Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 314742, 6 pages
http://dx.doi.org/10.1155/2014/314742

Research Article

Cluster Projective Synchronization of Fractional-Order Complex Network via Pinning Control

Li-xin Yang, Wan-sheng He, Fan-di Zhang, and Jin-ping Jia

School of Mathematic and Statistics, Tianshui Normal University, Tianshui 741001, China

Correspondence should be addressed to Li-xin Yang; jiaodayanglixin@163.com

Received 10 March 2014; Revised 20 April 2014; Accepted 22 April 2014; Published 12 May 2014

Academic Editor: Jun Hu

Synchronization is the strongest form of collective phenomena in complex systems of interacting components. In this paper, the problem of cluster projective synchronization of complex networks with fractional-order nodes based on the fractional-order differential equation stability theory is investigated. Only the nodes in one community which have direct connections to the nodes in other communities are controlled. Some sufficient synchronization conditions are derived via pinning control. Numerical simulations are provided to show the effectiveness of the theoretical results.

1. Introduction

In the past few decades, complex networks behaviors have attracted a great deal of attention in a variety of fields due to their wide and potential applications. Typical complex networks include the Internet, the World Wide Web, neural networks, and so on [1–5]. Synchronization, as a typical collective dynamical behavior of coupled dynamical systems, has been widely studied. Until now, several types of synchronization have been investigated, such as complete synchronization [6], phase synchronization [7], and generalized synchronization [8]. Projective synchronization (PS) [9], which was first proposed by Mainieri and Rehacek that the drive state vector and the response state vectors synchronize up to a constant scaling factor. The proportional feature of PS can be employed to extend binary digital to M-nary communication for achieving fast communication. Recently, Li and Chen [10] discussed projective synchronization of the random networks. After that, the authors studied the projective synchronization of time-delayed chaotic systems in a driven-response complex network. In [11], they investigated the projective and lag synchronization between general complex networks via impulsive control.

In particular, in many social and biological networks, which can be divided naturally into communities, nodes in the same community often have the same type of function. Cluster synchronization is an exact phrase that describes this important phenomenon [12]. Cluster synchronization is achieved when the dynamical nodes reach complete synchronization in each subgroup called cluster but no synchronization among the different clusters. Many results have been available for cluster synchronization of complex networks. For example, Ayati and Khaki-Sedigh studied cluster synchronization of a connected chaotic network and a star-like complex network in [13]. Wang and Song [14] investigated the cluster synchronization problem for linearly coupled networks. More recently, Hu et al. [15] studied the cluster synchronization for directed complex dynamical networks via pinning control. In [16, 17], cluster synchronization in community networks is studied with integer-order system nodes, and several sufficient conditions for synchronization are obtained analytically.

On the other hand, as we know, the well-studied integer-order complex networks are the special cases of the fractional-order ones. It has been revealed that, in interdisciplinary fields, various systems have been found to exhibit fractional dynamics. For example, viscoelasticity, dielectric polarization, quantum evolution of complex system, fractional kinetics, and anomalous attenuation can be described by fractional differential equations. To the best of our knowledge, most studies to date have been concerned with integer-order complex networks, and the corresponding research on fractional-order complex networks has received very little attention despite its practical significance [18–23]. Therefore, it is of great interest to investigate the synchronization in complex colored networks consisting of nodes with fractional-order dynamics.

Motivated by the above discussions, the cluster projective synchronization in complex dynamical networks with fractional-order dynamical nodes by pinning control is investigated in this paper. This method decreases the control cost to some extent by reducing the number of nodes. The pinning controllers are designed according to the nodes property, respectively. We derive some simple and useful criteria for cluster synchronization for any initial values through an effective control scheme.

This paper is organized as follows. In Section 2, the network model of fractional-order network and mathematical preliminaries is introduced. Section 3 is devoted to investigating the cluster projective synchronization of the complex coupled networks. In Section 4, illustrative examples are shown to support the theory results. Conclusions are drawn in Section 5.

2. Mathematical Preliminaries and Model for Community Networks

2.1. Fractional-Order Derivative. Fractional calculus is a generalization of integration and differentiation to a noninteger-order integrodifferential operator ${}_aD_t^q$ which is defined by

$$ {}_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & R(q) > 0 \\ 1 & R(q) = 0 \\ \displaystyle\int_a^t (d\tau)^{-q} & R(q) < 0, \end{cases} \quad (1)$$

where q is the fractional order which can be a complex number and $R(q)$ is the real part of q. The numbers a and t are the limits of the operator. There are many definitions for fractional differential equations. Three most frequently used definitions for general fractional differential equations are the Grunwald-Letnikov definition, the Riemann-Liouville definition, and the Caputo definitions.

The Grunwald-Letnikov definition (GL) derivative with fractional-order q is described by

$$ {}_a^{\mathrm{GL}}D_t^q f(t) = \lim_{h\to 0} f_h^{(q)} \sum_{j=0}^{[(t-a)/h]} (-1)^j \begin{bmatrix} q \\ j \end{bmatrix} f(t - jh), \quad (2)$$

where the symbol $[\cdot]$ means the integer part.

The Riemann-Liouville (RL) definition of fractional derivatives is given by

$$ {}_a^{\mathrm{GL}}D_t^q f(t) = \frac{1}{\Gamma(n-q)} \times \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q-n+1}} d\tau, \quad n-1 < q < n, \quad (3)$$

where $\Gamma(\cdot)$ is the gamma function.

The Caputo (C) fractional derivative is defined as follows:

$$ {}_a^{C}D_t^a f(t) = \frac{1}{\Gamma(n-q)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q-n+1}} d\tau, \quad n-1 < q < n. \quad (4)$$

It is well known that the initial conditions for the fractional differential equations with Caputo derivatives take on the same form as those for the integer-order ones, which is very suitable for practical problems. Therefore, we will use the Caputo definition for the fractional derivatives in this paper.

2.2. Model Description for Community Networks. Consider that the complex networks considered in this paper consist of N nodes and m communities, where $2 \le m \le N$. Let $\{C_1, C_2, \dots, C_m\}$ denote m communities of complex networks and $\cup_{i=1}^m C_i = \{1, 2, \dots, N\}$. Without loss of generality, let $C_1 = \{1, 2, \dots, r_1\}, C_1 = \{r_1+1, \dots, r_1+r_2\}, \dots, C_m = \{r_1+\cdots+r_{m-1}+1, \dots, r_1+\cdots+r_{m-1}+r_m\}$ with $r+\cdots+r_{m-1}+r_m = N$. If node i belongs to the jth community then let $\delta_i = j$. Denote by U_{δ_i} the set of all nodes in the δ_ith cluster. $\widetilde{U}_{\delta_i}$ represents all the nodes in the δ_i cluster, which have direct connections with the nodes in other communities.

A complex network consisting of N coupled identical nodes, with each node being a n-dimensional fractional-order dynamical system, can be described by

$$ D_*^q x_i(t) = f(x_i(t)) + \varepsilon \sum_{i=1}^N c_{ij} \Gamma x_j(t) \quad i = 1, 2, \dots, N, \quad (5)$$

where $0 < q < 1$ and $x_i = (x_{i1,} x_{i2}, \dots, x_{in})^T \in R^n$ is the state variable of the node i. $f : R^n \to R^n$ describes the dynamics of nodes and is differential and capable of performing abundant dynamical behaviors. $\varepsilon > 0$ is the coupling strength and $\Gamma = \mathrm{diag}(\beta_1, \beta_2, \dots, \beta_n)$ is the inner-coupling matrix. The matrix $C = (c_{ij})_{N\times N}$ is the coupling configuration diffusive matrix, which is defined as follows: if there is a connection between nodes i and j is connected, then $c_{ij} = c_{ji} > 0$ $(i \ne j)$; otherwise $c_{ij} = c_{ji} = 0$ $(i \ne j)$; let $c_{ii} = -\sum_{j=1, i\ne j}^N c_{ij}$. Further, assume that there are no isolated clusters in the network and the network is connected, so the coupling configuration C is symmetrical and irreducible. $u_i \in R^n$ are controllers to be designed later. For simplicity of further discussion, decompose the function $f(x_i(t))$ into two parts, $A(x_i(t)) + F(x_i(t))$, where A is an $n \times n$ constant matrix and $F : R^n \to R^n$ are nonlinear vector-valued functions.

Then the controlled network can be rewritten as

$$ D_*^q x_i(t) = A(x_i(t)) + F(x_i(t)) + \varepsilon \sum_{j=1}^N c_{ij} \Gamma x_j(t) + u_i \quad i = 1, 2, \dots, N. \quad (6)$$

Definition 1. Let $\{1, 2, \dots, N\}$ be the N nodes of the networks and let $\{C_1, C_2, \dots, C_m\}$ be the m communities, respectively.

A network with m communities is said to realize cluster projective synchronization if

$$\lim_{t\to\infty}\left\|x_i - Mx_j\right\| = 0, \quad \forall i, j \in C_n,\ n = 1, 2, \ldots, m,$$
$$\lim_{t\to\infty}\left\|x_i - Mx_j\right\| \neq 0, \quad i \in C_{n_1},\ j \in C_{n_2},\ n_1 \neq n_2 = 1, 2, \ldots, m. \tag{7}$$

Define the error variables

$$e_i(t) = x_i(t) - MS_{\delta_i} \quad (i = 1, 2, \ldots, N), \tag{8}$$

where $M = \operatorname{diag}(\lambda, \lambda, \ldots, \lambda)$ is the scaling matrix and $S_{\delta i}(t)$ is a solution of an isolated node in the δ_ith community; we assume that the smooth goal dynamics can be described by

$$D_*^q S_{\delta_i}(t) = BS_{\delta_i}(t) + g\left(S_{\delta_i}(t)\right), \tag{9}$$

where $S_{\delta_i}(t)$ may be an equilibrium point, a periodic orbit, or even a chaotic orbit. Where $B \in R^{n\times n}$ is a constant matrix, $g(s(t))$ is the nonlinear part of the isolated reference node dynamics, respectively.

Remark 2. It is easy to see that the definition of the cluster projective synchronization encompasses the cluster synchronization and cluster antisynchronization when the scaling matrix is selected to take the corresponding specific values, respectively.

Throughout this paper, the following assumption and lemma are needed to prove our main results.

Lemma 3 (see [22]). *For a given autonomous fractional-order linear system*

$$D_*^q x = Ax, \quad \text{with } x(0) = x_0, \tag{10}$$

where $x \in R^n$ is the state vector, $A \in R^{n\times n}$ is a constant matrix with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, the fractional order $q \in (0, 1)$, and system (9) is asymptotically stable if and only if $|\arg(\lambda_i)| > q\pi/2$, $i = 1, 2, \ldots, n$.

Lemma 4 (see [24]). *Assume $A \in R^{N\times N}$ satisfies the following conditions:*

(1) $a_{ij} \geq 0$ $(i \neq j)$, $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, $i, j, \ldots, N$,

(2) *A is irreducible.*

Then (i) real parts of the eigenvalues of A are all negatives except an eigenvalue 0 with multiplicity 1; (ii) A has the right eigenvector $(1, 1, \ldots, 1)^T$ corresponding to the eigenvalue 0; (iii) let $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$ be the left eigenvector of A corresponding to the eigenvalue 0; then we can let $\xi_i > 0$ hold for all $i = 1, 2, \ldots, N$.

Lemma 5 (see [24]). *If the matrix C is defined as in Lemma 4 and diagonal matrix $K = \operatorname{diag}(k_1, k_2, \ldots, k_n)$ with $k_i \geq 0$ $(i = 1, 2, \ldots, N)$, then all eigenvalues of the matrix $C - K$ are negative.*

3. Main Results

In this section, we present a scheme to make a complex network achieve cluster projective synchronization. First of all, we propose the concept of interlink and intralink nodes. Node i is said to be the interlink node if i belongs to $\widetilde{U}_{\delta_i}$, while i is said to be the intralink node if i belongs to $U_{\delta_i} - \widetilde{U}_{\delta_i}$.

According to the diffusive coupling condition of matrix A, we have

$$\varepsilon \sum_{j=1}^{N} c_{ij}\Gamma S_{\delta_i}(t) = 0, \quad U_{\delta_i} - \widetilde{U}_{\delta_i}. \tag{11}$$

So the error dynamical system is described as follows:

$$\begin{aligned} D_*^q e_i(t) &= Ae_i(t) + F\left(x_i(t)\right) + M(A - B)S_{\delta_i}(t) \\ &\quad - Mg\left(S_{\delta_i}(t)\right) + \varepsilon \sum_{j=1}^{N} c_{ij}\Gamma e_j(t) + u_i(t). \end{aligned} \tag{12}$$

In order to make the network achieve cluster synchronization, the control input u_i is designed as follows:

$$u_i = \begin{cases} -F\left(x_i(t)\right) - M(A - B)S_{\delta_i} + Mg\left(S_{\delta_i}(t)\right) - k_i e_i(t) & i \in \widetilde{U}_{\delta_i}, \\ 0 & i \in U_{\delta_i} - \widetilde{U}_{\delta_i}, \end{cases} \tag{13}$$

where $k_i > 0$ is the feedback control gain, which can adjust the synchronization speed.

Theorem 6. *For a certain fractional-order q, the fractional-order complex network (6) can achieve the cluster projective synchronization with controller (13) if*

$$\left|\arg\left(A + \lambda_i \varepsilon\Gamma\right)\right| > \frac{q\pi}{2}, \quad i = 1, 2, \ldots, N. \tag{14}$$

Proof. Combining (6) and (13), one has

$$D_*^q e_i(t) = \left(Ae_i(t)\right) + \varepsilon\sum_{i=1}^{N} c_{ij}\Gamma e_j(t) - k_i e_i(t), \quad 1 \leq i \leq N. \tag{15}$$

Denote $K = \operatorname{diag}(k_1, k_2, , \ldots, k_N)$ with $k_i = 0$ for all $U_{\delta_i} - \widetilde{U}_{\delta_i}$, $e(t) = (e_1(t), e_2(t), \ldots, e_N(t)) \in R^{n\times N}$; thus one can obtain the following equation:

$$D_*^q e(t) = Ae(t) + \varepsilon(C - K)e(t)\Gamma. \tag{16}$$

Since C is an irreducible matrix and $K = \operatorname{diag}(k_1, k_2, \ldots, k_N)$ with $k_i \geq 0$ $(i = 1, 2, \ldots, N)$, according to Lemmas 4 and 5, there exists unitary $\xi = (\xi_1, \xi_2, \ldots, \xi_N)$ such that

$$(C - K)\xi = \xi\Lambda, \tag{17}$$

where $\Lambda = \operatorname{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and λ_i is the eigenvalue of matrix $C-K$ and satisfies $0 > \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N = \lambda_{\min}(C-K)$.

It follows from (16) and (17) that

$$D_*^q e(t) = (Ae(t))\xi + \varepsilon\Gamma e(t)\xi\Lambda. \quad (18)$$

Denote $\beta(t) = e(t)\xi$, $\beta(t) = (\beta_1(t), \beta_2(t), \ldots, \beta_N(t))$; thus

$$D_*^q \beta(t) = A\beta(t) + \varepsilon\Gamma\beta(t)\Lambda; \quad (19)$$

that is,

$$D_*^q \beta_i(t) = (A + \lambda_i\varepsilon\Gamma)\beta_i(t), \quad i = 1, 2, \ldots, N. \quad (20)$$

According to Lemma 3, system (19) is asymptotically stable if and only if all the eigenvalues of $A + \lambda_i\varepsilon\Gamma$ satisfy $|\arg(A + \lambda_i\varepsilon\Gamma)| > q\pi/2$, which implies that system (6) can achieve the cluster projective synchronization. □

Based on Theorem 6, a corollary can be easily derived as follows.

Corollary 7. *If matrix $C - K$ is irreducible, for a certain fractional-order q, the fractional-order complex network* (6) *can achieve cluster projective synchronization via controller* (13) *if*

$$\left|\arg\left(\overline{\lambda}_{ik}(A + \lambda_i\varepsilon\Gamma)\right)\right| > \frac{q\pi}{2}, \quad k = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, N, \quad (21)$$

where $\lambda_{ik}(\cdot)$ is the eigenvalue of the ith node matrix $A + \lambda_i\varepsilon\Gamma$.

4. Numerical Example

In what follows, we take a representative example to demonstrate the effectiveness of the proposed approach for cluster projective synchronization.

Example 1. We arbitrarily set the network size as $N = 16$. Suppose this network consists of three communities. The size of three communities is $N_1 = 5$, $N_2 = 5$, and $N_3 = 6$. The local dynamics of nodes are described by Lorenz system. The fractional-order Lorenz system is given by

$$\begin{aligned} D_*^q x_1 &= a(x_2 - x_1), \\ D_*^q x_2 &= bx_1 - x_2 - x_1x_3, \\ D_*^q x_3 &= x_1x_2 - cx_3, \end{aligned} \quad (22)$$

where $a = 10$, $b = 28$, $c = 8/3$, parameters for which the system exhibits chaotic behavior. The chaotic attractor is depicted by Figure 1.

Without loss of generality, we suppose that $C_1 = \{1, 2, 3, 4, 5\}$, $C_2 = \{6, 7, 8, 9, 10\}$, and $C_3 = \{11, 12, 13, 14, 15, 16\}$.

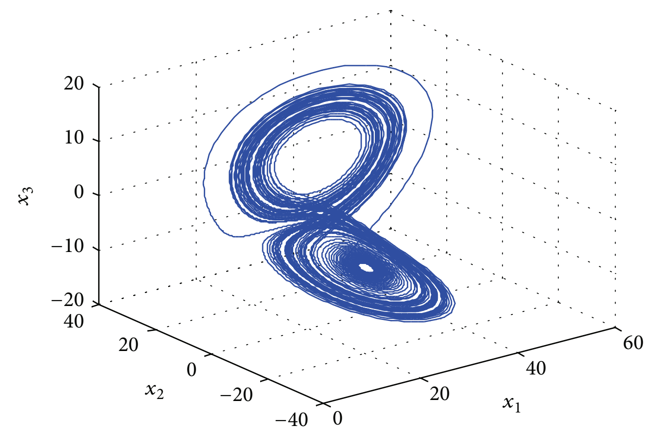

Figure 1: The chaotic orbit of the Lorenz system.

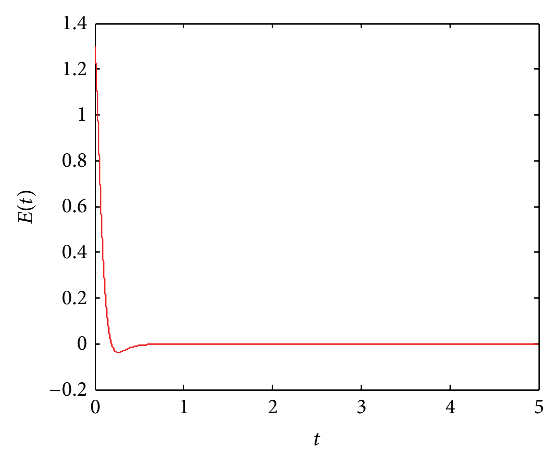

Figure 2: Time evolution of synchronization error $E(t)$.

In the following simulations, inner-coupling matrix Γ is chosen as identity matrix. The following quantities are used to investigate the process of cluster projective synchronization:

$$\begin{aligned} E(t) &= \sum_{i=1}^{N} \left\| x_i(t) - MS_{\delta_i}(t) \right\|, \\ E_{12}(t) &= \sum_{i=1}^{N} \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_1, \; v \in C_2, \\ E_{13}(t) &= \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_1, \; v \in C_3, \\ E_{23}(t) &= \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_2, \; v \in C_3, \end{aligned} \quad (23)$$

where $E(t)$ is the total error of cluster projective synchronization for all communities and $E_{12}(t)$, $E_{13}(t)$, $E_{23}(t)$ are the errors between two different communities. S is the states of Lorenz system. All the initial conditions are chosen randomly in $[0, 2]$. The results of simulation can be seen in Figures 2 and 3.

As can be seen from Figure 2, the synchronization errors $E(t)$ converge to zero. Figure 3 shows that the synchronization errors $E_{12}(t)$, $E_{13}(t)$, $E_{23}(t)$ do not converge to zero as

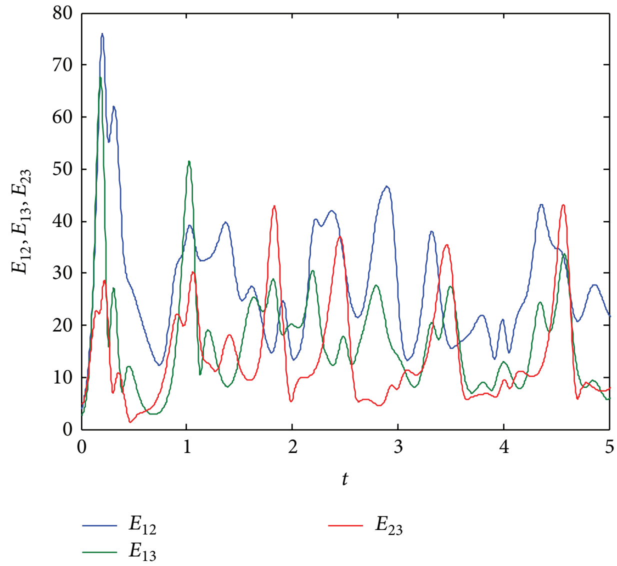

FIGURE 3: Time evolution of synchronization error $E(t)$.

$t \to \infty$. That is to say that the nodes in the same cluster reach synchronization and there is no synchronization among the different clusters, which implies that the desired cluster projective synchronization is achieved.

5. Conclusions

In this paper, we have investigated the cluster projective synchronization of complex networks with community structure. Based on the stability theory of the fractional-order differential system, the controllers are designed differently for the nodes in one community, which have direct connections to the nodes in the other communities and the nodes without direct connections to the nodes in the other communities. Several sufficient conditions for the network to achieve cluster projective synchronization are derived. Finally, a representative numerical example is provided to illustrate the effectiveness of the derived theoretical results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable comments and suggestions in helping them improve this paper.

References

[1] S. H. Strogatz, "Exploring complex networks," *Nature*, vol. 410, no. 6825, pp. 268–276, 2001.

[2] A. B. Horne, T. C. Hodgman, H. D. Spence, and A. R. Dalby, "Constructing an enzyme-centric view of metabolism," *Bioinformatics*, vol. 20, no. 13, pp. 2050–2055, 2004.

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, 2006.

[4] J. Hu, Z. Wang, B. Shen, and H. Gao, "Gain-constrained recursive filtering with stochastic nonlinearities and probabilistic sensor delays," *IEEE Transactions on Signal Processing*, vol. 61, no. 5, pp. 1230–1238, 2013.

[5] J. Hu, Z. Wang, H. Dong, and H. Gao, "Recent advances on recursive filtering and sliding mode design for networked nonlinear stochastic systems: a survey," *Mathematical Problems in Engineering*, vol. 2013, Article ID 646059, 12 pages, 2013.

[6] J. Lü, X. Yu, and G. Chen, "Chaos synchronization of general complex dynamical networks," *Physica A*, vol. 334, no. 1-2, pp. 281–302, 2004.

[7] X. Wang and G. Chen, "Synchronization in small-world dynamical networks," *International Journal of Bifurcation and Chaos*, vol. 12, no. 1, pp. 187–192, 2002.

[8] M. Yang, Y. Liu, Z. You, and P. Sheng, "Global synchronization for directed complex networks," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 3, pp. 2127–2135, 2010.

[9] K. Wang, X. Fu, and K. Li, "Cluster synchronization in community networks with nonidentical nodes," *Chaos*, vol. 19, no. 2, Article ID 023106, 2009.

[10] C. Li and G. Chen, "Synchronization in general complex dynamical networks with coupling delays," *Physica A*, vol. 343, pp. 263–278, 2004.

[11] S. Zheng, G. Dong, and Q. Bi, "Impulsive synchronization of complex networks with non-delayed and delayed coupling," *Physics Letters A*, vol. 373, no. 46, pp. 4255–4259, 2009.

[12] J.-W. Wang, Q. Ma, L. Zeng, and M. S. Abd-Elouahab, "Mixed outer synchronization of coupled complex networks with time-varying coupling delay," *Chaos*, vol. 21, no. 1, Article ID 013121, 2011.

[13] M. Ayati and A. Khaki-Sedigh, "Adaptive control of nonlinear in parameters chaotic system via Lyapunov exponents placement," *Chaos, Solitons and Fractals*, vol. 41, no. 4, pp. 1980–1986, 2009.

[14] X.-Y. Wang and J.-M. Song, "Synchronization of the fractional order hyperchaos Lorenz systems with activation feedback control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 8, pp. 3351–3357, 2009.

[15] J. Hu, Z. Wang, B. Shen, and H. Gao, "Quantised recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements," *International Journal of Control*, vol. 86, no. 4, pp. 650–663, 2013.

[16] J. Hu, D. Chen, and J. Du, "State estimation for a class of discrete nonlinear systems with randomly occurring uncertainties and distributed sensor delays," *International Journal of General Systems*, vol. 43, no. 3-4, pp. 387–401, 2014.

[17] J. Hu, Z. Wang, and H. Gao, "Recursive filtering with random parameter matrices, multiple fading measurements and correlated noises," *Automatica*, vol. 49, no. 11, pp. 3440–3448, 2013.

[18] L.-X. Yang and J. Jiang, "Adaptive synchronization of drive-response fractional-order complex dynamical networks with uncertain parameters," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 5, pp. 1496–1506, 2014.

[19] W. Yu, G. Chen, and J. Lü, "On pinning synchronization of complex dynamical networks," *Automatica*, vol. 45, no. 2, pp. 429–435, 2009.

[20] M. Porfiri and F. Fiorilli, "Experiments on node-to-node pinning control of Chua's circuits," *Physica D*, vol. 239, no. 8, pp. 454–464, 2010.

[21] L.-X. Yang and J. Jiang, "Hybrid projective synchronization of fractional-order chaotic systems with time delay," *Discrete Dynamics in Nature and Society*, vol. 2013, Article ID 459801, 8 pages, 2013.

[22] D. Matignon, "Stability results of fractional differential equations with applications to control processing," in *Proceedings of the IEEE-SMC International Association for Mathematics and Computers in Simulation (IMACS '96)*, pp. 963–968, Lille, France, 1996.

[23] K. Li, M. Small, and X. Fu, "Generation of clusters in complex dynamical networks via pinning control," *Journal of Physics A: Mathematical and Theoretical*, vol. 41, no. 50, Article ID 505101, 2008.

[24] W. Guo, "Lag synchronization of complex networks via pinning control," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 5, pp. 2579–2585, 2011.