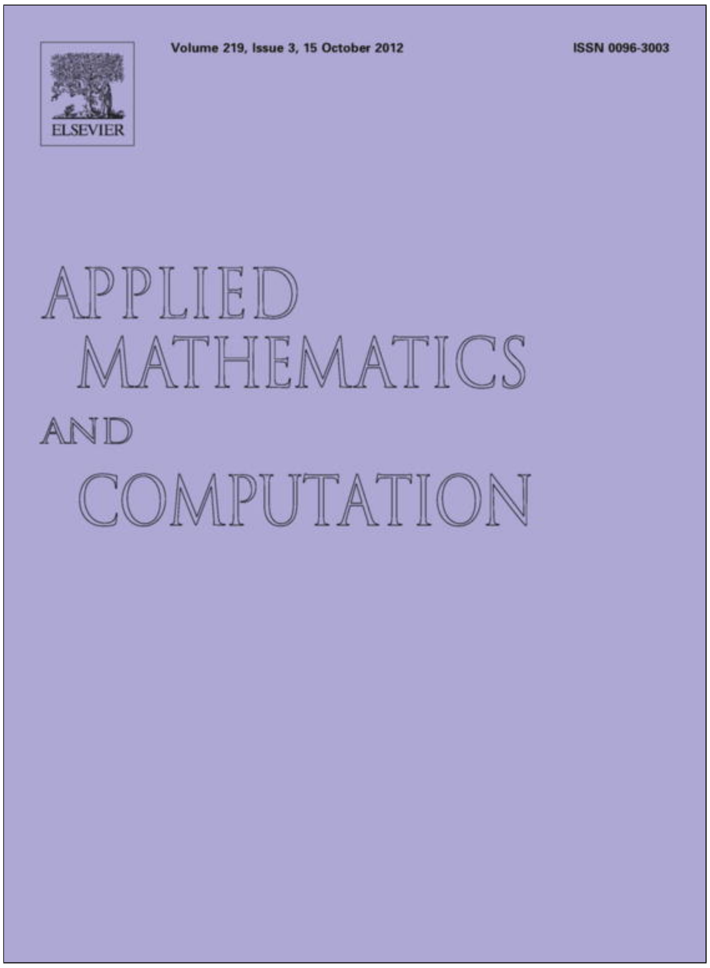
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Synchronization of a complex dynamical network with coupling time-varying delays via sampled-data control

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ABSTRACT

In this paper, the synchronization problem of a complex dynamical network with coupling time-varying delays via delayed sampled-data controller is investigated. In order to make full use of the sawtooth structure characteristic of the sampling input delay, a discontinuous Lyapunov functional is proposed based on the Extended Wirtinger Inequality. From a convex representation of the sector-restricted nonlinearity in system dynamics, the stability condition based on Lyapunov stability theory is obtained by utilization of linear matrix inequality formulation to find the controller which achieves the synchronization of a complex dynamical network with coupling time-varying delay. Finally, two numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

During the last decade, complex dynamical networks, which are a set of interconnected nodes with specific dynamics, have been attracted increasing attention in various fields such as physics, biology, chemistry and computer science [1–3]. As science and society have been developed, our everyday life has been closed to complex dynamical networks, for instance, transportation networks, World Wide Web, coupled biological and chemical engineering systems, neural networks, social networks, electrical power grids and global economic markets. Many of these networks exhibit complexity in the overall topological and dynamical properties of the network nodes and the coupled units. One of the significant and interesting phenomena in complex dynamical networks is the synchronization. Synchronization of complex dynamical networks can be divided into two points of view. One is the synchronization of a complex network that is called 'inner synchronization' [4–13]. It means that all the nodes in a complex network eventually approach to trajectory of a target node. The other is called 'outer synchronization' [14–16] which considers the synchronization between two or more complex networks. In this paper, a new control problem for inner synchronization will be investigated.

In real world situation, time delay is ubiquitous in many physical systems [17–21] due to the finite switching speed of amplifiers, finite signal propagation time in networks, finite reaction times, memory effects and so on. Furthermore, the time delay may cause undesirable dynamic behaviors such as oscillation, instability and poor performance. Therefore, the synchronization problem of complex dynamical networks with time delays has become a topic of both theoretical and practical importance. In this regard, some synchronization criteria for the general complex dynamical network with coupling delays which are both delay-independent and delay-dependent conditions are derived in [5]. Wang and Guan [6] presented several new exponential synchronization stability criteria for some general complex dynamical network models with time delays,

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which are less conservative than the work of [5]. But, those papers [5,6] considered the constant time delays. However, it is more general and desirable to consider time-varying delays instead of constant delays in the view of real applications. From this perspective, many results for the problem have been focused on time-varying delays. For example, the synchronization criterion of the neutral complex dynamical networks with time-varying delays was proposed in [7]. In addition, for Lur'e type complex dynamical networks with time-varying delays, Ji et al. [8] proposed a less conservative synchronization stability condition than existing ones in the literature.

In general, all the dynamic behaviors of the nodes in complex dynamical networks is not synchronized up to each other. Therefore, some attention of the problem, how to achieve the synchronization of asynchronous complex dynamical networks, has been increasing rapidly. Until now, in order to treat the synchronization control problem for complex dynamical networks, several control schemes are applied. For example, the impulsive control scheme has been applied to achieve the projective synchronization of a complex dynamical network in [9]. In [10], the state observer based control scheme has been proposed. In [11], the adaptive control scheme has been adopted to carry through the synchronization of a complex dynamical network, whereas in [16], the adaptive control for the synchronization between two complex dynamical networks has been investigated. Recently, in [12], the pinning-controllability and the method of choosing pinning nodes for the synchronization of a complex dynamical network are suggested. In addition, in [13], the synchronization of the complex dynamical network with linearly and nonlinearly coupling terms via pinning control has been studied.

Because of the rapid growth of the digital hardware technologies, the sampled-data control method, whose the control signals are kept constant during the sampling period and are allowed to change only at the sampling instant, has been more important than other control approaches. These discontinuous control signals which have stepwise form cause big trouble to control or analyze the system. In order to effectively deal with sampled-data control, Mikheev et al. [22] and Astrom and Wittenmark [23] introduced a concept that discontinuous sampled control inputs treat time-varying delayed continuous signals, although applied actual control signals are discontinuous. Since the works of [22,23], many types of the sampled-data control scheme by using the concept in [22,23] have been proposed. For instance, in [24], the robust $\mathcal{H}_\infty$ sampled-data control has been proposed. In [25], the sampled-data fuzzy controller has been proposed as well. Moreover, many researchers have adopted the sampled-data control scheme to solve control problems in various systems such as chaotic system [26], fuzzy system [27], neural networks [28] and so on. However, there are only a few papers for complex dynamical networks using the sampled-data control approach [29]. Besides, the sampled-data control with the signal transmission delay has not been discussed in [29]. The signal transmission delay is important factor because it cause serious situation such as instability and pure performance by combining sampling intervals. Therefore, it is very worth to consider the sampled-data control method in the presence of the signal transmission delays for complex dynamical networks.

From motivation mentioned above, this paper proposes a discontinuous Lyapunov functional approach to achieve asymptotic synchronization of a complex dynamical network with coupling time-varying delays using sampled-data control in the presence of the constant signal transmission delay. The discontinuous Lyapunov functional makes full use of the sawtooth structure characteristic of sampling input delays and thus get less conservative synchronization criterion for the system. A convex representation of the nonlinearity in system dynamics is introduced, and then a sector-bounded constraint of the nonlinearity is represented to an equality constraint. The derived sufficient condition for the stability is formulated by a linear matrix inequality that is easily solvable using various numerical convex optimization algorithms [30].

This paper is organized as follows. A problem statement is described in Section 2. Section 3 provides the design method of a stabilizing sampled-data controller in the presence of a constant input delay for the synchronization of a complex dynamical network with coupling time-varying delays. Two numerical examples are given in Section 4 to show the effectiveness of the derived results. Conclusions are drawn in Section 5.

Notation 1. $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrix. $X > 0$ (respectively, $X \geqslant 0$) means that the matrix X is a real symmetric positive definite matrix (respectively, positive semi-definite). I denotes the identity matrix. $diag\{\cdots\}$ denotes block diagonal matrix. $\star$ in a matrix represents the elements below the main diagonal of a symmetric matrix. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $\otimes$ stands for the notation of Kronecker product. Co denotes the convex hull.

2. Problem formulation

Consider a complex dynamical network consisting of N linearly coupled identical nodes as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{N} c_{ij}x_j(t-\tau(t)) + u_i(t), \quad i = 1,\ldots,N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the ith node, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$ are constant matrices, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear vector field and $u_i(t)$ is the control input of ith node, $\tau(t)$ is the coupling time-varying delay satisfying

$$0 \leqslant \tau(t) \leqslant \tau_M, \quad \dot{\tau}(t) \leqslant \mu,$$

where τ_M and μ are known constants. $C = (c_{ij})_{N\times N}$ is the coupling matrix of the network, where the coupling configuration parameter, c_{ij}, is defined as follows: if there is a connection from node i to node j $(i \neq j)$ then $c_{ij} = 1$; otherwise $c_{ij} = 0$ $(i \neq j)$, and the diagonal elements of matrix C are assumed by

$$c_{ii} = -\sum_{j=1,j\neq i}^{N} c_{ij} = -\sum_{j=1,j\neq i}^{N} c_{ji}, \quad i = 1,\ldots,N.$$

Throughout this paper, the following assumption is used.

Assumption 1. The smooth nonlinear function $f(\cdot)$ is satisfied the following sector and slop bound conditions:

$$\begin{aligned} &b_k \leqslant \frac{f_k(x_{ik}(t))}{x_{ik}(t)} \leqslant a_k, \\ &\beta_k \leqslant \frac{df_k(x_{ik}(t))}{dx_{ik}(t)} \leqslant \alpha_k, \quad k = 1,\ldots,n \end{aligned} \tag{2}$$

where $b_k,\ a_k$ are lower and upper sector bounds, and $\beta_k,\ \alpha_k$ are lower and upper slope bounds, respectively.

Our objective of the paper is to design stabilizing controllers $u_i(t)$ for asymptotical synchronization between all nodes of a complex dynamical network and a target node which is denoted by following definition.

Definition 1. A complex dynamical network is said to achieve the asymptotical inner synchronization, if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad \text{as} \quad t \to \infty,$$

where $s(t) \in \mathbb{R}^n$ is a solution of a target node, satisfying

$$\dot{s}(t) = Ax(t) + Bf(s(t)). \tag{3}$$

For our synchronization scheme, let us define the error vectors as follows:

$$e_i(t) = s(t) - x_i(t). \tag{4}$$

From Eq. (4), the error dynamics is given to

$$\dot{e}_i(t) = Ae_i(t) + B(f(s(t)) - f(x_i(t))) - \sum_{j=1}^{N} c_{ij}e_j(t-\tau(t)) - u_i(t) = Ae_i(t) + B\bar{f}_i(t) - \sum_{j=i}^{N} c_{ij}e_j(t-\tau(t)) - u_i(t), \quad i = 1,\ldots,N, \tag{5}$$

where $\bar{f}_i(t) = f(s(t)) - f(x_i(t))$.

By the well-known mean value theorem, there exists a constant $v \in (x_{ik}(t), s_{ik}(t))$ such that

$$f_k(s_k(t)) - f_k(x_{ik}(t)) = \frac{df_k(v)}{dv}(s_k(t) - x_{ik}(t)). \tag{6}$$

From the slope bounds given in Assumption 1, we have

$$\beta_k \leqslant \frac{df_k(v)}{dv} \leqslant \alpha_k. \tag{7}$$

By using Eqs. (6), (7) and $e_i(t) = s(t) - x_i(t)$, we have

$$\beta_k e_{ik}(t) \leqslant \bar{f}_{ik}(e_{ik}(t)) \leqslant \alpha_k e_{ik}(t). \tag{8}$$

Therefore, Eq. (8) can be represented the following equality condition by properties of the convex hull:

$$\bar{f}_i(e_i(t)) = \Delta_i e_i(t), \tag{9}$$

where Δ_i is an element of a convex hull $Co\{\alpha,\ \beta\}$.

Remark 1. The slope bound of nonlinear function, $f(\cdot)$, becomes new sector bound of the nonlinear function $\bar{f}(e_i(t)) = f(s(t)) - f(x_i(t))$. And this condition can be represented by a convex combination of the sector bounds α_k and β_k. This method was proposed in [8]. In general, most of nonlinear functions which consist of nonlinear system such as Chua's circuit [31], the rotational-translational actuator system [32] and so on, satisfy this condition. Also, this condition includes Lipschitz condition as a special case.

On the other hand, in order to design the controller with the sampled-data signal, the concept of the time-varying delayed control input which is proposed in [22,23], is adopted in this paper. For this, the following state feedback controller is considered

$$u_i(t) = K_i e_i(t_k), \quad t_k \leqslant t < t_{k+1}, \tag{10}$$

where K_i is the gain matrix of feedback controller to be determined.

Denote by t_k the updating instant time of the Zero-Order-Hold (ZOH), and suppose that the updating signal (successfully transmitted signal from the sampler to the controller and to the ZOH) at the instant t_k has transmitted with the constant signal transmission delay η. We assume that the sampling intervals satisfy

$$t_{k+1} - t_k = h_k \leqslant h, \tag{11}$$

for any integer $k \geqslant 0$, where h is a positive scalar and represents the largest sampling interval.

Thus, we have that

$$t_{k+1} - t_k + \eta \leqslant h + \eta = d. \tag{12}$$

Therefore, by defining $d(t) = t - t_k + \eta,\ t_k \leqslant t < t_{k+1}$, the controller (10) can be represented as following:

$$u_i(t) = K_i e_i(t_k - \eta) = K_i e_i(t - d(t)), \quad t_k \leqslant t \leqslant t_{k+1}. \tag{13}$$

From (12), we can find that $\eta \leqslant d(t) < t_{k+1} - t_k + \eta \leqslant d$ and $\dot{d(t)} = 1$ for $t \neq t_k$. Substituting (13) into (5) gives

$$\dot{e}_i(t) = Ae_i(t) + B\bar{f}_i(e_i(t)) - \sum_{j=i}^{N} c_{ij} e_j(t - \tau(t)) - K_i e_i(t - d(t)), \quad t_k \leqslant t < t_{k+1}; \quad i = 1, \ldots, N. \tag{14}$$

Then Eq. (14) can be rewritten as a vector–matrix form

$$\dot{e}(t) = A_N e(t) + B_N F(e(t)) - C_N e(t - \tau(t)) - Ke(t - d(t)), \tag{15}$$

where $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, $F(t) = [\bar{f}^T(e_1(t)), \ldots, \bar{f}^T(e_N(t))]^T$, $K = diag\{K_1, \ldots, K_N\}$, $A_N = I_N \otimes A$, $B_N = I_N \otimes B$ and $C_N = C \otimes I_n$.

Remark 2. Many systems such as the continuous-time systems with the digital control, the networked control systems and so on, can be modeled by sampled-data systems. Now days, most of controllers are the digital controller or networked to the system, so the sampled-data control approach is eligible to receive much attention. In addition, the signal transmission delay naturally rises with connections of each parts, but most papers about the sampled-data system [22–29] do not consider it. Hence, this paper investigates a control problem with the sampled-data control in the presence of the constant signal transmission delay. This idea is depicted in Fig. 1.

3. Main results

In this section, a design problem of the sampled-data feedback controller for the synchronization of a complex dynamical network with coupling time-varying delays will be investigated via a discontinuous Lyapunov functional approach. Before proceeding further, the following lemmas are given.

Lemma 1 (*Jensen Inequality [33]*). *For any matrix $M > 0$, scalars γ_1 and γ_2 satisfying $\gamma_2 > \gamma_1$, a vector function $x : [\gamma_1, \gamma_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_{\gamma_1}^{\gamma_2} x(s)ds\right)^T M \left(\int_{\gamma_1}^{\gamma_2} x(s)ds\right) \leqslant (\gamma_2 - \gamma_1) \int_{\gamma_1}^{\gamma_2} x^T(s)Mx(s)ds. \tag{16}$$

Lemma 2 (*Extended Wirtinger Inequality [34]*). *Let $x(t) \in W[a, b)$ and $x(a) = 0$. Then for any matrix $R > 0$ the following inequality holds:*

$$\int_a^b x(s)^T Rx(s)ds \leqslant \frac{4(b-a)^2}{\pi^2} \int_a^b \dot{x}(s)^T R\dot{x}(s)ds. \tag{17}$$

Now, the main result is given by the following theorem.

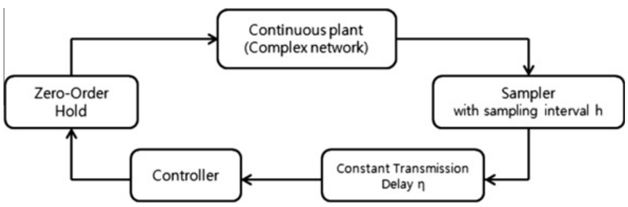

Fig. 1. The structure of the sampled-data control in this paper.

Theorem 1. *For given a scalar γ and a matrix Δ, there exists a sampled feedback controller (10) for synchronization of the complex dynamical network (1) if there exist positive-definite matrices $P, Q, R_1, R_2, R_3, Z_1, Z_2, Z_3, D \in \mathbb{R}^{nN\times nN}$, matrices $H, N_1, N_2 \in \mathbb{R}^{nN\times nN}$ and a block diagonal matrix $G = diag\{G_1,\ldots,G_N\}$ $(G_i \in \mathbb{R}^{n\times n})$ satisfying the following linear matrix inequality:*

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & 0 & \Gamma_{14} & Z_2 & -H & 0 & \Gamma_{18} \\ \star & \Gamma_{22} & Z_1 & 0 & 0 & 0 & 0 & -\gamma C_N^T G^T \\ \star & \star & \Gamma_{33} & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \Gamma_{44} & 0 & 0 & 0 & \gamma B_N^T G^T \\ \star & \star & \star & \star & \Gamma_{55} & \Gamma_{56} & 0 & 0 \\ \star & \star & \star & \star & \star & \Gamma_{66} & Z_3 & -\gamma H^T \\ \star & \star & \star & \star & \star & \star & \Gamma_{77} & 0 \\ \star & \star & \star & \star & \star & \star & \star & \Gamma_{88} \end{bmatrix}, \tag{18}$$

where

$$\begin{aligned}
&\Gamma_{11} = GA_N + A_N^T G^T + Q + R_1 + R_2 - Z_1 - Z_2 + N_1\Delta + \Delta N_1^T,\\
&\Gamma_{12} = Z_1 - GC_N,\\
&\Gamma_{14} = GB_N - N_1 + \Delta N_2^T,\\
&\Gamma_{18} = P - G + \gamma A_N^T G^T,\\
&\Gamma_{22} = -(1-\mu)Q - 2Z_1,\\
&\Gamma_{33} = -R_1 - Z_1,\\
&\Gamma_{44} = -N_2 - N_2^T,\\
&\Gamma_{55} = -R_2 + R_3 - \alpha D - Z_2 - Z_3,\\
&\Gamma_{56} = Z_3 + \alpha D,\\
&\Gamma_{66} = -2Z_3 - \alpha D,\\
&\Gamma_{77} = -R_3 - Z_3,\\
&\Gamma_{88} = \tau_M^2 Z_1 + \eta^2 Z_2 + h^2 Z_3 + h^2 D - \gamma G - \gamma G^T,\\
&\Delta = diag\{\Delta_1,\ldots,\Delta_N\}.
\end{aligned}$$

Also, the desired control gain matrix (10) can be given by $K = G^{-1}H$.

Proof. Consider the following discontinuous Lyapunov functional for the error system (14)

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad t \in [t_k, t_{k+1}), \tag{19}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t),\\
V_2(t) &= \int_{t-\tau(t)}^{t} e^T(s)Qe(s)ds + \int_{t-\tau_M}^{t} e^T(s)R_1 e(s)ds + \tau_M \int_{-\tau_M}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Z_1\dot{e}(s)dsd\theta,\\
V_3(t) &= \int_{t-\eta}^{t} e^T(s)R_2 e(s)ds + \int_{t-d}^{t-\eta} e^T(s)R_3 e(s)ds + \eta\int_{-\eta}^{0}\int_{t+\theta}^{t}\dot{e}^T(s)Z_2\dot{e}(s)dsd\theta + (d-\eta)\int_{-d}^{-\eta}\int_{t+\theta}^{t}\dot{e}^T(s)Z_3\dot{e}(s)dsd\theta,\\
V_4(t) &= (d-\eta)^2\int_{t_k-\eta}^{t}\dot{e}^T(s)D\dot{e}(s)ds - \frac{\pi^2}{4}\int_{t_k-\eta}^{t-\eta}(e(s)-e(t_k-\eta))^T D(e(s)-e(t_k-\eta))ds.
\end{aligned}$$

It is noted that $V_4(t)$ can be rewritten as

$$V_4(t) = (d-\eta)^2\int_{t-\eta}^{t}\dot{e}^T(s)D\dot{e}(s)ds + \widehat{V}_4(t), \tag{20}$$

where

$$\widehat{V}_4(t) = (d-\eta)^2\int_{t_k-\eta}^{t-\eta}\dot{e}^T(s)D\dot{e}(s)ds - \frac{\pi^2}{4}\int_{t_k-\eta}^{t-\eta}(e(s)-e(t_k-\eta))^T D(e(s)-e(t_k-\eta))ds.$$

According to Lemma 2, it is easy to find that $\widehat{V}_4(t) \geqslant 0$. In addition, it is correct that $\lim_{t\to t_k^-} V(t) \geqslant V(t_k)$, because $\widehat{V}_4(t)$ will disappear at $t = t_k$.

Here, the time derivative of $V(t)$ is

$$\dot{V}_1(t) = 2e^T(t)P\dot{e}(t), \tag{21}$$

$$\begin{aligned}
\dot{V}_2(t) &= e^T(t)Qe(t) - (1-\mu)e^T(t-\tau(t))Qe(t-\tau(t)) + e^T(t)R_1e(t) - e(t-\tau_M)R_1e(t-\tau_M) + \tau_M^2\dot{e}^T(t)Z_1\dot{e}(t) \\
&\quad - \tau_M\int_{t-\tau(t)}^{t}\dot{e}^T(s)Z_1\dot{e}(s)ds - \tau_M\int_{t-\tau_M}^{t-\tau(t)}\dot{e}^T(s)Z_1\dot{e}(s)ds \\
&\leqslant e^T(t)Qe(t) - (1-\mu)e^T(t-\tau(t))Qe(t-\tau(t)) + e^T(t)R_1e(t) - e(t-\tau_M)R_1e(t-\tau_M) \\
&\quad + \tau_M^2\dot{e}^T(t)Z_1\dot{e}(t) - \begin{bmatrix} e(t) \\ e(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} Z_1 & -Z_1 \\ \star & Z_1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \end{bmatrix} \\
&\quad - \begin{bmatrix} e(t-\tau(t)) \\ e(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} Z_1 & -Z_1 \\ \star & Z_1 \end{bmatrix} \begin{bmatrix} e(t-\tau(t)) \\ e(t-\tau_M) \end{bmatrix},
\end{aligned} \tag{22}$$

$$\begin{aligned}
\dot{V}_3(t) &= e^T(t)R_2e(t) - e^T(t-\eta)R_2e(t-\eta) + e(t-\eta)R_3e(t-\eta) - e^T(t-d)R_3e(t-d) + \eta^2\dot{e}^T(t)Z_2\dot{e}(t) + h^2\dot{e}^T(t)Z_3\dot{e}(t) \\
&\quad - \eta\int_{t-\eta}^{t}\dot{e}^T(s)Z_2\dot{e}(s)ds - h\int_{t-d(t)}^{t-\eta}\dot{e}^T(t)Z_3\dot{e}(s)ds - h\int_{t-d}^{t-d(t)}\dot{e}^T(s)Z_3\dot{e}(s)ds \\
&\leqslant e^T(t)R_2e(t) - e^T(t-\eta)R_2e(t-\eta) + e(t-\eta)R_3e(t-\eta) - e^T(t-d)R_3e(t-d) + \eta^2\dot{e}^T(t)Z_2\dot{e}(t) + h^2\dot{e}^T(t)Z_3\dot{e}(t) \\
&\quad - \begin{bmatrix} e(t) \\ e(t-\eta) \end{bmatrix}^T \begin{bmatrix} Z_2 & -Z_2 \\ \star & Z_2 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\eta) \end{bmatrix} - \begin{bmatrix} e(t-\eta) \\ e(t-d(t)) \end{bmatrix}^T \begin{bmatrix} Z_3 & -Z_3 \\ \star & Z_3 \end{bmatrix} \begin{bmatrix} e(t-\eta) \\ e(t-d(t)) \end{bmatrix} \\
&\quad - \begin{bmatrix} e(t-d(t)) \\ e(t-d) \end{bmatrix}^T \begin{bmatrix} Z_3 & -Z_3 \\ \star & Z_3 \end{bmatrix} \begin{bmatrix} e(t-d(t)) \\ e(t-d) \end{bmatrix},
\end{aligned} \tag{23}$$

$$\dot{V}_4(t) = h^2\dot{e}^T(t)D\dot{e}(t) - \frac{\pi^2}{4}\begin{bmatrix} e(t-\eta)^T \\ e(t-d(t))^T \end{bmatrix}\begin{bmatrix} D & -D \\ \star & D \end{bmatrix}\begin{bmatrix} e(t-\eta) \\ e(t-d(t)) \end{bmatrix}. \tag{24}$$

From the convex representation (9), we can obtain the following equation:

$$F(t) = \Delta e(t). \tag{25}$$

The constraint (25) is rewritten as

$$[\Delta \quad -I]\begin{bmatrix} e(t) \\ F(t) \end{bmatrix} = 0. \tag{26}$$

For matrices N_1 and N_2, the following equality are always satisfied:

$$2\begin{bmatrix} e(t) \\ F(t) \end{bmatrix}^T \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}[\Delta \quad -I]\begin{bmatrix} e(t) \\ F(t) \end{bmatrix} = 0. \tag{27}$$

Also, according to the error system (15), for any appropriately dimensioned matrix G, the following equation holds:

$$2\left[e^T(t)G + \gamma\dot{e}^T(t)G\right][-\dot{e}(t) + A_Ne(t) + B_NF(t) - C_Ne(t-\tau(t)) - Ke(t-d(t))] = 0. \tag{28}$$

By adding the left sides of (27) and (28) to $\dot{V}(t)$ and letting $H = GK$, we can obtain the following new upper bound of time derivative of Lyapunov functional $V(t)$

$$\dot{V}(t) \leqslant \zeta(t)^T\Gamma\zeta(t), \tag{29}$$

where

$$\zeta(t) = \left[e^T(t)\, e^T(t-\tau(t))\, e^T(t-\tau_M)\, F^T(t)\, e^T(t-\eta)\, e^T(t-d(t))\, e^T(t-d)\, \dot{e}^T(t)\right]^T.$$

Thus, Eq. (29) implies

$$\dot{V}(t) < -\epsilon\|e(t)\|^2 \tag{30}$$

where ϵ is the largest eigenvalue of Γ. This completes the proof. □

If the term $V_4(t)$ is neglected, then the discontinuous Lyapunov functional (19) becomes the continuous Lyapunov functional. If $D = 0$ in (19) choose zero matrix, then we can obtain the following theorem.

Corollary 1. *For given a scalar γ and a matrix Δ, if there exist positive-definite matrices $P, Q, R_1, R_2, R_3, Z_1, Z_2, Z_3 \in \mathbb{R}^{nN\times nN}$, any matrices $G, H, N_1, N_2 \in \mathbb{R}^{nN\times nN}$ and a scalar $\lambda > 0$ satisfying the LMI (18) such that $(18)|_{D=0}$, then there exists a sampled feedback controller (10) which achieves the synchronization of the complex dynamical network (1). Moreover, the desired control gain matrix in (10) can be given by $K = G^{-1}H$.*

Remark 3. The synchronization criteria for a complex dynamical network via the discontinuous and continuous Lyapunov functional come from Theorem 1 and Corollary 1, respectively. The point of two types of the Lyapunov functional is the existence of $V_4(t)$ which originates from [35] and makes full use of the sawtooth structure characteristic of sampling input delays. Thus, theoretically the conservatism of Theorem 1 is less than Corollary 1, which will be validated by numerical examples in the next section.

4. Numerical examples

In order to show the effectiveness of the proposed methods, we present two numerical examples which are the synchronization of a complex dynamical network with coupling time-varying delays using the sampled-data control in the absence of the constant signal transmission delay. In the both examples, MATLAB LMI toolbox is used to solve LMI problems and the parameter γ is given 0.01.

Example 1. The first example is about the synchronization of a complex dynamical network with five linearly coupled identical nodes which are Chua's chaotic circuit [31] and its chaotic behavior is displayed in Fig. 2. Thus, the complex dynamical network system consisting of five nodes is described by:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{5} c_{ij}x_j(t-\tau(t)) + u_i(t), \quad i = 1,\ldots,5, \tag{31}$$

where

$$A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -a(m_0 - m_1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$f(x_{ik}(t)) = \frac{1}{2}(|x_{ik}(t) + c| - |x_{ik}(t) - c|), \quad k = 1,\ldots,n$$

with the parameters are $a = 9,\ b = 14.28,\ c = 1,\ m_0 = -1/7,\ m_1 = 2/7$, and the nonlinear function $f(\cdot)$ belongs to sector $[0,1]$ and slope $[0,1]$.

Here, Chua's circuit is also chosen as a target node, $s(t)$. In this example, initial conditions of each nodes are chosen: $x_1(0) = [-0.1\ -0.5\ -0.7],\ x_2(0) = [-0.1\ -0.4\,0.3],\ x_3(0) = [0.6\ -1.5\,0],\ x_4(0) = [0.1\,0.1\,0.1],\ x_5(0) = [0\,0.5\ -0.4]$ and $s(0) = [0.1\,0.5\ -0.7]$. The coupling time-varying delay is assumed as $\tau(t) = 0.4 + 0.1\sin(t)$, then the parameters associated with $\tau(t)$ are obtained as $\tau_M = 0.5,\ \mu = 0.1$ and the coupling matrix, C, is given by

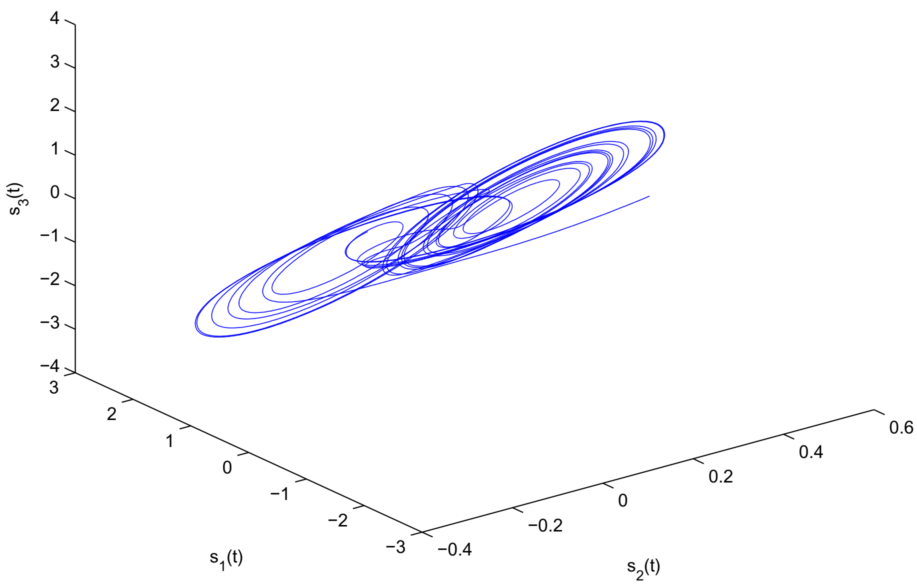

Fig. 2. The chaotic behavior of Chua's circuit.

Table 1
The maximum values of sampling intervals h for different signal transmission delays η in Example 1.

η	0.01	0.02	0.03	0.04	0.05
Theorem 1	0.17	0.16	0.16	0.14	0.12
Corollary 1	0.12	0.11	0.10	0.09	0.01

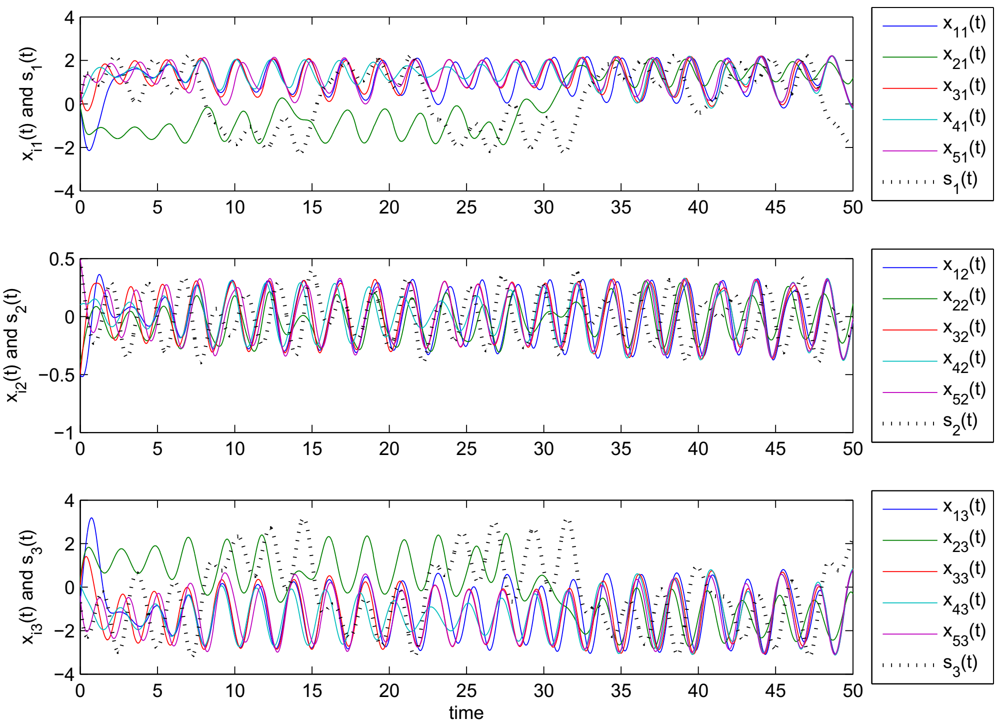

Fig. 3. The state orbits of the system (31) and the target node.

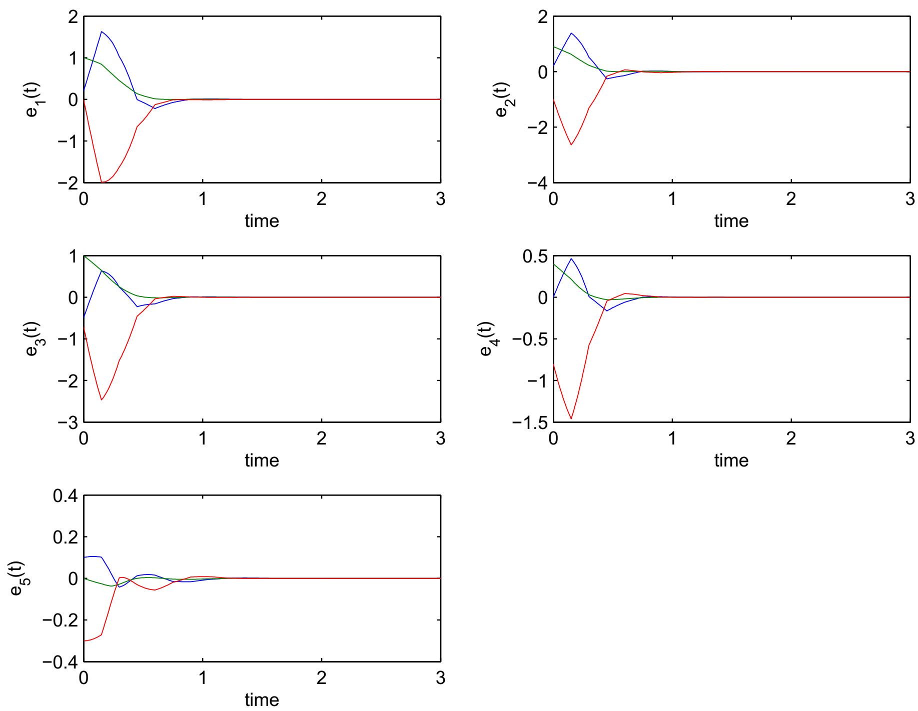

Fig. 4. The error signals of the controlled system (31).

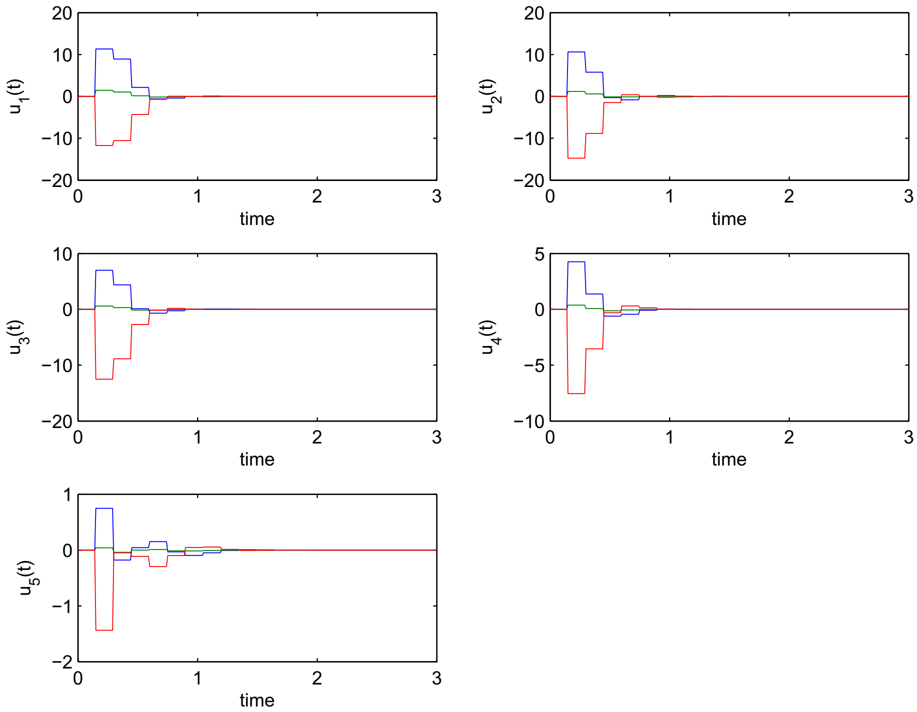

Fig. 5. The control input signals of Example 1.

$$C = 0.2 \times \begin{bmatrix} -3 & 1 & 1 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -3 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 1 & 0 & 1 & -3 \end{bmatrix}. \tag{32}$$

By Theorem 1 and Corollary 1, we can obtain the maximum sampling interval h for the signal transmission delay, η, given in Table 1. Table 1 shows the maximum values of h, for different η. It can be seen from Table 1 that Theorem 1 get the more improved solution than Corollary 1 as mentioned above in Remark 3.

In order to show effectiveness of the controller, the state orbits of the uncontrolled system (31) and the target system (dashed line) are depicted in Fig. 3. And by solving LMI problem (18) in Theorem 1 with $h = 0.15$ and $\eta = 0.02$, we can obtain the following control gains:

$$K = diag\{K_1, \ldots, K_5\}, \tag{33}$$

where

$$K_1 = \begin{bmatrix} 4.7881 & 2.8699 & -1.1342 \\ 0.7692 & 0.5493 & 0.0721 \\ -1.5394 & -1.7523 & 4.5927 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 4.8307 & 2.9759 & -1.0884 \\ 0.7823 & 0.5909 & 0.0772 \\ -1.3436 & -1.7391 & 4.8728 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 4.7306 & 2.9611 & -1.1360 \\ 0.7395 & 0.5521 & 0.0729 \\ -1.4960 & -1.8393 & 4.6115 \end{bmatrix}, \quad K_4 = \begin{bmatrix} 4.7843 & 2.9705 & -1.1296 \\ 0.7337 & 0.5511 & 0.0431 \\ -1.4765 & -1.8283 & 4.6594 \end{bmatrix},$$
$$K_5 = \begin{bmatrix} 4.7383 & 2.8358 & -1.1672 \\ 0.7674 & 0.5579 & 0.0893 \\ -1.4534 & -1.7825 & 4.7738 \end{bmatrix}.$$

Under the above control gains, the simulation result of the controlled system (31) and the sampled control inputs are presented in Fig. 4 and Fig. 5, respectively. As seen in Fig. 4, the trajectories of error systems are indeed well stabilized. It means that all states are synchronized up to the states of the target node by control inputs which are seen in Fig. 5. In order to show more clearly the performance of the proposed sampled-data control method, Fig. 6 is presented. Fig. 6 shows the state orbits of the system (31) and the target system (dashed line). In this figure, control inputs are applied to the system at 20 sec. From this figure, it is clear that the synchronization between each nodes and the target node do not achieved until 20 sec, however after activating the controller all states are synchronized up to the states of the target node.

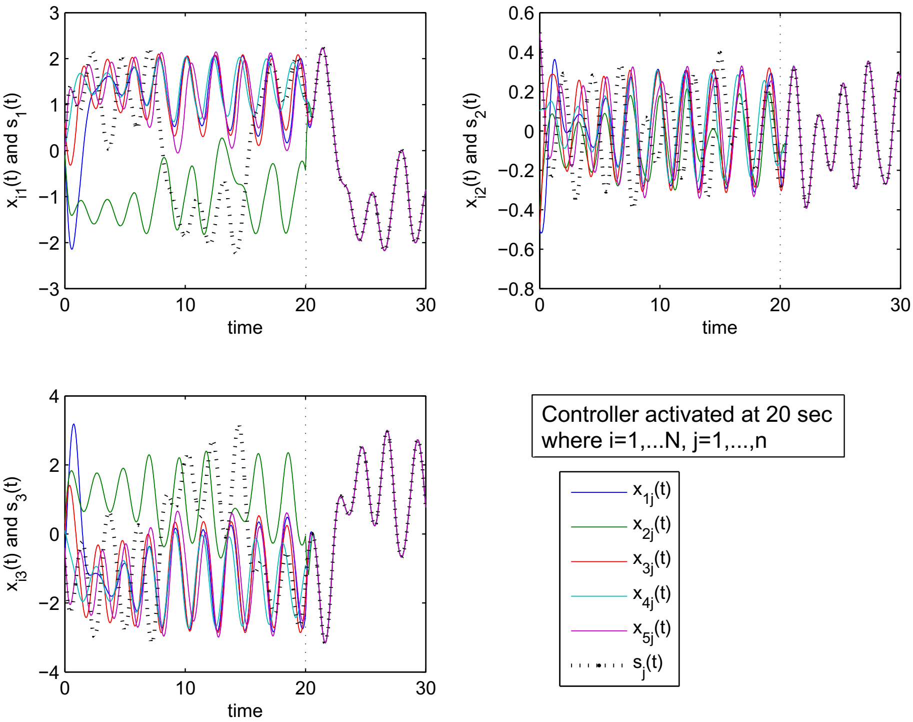

Fig. 6. The state orbits of Example 1.

Example 2. Next example is about the synchronization of a complex dynamical network with three linearly coupled identical nodes which are described by [29]:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + \sum_{i=1}^{3} c_{ij}x_j(t-\tau(t)) + u_i(t), \quad i = 1,\ldots,3, \tag{34}$$

where

$$A = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad C = 0.5 \times \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix},$$

$$f(t) = \begin{bmatrix} \tanh(0.2x_{i1}(t)) \\ \tanh(0.75x_{i2}(t)) \end{bmatrix}, \quad \tau(t) = 0.2 + 0.05\sin(10t).$$

So, we have $\tau_M = 0.25$, $\mu = 0.5$ and the sector and slope bound of the nonlinear function $f(\cdot)$ in (34) are [0, 1] and [0, 1], respectively. And a target node is also chosen as same system.

Under the above simulation setting, Table 2 which specifies the values of the maximum sampling interval, h, for different η can be obtained by Theorem 1 and Corollary 1.

When the sampling interval $h = 0.1$ and the signal transmission delay $\eta = 0.04$, the control gain matrix which is calculated by Theorem 1 is given by

$$K = diag\{K_1, K_2, K_3\}, \tag{35}$$

where

$$K_1 = \begin{bmatrix} 3.9837 & 0.1410 \\ 0.0735 & 4.8353 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 3.9837 & 0.1410 \\ 0.0735 & 4.8353 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 4.6674 & 0.0727 \\ 0.0382 & 4.9740 \end{bmatrix}.$$

Table 2
The maximum values of sampling intervals h for different signal transmission delays η in Example 2.

η	0.01	0.02	0.03	0.04	0.05
Theorem 1	0.15	0.13	0.11	0.10	0.08
Corollary 1	0.09	0.08	0.07	0.06	0.05

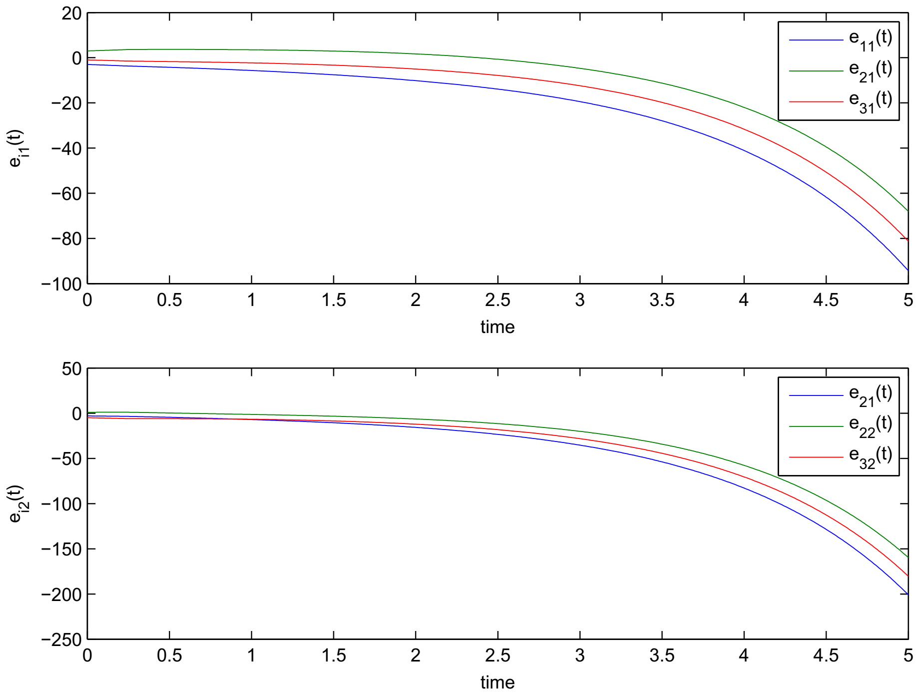

Fig. 7. The error signals of the system (34).

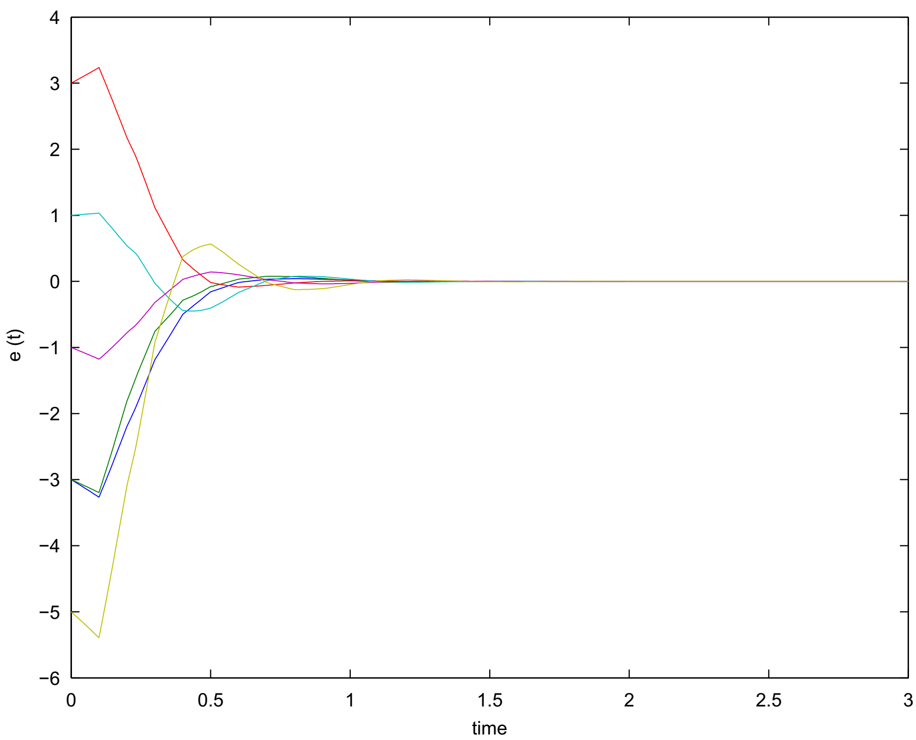

Fig. 8. The error signals of the controlled system (34).

The error signals of the uncontrolled and controlled system (34) by the control gain K are presented in Fig. 7 and Fig. 8, respectively. Comparing with Fig. 7 and Fig. 8, error signals of the controlled system become zero as time goes infinity, however, in the case of the uncontrolled system, error signals do not approach to zero. It implies that our proposed controller achieve the synchronization of a complex dynamical network (34). The applied control inputs which consisted of sampled signals are displayed in Fig. 9.

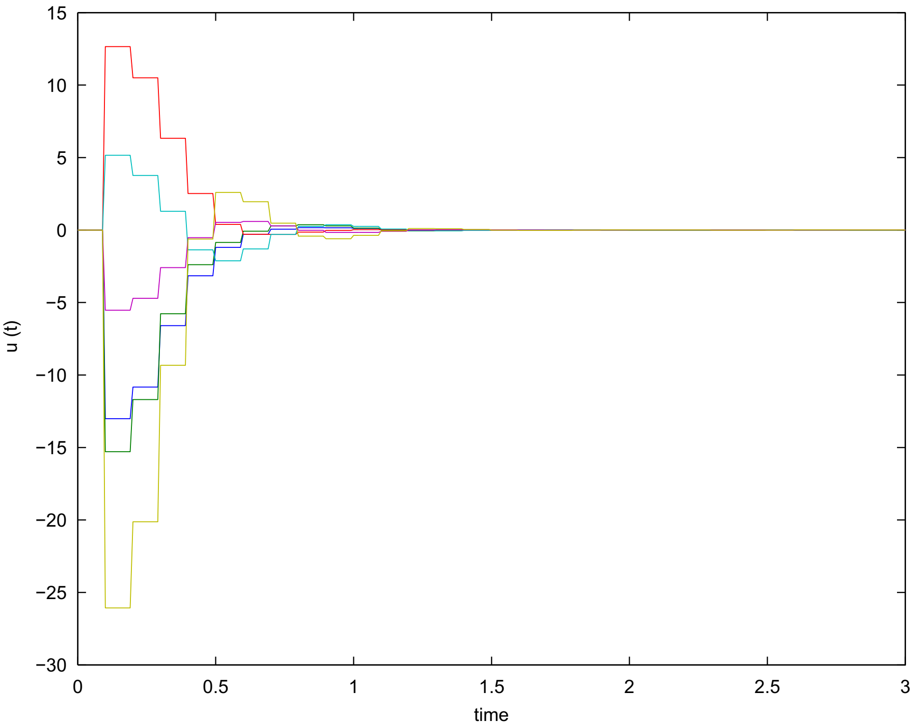

Fig. 9. The control input signals of Example 2.

5. Conclusions

In this paper, the sampled-data control for the synchronization of a complex dynamical network with time-varying delays has been discussed in the framework of the signal transmission delay. Based on Extended Wirtinger Inequality, a discontinuous Lyapunov functional which gives full information of sawtooth structure characteristic of the sampling delay has been proposed. The results have been shown that the use of the discontinuous Lyapunov functional gets less conservatism than the use of the continuous Lyapunov functional. Then existence criterion of the controller has been derived in terms of LMI which is based on Lyapunov stability theory and the sector-slope restricted nonlinearity conditions. Two numerical examples have been illustrated to show the performance of the proposed controller.

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