# Link Adaptation in Linearly Precoded Closed-Loop MIMO-OFDM Systems with Linear Receivers

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*Abstract*—Upcoming multi antenna systems such as 3GPP-LTE employ code book based multi-mode precoding in order to adapt to a wide range of channel conditions. Link adaptation, which includes the selection of precoding matrices, the number of spatially multiplexed layers as well as modulation coding schemes is a crucial task, carried out by the receiver. In this contribution<sup>1</sup> we propose link adaption based on the measure of mutual information between the channel input and the linear detector output and evaluate the behavior of the proposed algorithm in spatially correlated and uncorrelated propagation scenarios. The results highlight the importance of proper link adaption in order to mitigate the impact of spatial correlation.

#### I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems theoretically provide considerable gains regarding achievable rate and robustness as compared to single-input single-output (SISO) systems [1]. If perfect channel state information (CSI) is available at the transmitter, the water-filling strategy has been shown to be capacity achieving [2]. However, if up- and downlink of a wireless communication system are realized in frequency division duplex, CSI may only be available at the receiver since channel reciprocity cannot be assumed. This problem can be addressed by introducing a (finiterate) feedback channel which enables channel adaptive signal transmission. Recent research aimed at reducing the amount of feedback and a frequently proposed method is to let the receiver select matrices employed for precoding at the transmitter out of a finite code book and feedback only the code book indices. Code book design as well as precoding matrix selection criteria for spatial multiplexing with a fixed number of spatially multiplexed layers have been studied in e.g. [3] in the context of uncoded transmission over a narrow band channel. The results in [2] already indicate that allocating the transmit power to a fixed number of spatially multiplexed layers is clearly suboptimal. The work in [4] addresses this problem by introducing multi-mode precoding where the number of spatially multiplexed layers as well as the precoding matrices can be adaptively chosen.

Modern communication systems typically employ a large bandwidth in order to satisfy the data rate demands. In particular the combination of MIMO techniques with cyclic prefixed orthogonal frequency division multiplexing (OFDM) offers high spectral efficiency at reasonable computational complexity. Considering the combination of MIMO-OFDM and precoding another problem arises: the typically frequency selective channel requires different precoding matrices at different subcarriers. Thus, the required amount of feedback significantly increases. The central idea to counteract the feedback increase is to exploit the frequency correlation of the wireless channel by feeding back code book indices only on a subset of subcarriers. Precoding can be carried out e.g. by interpolating between precoding matrices at the transmitter [5] or by applying a single precoding matrix to a cluster of subcarriers [6].

Upcoming wireless communications standards such as 3GPP-LTE [13] transfer important ideas of the work in [3]–[6] into practice. In particular, 3GPP-LTE combines MIMO-OFDM, multi-mode precoding and rate adaptive, per spatially multiplexed layer coding in the downlink of the system. It is the receivers task to select a single number of spatially multiplexed layers for all occupied time-frequency resources and multiple precoding matrices which are applied to clusters of time-frequency resources. Jointly selecting the set of precoding matrices, the number of spatially multiplexed layers and the modulation coding schemes (MCS) such that the users data rate is maximized and at the same time a certain maximum error rate constraint is fulfilled is a problem which, to the best of the authors knowledge, has not been treated so far.

The contribution of this work is to derive an algorithm which, given a precoding code book and a set of available MCS, solves this problem. We therefore introduce the measure of mutual information between the discrete channel input, drawn e.g. from a MQAM constellation and the output of the linear detector in a MIMO-OFDM system as performance measure and apply this measure in order to select the three parameters of interest. Attention is payed to implementation and complexity aspects. The remainder of this work is organized as follows: In Section II we model the observed system. In Section III we derive the link adaptation scheme and discuss its performance in Section IV before we draw concluding remarks in Section V.

# II. SYSTEM MODEL

#### A. General Transmission Setup

We consider a MIMO-OFDM system with  $N_t$  transmit and  $N_r$  receive antennas. The transmission is organized in

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Fig. 1. Precoding and MCS configuration in the time-frequency-space plane

frames, each composed out of a number of subcarriers in frequency- and a number of OFDM symbols in time direction as sketched in Fig. 1. A number of adjacent subcarriers in all OFDM symbols within a frame is allocated to a user for transmission. The users time-frequency resources are grouped in  $N_{\rm Cl}$  resource clusters as shown in Fig. 1. The set  $\mathcal{P}_u$  contains the time-frequency indices of all resources allocated to a user and the set  $\mathcal{P}_a$  the time-frequency indices in the *a*-th resource cluster. A forward error correction scheme is employed and each codeword spans over a single spatially multiplexed layer and all time-frequency resources allocated to a user. The number of spatially multiplexed layers available for transmission  $N_l \in \{1 \dots \min\{N_t, N_r\}\}$  as well as the corresponding MCS are selected by a link adaption unit at the receiver and fed back to the transmitter.

# B. Signal Model

The equivalent base band frequency domain system model is sketched in Fig. 2. The vector of binary information bits



Fig. 2. Base band system model

 $\mathbf{b}_l$  on spatially multiplexed layer<sup>2</sup>  $l \in \{1 \dots N_l\}$  is encoded with code rate  $R_l$  resulting in a vector of coded bits  $\mathbf{u}_l$ . Subsequently  $\mathbf{u}_l$  is partitioned into blocks  $\mathbf{c}_l^p = [c_{l,1}^p \dots c_{l,N_b(l)}^p]$ which are associated with the time-frequency index  $p \in \mathcal{P}_u$ . Each block contains  $N_b(l)$  encoded bits. A modulator assigns each block of bits a complex valued symbol:  $\mathbf{c}_l^p \to x_l^p$  out of a set  $\mathcal{A}_{l}^{m}$  of  $|\mathcal{A}_{l}^{m}| = 2^{N_{b}(l)}$  constellation points (e.g. a QAM constellation) and  $\mathbb{E}[|x_{l}|^{2}] = \sigma_{x}^{2}$  holds. Note that the system supports  $m \in \{1 \dots M\}$  different modulation schemes. Each vector transmit symbol  $\mathbf{x}^{p} = [x_{1}^{p} \dots x_{N_{l}}^{p}]^{T}$  is mapped onto the  $N_{t}$  transmit antennas by a precoding matrix  $\mathbf{W}_{j}^{p} \in \mathbb{C}^{N_{t} \times N_{l}}$  which is chosen from a code book  $C_{N_{l}}$  with  $N_{\mathbf{W},N_{l}}$  entries. The index  $j \in \{1 \dots N_{\mathbf{W},N_{l}}\}$  denotes the *j*-th entry in the code book. The same precoding matrix is employed for a complete resource cluster but different precoding matrices can be applied to different resource clusters. Finally, all precoding matrices are designed to keep the sum transmit power constant regardless of  $N_{l}$  or  $N_{t}$ :  $\mathbb{E}[||\mathbf{W}_{j}^{p}\mathbf{x}^{p}||^{2}] = \sigma_{x}^{2}$ .

In cyclic prefixed OFDM the transmission over the wireless channel is conveniently modeled in frequency domain by the product of the transmit signal with the channel transfer function. This requires that the length of the time domain channel impulse response does not exceed the cyclic prefix length. Thus, the received frequency domain signal  $\mathbf{y}^p \in \mathbb{C}^{N_r \times 1}$  is obtained from:

$$\mathbf{y}^{p} = \underbrace{\mathbf{H}^{p} \mathbf{W}_{j}^{p} \mathbf{x}^{p}}_{\tilde{\mathbf{H}}^{p}} + \mathbf{v}^{p}.$$
 (1)

 $\mathbf{H}^{p} \in \mathbb{C}^{N_{r} \times N_{t}}$  denotes the MIMO channel matrix with jointly complex Gaussian distributed entries with zero mean and unit variance. The vector  $\mathbf{v}^{p} \in \mathbb{C}^{N_{r} \times 1}$  denotes complex additive white Gaussian noise (AWGN) with zero mean and covariance matrix  $\mathbb{E}[\mathbf{v}\mathbf{v}^{H}] = \sigma_{v}^{2}\mathbf{I}_{N_{r}}$ . The signal-to-noise-ratio (SNR) is defined by  $SNR = \sigma_{x}^{2}/\sigma_{v}^{2}$ .

The received signal vector  $y^p$  is fed into a MIMO detector, which computes log-likelihood ratios (LLR) required for decoding. In this work we consider a minimum mean square error (MMSE) linear detector (see Section II-C) which is likely to be applied in practice due to its robustness and low computational complexity. In the remainder of the paper we omit the time-frequency index p whenever it is clear from the context.

#### C. Linear Detection

Estimates  $\tilde{\mathbf{x}}$  of the transmitted signal  $\mathbf{x}$  are obtained from the received signal  $\mathbf{y}$  by a linear filter operation

$$\tilde{\mathbf{x}} = \mathbf{G}^{\cup \mathsf{B}} \mathbf{y} = \mathbf{x} + \tilde{\mathbf{v}} \tag{2}$$

where  $\mathbf{G}^{\text{UB}}$  denotes the unbiased Wiener filter [7]

$$\mathbf{G}^{\mathrm{UB}} = \mathrm{diag}\left\{\frac{1}{T_{1,1}}\dots\frac{1}{T_{N_l,N_l}}\right\}\mathbf{G}, \quad \mathbf{T} = \mathbf{G}\tilde{\mathbf{H}} \qquad (3)$$

and the (biased) Wiener filter matrix G is given by

$$\mathbf{G} = \left(\tilde{\mathbf{H}}^{H}\tilde{\mathbf{H}} + \frac{\sigma_{v}^{2}}{\sigma_{x}^{2}}\mathbf{I}_{N_{l}}\right)^{-1}\tilde{\mathbf{H}}^{H}.$$
 (4)

For complexity reasons we will solely consider the filter output  $\tilde{x}_l$  in order to obtain information about the channel input  $x_l$ . Effectively, we treat the MIMO channel after the linear filter operation as a set of  $N_l$  mutually independent channels with additive complex Gaussian distributed disturbance  $\tilde{v}_l$  with zero mean and variance  $\sigma_{\tilde{v}_l}^2$  which models the effect of residual

<sup>&</sup>lt;sup>2</sup>Boldface letters denote matrices and column vectors. Normal letters denote matrix elements or scalar values.  $I_A$ ,  $\Pr[\cdot]$ ,  $\mathbb{E}[\cdot]$ ,  $(\cdot)^H$ ,  $\operatorname{diag}\{\cdot\}$ ,  $||\cdot||$  and  $|\cdot|$  denote the identity matrix of dimension  $A \times A$ , a probability, the expectation operator, complex conjugate matrix transpose, a diagonal matrix composed from a vector, the column norm and the cardinality of a set or the absolute value of a number respectively.

spatial interference and AWGN. The effective noise variance  $\sigma_{\tilde{v}_l}^2$  at the *l*-th detector output can be computed by [7]:

$$\frac{1}{\sigma_{\tilde{v}_l}^2} = \frac{1}{\sigma_v^2} \frac{1}{\left[ \left( \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \frac{\sigma_v^2}{\sigma_x^2} \mathbf{I}_{N_l} \right)^{-1} \right]_{l,l}} - 1.$$
(5)

We define the post detection signal-to-interference-noise-ratio (SINR) of the *l*-th effective channel by  $SINR_l = \sigma_x^2 / \sigma_{\tilde{v}_l}^2$ . The probability density function of the filter output  $\tilde{x}_l$  conditioned on the channel input  $x_l$  and the MIMO channel matrix  $\tilde{\mathbf{H}}$  can be approximated with the complex Gaussian probability density function (pdf) as follows

$$p_{\tilde{x}_l|x_l,\tilde{\mathbf{H}}} \approx \frac{1}{\pi \sigma_{\tilde{v}_l}^2} \exp\left(-\frac{|\tilde{x}_l - x_l|^2}{\sigma_{\tilde{v}_l}^2}\right).$$
 (6)

Note that Eq. (6) becomes accurate if the conditions  $N_l \to \infty$ or  $x_l \in C\mathcal{N}(0, \sigma_x^2)$  hold. Without a priori information about the transmitted bits, the LLRs, required in the decoding step, can be approximated by [7]:

$$L(c_{l,i}|\tilde{x}_l) \approx \max_{x_l' \in \mathcal{A}_{l,i}^{+1}} \left\{ -\frac{1}{\sigma_{\tilde{v}_l}^2} \left| \tilde{x}_l - x_l' \right|^2 \right\} - \max_{x_l' \in \mathcal{A}_{l,i}^{-1}} \left\{ \dots \right\}$$
(7)

In (7),  $\mathcal{A}_{l,i}^{\pm 1}$  denotes the set of  $2^{N_b(l)-1}$  symbols for which the *i*-th bit takes the value  $\pm 1$ .

# **III. LINK ADAPTION**

Task of the receiver side link adaption unit is to select three parameters for feedback to the transmitter: the number of spatially multiplexed layers, the set of precoding matrices and a MCS per spatially multiplexed layer. The selection is carried out such that the users data rate is maximized and a certain maximum error rate constraint is fulfilled. We will first review how the channel quality of an MIMO-OFDM system can be measured in terms of mutual information, then proceed with mutual information based MCS selection before we combine both and derive a method which allows to jointly select the three parameters of interest.

# A. Mutual Information of MIMO-OFDM Systems with Linear Receivers

Throughout the mutual information analysis we assume an infinite OFDM frame length and a block fading (frequency selective channel) channel, i.e. the channel does not change during an OFDM frame. Moreover we assume mutually independent transmit vectors  $\mathbf{x}$  and transmission free of crosstalk between different subcarriers. With those assumptions (refer to [8] for a more detailed discussion) each subcarrier can be treated independent of other subcarriers and the mutual information of an OFDM frame equals the average mutual information of its subcarriers. Similarly, observing that the elements within a transmit vector  $\mathbf{x}$  are mutually independent and that the linear detector neglects any statistical dependency between different spatially multiplexed layers at the detector output, i.e.

$$p_{\tilde{\mathbf{x}}|\mathbf{x},\tilde{\mathbf{H}}_{\text{LD}}} = \prod_{l=1}^{N_l} p_{\tilde{x}_l|x_l,\tilde{\mathbf{H}}}$$
(8)

mutual information between a channel input vector  $\mathbf{x}$  and a detector output vector  $\tilde{\mathbf{x}}$  at a single subcarrier simplifies to

$$I\left(\tilde{\mathbf{x}};\mathbf{x}|\tilde{\mathbf{H}}\right)_{\mathrm{LD}} = \sum_{l=1}^{N_l} I\left(\tilde{x}_l;x_l|\tilde{\mathbf{H}}\right).$$
(9)

The right part of Eq. (9) can be expressed in terms of the difference between the entropy of the linear filter output  $\tilde{x}_l$  and the conditional entropy of the linear filter output given the channel input  $x_l$  [9]:

$$I(\tilde{x}_l; x_l | \tilde{\mathbf{H}}) = H(\tilde{x}_l | \tilde{\mathbf{H}}) - H(\tilde{x}_l | x_l, \tilde{\mathbf{H}})$$
(10)

Given that each element in the set  $\mathcal{A}_l^m$  occurs with the same probability:  $\Pr[x_l = x_l'] = 1/|\mathcal{A}_l^m| = 2^{-N_b(l)}, \forall x_l' \in \mathcal{A}_l^m$  the entropies are defined by

$$H(\tilde{x}_{l}|\tilde{\mathbf{H}}) = -\frac{1}{|\mathcal{A}_{l}^{m}|} \times \sum_{x_{l}' \in \mathcal{A}_{l}^{m}} \int_{\mathbb{C}} p_{\tilde{x}_{l}|x_{l}',\tilde{\mathbf{H}}} \log_{2} \left( \frac{1}{|\mathcal{A}_{l}^{m}|} \sum_{x_{l}'' \in \mathcal{A}_{l}} p_{\tilde{x}_{l}|x_{l}'',\tilde{\mathbf{H}}} \right) d\tilde{x}_{l}$$
(11)

and

$$H(\tilde{x}_l|x_l, \tilde{\mathbf{H}}) = -\frac{1}{|\mathcal{A}_l^m|} \sum_{x_l' \in \mathcal{A}_l^m} \int_{\mathbb{C}} p_{\tilde{x}_l|x_l', \tilde{\mathbf{H}}} \log_2\left(p_{\tilde{x}_l|x_l', \tilde{\mathbf{H}}}\right) d\tilde{x}_l.$$
(12)

From Eq. (11), Eq. (12) and Eq. (6) we obtain (see e.g. [10]):

$$I(\tilde{x}_l; x_l | \tilde{\mathbf{H}}) \approx \log_2 \left( |\mathcal{A}_l^m| \right) - \frac{1}{|\mathcal{A}_l^m|} \times \dots$$
$$\sum_{x' \in \mathcal{A}_l^m \tilde{v}_l} \mathbb{E} \left[ \log_2 \left( \sum_{x'' \in \mathcal{A}_l^m} \exp \left( -\frac{1}{\sigma_{\tilde{v}_l}^2} |\tilde{v}_l + x' - x''|^2 - |\tilde{v}_l|^2 \right) \right) \right]$$
(13)

Note that Eq. (13) only depends on  $\tilde{v}_l$  with known variance  $\sigma_{\tilde{v}_l}^2$ and a specific modulation scheme with  $\mathbb{E}[|x_l|^2] = \sigma_x^2$ . Thus,  $I(\tilde{x}_l; x_l | \hat{\mathbf{H}})$  is solely a function of the post detection SINR (denoted by  $I(SINR_l)$ ) and can thus be efficiently implemented e.g. by means of Monte Carlo methods or numerical integration and stored in a lookup table. It is emphasized that Eq. (13) is an approximation which will slightly underestimate the mutual information due to the assumption of complex gaussian distributed noise at the linear detector output.

# B. Mutual Information based MCS selection

The error rate performance achievable with a certain MCS in the SISO-AWGN channel is a function of the SNR which can be evaluated offline and stored in an error rate lookup table at receiver or transmitter. Thus, given the SNR at the receiver, the transmitter can directly access the lookup table and select the MCS which on the one hand maximizes the data rate and on the other hand guarantees a certain maximum error rate.

In case of a MIMO-OFDM system the set of post detection SINRs at the *l*-th detector output can be computed at the receiver for all  $p \in \mathcal{P}_u$ , i.e. all time-frequency resources occupied by a code word, using Eq. (5). Typically, in a

frequency selective channel, those SINR values will also be frequency selective. Thus, a mapping function is required which maps the frequency selective SINR values to a single equivalent SISO-AWGN-SNR  $(SNR_{map,l})$  at which the same error rate performance is achieved. Several mapping functions have been discussed in [11]. It turned out, that the measure of mutual information leads to the most accurate results as compared to other approaches. The mapping function can be stated as follows:

$$SNR_{\mathrm{map},l} = \beta I^{-1} \left( \frac{1}{|\mathcal{P}_u|} \sum_{p \in \mathcal{P}_u} I\left(\frac{SINR_l}{\beta}\right) \right).$$
(14)

where  $I(\cdot)$  refers to Eq. (13) and  $I^{-1}(\cdot)$  to its inverse. The parameter  $\beta$  in Eq. (14) is a MCS dependent calibration factor. During the calibration process (refer to [11] for details), it has been found that  $\beta = 1$  is convenient for most MCS if e.g. a turbo code is employed. Eq. (14) is computed for the Mmodulation schemes available in the system. Subsequently the MCS which offers the highest data rate and fulfills the error rate constraint can be selected by comparing (14) for each modulation scheme with the SISO-AWGN error rate lookup table.

#### C. Joint Selection of $N_l$ , Precoding Matrices and MCS

A multitude of criteria for selecting precoding matrices and the number of spatially multiplexed layers has been studied in [3]–[6] in the context of uncoded transmission. However, they cannot be applied straight forward since the link adaption problem considering coded transmission and the system setup introduced in Section II is quite different. On the one hand different precoding matrices can be selected at different resource clusters. On the other hand the number of spatially multiplexed layers  $N_l$  has to be kept constant for all resource clusters allocated to a user. Particularly the constraint that the MCS can typically only be adapted with a certain granularity has to be taken into account when deciding about  $N_l$  as will be subsequently shown.

1) Selection of Precoding Matrices: Assume for the moment that  $N_l$  is kept fix and that the precoding matrix optimal in terms of the average mutual information of the *a*-th resource cluster shall be found. The link adaption unit therefore has to compute the post detection SINRs for the spatially multiplexed layers  $l = \{1 ... N_l\}$ , all time-frequency indices  $p \in \mathcal{P}_a$  and each precoding matrix  $\mathbf{W}_j$ ,  $j \in \{1 ... N_{\mathbf{W},N_l}\}$  using Eq. (5). These SINR values are denoted  $SINR_{N_l,l,j}^{a,p}$ . Bearing in mind the discussion in Section III-A, those SINR values can be employed to compute the average mutual information at the *a*-th ressource cluster by using Eq. (9) and Eq. (13)

$$\overline{I}^{a}_{N_{l},j} = \frac{1}{|\mathcal{P}_{a}|} \sum_{p \in \mathcal{P}_{a}} \sum_{l=1}^{N_{l}} I(SINR^{a,p}_{N_{l},l,j}).$$
(15)

Eq. (15) allows to compare the performance with different precoding matrices and the precoding matrix is selected which

maximizes the average mutual information:

$$J_{N_l}^a = \arg \max_j \left\{ \overline{I}_{N_l,j}^a \right\}$$
(16)

Note that it is sufficient to carry out the precoding matrix selection (Eq. (15) and Eq. (16)) for the modulation scheme with the highest cardinality available in the system since the selected precoding matrix is inherently also optimal for any modulation scheme of lower cardinality. By applying this strategy, the optimal precoding matrices can be selected for each resource cluster and each  $N_l$ .

2) Selection of  $N_l$  and MCS: In order to select the  $N_l$  which maximizes the users data rate first consider a toy example with the parameters  $N_l \in \{1, 2\}$ ,  $N_l = 1$ :  $\overline{I}_{1,J_1^1}^1 = 5b/s/Hz$  and  $N_l = 2$ :  $\overline{I}_{2,J_2^1}^1 = 5.3b/s/Hz$  where l = 1 contributes 4.8 bit/s/Hz and l = 2 contributes 0.5 b/s/Hz. Deciding for  $N_l = 2$  only based on the measure of mutual information without verifying that there is a MCS available which supports e.g. the low rate at l = 2 might cause an erroneous transmission on the second spatially multiplexed layer and thus cause a lower data rate with  $N_l = 2$  as compared to  $N_l = 1$ .

A simple method to counteract this problem is to test which of the available modulation coding schemes is supported on each spatially multiplexed layer for each  $N_l$  using the method introduced in Section III-B. Again assume that  $N_l$  is kept fix. Note that the optimal precoding matrices for that  $N_l$  as well as the associated post detection SINRs are already available. By computing  $SNR_{map,l}$  (Eq. (14)) for each available modulation scheme  $m \in \{1 \dots M\}$  and comparing it to the SISO-AWGN error rate lookup table the highest supported code rate  $\hat{R}_{N_l,l}^m$ for each modulation scheme can be selected. The modulation scheme which maximizes the data rate is

$$\hat{m}_{N_l,l} = \arg \max_{m} \left\{ \hat{R}_{N_l,l}^m \times \log_2\left(\mathcal{A}_l^m\right) \right\}.$$
(17)

Consequently, the maximum data rate  $\hat{D}_{N_l}$  achievable with a certain  $N_l$  is

$$\hat{D}_{N_{l}} = \sum_{l=1}^{N_{l}} \hat{R}_{N_{l},l}^{m} \times \log_{2}\left(\mathcal{A}_{l}^{m}\right) \bigg|_{m=\hat{m}_{N_{l},l}}$$
(18)

and the  $\hat{N}_l$  which maximizes the data rate can be chosen according to

$$\hat{N}_{l} = \arg \max_{N_{l}} \left\{ \sum_{l=1}^{N_{l}} \hat{D}_{N_{l},l} \right\}.$$
 (19)

The complete algorithm in pseudo notation is stated in Algorithm. 1 and the results are highlighted in gray.

#### D. Implementation and Complexity Aspects

The algorithm basically requires two components: 1. Complex valued matrix multiplications and inversions for the computation of post detection SINR values (Algorithm 1, line 6) and 2. Lookup table operations which assign a mutual information value to the corresponding SINR value (Algorithm 1, lines 9 and 15), whereby the first component is considered

	Algorithm	1	Algorithm	for	Link	Adaption
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1:	for $N_l = 1 \dots min\{N_t, N_r\}$ do
2:	for $a = 1 \dots N_{Cl}$ do
3:	for $j = 1 \dots N_{\mathbf{W}, N_l}$ do
4:	for $p \in \mathcal{P}_a$ do
5:	for $l = 1 \dots N_l$ do
6:	compute $SINR^{a,p}_{N_{i},l,i}$ , Eq. (5)
7:	end for
8:	end for
9:	compute $\overline{I}_{N_l,j}^a$ , Eq. (15)
10:	$J_{N_l}^a = rg \max_{j} \left\{ \overline{I}_{N_l,j}^a \right\}$
11:	end for
12:	end for
13:	for $l = 1 \dots N_l$ do
14:	for $m = 1 \dots M$ do
15:	select $\hat{R}_{N_l,l}^m$ , Section III-B
16:	end for
17:	$\hat{m}_{N_l,l} = \arg \max_{m} \left\{ \hat{R}_{N_l,l}^m \times \log_2\left(\mathcal{A}_l^m\right) \right\}$
18:	end for
19:	$\hat{D}_{N_l} = \sum_{l=1}^{N_l} \max_m \left\{ \hat{R}_{N_l,l}^m  imes \log_2(\mathcal{A}_l^m) \right\}$
20:	end for
21:	$\hat{N}_l = rg\max_{N_l} \left\{ \sum_{l=1}^{N_l} \hat{D}_{N_l,l}  ight\}$

most complex in hardware implementation. Keeping in mind that the channel coherence bandwidth covers a large number of subcarriers in typical OFDM systems the algorithms complexity can be significantly decreased. One possible strategy would be to compute the post detection SINRs not at every subcarrier in a resource cluster, but only in frequency spacings of the channel coherence bandwidth which reduces both, the number of matrix inversions and the number of lookup table operations. Defining the set  $\mathcal{P}_{a,red}$  which contains the time frequency indices in the *a*-th resource cluster with a certain spacing in frequency direction, the number of lookup table operations for the precoding matrix selection  $N_{PS}$  and the number of lookup table operations required to compute the equivalent SISO-AWGN-SNR  $N_{SNR,max}$  can be stated as

$$N_{\text{SINR}} = \sum_{N_{l}=1}^{\min\{N_{t},N_{r}\}} |\mathcal{C}_{N_{l}}| \times \sum_{a=1}^{N_{Cl}} |\mathcal{P}_{a,\text{red}}|$$

$$N_{\text{PS}} = \sum_{N_{l}=1}^{\min\{N_{t},N_{r}\}} |\mathcal{C}_{N_{l}}| \times \sum_{N_{l}=1}^{\min\{N_{t},N_{r}\}} N_{l} \times \sum_{a=1}^{N_{Cl}} |\mathcal{P}_{a,\text{red}}| \quad (20)$$

$$N_{\text{SNR}_{\text{map}}} = \sum_{N_{l}=1}^{\min\{N_{t},N_{r}\}} N_{l} \times \sum_{a=1}^{N_{Cl}} |\mathcal{P}_{a,\text{red}}| \times M.$$

An illustrative example will be given in Section IV .

#### **IV. PERFORMANCE EVALUATION**

# A. Simulation Setup

We consider the downlink of a cellular system with  $N_t = 2$ and  $N_r = 2$ . The wireless channel is modeled by the typically urban (TU) channel model [15]. We account for spatial correlation with the Kronecker model (refer to the discussion in [12] for details). The correlation coefficients  $\rho$  between antenna elements at the base station (BS) and mobile terminal (MT) are chosen with  $\rho_{BS} = \rho_{MT} = 0$  (uncorrelated case) and  $\rho_{BS} = 0.3, \rho_{MT} = 0.9$  (correlated case [16]). The main system parameters were drawn from the 3GPP-LTE standard [13], [14]. The sampling rate and subcarrier spacing are 7.68 MHz and 15 kHz respectively. A frame is composed of 14 OFDM symbols and 512 subcarriers per OFDM symbol. In the example setup an user is allocated 120 adjacent subcarriers grouped into 10 equally sized resource clusters. The following MCS (a parallel concatenated turbo code is employed [14]) can be selected: 4 QAM and  $R = \{1/2, 2/5, 1/2, 4/7, 2/3, 3/4, 4/5\},\$ 16 QAM and  $R = \{1/2, 4/7, 2/3, 3/4, 4/5\}, 64$  QAM and R = $\{4/7, 2/3, 3/4, 4/5, 0.85, 9/10\}$  w.r.t. a target code word error rate (CWER) of  $10^{-2}$ . A small code book as defined in [13] is employed, namely  $C_1$  with six entries which are columns of scaled unitary matrices in  $\mathcal{C}_2$  with three entries. Throughout the discussion we assume perfect CSI at the receiver, perfect synchronization and static MT and BS, i.e. a constant wireless channel within a frame.

# B. Results

1) Complexity: The delay spread of the TU channel model is  $T_d = 0.5\mu s$  and the coherence bandwidth can be approximated by  $B_{c,0.5} \approx 1/(5T_d) = 400kHz$ . Thus it would be sufficient to consider only a single subcarrier in each ressource cluster ( $|\mathcal{P}_{a,\text{red}}| = 1$ ) in the link adaption scheme. With the aforementioned system setup, the algorithm requires (see Section III-D):  $N_{\text{SINR}} = 90$  post detection SINR computations,  $N_{\text{PS}} = 270$  lookup table operations for precoding matrix selection and  $N_{\text{SNR}_{\text{map}}} = 90$  lookup table operations for computing the equivalent SISO-AWGN-SNR per OFDM frame.

2) Performance Results: For performance assessment we compute the average rate for a spatially uncorrelated (Fig. 3) and for a spatially correlated propagation scenario (Fig. 4). Both plots show the achieved average rate with the proposed link adaption scheme. For comparison we also plotted the average rate achieved without precoding by always employing two spatially multiplexed layers and adapting only the MCS as described in Section III-B. Those rates will be referred to as 'achieved rates' in the following. In order to measure the performance without the impact of suboptimal LLR value computation and suboptimal encoding/decoding, we also computed the average mutual information (MI, see Section III-A) achievable by transmitting 64 QAM modulated symbols over the effective channel H which includes the selected precoding matrices. Finally, for comparison we also computed the average MI with gaussian signaling and spatial waterfilling [2] independently carried out for each subcarrier as an upper bound.



Fig. 3. Average rate versus SNR without spatial correlation

Regarding the spatially uncorrelated case it can be observed that the achieved rate without precoding in the low SNR regime (SNR = { $0dB \dots 10dB$ }) is only 40% - 80% of the achieved rate with precoding which is mainly enabled by switching the second spatially multiplexed layer on or off. This effect becomes negligible with an further increasing SNR. Under the impact of spatial correlation an even larger gap between the achieved rate with and without precoding can be observed. Below an SNR of 5 dB the target CWER could not be met without precoding. In an SNR Range from 5 dB to 15 dB only  $\approx$ 50% of the achieved rate with precoding is achieved without precoding.

Considering only the achieved rate without precoding it can be observed that under the impact of spatial correlation a rate loss > 40% occurs in a SNR range from 5 dB - 20 dB as compared to the achieved rate without the impact of spatial correlation. The same comparison for a transmission with precoding reveals that up to an SNR of 10 dB the same rate is achieved independent of spatial correlation. The reason is that in the low SNR regime only one of two spatial layers is selected for transmission in the majority of transmissions. In the higher SNR regime spatial correlation causes a non negligible rate loss (e.g. 20 % at an SNR of 20 dB). Those results stress that carefully applied link adaptation can help mitigate the impact of spatial correlation on the performance of suboptimal linear MIMO receivers.

#### V. SUMMARY AND CONCLUSIONS

In this contribution we proposed a link adaptation strategy for multi-mode precoded MIMO-OFDM systems with linear receivers. The strategy is based on the measure of mutual information between the discrete channel input and the output of the linear MIMO detector which can be computed from the post detection SINR. It allows to predict the performance with various transmission setups and enables the receiver to jointly select the number of spatially multiplexed layers, precoding matrices and modulation coding schemes. The method presented is not limited to linear receivers and can be easily applied to e.g. successive interference cancelation receivers as

2x2, TU channel, spatially correlated



Fig. 4. Average rate versus SNR with spatial correlation

well. Numerical results stress the importance of proper link adaption and demonstrate gains up to a factor of two in terms of achieved rate compared to not applying adaptive precoding.

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