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Designing Cellular Mobile Network Using Lagrangian Based Heuristic

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라그랑지안 기반의 휴리스틱 기법을 이용한 셀룰러 모바일 네트워크의 설계

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Cellular network is comprised of several base stations which serve cellular shaped service area and each base station (BS) is connected to the mobile switching center (MSC). In this paper, the configuration modeling and algorithm of a cellular mobile network with the aim of minimizing the overall cost of operation (handover) and network installation cost (cabling cost and installing cost of mobile switching center) are considered. Handover and cabling cost is one of the key considerations in designing cellular telecommunication networks. For real-world applications, this configuration study covers in an integrated framework for two major decisions: locating MSC and assigning BS to MSC. The problem is expressed in an integer programming model and a heuristic algorithm based on Lagrangian relaxation is proposed to resolve the problem. Searching for the optimum solution through exact algorithm to this problem appears to be unrealistic considering the large scale nature and NP-Completeness of the problem. The suggested algorithm computes both the bound for the objective value of the problem and the feasible solution for the problem. A Lagrangian heuristics is developed to find the feasible solution. Numerical tests are performed for the effectiveness and efficiency of the proposed heuristic algorithm. Computational experiments show that the performance of the proposed heuristics is satisfactory in the quality of the generated solution.

Keywords: Cellular Network Design, BS Assignment, Lagrangian Relaxation, Heuristic

1. Introduction

Cellular phone has become an absolute necessity for the people in any developed countries. Even in the developing country, it is very easy to see people using cellular phones in the street. But the technical and design issues behind the cellular service are not so easy. Especially, the design of large cellular networks is a complex task with a great impact on the quality of service and the cost of the network.

Cellular systems can cover a large number of users over a large geographic area with a limited frequency spectrum. This can be possible by dividing a large geographic area into a small area called cell. Each base station (BS) which manages the wireless call within the cell limits the power of signal to a small geographic area so that the same radio channels may be reused with another BS located some distance away. Moreover, a sophisticated switching technique, called

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handover, enables a wireless call to proceed uninterruptedly when the mobile user moves from one cell to another, where handover is the process managed by a mobile switching center (MSC) to maintain the call quality as any mobile user moves in and out of the range of each BS (Rappaport, 1996). The BS is employed to serve as a bridge between mobile users in the cell and its associated MSC. The MSC has the functions of coordinating the activities of all BSs and of connecting the cellular system to the public communication network (Rappaport, 1996). The handover between two cells connected to the same MSC is managed by the common MSC without involving any backbone network. On the other hand, the handover between two cells connected to the different MSCs goes through a complicated handover procedure.

Handling a user's mobility is a very important issue in cellular network. Therefore, it is also very important, in designing a cellular network, to connect cells having very high handover frequencies to the same MSC (Jabbari *et al*., 1995), while the cabling cost between BSs and the MSCs also should be considered. This leads to an issue of taking care of a trade-off between the installation cost, which includes cabling and MSC setup cost, and the handover cost, which is explicitly treated in this paper to configure a cellular network optimally under the associated constraints.

This type of design problem for cellular network has been considered in the previous works (Merchnat and Sengupta, 1995; Kim and Kim, 1997; Rajalakshmi *et al*., 2010). In Merchant and Sengupta (1995)'s paper, the problem was first developed as an integer programming model, and a heuristic algorithm was proposed. Kim and Kim (1997) dealt with a somewhat relaxed problem where handover cost was set to a constant for every adjacent pair of cells, and the simulated annealing method was used to solve the problem. In Rajalakshmi *et al*. (2010)'s paper, a hybridized heuristic approach based on iterative local search and simulated annealing approach was proposed to solve the assignment problem of cellular mobile network.

Several other heuristic techniques have been applied to the similar problem, such as Tabu Search (TS) (Pierre and Houéto, 2002), Simulated Annealing (Menson and Gupta, 2004), Genetic algorithm (Salcedo-Sanza and Yaob, 2008). Dianati *et al*. (2003) suggested a Genetic based solution procedure for the problem. They also proposed a new formulation more convenient for Genetic algorithm implementation. The computational results showed that the Genetic based heuristic outperformed the Tabu Search approach. The Genetic algorithm and Tabu Search have been applied to find a good topology of the various communication networks (Oh and Kim, 2008; Lee *et al*., 2008). Andre *et al*. (2005) suggested a hybrid solution approach of Tabu Search and variable neighborhood search methods. The problem was formulated as a comprehensive integer programming model integrating MSC location problem and BS-to-MSC assignment problem. Saha *et al*. (2007) considered cell-to-switch assignment (CSA) problem in which the hybrid cost, comprising handover cost between adjacent cells, and the cable cost between cells and switches, was minimized subject to the constraint that the call volume to be handled by a switch should not exceed its traffic handling capacity. The problem assumed quasi-static/dynamic assignment environment. In a quasi-static assignment, clustering is relatively fixed over a long period of time, but it changes with major (i.e., diurnal) variation in system parameters. In a dynamic assignment, clustering varies on the fly with changes in parameters (i.e., traffic demand). In the quasi-static or dynamic assignment, clustering must be optimized whenever traffic pattern is changed. So they concentrated on the time efficiency rather than optimality.

In Hung and Song (2002)'s paper, a combinatorial search method was adopted as a solution method. Fournier and Pierre (2005) applied the ant colony optimization meta-heuristic to the CSA assignment problem. But the algorithm shows lacks of robustness with respect to parameter values. Certain values significantly increase the algorithm execution time without necessarily generating better results. Mandal *et al*. (2007) also dealt with CSA problem and formulated the problem as a state-space search problem. A lower bound heuristic for the block depth first search (BDFS) algorithm was developed to solve the CSA problem. The algorithm considered the use of preferred (or potential) locations for switches. Quintero and Pierre (2002) proposed Memetic algorithm to solve the CSA assignment problem. They compared their algorithm with Tabu Search and other heuristic. Quintero and Pierre (2003a) proposed an evolutionary approach (Memetic algorithms) to solve the problem of assigning cells to switches in cellular mobile networks. They compare the solution from Memetic algorithms with those from Genetic algorithm and Tabu Search. Quintero and Pierre (2003b) compared three meta-heuristic algorithms-Tabu Search, Simulated annealing and Parallel Genetic algorithm with migrations-for assigning cell to switches in cellular mobile network. Saha *et al*. (2000) proposed simple assignment heuristic for assignment of cells to switches problem. Goudos *et al*. (2010) suggested Discrete Particle Swarm optimization algorithms to solve cell assignment problem. Particle Swarm optimization is a population-based stochastic optimization technique inspired by the social behavior of birds flocking, where a swarm of individuals (particles) flies through the search space. The particles move into the search space by following the current optimum ones. The system is at first initialized with a population of random particles (solutions) and searches for optimum by updating the positions of the particles in any iteration. Each particle position is updated by finding two optimum values, the particle's best solution (fitness) achieved and the global best value obtained so far by any particle. After finding two optimum values, each particle updates its position and velocity. The algorithm is executed repeatedly until the number of iterations reaches a specified number or the velocity updates are close to zero.

In this paper, a solution algorithm using Lagrangian relaxation based heuristic method is proposed for solving this complex cellular network design problem. This method may have the advantage of producing a (Lagrangian) bound on the optimal solution, which can be used to evaluate the quality of any feasible solution. In the problem analysis, a problem reduction property with valid inequality is characterized to reduce the solution space of Lagrangian sub-problem. In the solution algorithm, we suggest the algorithm to find a violated valid inequality. Our solution algorithm is based on a Lagrangian relaxation technique and subgradient method. The bound for the optimal solution is derived by the solution of the Lagrangian dual problem, and the feasible solution is computed through a Lagrangian heuristic. Computational results show that the algorithm can solve practical size problems with good solution quality.

The contribution of this paper is that the suggested Lagrangian based algorithm can be used to obtain a bound for the optimal solution of CSA problem and moreover, the suggested Lagrangian based heuristic algorithm can be used to solve real problems with large number of cells.

Our problem of locating MSC and assigning BS to MSC, simultaneously is now specifically described as the following problem environments. First, we are given the site location of BS offices, candidate MSC offices, and also the potential links from each BS to every MSC. Second, we are given the input data of traffic requirements measured in Erlang for all BS. Accordingly, transmission rate of the cable to be installed on each potential link is known. Third, the handover traffic for a pair of BSs is initially given as an input data in Erlang. This traffic data is then converted into the handover (operation) cost by any conversion process which considers the various administrative constraints such as routing information and physical location. We assume that the operation cost function, which converts the handover traffic to handover cost, is known in advance. The described problem considers three major cost elements-the cost of establishing a MSC, the cost of placing a cable, and the operation cost of the handover. Thus, two kinds of decisions have to be made to minimize the associated total cost. One is to determine which potential MSCs to open and the other is to find a link from each BS to the selected MSC under the MSC capacity constraints. Note that the handover cost also has to be considered when assigning of each BS to MSC.

Li *et al*. (1997) dealt with the same model described above. In the paper, they suggested local-search-type heuristic algorithm but they did not consider the bound for the algorithm with which effectiveness of the algorithm can be measured.

The complex interrelationship among the costs of these elements, together with the huge number of possible network configurations, makes it extremely hard to find the overall network design plan, where all three costs are optimally traded off. For this reason, a solution procedure searching for the optimal solution may not terminate within a reasonable amount of computing time. Therefore, to deal with realistic-sized problems, it is worth developing effective heuristic procedure. The solution quality of the heuristic procedure can be measured by calculating the gap between the heuristic solution value and the Lagrangian lower bound.

The organization of this paper is briefed as follows. In Section 2, an integer programming model for the problem is formulated. The Lagrangian relaxation method is proposed in Section 3. In Section 4, efficient heuristic procedures to solve the problem are derived and applied to make the associated numerical experiments. In Section 5, the computational results of the suggested algorithm for the network planning of a cellular mobile network are reported. We conclude our work in Section 6.

2. Problem Modeling

2.1 Notation

We refer to the Merchant and Sengupta (1995)'s paper for the basic formulation of the problem. Given a service area with a set of BSs, *N* and a set of MSCs, *M* representing all candidates MSCs that may support BSs, where the locations of BS and candidate MSC are fixed and known in advance. A cellular mobile network can be represented by a graph, where each BS is a node and any two adjacent BSs are connected by an undirected link. In this graph, a link represents that there exists any handover traffic between two adjacent BSs connected by the link. Let *E* be the set of such undirected link set.

With our network definition, the following additional notations are needed.

h_e^1 : Handover cost occurring between two adjacent BSs *i* and *j* that are connected to the different MSCs, $e=(i, j)\in E$

h_e^2 : Handover cost occurring between two adjacent BSs *i* and *j* that are connected to the same MSCs, $e=(i, j)\in E$

c_{im} : Cabling cost between BS *i* and MSC *m*

λ_i : Number of calls that cell *i* handles per unit time

M_m : Call handling capacity of MSC *m* per unit time

s_m : Installing cost of MSC *m*

$$x_{im}=\begin{cases}1 & \text{if cell } i \text{ is assigned to MSC } m\\ 0 & \text{otherwise}\end{cases}$$

$$y_{em}=\begin{cases}1 & \text{if both cell } i \text{ and } j \text{ are assigned to the same MSC } m,\ e=(i,\ j)\in E\\ 0 & \text{otherwise}\end{cases}$$

$$y_{e}=\begin{cases}1 & \text{if both cell } i \text{ and } j \text{ are assigned to the same MSC},\ e=(i,\ j)\in E\\ 0 & \text{otherwise}\end{cases}$$

$$z_{m}=\begin{cases}1 & \text{if MSC } m \text{ is installed}\\ 0 & \text{otherwise}\end{cases}$$

As described in the previous section, there are two types of handover cost, one is inter-MSC handover cost, h_e^1, the other is intra-MSC handover cost, h_e^2. Generally, inter-MSC handover cost is much greater than intra-MSC handover cost. It is because inter-MSC handover has more complicate handover procedure. And the inter-MSC handover results in a longer handover delay which may result in the greater call drop probability and service degradation possibility.

2.2 Mathematical Formulation

The problem can be defined as assigning a BS to an MSC such that the sum of the cabling cost for connecting the BS to the MSC, installing cost for MSC, and the handover cost between BSs is minimized under the MSC capacity constraints. Then, the problem can be modeled as an integer programming by reformulating the model of the Merchant and Sengupta (1995). The formulated model is expressed as follows :

Problem *ABM* :

$$Z_p = Minimize \sum_{i \in N} \sum_{m \in M} c_{im} x_{im} + \sum_{e \in E} h_e^1 (1 - y_e) + \sum_{e \in E} h_e^2 y_e + \sum_{m \in M} s_m z_m$$

subject to

$$\sum_{m \in M} x_{im} = 1 \qquad \forall i \in N \qquad (1)$$

$$x_{im} \le z_m \qquad \forall i \in N, \forall m \in M \qquad (2)$$

$$\sum_{i \in N} \lambda_i x_{im} \le M_m z_m \qquad \forall m \in M \qquad (3)$$

$$y_{em} \le x_{im}, y_{em} \le x_{jm} \qquad e = (i, j) \in E, \forall m \in M \qquad (4)$$

$$x_{im} + x_{jm} \le y_{em} + 1 \qquad e = (i, j) \in E, \forall m \in M \qquad (5)$$

$$y_e = \sum_{m \in M} y_{em} \qquad e \in E \qquad (6)$$

$$x_{im}, y_{em}, y_e, z_m \in \{0, 1\} \qquad \forall i \in N, \forall m \in M, \forall e \in E \qquad (7)$$

The objective function of problem *ABM* is composed of two handover cost terms, one cabling cost term, and MSC installing cost term. One of the handover costs is concerned with the inter-MSC handover cost and the other one is with the intra-MSC handover cost. The intra-MSC handover cost represents the handover cost between two BSs that are connected to the same MSC. The inter-MSC handover cost represents the handover cost between BSs that are connected to the different MSCs. The constraints (1) imply that each BS should be assigned to only one MSC. The constraints (2) are to tighten LP (Linear Programming) bound. The constraints (3) are about MSC capacity constraints. The constraints (4) and (5) imply that an inter-MSC handover cost term in the objective function equals to zero only if both cells *i* and *j* are assigned to the same MSC *k*.

The objective function of the problem *ABM* has a characterization that can be reduced to a simpler form by manipulating its coefficients. Two handover cost terms can be reduced to $-\sum_{e \in E} (h_e^1 - h_e^2) y_e + \sum_{e \in E} h_e^1$. Now, letting $h_e = h_e^1 - h_e^2 \ge 0$ and incorporating constraints (6) into the objective function, the problem can be rewritten as :

$$Min \sum_{i \in N} \sum_{m \in M} c_{im} x_{im} - \sum_{e \in E} \sum_{m \in M} h_e y_{em} + \sum_{m \in M} s_m z_m + \sum_{e \in E} h_e^1$$

subject to

(1), (2), (3), (4), (5), (7)

3. Lagrangian Relaxation Method

3.1 Lagrangian relaxation and subproblems

The model that we are considering is a very large-sized zero-one integer programming problem. An instance of this model with 400 BSs and 20 candidate MSC locations has at least 48,020 integer variables and 128,420 constraints. For this reason, we employ the Lagrangian relaxation method and decomposition approach.

In this section, the Lagrangian decomposition and the solution approach are presented. By relaxing constraint set (1) with unrestricted Lagrange multipliers $\alpha_i, \ \forall i \in N$, we arrive at the following relaxed formulation :

Problem *LABM* (α) :

$$Z_{LABM}(\alpha) = Min \sum_{i \in N} \sum_{m \in M} c_{im} x_{im} - \sum_{e \in E} \sum_{m \in M} h_e y_{em} + \sum_{m \in M} s_m z_m + \sum_{e \in E} h_e^1 + \sum_{i \in N} \alpha_i \Big(1 - \sum_{m \in M} x_{im}\Big)$$

subject to

(2), (3), (4), (5), (7)

The objective function of *LABM* (α) can be manipulated algebraically, and be rewritten as :

$$Z_{LABM}(\alpha) = Min \sum_{m \in M} \Big(\sum_{i \in N} (c_{im} - \alpha_i) x_{im} - \sum_{e \in E} h_e y_{em} + s_m z_m \Big) + \sum_{e \in E} h_e^1 + \sum_{i \in N} \alpha_i$$

The last two terms in the objective function of *LABM* (α) can be omitted because they are constant value. Then the problem *LABM* (α) can be decomposed into following $|M|$ subproblems :

Problem $LABM_m(\alpha)$:

$$Z_{LABM}^m(\alpha) = Min \sum_{i \in N} (c_{im} - \alpha_i) x_{im} - \sum_{e \in E} h_e y_{em} + s_m z_m$$

subject to

$$x_{im} \le z_m \qquad \forall i \in N \qquad (8)$$

$$\sum_{i \in N} \lambda_i x_{im} \le M_m z_m \qquad (9)$$

$$y_{em} \le x_{im}, y_{em} \le x_{jm} \qquad e=(i,j) \in E \qquad (10)$$

$$x_{im} + x_{jm} \le y_{em} + 1 \qquad e=(i,j) \in E \qquad (11)$$

$$x_{im}, y_{em}, z_m \in \{0,1\} \qquad \forall i \in N, \forall e \in E \qquad (12)$$

Note that the constraints (11) can be dropped since the costs h_e are non-negative so that an optimal solution will automatically satisfy it.

The problem $LABM_m(\alpha)$ without variables z_m is referred to the Knapsack Quadratic Problem (KQP) (Johnson *et al*., 1993). We can show that the subproblem $LABM_m(\alpha)$ can be reduced to the KQP. Let $KQP_m(\alpha)$ be the problem $LABM_m$ (α) without variables z_m. The solution space of $LABM_m(\alpha)$ can be divided into two mutually disjoint sets, one is the solution space with the variable z_m set to zero and the other is set to one. When the variable z_m is set to zero, all other variables are automatically to be zero. Accordingly, the objective function value is to be zero. When the variable z_m is set to one, the problem is the same as $KQP_m(\alpha)$.

Problem $KQP_m(\alpha)$

$$Z^m_{KQP}(\alpha) = Min \sum_{i \in N} (c_{im} - \alpha_i) x_{im} - \sum_{e \in E} h_e y_{em}$$

subject to

$$\sum_{i \in N} \lambda_i x_{im} \le M_m \qquad (13)$$

$$y_{em} \le x_{im}, y_{em} \le x_{jm} \qquad e=(i,j) \in E \qquad (14)$$

$$x_{im}, y_{em}, z_m \in \{0,1\} \qquad \forall i \in N, \forall e \in E \qquad (15)$$

Then, the relation between $Z^m_{KQP}(\alpha)$ and $Z^m_{LABM}(\alpha)$ is as follow:

$$Z^m_{LABM}(\alpha) = \begin{cases} Z^m_{KQP}(\alpha) + s_m, & \text{if } Z^m_{KQP}(\alpha) < -s_m \\ 0 & otherwise \end{cases}$$

Although the *KQP* is known to be NP-hard, Johnson *et al*. (1993) have developed an efficient branch and bound algorithm using cutting planes. In the usual branch and bound process, the linear programming relation of KQP, in which constraints (15) is replaced by $0 \le x_{im}, y_{em} \le 1$ provides a lower bound on the objective function value of $KQP_m(\alpha)$. For the detail of cutting planes refers to Johnson *et al*. (1993).

We develop an additional cutting plane for problem $LABM_m(\alpha)$. Let node set *C* be an independent set if $\sum_{i \in C} \lambda_i \le M_m$, otherwise *C* is a dependent set for $C \subseteq N$. A dependent set is minimal if all of its subsets are independent.

Proposition :

If we assume the graph *G* (*N*, *E*) is complete. For $C \subseteq N$, let $CE(C) = \{(i,j) \mid i \in C, j \in C, i \ne j\}$ and *C* be a minimal dependent set. Then, the following inequality is valid.

$$|C| \sum_{i \in C} x_{im} - \frac{(|C|+2)(|C|-1)}{2} z_m \le \sum_{e \in CE(C)} y_{em} \qquad (16)$$

where, $|C|$ is the cardinality of a set *C*.

Proof :

If $z_m = 0$, then all other variables have zero values. Therefore, the inequality (16) is valid. Otherwise, i.e., $z_m = 1$, then for an arbitrary feasible solution,

$$x_{im} = \begin{cases} 1 & i \in S \\ 0 & otherwise \end{cases}, \; y_{em} = \begin{cases} 1 & e \in CE(S) \\ 0 & otherwise \end{cases},$$

$$\sum_{i \in S} x_{im} = |S| \le |C| - 1, \text{ and } \sum_{e \in CE(S)} y_{em} = \frac{|S|(|S|-1)}{2}$$

where *S* is the set of BSs that are connected to MSC *m*, given that $z_m = 1$. Then the validity of the (16) can be established as

$$\begin{aligned} |C| \sum_{i \in S} x_{im} - \sum_{e \in CE(S)} y_{em} &= |C||S| - \frac{|S|(|S|-1)}{2} \\ &= \frac{-|S|^2 + (1+2|C|)|S|}{2} = \frac{-(|S| - (|C|+1/2))^2}{2} \\ &\quad + \frac{(|C|+1/2)^2}{2} \le \frac{(|C|+2)(|C|-1)}{2} \\ &= \frac{(|C|+2)(|C|-1)}{2} z_m \end{aligned}$$

This completes the proof.

But, as explained in the introduction section, the graph we are considering is not a complete graph. To incorporate the inequality into our solution algorithm, we modify the inequality into a non-complete graph case with constraints (10). For $C \subseteq N$, let *NoE* (*C*)= $\{(i, j) \mid i \in C, j \in C, i \ne j, (i, j) \notin E\}$ and *E* (*C*) $= \{(i,j) \mid i \in C, j \in C, (i,j) \in E\}$. Then, for a dependent set *C* the following inequality is valid.

$$|C| \sum_{i \in C} x_{im} - \frac{(|C|+2)(|C|-1)}{2} z_m \le \sum_{e \in E(C)} y_{em} + \frac{1}{2} \sum_{e \in NoE(C)} (x_{im} + x_{jm}) \qquad (17)$$

The separation problem for the inequality involves finding a minimal dependent set that minimizes a linear function. Since this problem is NP-hard, it follows that the separation problem for inequality (17) is NP-hard. Therefore, we suggest heuristics procedures to find the violated inequality. To find a violated inequality, we use a greedy approach. The greedy algorithm starts with an x_{im} corresponding to the biggest fractional value. At each subsequent iteration, x_{im} with fractional value is selected. The process stops as soon as the BSs in the set construct a dependent set or when there is no remaining fractional x_{im}. If there is no fractional x_{im} remained but the set of all fractional x_{im} does not dependent set, then x_{im} corresponding to the one is chosen until the set of selected BSs are corresponding to a dependent set.

To optimize the subproblem $LABM_m(\alpha)$, we use the branch and bound with cutting plane. The LP relaxation of problem $LABM_m(\alpha)$ provides a lower bound on $Z^m_{LABM}(\alpha)$. Since the LP relaxation of the problem $LABM_m(\alpha)$ may provide a poor bound, cutting plane algorithm is devised to improve the lower bound. In the cutting plane algorithm, we add the violated inequality (17) found by the above separation heuristic or tree inequality (Johnson *et al*., 1993) to the current LP, and solve the resulting LP to get a new optimal LP solution. We repeat this process until the current optimal LP solution is integral or no further inequality is found. In the second case, the branch and bound phase is activated.

In the LP based branch and bound procedure, best bound rule is used for node selection. For a given fractional solution to LP, we select the biggest fractional x_{im} variables as branching variable. Then, we make two new nodes in the enumeration tree, one with $x_{im}=0$, the other with $x_{im}=1$.

3.2 Lagrangian dual search procedure

For the dual problem, a subgradient optimization method is applied to obtain lower bounds on Z_p. Finding the optimal Lagrangian multipliers is known to be a very difficult problem. Subgradient optimization method usually gives a good, not necessarily optimal, set of multipliers. Given an initial multiplier, a sequence of multipliers is generated using the following rule:

Let $x_{im}(\alpha^t)$, $y_{em}(\alpha^t)$, $z_m(\alpha^t)$ be the optimal solutions to the Lagrangian problem for a fixed (α^t) at the *t*-th iterations. These values can be used to calculate the subgradient directions by the following formula:

$$\gamma_i(\alpha^t)=1-\sum_{m\in M}x_{im}(\alpha^t) \qquad \forall i\in N$$

Each multiplier for the (*t*+1)th iteration is given by

$$\alpha_i^{t+1}=\alpha_i^t+\tau^t\omega_i(\alpha_i^{t+1})$$

where

$$\omega_i(\alpha^t)=\gamma_i(\alpha^t)+\theta^t\omega_i(\alpha^{t-1}),$$

$$\theta^t=\eta\frac{\sum_{i\in N}\omega_i(\alpha^t)\gamma_i(\alpha^t)}{\sum_{i\in N}|\omega_i(\alpha^t)|^2},\ \tau^t=s^t\frac{Z_f-Z_{LABM}(\alpha^t)}{\sum_{i\in N}|\gamma_i(\alpha^t)|}$$

Z_f is the objective function value of a feasible solution and η, s^t are scalar values (Camerini *et al*., 1975). η is initially set to 1.0 and s^t is a scalar initially set to 0.5, and is then halved whenever the lower bound does not improve in 6 consecutive iterations.

Lagrangian solution obtained from the relaxed problem is rarely feasible to the original problem. If the Lagrangian solution is feasible to the original problem, the solution is the optimal solution. Although it is infeasible to the original problem, a heuristic procedure can be used to obtain a feasible solution. In the next section, we suggest the heuristic procedures.

4. Lagrangian Heuristic Solution Procedures

In order to generate a feasible solution, the heuristic uses the solution of *LABM*. The heuristic algorithm is invoked just after each iteration of the subgradient optimization algorithm. For each violated one among the constraints (1), we make it be feasible by modifying *x* solution. The modifications are done by comparing between the value of cabling cost coefficient and that of handover cost coefficient in the objective function.

Let (x^*,y^*,z^*) be the solution of *LABM*. Then, unfeasibility in constraints (1) is divided into two cases. One is under assigned case, $\sum_{m\in M}x^*_{im}=0$, the other is over assigned case, $\sum_{m\in M}x^*_{im}\geq 2$. Let *A* be the set of BSs corresponding to the under assigned case and *B* be the set of BSs corresponding to over assigned case, i.e., $A=\left\{i|\sum_{m\in M}x^*_{im}=0, i\in N\right\}$, $B=\left\{i|\sum_{m\in M}x^*_{im}\geq 2, i\in N\right\}$. Let C_m be the set of BSs that are assigned to the MSC *m*, $C_m=\left\{i|x^*_{im}=1, \forall i\in N\right\}$ and *D* be the set of MSCs that all of its assigned BSs, C_m, are in the set *B*, $D=\{m|i\in B, \forall i\in C_m\}$.

The first step of the heuristic procedure is to reduce surplus MSC. The procedure selects $m\in D$ corresponding to the maximum Z^m_{LABM} value. Let m^* be the selected MSC, $m^*=\arg\{\max_{m\in D}\{Z^m_{LABM}\}\}$, then set $z^*_m=0$ and all of its assigned BSs, x^*_{im}, to zero, and update the associated set *A*, *B*, C_m, *D*. Repeat this procedure until there is no surplus MSC, $|D|=0$.

At the end of the first step of the heuristic, there may exist any over assigned BSs. The second step of the heuristic procedure is concerning the modification of the over assigned BSs. For $i\in B$, let E_i be the set of MSCs that the BS *i* is assigned, $E_i=\{m|x^*_{im}=1 \text{ for } m\in M\}$. The algorithm selects the minimum cost MSC $m^*\in E_i$ for each $i\in B$ and reassigns BS *i* only to MSC m^*. Repeat this procedure until there is no over assigned BSs.

The third step of the procedure is concerning the modification of the under assigned BSs. Let F_i be the set of MSCs that are selected to be installed and have a remaining capacity for BS i, $F_i = \left\{m | z_m^* = 1, \sum_{i \in N} \lambda_i x_{im}^* + \lambda_i \le M_m \right\}$. For $i \in A$, the algorithm selects the minimum cost MSC $m^* \in F_i$ and assigns BS i to MSC m^*. And at the end of each iteration, update set F_i for all $i \in A$. Repeat this procedure until there is no under assigned BS.

<Figure 1> describes the structure of the overall solution procedure.

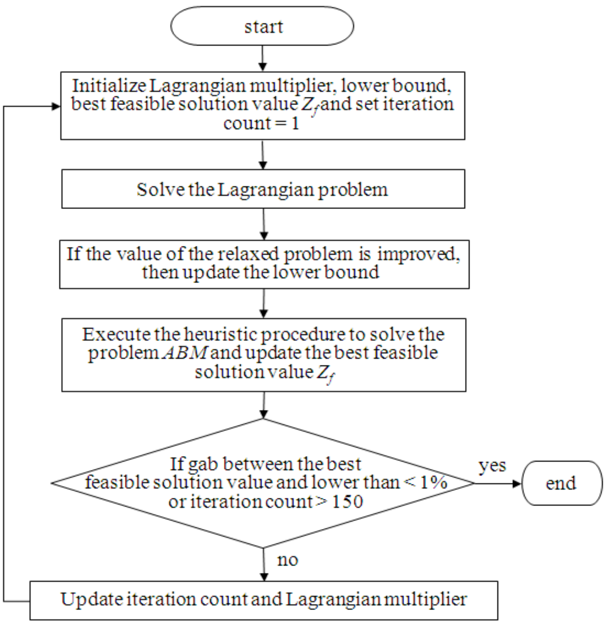

Figure 1. Overall Procedure of the Solution Algorithm

5. Computational Experiments

For the performance test, the proposed algorithm with sets of computational experiments was coded in C and carried out on a laptop computer. The problem data used in these experiments were generated systematically to find a range of different problem structures. Test problems were also generated by changing some parameters associated with the problem size.

Cellular network was assumed to be a hexagonal cell array and each BS was located at the center of its cell. Candidate MSC locations were drawn from a uniform distribution over a cell array. The capacity per MSC was assumed to be 5,000 Erlang (Rappaport, 1996) and the setup costs of MSC were generated from the uniform distribution from $s^{\min}$ to $s^{\max}$. The demand per cell, λ, was assumed to occur according to an exponential distribution. We assumed that the mean of the demand per cell is 50 Erlang. The cabling cost was assumed to be in proportional to Euclidean distances between BS and MSC. Let d_{im} be the Euclidean distance between BS i and MSC m, then the cabling cost c_{im} is defined as $c_{im} = p \cdot d_{im}$, where p is a positive multiplier. <Figure 2> and <Table 1> show the examples of the distance d_{im} between BS and MSC (Salcedo-Sanza and Yaob, 2008).

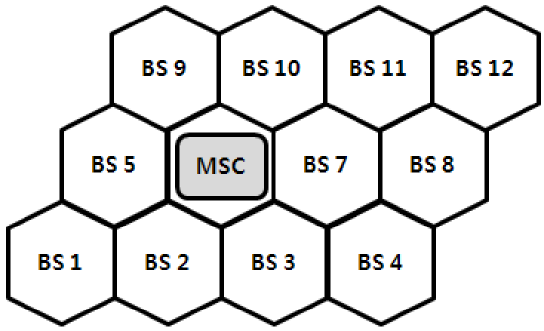

Figure 2. A Cellular Mobile Network

Table 1. Examples of BS and MSC Distances of <Figure 2>

#	x-coordinate	y-coordinate	d_{im}
BS 1	1	1	3.61
BS 2	3	1	2.24
BS 3	5	1	2.24
BS 4	7	1	3.61
BS 5	2	3	2.00
BS 7	6	3	2.00
BS 8	8	3	4.00
BS 9	3	5	2.24
BS 10	5	5	2.24
BS 11	7	5	3.61
BS 12	9	5	5.39
MSC	4	3	0.00

We assumed that the handover cost is in proportional to the handover traffic. The handover cost h_e can be calculated as follow :

$$h_e = h \cdot \mu_e$$

where h is the handover cost factor and μ_e is the handover traffic between two adjacent cell i and j and given by the following formula (Markoulidakis and Sykas, 1993) :

$$\mu_e = L_c \cdot \frac{a_c}{b_c} \cdot \sigma_c \cdot v_c \text{ (mobile users/}h\text{)}$$

where c is the border between two cell i and j, L_c is the length of the border c (km), a_c is the percentage of L_c corresponding to streets, b_c is the percentage of the border area covered with streets, σ_c is the density of power-on mobile user around border *c* and v_c is the average speed for mobile users moving near border (km/h). Without loss of generality, we assume that L_c, a_c, b_c and v_c for all BSs have the constant value. The density of power-on mobile users, σ_c, is assumed to be variable value which is proportional to the sum of demands of cells *i* and *j*, and is inverse proportional to the size of cell area if we assume that mobile users are distributed uniformly within a cell. The value $\frac{a_c}{b_c}$ is the ratio of each border to all borders in a hexagonal cell. If we assume that streets are distributed uniformly in a cell, the ratio $\frac{a_c}{b_c}$ can be set as 1/6.

In the urban area, the cell is small in size to cover high traffic density and because of small cell size even a pedestrian can cross the border of cell, which means that L_c can be 1 km and v_c would be less than 4 km. In the rural area, the cell is large enough to cover large area with small number of mobile users. Therefore, mobile users in a large cell should move faster in oder to cross the border of the cell than ones in a small cell. For a large cell size, L_c is less than 10 km and v_c would be around 40 km. The density of the mobile user near border in large cell will be inverse proportional to cell area because we assumed that density of user is uniformly distributed within cell. We can infer from this that $L_c \cdot \sigma_c \cdot v_c$ is linear function of the sum of demands of cells *i* and *j* independent of cell size. In our experiments, we assume $L_c \cdot \sigma_c \cdot v_c$ as $3.6 \cdot \frac{(\lambda_i + \lambda_j)}{60}$, where λ_i and λ_j are demands of cell *i* and *j*, respectively. With this parameter, we can show that the handover traffic, μ_e, between two cells has an exponential distribution and its expected value is to be 1 Erlang. In generating test problems, we calculated the μ_e from the demands of cells with the parameter values above.

In the computational experiment, four kinds of parameters were changed to test the consistency of the algorithm performance. The first is the size of the network that can be represented by the number of cells and the number of candidated MSCs. The second is the setup cost of MSC that can be represented by $s^{\min}$ and $s^{\max}$. The third is the weight of handover cost that can be represented by *h*. And the last is the cabling cost factor that can be represented by the p. The experimental test result is the averaged value over the 30 test problems for each fixed parameter. In the computational experiment, the result of the suggested algorithm is compared with other heuristic algorithm (Kameda-Itoh's algorithm) suggested by Li *et al*. (1997).

<Table 2> shows the computational results for various number of candidate MSCs. In the computational experiment of <Table 2>, the parameters were set as the handover cost factor *(h)* = 2, the cabling cost factor (p) = 4 and $s^{\min}$ = 1000, $s^{\max}$ = 1200 for the MSC setup cost. Note that the cabling cost in Lagrangian heuristic is decreased as the number of the candidate MSCs increases. This means that the algorithm can select the optimal MSCs among many candidate MSCs. Whereas the handover cost and the MSC setup cost are not affected by the candidate MSCs.

In the problem, the total traffic which should be covered by the MSCs will be the sum of the traffic from 200 cells. The traffic from each cell has an exponential distribution with mean 50 Erlang which means that the total traffic will have a normal distribution with mean 10,000 Erlang. From this, the expected value of the minimum number of MSCs to satisfy the demand from all the cells can be deduced. In the 200 cells problem, the expected value of the minimum number of MSCs to satisfy the total traffic is to be 2.5. In <Table 2>, the average number of MSCs opened from Lagrangian heuristic is approximately 2.5 which is the expected value of the minimum number of MSCs to satisfy 200 cells.

The solution quality of the heuristic procedure can be measured by calculating the gap between the heuristic solution value and the Lagrangian bound. The values in the Average gap columns are the ratio of the gap divided by Lagrangian bound. Our results indicate that the average performance does not increase as the problem size increases, but the result of the Kameda-Itoh's algorithm is significantly af-

Table 2. Computational Results for Various Number of Candidate MSCs(the Number of Cells = 200)

MSC	Lagrangian Heuristic						Kameda-Itoh's algorithm					
	LAG	MO	CC	HC	MC	TC	KAG	MO	CC	HC	MC	TC
3	5.77%	2.53	3852	76	2789	6716	51.96%	3.00	5682	605	3312	9598
4	4.46%	2.47	3423	74	2726	6223	80.42%	4.00	5559	743	4435	10737
5	3.60%	2.43	3381	75	2653	6109	100.91%	5.00	5519	802	5506	11827
6	4.34%	2.60	3237	79	2801	6117	121.41%	5.97	5553	862	6540	12955
7	4.41%	2.60	3102	75	2822	6000	146.36%	7.00	5586	876	7691	14153

MSC : the number of candidate MSCs MO : the average number of MSCs opened
CC : the average cabling cost HC : the average handover cost
MC : the average MSC setup cost TC : the total cost
LAG : the average gap between the best feasible solution obtained from Lagrangian heuristic and the Lagrangian bound
KAG : the average gap between the best feasible solution obtained from Kameda-Itoh's algorithm and the Lagrangian bound

fected by the number of candidate MSCs.

<Table 3> shows the computational results for various number of cells. In the computational experiment of <Table 3>, the parameters were set as $s^{\min}$ = 1000, $s^{\max}$ = 1200, h = 2, and p = 4. The results show that the average gaps do not degrade as the problem size increases. In <Table 3>, the average number of MSCs opened from Lagrangian heuristic is almost same to the expected value of the minimum number of MSCs to satisfy each number of cells.

<Table 4> shows the computational results for different MSC setup costs of the same problem size. The results show that the number of MSCs opened decreases as the MSC setup cost increases.

<Table 5> shows the computational results for different handover cost factors of the same problem size. The results show that the number of opened MSC in the Lagrangian solution decreases as the handover cost factor increases. Result shows that cells tend to belong to the same MSC as the handover cost factor increases if the other cost factors are fixed.

<Table 6> shows the computational results for different

Table 3. Computational Results for Various Number of Cells (the Number of Candidate MSCs = 6)

Cells	Lagrangian Heuristic						Kameda-Itoh's algorithm					
	LAG	MO	CC	HC	MC	TC	KAG	MO	CC	HC	MC	TC
100	18.80%	1.50	1480	19	1628	3126	241.00%	6.00	1909	388	6641	8939
200	4.34%	2.60	3237	79	2801	6117	121.41%	5.97	5553	862	6540	12955
300	3.21%	3.57	5128	148	3896	9172	103.48%	6.00	10212	1275	6570	18056
400	2.65%	4.53	7463	235	4976	12674	97.39%	6.00	16025	1712	6569	24306
500	3.41%	5.62	10522	414	6197	17133	91.38%	6.00	22734	1972	6626	31332

Table 4. Computational Results for Different MSC Setup Costs(the Number of Cells = 200, the Number of Candidate MSCs = 6)

$s^{\min}$	$s^{\max}$	Lagrangian Heuristic						Kameda-Itoh's algorithm					
		LAG	MO	CC	HC	MC	TC	KAG	MO	CC	HC	MC	TC
0	200	0.48%	4.60	2513	119	450	3082	132.20%	6.00	5540	852	659	7051
100	300	0.48%	3.83	2666	99	700	3465	116.39%	6.00	5400	842	1176	7418
200	400	0.48%	3.40	2738	89	968	3795	116.70%	5.97	5543	862	1761	8167
300	500	0.47%	3.20	2820	84	1197	4102	112.36%	5.93	5493	851	2307	8652
400	600	0.80%	2.97	2941	81	1467	4488	111.87%	6.00	5542	865	3010	9417
500	700	1.31%	2.73	3035	79	1632	4746	112.48%	6.00	5461	839	3648	9948
600	800	2.45%	2.80	3166	83	1940	5189	110.75%	5.97	5648	841	4178	10667
700	900	1.77%	2.43	3324	77	1917	5318	115.02%	6.00	5576	838	4812	11225
800	1000	2.45%	2.50	3211	86	2211	5508	118.31%	5.93	5573	827	5321	11721
900	1100	3.71%	2.60	3147	77	2572	5796	122.25%	6.00	5529	862	6023	12415
1000	1200	4.34%	2.60	3237	79	2801	6117	121.41%	5.97	5553	862	6540	12955

Table 5. Computational Results for Various Handover Cost Factors(the Number of Cells = 200, the Number of Candidate MSCs = 6)

h	Lagrangian Heuristic						Kameda-Itoh's algorithm					
	LAG	MO	CC	HC	MC	TC	KAG	MO	CC	HC	MC	TC
2	1.31%	2.73	3035	79	1632	4746	112.48%	6.00	5461	839	3648	9948
4	1.64%	2.83	3024	143	1669	4835	126.46%	6.00	5471	1714	3575	10759
6	1.96%	2.70	3106	188	1586	4880	142.97%	6.00	5498	2511	3616	11624
8	2.57%	2.83	3052	258	1665	4975	157.88%	6.00	5527	3349	3617	12493
10	3.68%	2.57	3213	289	1500	5002	175.34%	5.97	5495	4222	3539	13256

Table 6. Computational Results for Various Cabling Cost Factors(the Number of Cells = 200, the Number of Candidate MSCs = 6)

p	Lagrangian Heuristic						Kameda-Itoh's algorithm					
	LAG	MO	CC	HC	MC	TC	KAG	MO	CC	HC	MC	TC
2	9.90%	2.57	1560	69	2768	4398	153.20%	5.97	2731	847	6526	10103
4	4.34%	2.60	3237	79	2801	6117	121.41%	5.97	5553	862	6540	12955
6	1.68%	2.67	4843	75	2900	7818	106.31%	5.97	8438	844	6572	15853
8	0.98%	2.87	6063	89	3115	9267	101.03%	6.00	11003	854	6568	18425
10	0.51%	3.10	7295	93	3401	10789	98.81%	6.00	13850	859	6590	21298

cabling cost factors of the same problem size. The results show that the number of opened MSC in the Lagrangian solution increase as the cabling cost factor increases. This means that cells have a tendency of connecting to the nearest MSC as the cabling cost factor increases.

<Table 7> shows the computation time comparisons with and without inequality (17), respectively. Note that the computational results with valid inequality (17) have less computation time. To get the Lagrangian bound, the subproblem $LABM_m(\alpha)$ should be optimally solved. With the valid inequality (17), we can expect the subproblem $LABM_m(\alpha)$ can be solved more efficiently. The effect of valid inequality is more important when the problem size is large enough.

Table 7. Computation Time in Seconds with/without Inequality in the Different Problem Size

Problem Size (Cell)	Average Computation Time (sec)	Average Computation Time without Inequality (17)
100	13.0	64.3
200	49.0	193.5
300	161.2	5601.3
400	2383.0	N/A
500	12978.8	N/A

It can be deduced from the test results that our solution procedures are successful in generating good feasible solutions for a wide variety of problem instances and their performances do not degrade as the problem size increases.

6. Conclusion

This paper investigates a base station allocation problem. For the problem, a heuristic solution procedure is exploited based on Lagrangian relaxation, and tested extensively with many problems to show how efficient and effective it is. Based on the test results, it is suggested that the exploited algorithm may practically be used for BS allocation. This paper may be extended to considering both BS and MSC location-allocation together, and by considering more complex problems subject to stochastic failures of network elements.

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