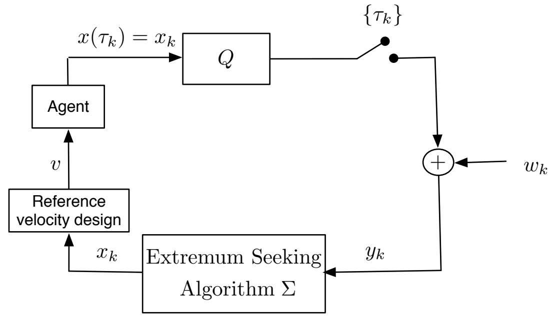

Fig. 3. Source seeking with a single agent.

The main result of this section is stated next. Let $Q : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ be a Lipschitz continuous field distribution with the set of maximisers $\mathcal{C}$. Given a vehicle satisfying Assumption 2 and an extremum seeking controller satisfying Assumption 1, consider the extreme source seeking setup in Figure 3, where the vehicle samples Q at the user-defined time instants $\{\tau_k\}_{i=1}^{\infty}$. Let w_k be the additive measurement noise. Suppose a piecewise continuous reference velocity v has been designed so that the position trajectory of the vehicle satisfies $x(\tau_k) = x_k$ for $k = 1, 2, \ldots$, where x_k is the output of the extremum seeking controller with its input being $y_k = Q(x_k) + w_k$, the noise-corrupted samples of Q.

Theorem 4. Let the extremum seeking configuration be as described above. Given any (Δ, μ) such that $\Delta, \mu > \delta$, where $\delta \geq 0$ is as in Assumption 1(ii), there exist a bound $\nu > 0$ and a $\bar{\beta} \in \mathcal{KL}$ such that for any $\|x_0\|_{\mathcal{C}} \leq \Delta$, if $|w_k| \leq \nu$, then

$$\|x_k\|_{\mathcal{C}} \leq \bar{\beta}(\|x_0\|_{\mathcal{C}}, k) + \mu \quad (6)$$

for all $k = 0, 1, \ldots$.

Proof. The proof can be established using similar arguments in (Khong et al., 2013, Thm. 19). It exploits the multi-step consistency and time-invariance of the extremum seeking algorithm. □

Theorem 4 demonstrates that semi-global practical asymptotic convergence of the extremum seeking scheme can be achieved with a single controllable agent. In the succeeding section, a network of multiple agents is examined within the context of extremum seeking control.

4. MULTI-AGENT EXTREMUM SEEKING

While the problem of extreme source seeking can be tackled using a single expensive mobile agent as in the previous section, there are cases where multiple sensor-enabled vehicles are needed to move slowly in a formation towards the source (Ögren et al., 2004). For instance, a fleet of economical AUV's may be deployed to locate a source in the ocean. By working collaboratively, they are less prone to failures due to vehicular malfunction as opposed to the single-agent case. This section considers the problem of steering a group of sensor-equipped agents to the source of the signal field using the extreme seeking framework described in Section 2, together with a generic set of distributed coordination rules which maintain the agents' formation. It is assumed that the extremum seeking controller has access to all the samples collected by the sensing agents. However, it does not function as a supervisory controller and the agents are coordinated in a distributed fashion (Bai et al., 2011; Ren and Beard, 2008). As in the single-agent section, semi-global practical asymptotic stability is established by exploiting the uniform asymptotic stability of the fleet formation.

The formation of the mobile sensors can be chosen in such a way that facilitates the estimation of the gradients of the environment field. For example, if (4) is used as a derivative estimator, a network of three agents can be deployed and their formation selected to be the vertices of a right-angled triangle. Likewise, in $\mathbb{R}^3$, four agents may be deployed to form a tetrahedron. If more complicated gradient approximation methods are employed such as the family of Runge-Kutta, more samples points would be needed and the number and formation of the agents can be decided accordingly.

Let the topology of information exchange between N number of dynamic agents be modelled by a graph. A desired formation, which may be time-varying, for the purpose of derivative estimation may be given by

$$\mathcal{P} = \left\{ z^i \;\middle|\; |z^i| = b^i;\ i = 1, 2, \ldots, \ell \right\}, \tag{7}$$

where $b^i > 0$, z^i denotes the distance between two agents connected by a link, and ℓ the total number of links in the graph. Note that $\mathcal{P}$ can also be chosen to be position-based and/or updated over time; see Section 4.1 for an example. The dynamics of the agents are assumed to satisfy Assumption 2 and are given by

$$\dot{x}^i = f(\xi^i, u^i, v^i); \qquad \dot{\xi}^i = g(\xi^i, u^i) \quad i = 1, 2, \ldots, N.$$

The cooperative control u^i is a function of x^j and $\dot{x}^j$ if the j^{th} agent is linked to the i^{th} agent or in certain consensus control frameworks where virtual leaders are exploited, if the j^{th} agent is a virtual leader. It is nominally zero when the vehicle network achieved the desired formation. The following assumption prescribes an objective for a formation coordination framework to be used in conjunction with extremum seeking.

Assumption 5. Given N number of mobile agents satisfying Assumption 2 and a desired formation, the distributed coordination control law is such that the formation is a uniformly asymptotically stable equilibrium. In the case where (7) is used, this means there exist a $c > 0$ and a $\beta \in \mathcal{KL}$ such that

$$\|z^i(t)\|_{\mathcal{P}} \le \beta(\|z^i(0)\|_{\mathcal{P}}, t) \quad \forall \|z^i(0)\|_{\mathcal{P}} \le c, i = 1, \ldots, \ell.$$

Examples of consensus algorithms that satisfy the assumption above are provided in the next subsections. Extremum seeking with a group of vehicles is examined below.

Given a Lipschitz continuous $Q : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ with Lipschitz contant $L > 0$, consider the gradient-based multi-agent extremum seeking scheme illustrated in Figure 4, where $x^1, \ldots, x^N$ represent the state trajectory of agent 1 to N respectively and $d_1, \ldots d_N$ denote the dither functions as in (3). In particular, it is assumed that given any $x \in \mathbb{R}^n$, $x + d_1(x), \ldots, x + d_N(x)$ define

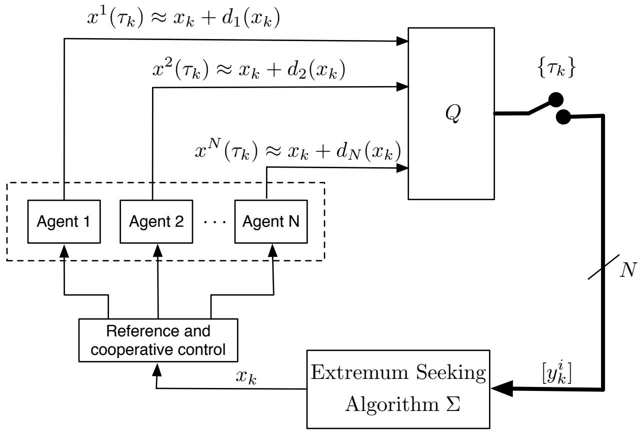

Fig. 4. Source seeking with multiple agents.

a formation (e.g. vertices of a triangle/tetrahedron) by which the derivatives of Q at x can be well-approximated with the information $Q(x + d_1(x)), \ldots, Q(x + d_N(x))$. The dithers can be constant functions as in Biyik and Arcak (2008). On the other hand, Ögren et al. (2004) studies dither functions that vary with the positions of the vehicles and adapt their configuration in response to the measurement noise to optimise gradient climbing. Reference velocities $v^1, \ldots, v^N$ are assumed to be designed according to Assumption 2 to steer the vehicles' positions $x^1, \ldots, x^N$ towards $x + d_1(x), \ldots, x + d_N(x)$, similarly to the single-vehicle case. The formation of the vehicles is maintained with a consensus algorithm satisfying Assumption 5.

Because the agents are driven by both the reference velocity v and the cooperative control laws u, in general $x^i(\tau_k)$ will lie close to but not precisely on $x_k + d_i(x_k)$, by contrast with the single-agent case. Indeed, $x^i(\cdot)$ converges to $x_k + d_i(x_k)$ as time tends to infinity by the uniformly asymptotically stable property of the equilibria defining the desired formation. Nevertheless, this is sufficient to give rise to practical convergence to the extrema of the field distribution as demonstrated by the next result.

Theorem 6. Suppose the extremum seeking controller in Figure 4 satisfies Assumption 1 and the agents with dynamics in Assumption 2 are coordinated with a control law satisfying Assumption 5 with respect to the formation defined by the dither functions. Also assume that the vehicles commence within the cooperative control law's region of attraction and are driven by appropriate reference velocities/forces towards destinations specified by the extremum seeking controller. Then given any (Δ, μ) such that $\Delta, \mu > \delta$, where $\delta \ge 0$ is as in Assumption 1(ii), there exist a $T > 0$ and a $\beta \in \mathcal{KL}$ such that for any $\|x_0\|_{\mathcal{C}} \le \Delta$, if $\tau_{i+1} - \tau_i \ge T\ \forall i = 0, 1, \ldots$, then

$$\|x_k\|_{\mathcal{C}} \le \beta(\|x_0\|_{\mathcal{C}}, k) + \mu \tag{8}$$

for all $k = 0, 1, \ldots$.

Proof. First note that the gradient motions of the vehicles as determined by the reference velocity need to be sufficiently slow so that the vehicles never leave the local region of attraction of the formation during manoeuvres. By Assumption 2, this can be guaranteed by an $T_1 > 0$ such that $\tau_{i+1} - \tau_i \ge T_1\ \forall i = 0, 1, \ldots$.

Now following the arguments in Theorem 4, there exists a $\nu > 0$ such that (8) holds if $|x_k + d_i(x_k) - x^i(\tau_k)| \leq \nu$ for all $i = 1, \ldots, N$ and $k = 1, 2, \ldots$. By the Lipschitz continuity of the environment distribution Q, the latter holds if $x^i(\tau_k)$ lies within a ball of radius $\frac{\nu}{L}$ centered at $x_k + d_i(x_k)$ for all i and k, where L is a Lipschitz bound for Q. In other words,

$$x^i(\tau_k) \in x_k + d_i(x_k) + \frac{\nu}{L}\bar{\mathcal{B}}, \tag{9}$$

where $\bar{\mathcal{B}}$ denotes the closed unit ball in $\mathbb{R}^n$. Since $x_k + d_i(x_k)$ is a uniformly asymptotically stable equilibrium for the i^{th} agent by Assumption 5, it follows that (9) can be ensured if the network is given enough time to manoeuvre towards its final formation during $[\tau_{k-1}, \tau_k]$. That is, there needs a sufficiently large gap $T_2 > 0$ between the time instants τ_{k-1} and τ_k. Defining $T := \max\{T_1, T_2\}$ completes the proof. □

Theorem 6 identifies general conditions on an optimisation algorithm (Assumption 1) and a cooperative control law (Assumption 5), combining which guarantees convergence towards an extreme source of a field distribution for a class of controllable mobile agents satisfying Assumption 2. Examples of such optimisation algorithms in Section 2 and examples of such cooperative control laws in subsequent subsections demonstrate that a wide range of options are at the user's disposal. Various properties of these algorithms and control laws can be investigated and exploited at the design stage to decide which to implement on a given problem.

4.1 An example of formation manoeuvre

This subsection examines the decentralised approach to formation manoeuvres of Lawton et al. (2003); Ren and Beard (2008) and demonstrates that it fits within the general framework of Theorem 6. It is also used as a basis of simulations in Section 5. Lawton et al. (2003) considers N number of wheeled robots with the following dynamics:

$$\begin{bmatrix} \dot{r}_x^i \\ \dot{r}_y^i \\ \dot{\theta}^i \\ \dot{v}^i \\ \dot{\omega}^i \end{bmatrix} = \begin{bmatrix} v^i \cos(\theta^i) \\ v^i \sin(\theta^i) \\ \omega_i \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m^i} & 0 \\ 0 & \frac{1}{J^i} \end{bmatrix} \begin{bmatrix} F^i \\ \tau^i \end{bmatrix},$$

where $r^i = [r_x^i, r_y^i]^T$ is the inertial position of the i^{th} robot, θ^i the orientation, v^i the linear speed, ω^i the angular speed, τ^i the applied torque, F^i the applied force, m^i the mass, and J^i the moment of inertia for $i = 1, 2, \ldots, N$. Applying output feedback linearisation about the hand position yields an approximate model for the position: $\ddot{x}^i = u^i$ (see Lawton et al. (2003) for details), which satisfies Assumption 2. A formation pattern is a set

$$\mathcal{P} := \{x_*^1, \ldots, x_*^N\},$$

where x_*^i denotes the desired constant location of the hand position of the i^{th} robot. Suppose the group of robots is required to transition through a sequence of formation patterns $\mathcal{P}_j$ which have been designed to avoid collisions. Lawton et al. (2003); Ren and Beard (2008) consider a bidirectional ring topology and propose the following control law which maintain the robots in the same shape as the destination pattern during the transition from one formation pattern to another:

$$\begin{aligned} u^i = &- K_g\tilde{x}^i - D_g\dot{x}^i \\ &- K_f(\tilde{x}^i - \tilde{x}^{i-1}) - D_f(\dot{x}^i - \dot{x}^{i-1}) \\ &- K_f(\tilde{x}^i - \tilde{x}^{i+1}) - D_f(\dot{x}^i - \dot{x}^{i+1}), \end{aligned} \tag{10}$$

where $\tilde{x}^i := x^i - x_*^i$, K_g and D_g are symmetric positive-definite matrices while K_f and D_f are symmetric positive-semidefinite matrices. Note that the indices are defined modulo N to observe the ring structure of the topology, whereby $x^{1-1} = x^N$ and $x^{N+1} = x^1$. It can be seen that the first two terms in (10) serve to drive the robot to reach its final position in the formation pattern. This accomplishes the reference forces design requirements of Theorem 6. The second two terms maintain the formation with the $i-1$ robot and the last two terms with the $i+1$ robot. These four terms guarantee that the formation pattern is an asymptotically stable equilibrium, i.e. Assumption 5 is satisfied. The coupled dynamics formation control law of (10) can be modified to take into account interrobot damping and actuator saturation constraints (Lawton et al., 2003; Ren and Beard, 2008).

Following the idea of the hardware experimental results of Lawton et al. (2003), the network manoeuvre from one formation pattern to another can be regarded as complete when each vehicle is within a pre-selected error distance tolerance ϵ of its destination; see (9). This error decreases with the increase of the time the fleet of vehicles is allocated to converge towards the destination's formation pattern. In the case of extreme source seeking, the formation pattern is updated recursively with the measurements of the field distribution by the extremum seeking controller.

Procedure 7. Suppose the transition to the formation pattern $\mathcal{P}_j$ has been completed, i.e. all the vehicles collected measurements of the field distribution Q at a distance no greater than $\epsilon := \frac{\nu}{L} > 0$ from their destinations; see Theorem 6. The measurements are exploited by the extremum seeking controller to estimate the gradient of Q around the local neighbourhood of where the vehicles are current situated. The next formation pattern $\mathcal{P}_{j+1}$ is then supplied by the controller to the vehicles. The reference and cooperative control law applied to each of the vehicle is adjusted accordingly to (10). The fleet of vehicles then proceeds to $\mathcal{P}_{j+1}$ in the prescribed formation.

5. SIMULATION

Consider the following quadratic scalar field distribution

$$Q(x, y) := -(x + 10)^2 - (y + 8)^2 + 5,$$

which has a unique global maximum at $[-10, -8]^T$. Suppose three mobile vehicles with single integrator dynamics are available, i.e. $\dot{x}^i = u^i$ for $i = 1, 2, 3$, and assume an undirected ring communication topology. The cooperative control and formation manoeuvre laws from Section 4.1 are adapted/simplified for these vehicles so that

$$u^i = 5\tilde{x}^i - (\tilde{x}^i - \tilde{x}^{i-1}) - (\tilde{x}^i - \tilde{x}^{i+1}),$$

where $\tilde{x} := x - x_*^i$ and x_*^i denotes the desired position of the i^{th} vehicle. The gradient descent method is employed

$$x_{k+1} = x_k + 0.2\nabla Q(x_k), \quad x_0 := [1, 1]^T, \tag{11}$$

together with the derivative estimator (4) with $h := 0.1$ as the extremum seeking controller of the form illustrated in Figure 2. Assume that the vehicles are initialised at the vertices of the right-angle triangle: $[1, 1]^T$, $[1 + h, 1]^T$, and $[1, 1 + h]$. The subsequent formation patterns are given by x_k, $x_k + [h, 0]^T$, and $x_k + [0, h]^T$ for $k = 1, 2, \ldots$. A duration of 0.7s is allocated for the network of vehicles to manoeuvre from one formation pattern to another. The vehicles collect measurements of the field distribution at the end of the 1s and return this information to the extremum seeking controller as in Figure 4, which is used in the output update (11). The simulation results are as follow. Figure 5 shows the 2D positions of the vehicles as they converge to the global maximum point $[-10, -8]^T$. The x and y components over time of the vehicles are plotted in Figure 6. These figures also demonstrate that no collision has occurred and the right-triangle formation is maintained during the formation manoeuvres.

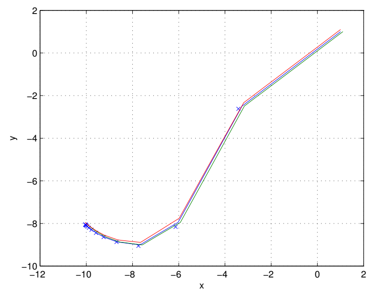

Fig. 5. Vehicles' positions

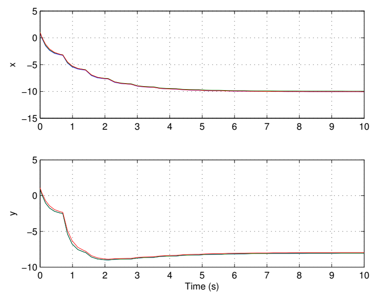

Fig. 6. Vehicles' position components over time

6. CONCLUSIONS

This paper proposes a unified approach to the extreme source seeking problem with multiple agents. Gradient-based local optimisation is considered in which a network of vehicles gradually converges to the source of a field environment. The formation is cooperatively maintained for gradient estimation with bounded errors. Some examples of optimisation algorithms and cooperative control algorithms are provided.

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