Optimum linear decoding of vector quantisation transmitted over a CDMA channel

H.H. Nguyen and W.A.C. Fernando

Abstract: The problem of joint source–channel and multiuser decoding for code division multiple access channels is considered. The block source–channel encoder is defined by a vector quantiser (VQ). The jointly optimum solution to such a problem has been considered before, but its extremely high complexity makes it impractical for systems with medium to large number of users and/or medium to large size of VQ codebook. Instead, the optimum linear decoder with a much lower complexity that minimises the mean-squared error is introduced. The optimum linear decoder is soft in the sense that it utilises all the soft information available at the receiver. Analytical and simulation results show that at low channel signal-to-noise ratio region, the proposed decoder's performance is almost the same as that of the jointly optimum decoder and significantly better than that of the tandem approaches that use separate multiuser detection and table-lookup decoding.

1 Introduction

The source and channel codings of a communication system are often designed and implemented separately. This common practice is mainly due to the work by Shannon [1], in which it was shown that such a separation can perform optimally. However, the positive coding theorems of information theory [1, 2] only show such separability in the limit of infinite codeword length and, hence, infinite delay. Furthermore, there exist channels for which the separation theorem is not valid, even asymptotically. One important class of such channels is the class of multiple access (or multiuser) channels [2]. These facts justify the study of combined source–channel coding, for example, when delay is a limiting factor (such as in two-way communications) for multiuser channels.

In this paper, the term 'combined source–channel coding' is used to denote approaches where the source and channel codes are joined into one overall code. The study is also restricted to block coding where the code is defined by a robust vector quantiser (RVQ), which is an encoding method where channel imperfections are taken into account in the design of the source encoder–decoder pair, with no introduction of 'additional' error protection redundancy. One important example is the case when a noisy channel is known and the VQ codevectors are given a careful assignment of transmission codewords [3]. The codeword assignment problem is generally referred to as the index assignment (IA) problem [4–6].

Most previous works on VQ for noisy channels have concentrated on discrete memoryless channels with an emphasis on the binary symmetric channel [5–7]. Some works [8–10] have, however, studied VQ over waveform channels using the so-called soft decoding. In soft VQ decoding, the operation of the decoder is not defined by a lookup in a finite decoder codebook. Instead, all of the received soft information is utilised for decoding and the decoder, in effect, has an infinite output alphabet. Such decoding was studied for the additive white Gaussian noise (AWGN) channel in [3, 8–10] and for Rayleigh-fading channels in [10–12]. The present work also takes on the approach of soft decoding. In particular, a suboptimal approach to joint multiuser detection and VQ decoding for code division multiple access (CDMA) channels based on linear filtering is presented. The proposed linear decoder is optimum within the class of linear decoders since it is designed to minimise the mean-squared error. It is demonstrated that when the channel is noisy [i.e. when the channel signal-to-noise (CSNR) ratio is low], the optimum linear decoder can achieve almost the same performance as that of the jointly optimum decoder and a considerable performance gain over the tandem approaches that use separate multiuser detection and table-lookup decoding. The chief advantage of the optimum linear decoder is, of course, its lower complexity and simplicity in implementation.

The paper is organised as follows. Section 2 introduces the CDMA system model. It also reviews the previously proposed decoding techniques for VQ over a CDMA channel. Section 3 then presents the optimum linear decoder. Section 4 provides the numerical and simulation results and compares the proposed decoder with previously known decoders. Finally, some conclusions are drawn in Section 5.

2 System model

Fig. 1 shows the model of a synchronous CDMA communication system under consideration. There are K users in the system where each user transmits its source vectors by means of VQ. The model applies well to the downlink where the assumption of synchronisation comes naturally,

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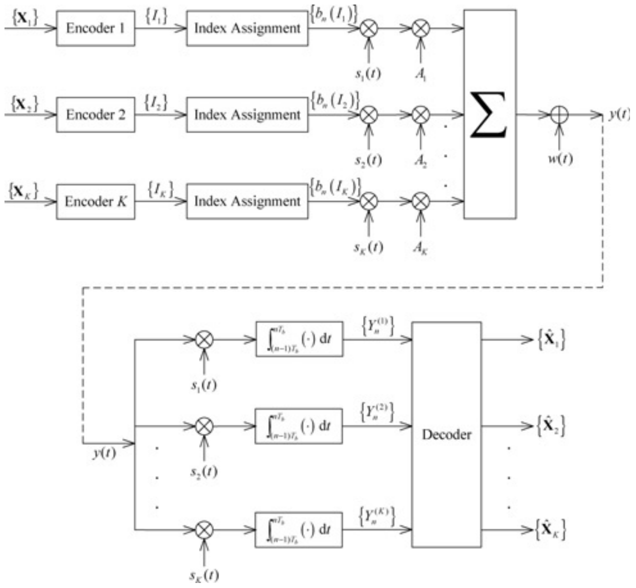

Fig. 1 *Model of a synchronous CDMA system*

but it can also be applied for the uplink in a synchronised CDMA system [3]. The kth user produces a d-dimensional random vector $\boldsymbol{X}_k \in \mathbb{R}^d$. The vector $\boldsymbol{X}_k$ is then encoded into an index $I_k \in \{0, 1, \ldots, N-1\}$ by the encoder of user k, where $N = 2^L$ for some integer L. Note that the encoder in Fig. 1 may include channel coding and source coding, either explicitly as in tandem source–channel encoding or implicitly as in combined source–channel coding (COVQ). The transmission rate of the system is thus $R = L/d$ (bits per source dimension).

The kth encoder is described by a partition $\left\{\mathcal{S}_i^{(k)}\right\}_{i=0}^{N-1}$ of the Euclidean source space $\mathbb{R}^d$ such that if $\boldsymbol{X}_k \in \mathcal{S}_i^{(k)}$, then $I_k = i$. Let $P_i^{(k)} = \Pr(I_k = i) = \Pr\left(\boldsymbol{X}_k \in \mathcal{S}_i^{(k)}\right)$. Then the encoder entropy of the kth user is defined as $H_k = -\sum_{j=0}^{N-1} P_j^{(k)} \log_2 P_j^{(k)}$. If the encoder includes channel coding, the entropy describes the 'redundancy' content in the transmitted data. Low entropy implies a high redundancy, and hence high error protection capability. The maximum possible value of the encoder entropy is $\log_2 N = L$. When $H_k = L$, the encoder entropy is said to be full. Also define the ith encoder centroid of user k as $\boldsymbol{c}_i^{(k)} = E[\boldsymbol{X}_k | I_k = i]$. Note that $\boldsymbol{c}_i^{(k)}$ is the minimum mean-squared error (MMSE) estimate of $\boldsymbol{X}_k$ given $I_k = i$. Thus, for a VQ with a mean-squared error distortion measure, the centroids, $\boldsymbol{c}_i^{(k)}$, are the optimal reconstruction vectors for a noiseless channel [13]. For a noisy channel, the optimal hard-decision reconstruction vectors are formed as linear combinations of the centroids [14].

For binary phase shift keying transmission, the index I_k is converted into a block $(b_L(I_k), \ldots, b_1(I_k))$ of L bits, where $b_n(I_k) \in \{\pm 1\}$. For simplicity, it is assumed that all users have the same block length L. The bits of the kth user's indices are transmitted over a synchronous CDMA channel by modulating the kth user's distinct signature waveform. Let T_b be the bit duration and $s_k(t)$, $0 \le t \le T_\mathrm{b}$, be the signature waveform of the kth user whose energy is normalised to unity. Because the system is synchronous, it suffices to consider the transmission of a single index of every user. Thus the received signal in one index interval (i.e. L bit intervals) can be expressed as

$$y(t) = \sum_{n=1}^{L} \sum_{k=1}^{K} A_k b_n(I_k) s_k(t - (n-1)T_\mathrm{b}) + w(t), \quad 0 \le t \le LT_\mathrm{b} \tag{1}$$

where A_k is the kth user's received amplitude and $w(t)$ is AWGN of spectral density $\sigma^2 = N_0/2$ (W/Hz).

It is well known [15] that the sufficient statistic for decoding the source vectors of K users can be obtained by a bank of K matched filters (or equivalently, a bank of K correlators) as shown in Fig. 1. The outputs from the bank of matched filters at time n can be written as

$$\boldsymbol{Y}_n = [Y_n^{(1)}, Y_n^{(2)}, \ldots, Y_n^{(K)}]^\mathrm{T} = \boldsymbol{RAb}_n + \boldsymbol{W} \tag{2}$$

where $\boldsymbol{A} = \mathrm{diag}(A_1, A_2, \ldots, A_K)$, $\boldsymbol{b}_n = [b_n(I_1), b_n(I_2), \ldots, b_n(I_K)]^\mathrm{T}$, $\boldsymbol{R}$ the correlation matrix of the signature waveforms with $\boldsymbol{R}_{ij} = \int_0^{T_\mathrm{b}} s_i(t) s_j(t)\,\mathrm{d}t$ and $\boldsymbol{W}$ a Gaussian vector of zero-mean and covariance matrix $\sigma^2 \boldsymbol{R}$ and independent of the transmitted bits.

On the basis of the the sufficient statistic $\{\boldsymbol{Y}_n\}_{n=1}^L$, the decoder in Fig. 1 needs to make the decision on the transmitted source vectors of all K users. Of course, different processing algorithms on $\{\boldsymbol{Y}_n\}_{n=1}^L$ yield different decoders. In the remaining of this section, two such decoders shall be discussed as they will be used as benchmark decoding schemes to compare with the decoder proposed in this paper.

2.1 Jointly optimum multiuser-VQ decoder

As in [3], it is convenient to introduce notations that simplify the handling of all K users in the system. Thus, let $\boldsymbol{I}^K = [I_1, I_2, \ldots, I_K]^\mathrm{T}$ denote the vector consisting of all users' random indexes having sample values $\boldsymbol{i}^K$. Let $P_{\boldsymbol{i}^K}$ denote the probability $\Pr(\boldsymbol{I}^K = \boldsymbol{i}^K)$. Also define the augmented source vector $\boldsymbol{X}^K = [\boldsymbol{X}_1^\mathrm{T}, \boldsymbol{X}_2^\mathrm{T}, \ldots, \boldsymbol{X}_K^\mathrm{T}]^\mathrm{T} \in \mathbb{R}^{Kd}$ and the augmented centroid vector $\boldsymbol{c}_{\boldsymbol{i}^K}^K = \left[\left(\boldsymbol{c}_{i_1}^{(1)}\right)^\mathrm{T}, \left(\boldsymbol{c}_{i_2}^{(1)}\right)^\mathrm{T}, \ldots, \left(\boldsymbol{c}_{i_K}^{(K)}\right)^\mathrm{T}\right]^\mathrm{T} \in \mathbb{R}^{Kd}$.

The soft decoder shown in Fig. 1 measures $\boldsymbol{Y} \triangleq \left[\boldsymbol{Y}_1^\mathrm{T}, \boldsymbol{Y}_2^\mathrm{T}, \ldots, \boldsymbol{Y}_L^\mathrm{T}\right]^\mathrm{T} \in \mathbb{R}^{KL}$ and forms estimates $\hat{\boldsymbol{X}}_k(\boldsymbol{Y}) \in \mathbb{R}^d$ of the transmitted source vectors for all users. If the MMSE criterion [16] is used, then the soft decoder for user k is an MMSE estimator $\hat{\boldsymbol{X}}_k(\boldsymbol{y})$, which is a continuous function of the received data $\boldsymbol{Y} = \boldsymbol{y} \triangleq \left[\boldsymbol{y}_1^\mathrm{T}, \boldsymbol{y}_2^\mathrm{T}, \ldots, \boldsymbol{y}_L^\mathrm{T}\right]^\mathrm{T}$. Assume that the encoders and the sources of all users are known (i.e. the centroids and the index probabilities are specified). Also assume that the sources of the different users are statistically independent of each other, which implies that $P_{\boldsymbol{i}^K} = P_{i_1} P_{i_2} \cdots P_{i_K}$. Furthermore, it is assumed that the decoder has a perfect knowledge of the amplitudes and the cross-correlations between users (i.e. a knowledge of $\boldsymbol{A}$ and $\boldsymbol{R}$). The jointly optimum MMSE decoder minimises the distortion $E\{\|\boldsymbol{X}_k - \hat{\boldsymbol{X}}_k(\boldsymbol{Y})\|^2\}$ for each user k, $k = 1, 2, \ldots, K$. From estimation theory [16], the optimal estimate of the augmented sample vector $\boldsymbol{X}^K$ is the conditional mean $\hat{\boldsymbol{X}}^K(\boldsymbol{Y}) = [(\hat{\boldsymbol{X}}_1(\boldsymbol{Y}))^\mathrm{T}, \ldots, (\hat{\boldsymbol{X}}_K(\boldsymbol{Y}))^\mathrm{T}]^\mathrm{T} = E\{\boldsymbol{X}^K | \boldsymbol{Y}\}$. It is straightforward to show that the corresponding estimator is [3]

$$\hat{\boldsymbol{X}}^K(\boldsymbol{y}) = E\{\boldsymbol{c}_{\boldsymbol{I}^K} | \boldsymbol{y}\} = \sum_{\boldsymbol{i}^K} \boldsymbol{c}_{\boldsymbol{i}^K} \Pr(\boldsymbol{I}^K = \boldsymbol{i}^K | \boldsymbol{Y} = \boldsymbol{y})$$

$$= \frac{\sum_{\boldsymbol{i}^K} \boldsymbol{c}_{\boldsymbol{i}^K}^K p_{\boldsymbol{Y}}(\boldsymbol{y}|\boldsymbol{i}^K) P_{\boldsymbol{i}^K}}{\sum_{\boldsymbol{i}^K} p_{\boldsymbol{Y}}(\boldsymbol{y}|\boldsymbol{i}^K) P_{\boldsymbol{i}^K}} \tag{3}$$

That is, the soft estimate $\hat{X}^K(Y)$ is formed as a weighted sum over the encoder centroids. For the CDMA channel model in (2), the probability density function $p_Y(y|i^K)$ is given by

$$p_Y(y|\boldsymbol{i}^K) = \frac{1}{(2\pi\sigma^2|\boldsymbol{R}|)^{-KL/2}} \times \exp\left[-\frac{1}{2\sigma^2}\sum_{n=1}^{L}(\boldsymbol{RAb}_n - \boldsymbol{y}_n)^{\mathrm{T}}\boldsymbol{R}^{-1}(\boldsymbol{RAb}_n - \boldsymbol{y}_n)\right] \quad (4)$$

where $|\boldsymbol{R}|$ is the determinant of the correlation matrix $\boldsymbol{R}$.

In [3], the implementation of the above optimal decoder based on Hadamard matrix description of the VQs is presented. Such an optimal decoder is named Hadamard-based multiuser decoder (HMD). Although the Hadamard-based decoder is equivalent, in terms of both performance and complexity, to the optimal decoder in (4), its implementation offers a clear interpretation of the jointly optimum soft multiuser-VQ decoding. In particular, it shows how to utilise the a priori and channel information in an optimal fashion to counteract channel noise and multi-user interference. The total decoding complexity of HMD is about $\mathcal{O}(KL\cdot 2^{KL}) + \mathcal{O}(K\cdot 2^{L}\cdot d)$ operations.

2.2 Suboptimum decoders based on table-lookup

The alternative approach for the decoder shown in Fig. 1 is based on the combination of separate multiuser detection and table-lookup VQ decoding. The multiuser detection can be, for example, the maximum likelihood (ML) or the suboptimum MMSE receiver [15]. Such a tandem approach first gives the hard decision for the transmitted vector of bits $\boldsymbol{b}_n$. For each user k, the bits are then converted to the corresponding estimated index $\hat{i}_k$. The VQ decoder of the kth user then finds and outputs the centroid $\boldsymbol{c}^{(k)}_{\hat{i}_k}$ for VQ decoding. If the ML multiuser detection is used, then the complexity of the suboptimum decoder is about $\mathcal{O}(L\cdot 2^{K})$ operations per user. On the other hand, if the MMSE multiuser detection is employed, then the decoder complexity is about $\mathcal{O}(L\cdot K^2)$ operations.

3 Optimum linear decoder

This section introduces a suboptimal decoder based on linear filtering of the sufficient statistic $\boldsymbol{Y}$. The proposed decoder is a soft decoder since it accepts the soft values of the matched filters' outputs. It is also a joint multiuser detection and VQ decoding scheme since the two functions are jointly handled by the proposed decoder. The proposed decoder is suboptimal since it is constrained to be a linear receiver (which implies a low complexity). In contrast, it is the optimum decoder among all the linear decoders since it is designed to minimise the mean-squared error.

A linear decoder is defined by a $KL\times Kd$ matrix $\boldsymbol{G}$ as follows

$$\hat{X}^K(\boldsymbol{Y}) = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{Y} \quad (5)$$

It then follows that the optimum linear decoder is specified by matrix $\boldsymbol{G}^{\mathrm{T}}$ that minimises the mean-squared error $E\{\|\boldsymbol{X}^K - \boldsymbol{G}^{\mathrm{T}}\boldsymbol{Y}\|^2\}$. To obtain the solution to the above problem, recognise that $\boldsymbol{Y}$ can be written as

$$\boldsymbol{Y} = \boldsymbol{S} + \boldsymbol{Z} \quad (6)$$

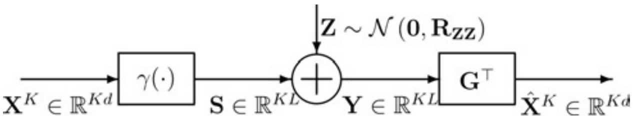

Fig. 2 *System block diagram for the linear decoder*

where

$$\boldsymbol{S} = \underbrace{\begin{bmatrix} \boldsymbol{RA} & 0 & \cdots & 0 \\ 0 & \boldsymbol{RA} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{RA} \end{bmatrix}}_{KL\times KL\ \text{matrix}} \cdot \underbrace{\left[\boldsymbol{b}_1^{\mathrm{T}}, \boldsymbol{b}_2^{\mathrm{T}}, \ldots, \boldsymbol{b}_L^{\mathrm{T}}\right]^{\mathrm{T}}}_{\boldsymbol{b}\in\mathbb{R}^{KL}} \quad (7)$$

and $\boldsymbol{Z}$ is a $KL\times 1$ zero-mean Gaussian vector with the following $KL\times KL$ covariance matrix

$$\boldsymbol{R_{ZZ}} = \sigma^2\begin{bmatrix} \boldsymbol{R} & 0 & \cdots & 0 \\ 0 & \boldsymbol{R} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{R} \end{bmatrix} \quad (8)$$

Thus, the decoding problem can be described using a block diagram shown in Fig. 2, where the mapping $\gamma(\cdot)$ includes the functions of VQ encoders, IAs and CDMA channel. Since $\boldsymbol{X}^K$ and $\boldsymbol{Z}$ are independent and have zero means, the following relationships of covariance matrices can be easily verified

$$\boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{Y}} = \boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{S}} = \sum_{\boldsymbol{i}^K} P_{\boldsymbol{i}^K}\boldsymbol{c}_{\boldsymbol{i}^K}\boldsymbol{s}^{\mathrm{T}}_{\boldsymbol{i}^K} \quad (9)$$

$$\boldsymbol{R}_{\boldsymbol{X}^K\hat{\boldsymbol{X}}^K} = \boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{S}}\boldsymbol{G} \quad (10)$$

$$\boldsymbol{R_{SS}} = \sum_{\boldsymbol{i}^K} P_{\boldsymbol{i}^K}\boldsymbol{s}_{\boldsymbol{i}^K}\boldsymbol{s}^{\mathrm{T}}_{\boldsymbol{i}^K} \quad (11)$$

$$\boldsymbol{R_{YY}} = \boldsymbol{R_{SS}} + \boldsymbol{R_{ZZ}} \quad (12)$$

$$\boldsymbol{R}_{\hat{\boldsymbol{X}}^K\hat{\boldsymbol{X}}^K} = \boldsymbol{G}^{\mathrm{T}}\boldsymbol{R_{YY}}\boldsymbol{G} \quad (13)$$

where $\boldsymbol{s}_{\boldsymbol{i}^K}$ is the vector $\boldsymbol{S}$ that corresponds to the users' indexes $\boldsymbol{i}^K$.

Using the orthogonality principle [16], the optimum linear decoder $\boldsymbol{G}^{\mathrm{T}}$ can be found by solving the following equation

$$E\left\{(\boldsymbol{X}^K - \boldsymbol{G}^{\mathrm{T}}\boldsymbol{Y})(\mathcal{L}\boldsymbol{Y})^{\mathrm{T}}\right\} = 0 \quad (14)$$

where $\mathcal{L}$ is any linear maps from $\mathbb{R}^{KL}$ to $\mathbb{R}^{Kd}$. The solution is given by

$$\boldsymbol{G}^{\mathrm{T}} = \boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{Y}}\boldsymbol{R}^{-1}_{\boldsymbol{YY}} = \boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{S}}\left(\boldsymbol{R_{SS}} + \boldsymbol{R_{ZZ}}\right)^{-1} \quad (15)$$

With the above solution for $\boldsymbol{G}^{\mathrm{T}}$, define the following $Kd\times 1$ vector

$$\begin{aligned}\boldsymbol{\Lambda} &= \mathrm{diag}\left(E\left\{\left(\boldsymbol{X}^K - \hat{\boldsymbol{X}}^K\right)\left(\boldsymbol{X}^K - \hat{\boldsymbol{X}}^K\right)^{\mathrm{T}}\right\}\right) \\ &= \mathrm{diag}\left(\boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{X}^K} - 2\boldsymbol{R}_{\boldsymbol{X}^K\hat{\boldsymbol{X}}^K} + \boldsymbol{R}_{\hat{\boldsymbol{X}}^K\hat{\boldsymbol{X}}^K}\right) \\ &= \mathrm{diag}\left(\boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{X}^K} - \boldsymbol{R}_{\boldsymbol{X}^K\boldsymbol{S}}\boldsymbol{G}\right)\end{aligned} \quad (16)$$

It then follows that the resulting MMSE corresponding to user k is given by

$$E\{\|\boldsymbol{X}_k - \hat{\boldsymbol{X}}_k\|^2\} = \sum_{j=1}^{d}\boldsymbol{\Lambda}_{(k-1)d+j} \quad (17)$$

and the signal to quantisation noise ratio of the kth user can be computed as

$$\mathrm{SNR}_k = \frac{E\{\|\boldsymbol{X}_k\|^2\}}{E\{\|\boldsymbol{X}_k - \hat{\boldsymbol{X}}_k\|^2\}} = \frac{E\{\|\boldsymbol{X}_k\|^2\}}{\sum_{j=1}^{d} \boldsymbol{\Lambda}_{(k-1)d+j}} \tag{18}$$

Finally, it is noted that the complexity of the optimum linear decoder is about $\mathcal{O}(KL \cdot Kd)$ operations.

Before closing this section, it should be pointed out that the principle of the optimum linear receiver can be readily extended to a frequency-selective Rayleigh-fading channel with asynchronous users. This is because the input/output model of such a more complicated CDMA model can also be represented in the form of (2), as shown in (5) of [12].

4 Numerical results and comparison

This section presents the performance results and compares the proposed linear decoder with other decoders described in Section 2. The performance of VQ decoders is measured in terms of the output SNR, $\mathrm{SNR}_k = E\{\|\boldsymbol{X}_k\|^2\}/E\{\|\boldsymbol{X}_k - \hat{\boldsymbol{X}}_k\|^2\}$, against the CSNR E_b/N_0. For simplicity, it is assumed that all users' amplitudes are equal ($A_k = A$).

The individual user's source is modelled as a zero-mean, unit-variance, stationary and first-order Gauss–Markov random process with correlation coefficient ρ. Thus, the source is described by

$$X_n = \rho X_{n-1} + W_n \tag{19}$$

where W_n is an independent and identically distributed Gaussian process with variance $1-\rho^2$. For the kth user's source $\boldsymbol{X}_k$, the $d \times d$ covariance matrix $\boldsymbol{R}_{X_k X_k} = (r_{ij})$, is given by

$$r_{ij} = \begin{cases} \rho^{|i-j|}, & i \neq j \\ 1, & i = j \end{cases} \tag{20}$$

Note that the SNR of the proposed linear decoder can be evaluated numerically from (18). In contrast, the performance of other decoders described in Section 2 is obtained by computer simulation. Different VQs were trained for the Gauss–Markov source and a noiseless channel and then given good IAs based on the LISA–algorithm [17]. The encoders of these VQs were then used in the simulations. The parameters of the VQs are given in Table 1. The last column of Table 1 shows the signal-to-distortion ratio, which is the highest achievable value of the SNR corresponding to a given VQ. Furthermore, two channel models with two and four users and the following cross-correlation matrices

Table 1: VQ encoders

VQ	d	L, bits	N, codewords	Entropy, bits	SDR, dB
VQ1	3	3	8	2.88	9.4
VQ2	6	6	64	5.87	11.0

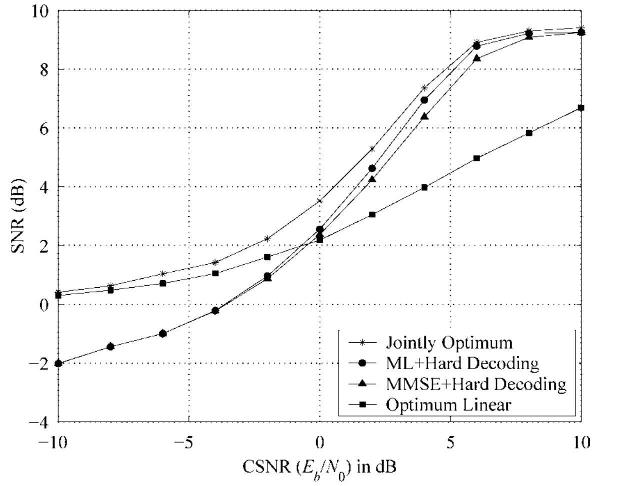

Fig. 3 *Performance of a CDMA system with two users (averaged over the users): both users employ VQ1 and the correlation matrix is* $\boldsymbol{R}_2$

$$\boldsymbol{R}_2 = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \tag{21}$$

$$\boldsymbol{R}_4 = \frac{1}{7}\begin{pmatrix} 7 & -1 & 3 & 3 \\ -1 & 7 & 3 & -1 \\ 3 & 3 & 7 & -1 \\ 3 & -1 & -1 & 7 \end{pmatrix} \tag{22}$$

are considered.

Figs. 3 and 4 present the performance of different decoders for a two-user CDMA system and when VQ1 and VQ2 are employed, respectively. Similarly, Figs. 5 and 6 present the results for a four-user CDMA system with the only exception that the performance of the jointly optimum decoder is absent from Fig. 6. This absence is due to the fact that the complexity of the jointly optimum decoder for a four-user system and VQ2 is already very high and its performance could not be obtained reliably in a reasonable amount of time (If the jointly optimum decoder based on Hadamard matrix is implemented, a Hadamard matrix of size $64^4 \times 64^4$ is involved. The size of this matrix is too big to handle by any computing software.). More precisely, the performance in Figs. 3 and 4

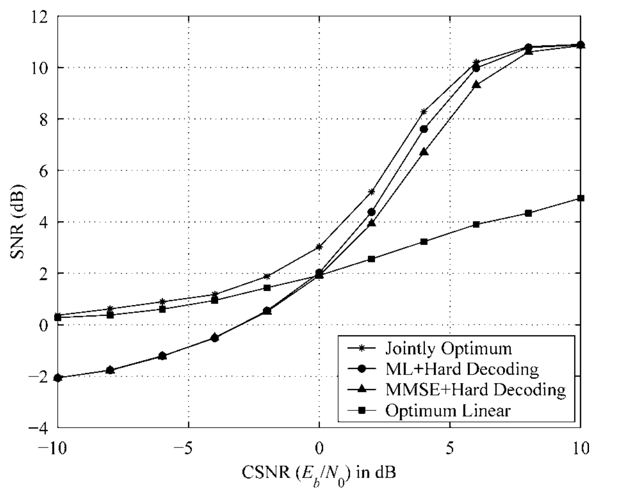

Fig. 4 *Performance of a CDMA system with two users (averaged over the users): both users employ VQ2 and the correlation matrix is* $\boldsymbol{R}_2$

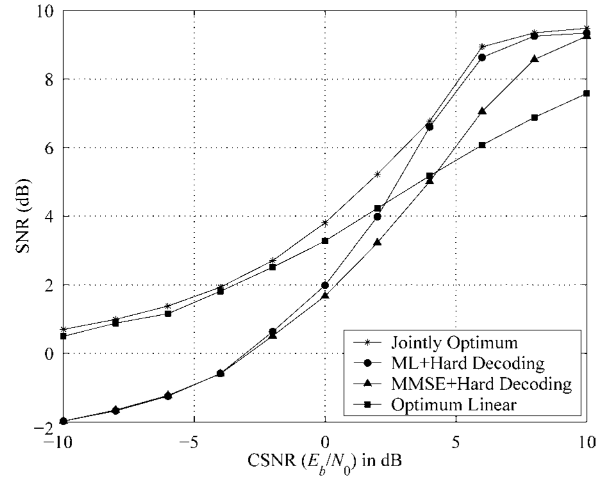

Fig. 5 *Performance of a CDMA system with four users (the first user is illustrated): all users employ VQ1 and the correlation matrix is* $\boldsymbol{R}_4$

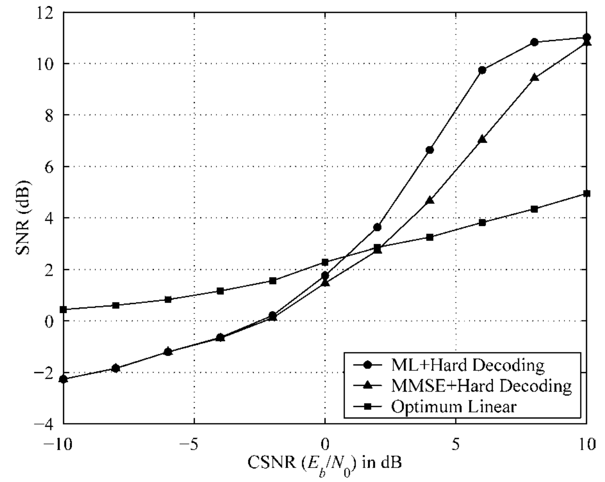

Fig. 6 *Performance of a CDMA system with four users (the first user is illustrated): all users employ VQ2 and the correlation matrix is* $\boldsymbol{R}_4$

is averaged over the two users, whereas it is the performance of the first user that is shown in Figs. 5 and 6.

Observe from Figs. 3–6 that among the two tandem decoders, the one based on ML multiuser detection only outperforms the one based on MMSE multiuser detection at medium CSNR region and for the four-user channel. Both the tandem decoders perform identically at low CSNR region and they approach the performance of the jointly optimum decoder at high CSNR region. In other words, the advantage of the jointly optimum decoder over the tandem approaches is only prominent at low CSNR region. The advantage is ~2.5 dB in SNR. Interestingly, Figs. 3–5 show that the optimum performance of the

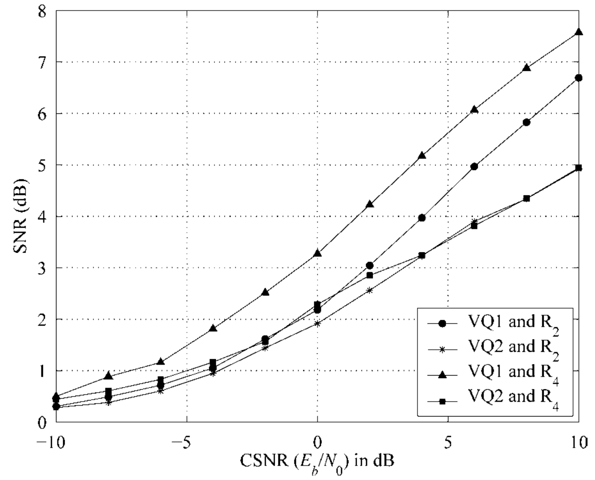

Fig. 7 *Performance comparison of the optimum linear decoder for different combinations of VQs and channel models*

jointly optimum decoder at low CSNR can be closely achieved by the proposed liner decoder.

It is important to point out again that the performance of the optimum linear decoder at low CSNR region is achieved by a linear decoder with a much lower complexity. This can be clearly seen from Table 2 where it compares the complexity of different decoders considered in this paper. Note that, although it appears from Table 2 that the complexity of the two tandem approaches are exactly the same and is less than that of the optimum linear decoder, this only applies for the selection of parameters *d*, *L* and *K* in this section. In general, for CDMA systems with larger number of users, the complexity of the ML multiuser detection/table-lookup VQ decoding is much higher than that of the MMSE multiuser detection/table-lookup VQ decoding and optimum linear decoder. The more important observation, however, is that the complexity of the jointly optimum decoder quickly become unacceptably high as *d*, *L* and *K* increase.

The results in Figs. 3–6 and Table 2 also suggest that, instead of using the jointly optimum decoder with very high computational complexity, the tandem approaches can be used at high CSNR and the optimum linear decoder can be used at low CSNR. With such a combination, the performance of the jointly optimum decoder can be closely approached at any CSNR region with much lower computational complexity.

Finally, Fig. 7 compares the performance of the optimum linear decoder for different combinations of VQs and channel models. It can be observed from Fig. 7 that the performance of the proposed linear decoder is quite robust to both VQ and CDMA channel model at low CSNR region. However, at high CSNR, its performance depends on the choice of VQ and channel model. In particular, Fig. 7 shows that the optimum linear decoder is more effective for VQ1 (with a smaller codebook) than VQ2 (with a bigger codebook) at high CSNR region.

Table 2: Complexity comparison of different decoders

VQ	*d*	*L*	*R*	*K*	JO	ML + TL	MMSE + TL	OL
VQ1	3	3	$\boldsymbol{R}_2$	2	≃384	12	12	36
VQ2	6	6	$\boldsymbol{R}_2$	2	≃49 152	24	24	144
VQ1	3	3	$\boldsymbol{R}_4$	4	≃49 152	48	48	144
VQ2	6	6	$\boldsymbol{R}_4$	4	≃402 653 184	96	96	576

JO, Jointly optimum; ML + TL, ML + table-lookup; MMSE + TL, MMSE + table-lookup; OL, optimum linear

5 Conclusions

An optimum linear decoder is proposed for joint source–channel and multiuser decoding in CDMA channels. Numerical and simulation results show that the proposed decoder's performance is almost the same as that of the jointly optimum decoder at low CSNR region. Such an excellent performance is achieved with a much lower computational complexity of the linear decoder. With the proposed optimum linear decoder and the tandem decoding approach (which is based on the ML or MMSE multiuser detection and table-lookup VQ decoding), the performance of the jointly optimum decoder at any CSNR region can be closely approached with a much lower decoding complexity.

The analysis and numerical results were reported for the case of binary antipodal modulation. Although such a modulation scheme is popular for CDMA systems, a further interesting study is to analyse and compare the performance of the optimum linear receiver with that of the optimum nonlinear receiver when higher-order modulation schemes are employed.

6 Acknowledgment

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