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Stable Adaptive Reference Trajectory Modification for Saturated Control Applications

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Abstract— **This paper will address the issue of reference trajectory modification in the presence of actuator saturation constraints. A stable adaptive trajectory modification scheme is proposed that ensures the stability of the closed loop system under actuator constraints. Also, the specific issues of the effects of actuator constraints on controller performance and stability is studied in detail. In particular, the performance of the control law is evaluated by considering the spacecraft rendezvous problem with realistic actuator constraints. The essential ideas and results from computer simulations are presented to illustrate the algorithm developed in paper.**

I. Introduction

Control saturation is one of the major challenges in in design of feedback control systems, wherein physical limitations on actuators and/or the plant generally dictate the control input constraints either to avoid damage to or deterioration of the process. Thus, it is imperative that the control input does not exceed its bounds while simultaneously realizing the performance objectives. However, there may be some instances where input saturation may even be desired from an optimality point-of-view as in the case of bang-bang control for time optimal purposes [1].

As such, the actuator saturation issues and their effects on closed loop stability as well as performance is well-recognized and has received much attention from the control community. [2] presents an extensive summary of recent research results in designing and analyzing control schemes for systems with unknown actuator failures and unknown parameters. The main focus in [2] is an adaptive actuator failure compensation approach that does not need an explicit fault detection and diagnosis procedure for failure compensation. Adaptive laws are designed that automatically adjust controller parameters based on system response errors. This allows an easy reconfiguration of the remaining functional actuators to accommodate a wide range of actuator failures and systems parameter uncertainties (see [2] for more details).

We further note that typical control law formulations normally do not incorporate any information about actuator position or rate constraints [3]–[5] a priori. Of late however, a lot of work has been done to incorporate actuator constraints in control formulations, however, the stability of the resultant controller is still an issue [6]–[9]. In [10], the effect of input saturation was analyzed on feedback linearization and in design of controllers for stabilization and tracking. Also, for the feedback linearization controllers, the regions of attraction of the controllers were characterized ([10])in addition to the space of feasible trajectories. The application of these controllers included aircraft flight control. Typical scenarios of trajectory tracking wherein demanding reference trajectories resulted in actuator saturation were studied. In [11], a technique for avoiding input saturation was proposed by re-parameterizing the reference trajectory on a slower time scale. In that sense, this paper tries to achieve a similar objective albeit differently.

The objective of this paper is to consider the effect of actuator position and rate constraints on the stability and performance of a model reference adaptive control system. The results of this paper can then be used as a basis for adaptive control formulation for a general nonlinear system under actuation constraints. In particular, the performance of the control law will be evaluated by considering the relative spacecraft position and attitude control problems with realistic actuator constraints.

The rest of the paper is organized as follows. We first introduce the class of nonlinear dynamical systems under study. Nominal control laws to meet a desired tracking trajectory are outlined. This is followed by a detailed discussion on an adaptive reference trajectory modification algorithm when control saturation occurs. Simulation results are presented for a spacecraft rendezvous problem followed by the summary and conclusions.

II. Problem Formulation

In this paper, we seek to design a novel adaptive reference trajectory modification scheme integrated with a feedback control law to track some nominal reference trajectory in space. The reference trajectory is specified as $\mathbf{x}_r \in \mathbb{R}^3$ and is assumed to be twice differentiable with respect to time. For the developments in this paper, we assume a generic mechanical system whose kinematics is characterized by three position coordinates and three velocity coordinates. These are represented as $\mathbf{x}_1 \in \mathbb{R}^3$ and $\mathbf{x}_2 \in \mathbb{R}^3$ respectively. Then, the general nonlinear system representation can be summarized as follows:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 &= \mathbf{A}_1\mathbf{x}_1 + \mathbf{A}_2\mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1,\mathbf{x}_2) + \mathbf{F} \end{aligned} \tag{1}$$

where, $\mathbf{F} \in \mathbb{R}^3$ represents the control forces. $\mathbf{g}(\mathbf{x}_1,\mathbf{x}_2) \in \mathbb{R}^3$ such that the nonlinearity $\|\mathbf{g}(\mathbf{x}_1,\mathbf{x}_2)\| \leq \mathbf{g}_0$, $(\mathbf{g}_0 \in \mathbb{R} > 0)$ is a vector function of position and velocity coordinates.

If we denote the relative position and velocity tracking errors by $\mathbf{e}_1 \triangleq \mathbf{x}_1 - \mathbf{x}_{1r}$ and $\mathbf{e}_2 \triangleq \mathbf{x}_2 - \mathbf{x}_{2r}$ respectively, then it is easy to show that the error dynamics can be written as:

$$\begin{aligned} \dot{\mathbf{e}}_1 &= \mathbf{e}_2 \\ \dot{\mathbf{e}}_2 &= \mathbf{A}_1\mathbf{x}_1 + \mathbf{A}_2\mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1,\mathbf{x}_2) + \mathbf{F} - \ddot{\mathbf{x}}_r \end{aligned} \tag{2}$$

To compute the control effort $\mathbf{F}$ to track the nominal base trajectory $\mathbf{x}_r(t)$, we impose the following error dynamics:

$$\ddot{\mathbf{e}}_1 + \mathbf{C}_d\dot{\mathbf{e}}_1 + \mathbf{K}_d\mathbf{e}_1 = 0 \tag{3}$$

Now, since nominal base trajectory is assumed to be twice differentiable, on substituting Eq. (2) in Eq. (3), we get:

$$\mathbf{F} = -\mathbf{A}_1\mathbf{x}_1 - \mathbf{A}_2\mathbf{x}_2 - \mathbf{g}(\mathbf{x}_1,\mathbf{x}_2) + \ddot{\mathbf{x}}_r - \mathbf{C}_d\dot{\mathbf{e}}_1 - \mathbf{K}_d\mathbf{e}_1 \tag{4}$$

Remarks:
(1) The above control law can be shown to be a straightforward dynamic inversion based control algorithm that essentially seeks to cancel the nonlinearities and render the tracking error dynamics linear.
(2) While there are several choices for the control algorithm, we just pick this control law for this paper. The adaptive reference trajectory modification and the actuator saturation issues are pertinent to the applications irrespective of what sort of a control algorithm chosen.
(3) Note, the above-mentioned control law assumes that there are no position as well as rate constraints on the control vector $\mathbf{F}$.

Since control saturation could occur in practice, the applied control ($\mathbf{F}_a$) is limited by the actuator saturation constraints and therefore will be different from the computed control $\mathbf{F}$. This applied control is thus a restriction of the computed control to the set of feasible control inputs $\mathbf{F}_a(t) = \lfloor \mathbf{F}(t) \rfloor$. *Certainly, the choice of the restriction operator $\lfloor . \rfloor$ is an important part of the problem, and this issue will be discussed in the full paper.*

Let δ be the difference between the computed and the applied control,

$$\delta(t) = \mathbf{F}(t) - \mathbf{F}_a(t) \tag{5}$$

Substituting for the applied control vector $\mathbf{F}_a$ in the equations of motion, given by Eq. (2), we get the following expression for the closed-loop dynamics:

$$\ddot{\mathbf{e}}_1 + \mathbf{C}_d\dot{\mathbf{e}}_1 + \mathbf{K}_d\mathbf{e}_1 = -\delta(t) \tag{6}$$

Note, however that the RHS of Eq. (6) i.e., $\delta(t)$ is essentially a state dependent disturbance forcing the second order tracking error dynamics. This is because $\delta(t) = \mathbf{F}(t) - \mathbf{F}_a(t)$ and $\mathbf{F}(t)$ is a function of the states as well as the reference trajectory as seen in Eq. (4). Beside canceling the nonlinearities of the system, the terms that influence the magnitude of $\mathbf{F}(t)$ are the stabilizing terms $\mathbf{C}_d\dot{\mathbf{e}}_1$, $\mathbf{K}_d\mathbf{e}_1$ and the reference trajectory demand $\ddot{\mathbf{x}}_r$.

There is a significant amount of prior work in the design of the control gain matrices that ensure stability of the closed loop when actuator saturation happens [12]–[16]. Further [9], [17] has addressed the issue of reference trajectory modification using a "pseudo-control-hedging" (PCH) technique. The PCH component designed by Johnson et al. utilizes an estimate of actuator position based on actuator command and possibly vehicle state.

The work presented in this paper is clearly different from any of the earlier works as we present a direct adaptation of the gain matrices $\mathbf{C}_d$ and $\mathbf{K}_d$ to alleviate the problems due to saturation.
Note, in the absence of any control saturation, $\delta(t) = 0$ and the gain matrices $\mathbf{C}_d$ and $\mathbf{K}_d$ are constant. When $\delta(t) \neq 0$, we propose time varying gain matrices $\mathbf{C}(t)$ and $\mathbf{K}(t)$ to compensate for the non-zero $\delta(t)$. Hence, we modify the control law of Eq. (4) as follows:

$$\mathbf{F} = -\mathbf{A}_1\mathbf{x}_1 - \mathbf{A}_2\mathbf{x}_2 - \mathbf{g}(\mathbf{x}_1,\mathbf{x}_2) + \ddot{\mathbf{x}}_r - \mathbf{C}(t)\dot{\mathbf{e}}_1 - \mathbf{K}(t)\mathbf{e}_1 \tag{7}$$

Now, we can write the closed loop error dynamics of Eq. (6) with the gain adaptation as:

$$\ddot{\mathbf{e}}_{1s} + \mathbf{C}(t)\dot{\mathbf{e}}_{1s} + \mathbf{K}(t)\mathbf{e}_{1s} = -\delta(t) \tag{8}$$

where, $\delta(t)$ denotes the difference between computed control of Eq. (7) and its restriction $\mathbf{F}_a = \lfloor \mathbf{F} \rfloor$. Now, we seek adaptation/update laws for the gain matrices $\mathbf{C}(t)$ and $\mathbf{K}(t)$ such that closed loop error dynamics of Eq. (8) follows the exponentially stable desired error dynamics of Eq. (3).

Let us define

$$\mathbf{e} = \mathbf{e}_1 - \mathbf{e}_{1s} \tag{9}$$

The second time derivative of $\mathbf{e}$ can be written as:

$$\begin{aligned}
\ddot{\mathbf{e}} &= -\mathbf{C}_d\dot{\mathbf{e}}_1 - \mathbf{K}_d\mathbf{e}_1 + \mathbf{C}(t)\dot{\mathbf{e}}_{1s} + \mathbf{K}(t)\mathbf{e}_{1s} + \delta(t) \\
&= -\mathbf{C}_d\dot{\mathbf{e}} - \mathbf{K}_d\mathbf{e} + \delta(t) + (\mathbf{C}(t) - \mathbf{C}_d)\,\dot{\mathbf{e}}_{1s} \\
&\quad + (\mathbf{K}(t) - \mathbf{K}_d)\,\mathbf{e}_{1s} \\
&= -\mathbf{C}_d\dot{\mathbf{e}} - \mathbf{K}_d\mathbf{e} + \tilde{\mathbf{C}}\dot{\mathbf{e}}_{1s} + \tilde{\mathbf{K}}\mathbf{e}_{1s} + \delta(t)
\end{aligned}$$

The above can be re-cast as follows:

$$\ddot{\mathbf{e}} + \mathbf{C}_d\dot{\mathbf{e}} + \mathbf{K}_d\mathbf{e} = \tilde{\mathbf{C}}\dot{\mathbf{e}}_{1s} + \tilde{\mathbf{K}}\mathbf{e}_{1s} + \delta(t) \tag{10}$$

where, $\tilde{\mathbf{C}} = \mathbf{C}(t) - \mathbf{C}_d$ and $\tilde{\mathbf{K}} = \mathbf{K}(t) - \mathbf{K}_d$.

Note: The above essentially amounts to incorporation of a parallel error dynamical system that has within it two time varying gain matrices that are updated based on the control defect term, $\delta(t)$.

To obtain the adaptation/update laws for the gain matrices, we define a candidate Lyapunov function as:

$$\begin{aligned}
V = \quad & \frac{1}{2}\Big[\dot{\mathbf{e}}^T\dot{\mathbf{e}} + \mathbf{e}^T\mathbf{K}_d\mathbf{e} + Tr\left(\tilde{\mathbf{C}}^T\Gamma_C^{-1}\tilde{\mathbf{C}}\right) + \\
& Tr\left(\tilde{\mathbf{K}}^T\Gamma_K^{-1}\tilde{\mathbf{K}}\right)\Big]
\end{aligned} \tag{11}$$

where, Γ_C^{-1} and Γ_K^{-1} are symmetric positive definite matrices of appropriate dimension. They will serve as tuning parameters to speed up (or slow down) the adaptive laws. The time derivative of V evaluated along the trajectories of Eq. (10)can be written as:

$$\begin{aligned}
\dot{V} &= \dot{\mathbf{e}}^T\left(-\mathbf{C}_d\dot{\mathbf{e}} - \mathbf{K}_d\mathbf{e} + \tilde{\mathbf{C}}\dot{\mathbf{e}}_{1s} + \tilde{\mathbf{K}}\mathbf{e}_s + \delta(t)\right) \\
&\quad + \dot{\mathbf{e}}^T\mathbf{K}_d\mathbf{e} + Tr\left(\tilde{\mathbf{C}}^T\Gamma_C^{-1}\dot{\tilde{\mathbf{C}}}\right) + Tr\left(\tilde{\mathbf{K}}^T\Gamma_K^{-1}\dot{\tilde{\mathbf{K}}}\right) \\
&= -\dot{\mathbf{e}}^T\mathbf{C}_d\dot{\mathbf{e}} + \dot{\mathbf{e}}^T\tilde{\mathbf{C}}\dot{\mathbf{e}}_{1s} + \dot{\mathbf{e}}^T\tilde{\mathbf{K}}\mathbf{e}_{1s} + \dot{\mathbf{e}}^T\delta(t) \\
&\quad + Tr\left(\tilde{\mathbf{C}}^T\Gamma_C^{-1}\dot{\tilde{\mathbf{C}}}\right) + Tr\left(\tilde{\mathbf{K}}^T\Gamma_K^{-1}\dot{\tilde{\mathbf{K}}}\right) \\
&= -\dot{\mathbf{e}}^T\mathbf{C}_d\dot{\mathbf{e}} + Tr\left(\tilde{\mathbf{C}}^T\Gamma_C^{-1}\dot{\tilde{\mathbf{C}}} + \tilde{\mathbf{C}}^T\dot{\mathbf{e}}\dot{\mathbf{e}}_{1s}^T\right) \\
&\quad + Tr\left(\tilde{\mathbf{K}}^T\Gamma_K^{-1}\dot{\tilde{\mathbf{K}}} + \tilde{\mathbf{K}}^T\dot{\mathbf{e}}\mathbf{e}_{1s}^T\right) + \dot{\mathbf{e}}^T\delta(t) \\
&= Tr\left(\tilde{\mathbf{C}}^T\Gamma_C^{-1}\dot{\tilde{\mathbf{C}}} + \tilde{\mathbf{C}}^T\dot{\mathbf{e}}\dot{\mathbf{e}}_{1s}^T + \alpha\dot{\mathbf{e}}\delta^T(t)\right) \\
&\quad + Tr\left(\tilde{\mathbf{K}}^T\Gamma_K^{-1}\dot{\tilde{\mathbf{K}}} + \tilde{\mathbf{K}}^T\dot{\mathbf{e}}\mathbf{e}_{1s}^T + (1-\alpha)\dot{\mathbf{e}}\delta^T(t)\right) \\
&\quad - \dot{\mathbf{e}}^T\mathbf{C}_d\dot{\mathbf{e}}
\end{aligned} \tag{12}$$

Thus, from Eq. (12) if we choose the following adaptation laws:

$$\begin{aligned}
\dot{\tilde{\mathbf{C}}} = \dot{\mathbf{C}} &= -\Gamma_C\left[\dot{\mathbf{e}}\dot{\mathbf{e}}_{1s}^T + \tilde{\mathbf{C}}^{-T}\alpha\dot{\mathbf{e}}\delta(t)^T\right] \\
\dot{\tilde{\mathbf{K}}} = \dot{\mathbf{K}} &= -\Gamma_K\left[\dot{\mathbf{e}}\mathbf{e}_{1s}^T + \tilde{\mathbf{K}}^{-T}(1-\alpha)\dot{\mathbf{e}}\delta(t)^T\right]
\end{aligned} \tag{13}$$

Then substituting the adaptation laws in Eq. (12) we obtain,

$$\dot{V} = -\dot{\mathbf{e}}^T\mathbf{C}_d\dot{\mathbf{e}}$$

which is negative semi-definite and ensures that in the presence of actuator saturation the actual tracking errors *are close to* the desired tracking error states (in the absence of saturation) asymptotically.

By repeated evaluation of the higher order derivatives of V and the application of the asymptotic stability theorem given by Mukherjee and Chen [18] we conclusively prove that even in the presence of actuator saturation, the actual tracking error dynamics asymptotically approach the ideal desired tracking error dynamics (in the absence of saturation). As shown, this is achieved by adaptively modifying the gains $\mathbf{C}(t)$ and $\mathbf{K}(t)$.

Remark:
It is to be noted that the structure of control law has not been re-designed in this case. It is the dynamics of the parallel error reference system that is modified by introducing the time varying matrices. This dynamics is forced by the control defect between the applied and the saturated values. In essence, the net effect is to have modified the $\ddot{\mathbf{x}}_r$ values in the overall control law to facilitate the actuator saturation.

III. Numerical Simulations

To illustrate the effectiveness of the proposed adaptation laws in presence of control saturation, we consider the problem of spacecraft rendezvous. Let us consider the chaser spacecraft motion relative to the target spacecraft, in the Local-Vertical-Local-Horizontal (LVLH) frame as shown in Fig. 1. The LVLH reference frame is attached to the center of mass of target space vehicle with X-axis pointing radially outward of its orbit, Y-direction perpendicular to X along its direction of motion and Z completes the right handed co-ordinate system. Usually in rendezvous and docking problems, the trajectory of target spacecraft is described in the LVLH coordinate system, and this frame is taken as the reference target trajectory for the chaser spacecraft. The relative dynamics between two spacecrafts is governed by fully non-linear Clohessy-Wiltshire equations, given as follows [19]:

$$\begin{aligned}\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x &= -\frac{\mu(r_c + x)}{\rho^{3/2}} + \frac{\mu}{r_c^2} + \frac{F_x}{m} \\ \ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y + \frac{\mu}{r_c^3}y &= -\frac{\mu y}{\rho^{3/2}} + \frac{\mu}{r_c^3}y + \frac{F_y}{m} \\ \ddot{z} + \frac{\mu}{r_c^3}z &= -\frac{\mu z}{\rho^{3/2}} + \frac{\mu}{r_c^3}z + \frac{F_z}{m}\end{aligned} \tag{14a}$$

$$\begin{aligned}\ddot{r_c} &= r_c\dot{\theta}^2 - \frac{\mu}{r_c^2} \\ \ddot{\theta} &= -2\frac{\dot{r_c}\dot{\theta}}{r_c} \\ \rho &= \sqrt{(r_c + x)^2 + y^2 + z^2}\end{aligned}$$

where x, y, z represents the relative position of chaser spacecraft w.r.t. the target, r_c and ρ refer to the scalar radius of the target and chaser from the center of the Earth, respectively, θ represents the latitude angle of the target, and μ is the gravitational parameters. F_x, F_y and F_z are the control forces and m is the mass of chaser spacecraft. These equations amount to the classical nonlinear Encke [19] relative motion differential equations of the chaser vehicle written in the rotating LVLH coordinate system, centered in the target vehicle. The matrices $\mathbf{A}_1$, $\mathbf{A}_2$ and vector $\mathbf{g}$ in Eq. 1 can be constructed as follows:

$$\mathbf{A}_1 = \begin{bmatrix} \dot{\theta}^2 + 2\frac{\mu}{r_c^3} & \ddot{\theta} & 0 \\ -\ddot{\theta} & \dot{\theta}^2 - \frac{\mu}{r_c^3} & 0 \\ 0 & & 0 \end{bmatrix} \tag{15}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 2\dot{\theta} & 0 \\ -2\dot{\theta} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$\mathbf{g} = \begin{Bmatrix} -\frac{\mu(r_c+x)}{\rho^{3/2}} + \frac{\mu}{r_c^2} - 2\frac{\mu}{r_c^3}x \\ -\frac{\mu y}{\rho^{3/2}} + \frac{\mu y}{r_c^3} \\ -\frac{\mu z}{\rho^{3/2}} + \frac{\mu z}{r_c^3} \end{Bmatrix} \tag{17}$$

For simulation purposes, it is assumed that chaser spacecraft is at a distance of $(-50,\ -50,\ -50)^T$ m from the target spacecraft. The target spacecraft is assumed to be in a circular orbit at an altitude of 400 km. The actual mass of chaser spacecraft is assumed to be 400 kg. The reference target trajectory for the translation motion of the chaser is generated by connecting a 3rd order spline curve between the initial state to the final desired position of the chaser. To fit the smooth curve, the rendezvous time is assumed to be 60 seconds. We assume the control limits to be:

$$-1 \leq F_x \leq 1, \quad -0.2 \leq F_y \leq 0.2, \quad -0.1 \leq F_z \leq 0.1$$

Nominal stiffness matrix $\mathbf{K}_d$ and damping matrix $\mathbf{C}_d$ were chosen to be:

$$\mathbf{K}_d = \mathbf{C}_d = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

The various adaptation gains were assigned following values:

$$\mathbf{\Gamma}_K = 10^{-7}\mathbf{I},\ \mathbf{\Gamma}_C = 10\mathbf{I},\ \alpha = 0.7,\ \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We deliberately choose very low value for adaptation gain corresponding to the adaptation of stiffness matrix since higher value of adaptation gain $\mathbf{\Gamma}_K$ can lead to high bandwidth for error dynamics. Fig. 2(a) shows the position level tracking error $\|\mathbf{e}_{1_s}\|$ with respect to time while Fig. 2(b) shows the velocity level tracking error $\|\dot{\mathbf{e}}_{1_s}\|$ with respect to time. The red solid line represents the case without any adaptation of controller gain parameters while blue dotted line represents the case with adaptation of controller gain parameters. From these plots, it is clear that nominal error dynamics of Eq. (2) is unstable in presence of saturation constraint and with constant controller gain parameters while the tracking errors are still bounded with the adaption of controller gain parameters. Further, Fig. 2(c) shows the plot for vehicle state trajectory for both the cases (with and without adaption) and actual reference trajectory. Once again, it is clear that with the adaptation of controller gain the vehicle state trajectory is bounded and remains in the neighborhood of true reference trajectory. Fig. 2(d) shows the plot of the applied control effort with respect to time. As expected the adaptation of controller gain matrices $\mathbf{C}_d$ and $\mathbf{K}_d$ in presence of saturation changes the control profile and hence, results in bounded tracking errors. Further, Figs. 2(e) and 2(f) show the plots of adaptation of controller gain matrices $\mathbf{K}$ and $\mathbf{C}$, respectively. From these plots, it is clear that with the adaptation of controller gain matrices the tracking errors can be reduced even in presence of control saturation.

Finally, we mention that the controller parameter adaption is shown to work well in the presence of bounded control saturation, fully consistent with the bounded stability analysis presented. While, the simulation results presented in this paper merely illustrate formulations for a particular rendezvous and docking maneuver, further testing would be required to reach any conclusions about the efficacy of the control and adaptation laws for tracking arbitrary maneuvers. In particular, optimization of the remaining parametric degrees of freedom to extremize some measure of performance or

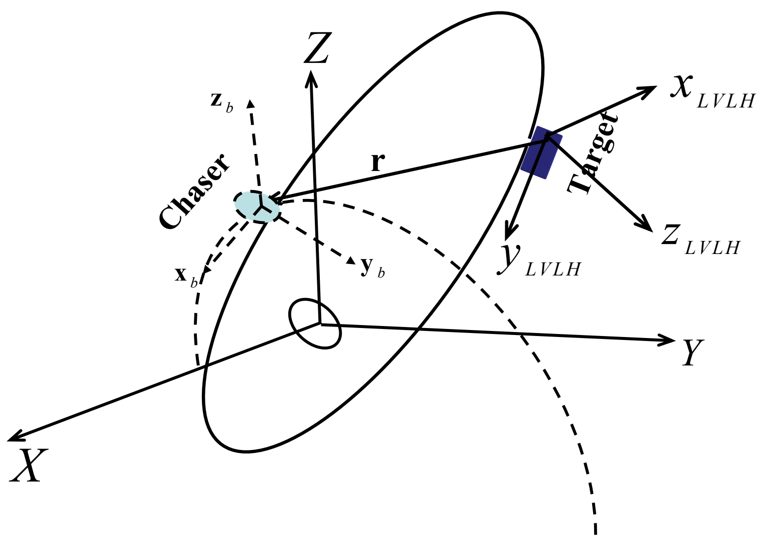

Fig. 1. The LVLH and Body Frame

robustness should be considered, subject to the stability constraints derived herein.

IV. CONCLUSIONS

This paper addressed the issue of reference trajectory modification in the presence of actuator saturation constraints. A stable adaptive trajectory modification scheme was developed that ensured the stability of the nominal control algorithms under actuator constraints. Issues concerning the effects of actuator constraints on controller performance and stability are studied. In particular, the performance of the control law is evaluated by considering the spacecraft rendezvous problem with realistic actuator constraints. The essential ideas and results from computer simulations illustrate the effectiveness of the algorithm developed in paper.

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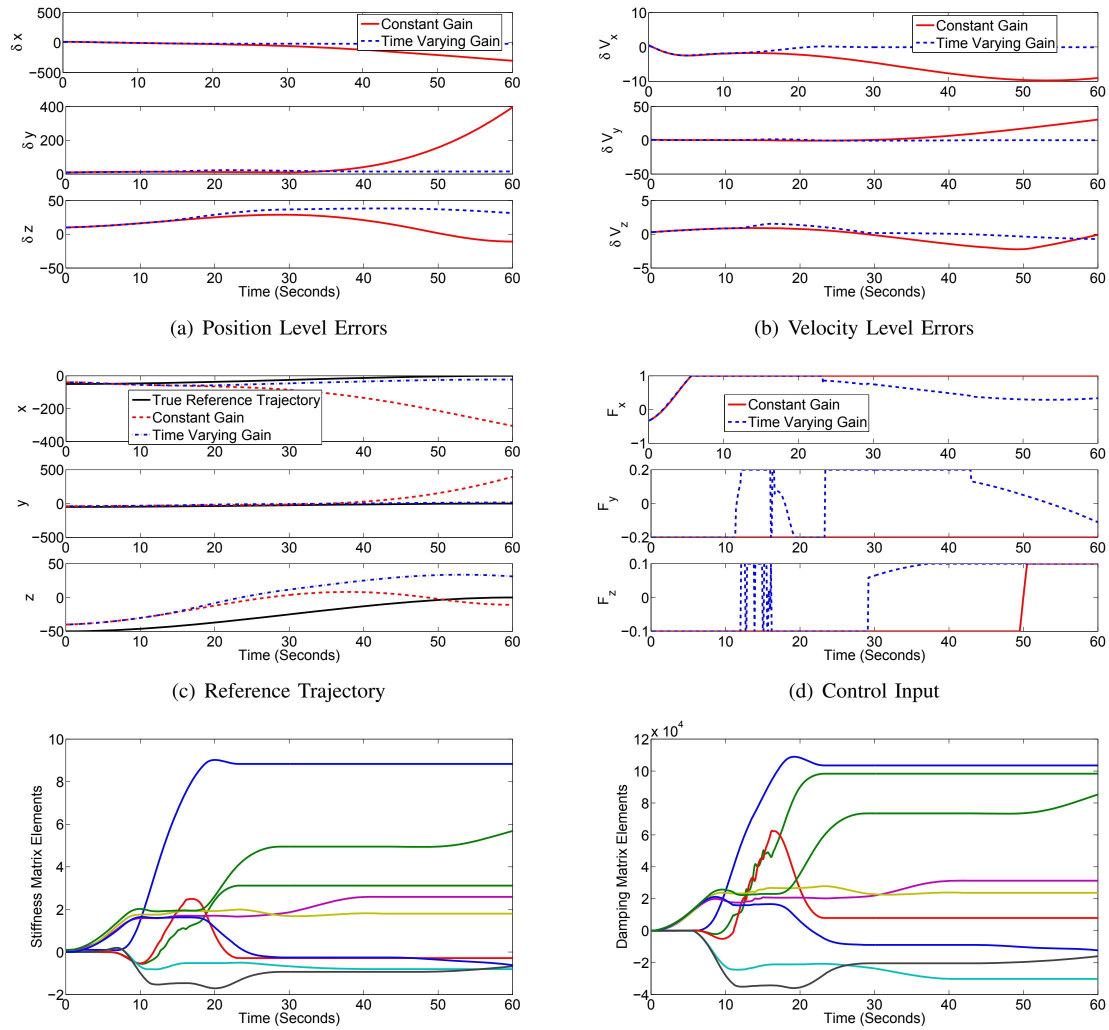

(a) Position Level Errors

(b) Velocity Level Errors

(c) Reference Trajectory

(d) Control Input

(e) Adaptation of Stiffness Matrix $\mathbf{K}$

(f) Adaptation of Damping Matrix $\mathbf{C}$

Fig. 2. Simulation Results

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