One Bit Feedback for Quasi-Orthogonal Space-Time Block Codes Based on Circulant Matrix

Zhu Chen and Moon Ho Lee, *Senior Member, IEEE*

***Abstract*—During the last few years, a number of Quasi-Orthogonal Space-Time Block Codes (QOSTBC) have been proposed for using in multiple transmit antennas systems. In this letter, based on circulant matrix, we propose a novel method of extending any QOSTBC constructed for 4 transmit antennas to a closed-loop scheme. We show that with the aid of multiplying the entries of QOSTBC code words by the appropriate phase factors which depend on the channel information, the proposed scheme can improve its transmit diversity with one bit feedback. The performances of the proposed scenario extended from Jafarkhani's QOSTBC as well as its optimal constellation rotated scheme are analyzed. The simulation results suggest that there is a significant Eb/No advantage in the proposed scheme which is able to be designed easily.**

***Index Terms*—QOSTBC, feedback, closed-loop, rotation, circulant.**

I. Introduction

SPACE-time coding is a transmit diversity scheme with optional receive diversity to achieve high data rate and to improve the reliability of a wireless channel. Since the work of Alamouti orthogonal space-time block coding (OSTBC) [1] has been an intensive area of research due to their low decoding complexity, However, for complex constellations, rate-one code (one symbol transmitted in one symbol duration) exists only for two transmit antennas, when three or four transmit antennas were considered, the maximum symbol transmission rate of the complex OSTBC with the linear processing was 3/4 [2]. Due to this drawback, various QOSTBC have been proposed to achieve a full rate (R=1) for more than 2 transmit antennas at the expense of losing the diversity gain and increasing the decoding complexity [3][4].

Recently, a lot of researches have been put into designing the STBC with full rate and full diversity for four transmit antennas [5]-[10]. For open-loop communication systems, the optimum constellation rotation proposed for QOSTBC with different modulation schemes is the one of good diversity improvement approaches [5]. Although a lot of partial feedback methods can be adopted to improve the closed-loop system performance [6][7][10], the major problems of such systems are high cost and high complexity due to the more feedback information. For practical interests of the design of the closed-loop transmission schemes, it is desirable to have features such as a limited amount of feedback information, low decoding delay, low cost and simple decoding processing.

In this letter, we present a novel closed-loop scenario extended from Jafarkhani's QOSTBC as well as its optimal rotated scheme for the quasi-static flat fading channels with four transmit antennas. We show that, by feeding back one bit channel information, our proposed scheme can increase the transmit diversity and reduce the self interference from adjacent symbols in QOSTBC scheme. The proposed approach, unlike [11] which need to sacrifice the optimal rotated phase in open-loop system for the feedback variable in closed-loop system, can avoid the damage on optimal rotated phase by employing circulant matrix. It, therefore, is able to offer not only more flexibility but also a performance advantage over exiting methods.

Notation: Throughout this paper, by A^*, A^T and A^H we mean the conjugate, transpose and Hermition of matrix A .By a^* and $Re(a)$ we mean the conjugate and the real part of element a. $A.*B$ means the element-by-element product of the matrices A and B.

II. The Proposed Closed-loop Scheme Extended from Jafarkhani's QOSTBC

In this section, a quasi-static flat fading channel with four transmit antennas and one receive antenna is considered. With this assumption, Jafarkhani's QOSTBC is first described in order to facilitate the introduction of the new scheme. The (4×4) QOSTBC is given by

$$S_J = \begin{bmatrix} S_{12} & S_{34} \\ -S_{34}^* & S_{12}^* \end{bmatrix}, \tag{1}$$

where S_{12} and S_{34} are the two (2×2) building blocks based on the Alamouti scheme of two transmit antennas, $S_{12} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$ and $S_{34} = \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix}$, thus $S_J = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}$. The received signals during four successive time slots can be expressed as:

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} = H_J S + N, \tag{2}$$

where the noise samples and the entries of H_J are independent samples of a zero-mean complex Gaussian random variable with variance 1.

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Z. Chen and M. H. Lee (corresponding author) are with the Institute of Information and Communication, Chonbuk National University, Jeonju, 561-756 Korea (e-mail: {chenzhu, moonho}@chonbuk.ac.kr).

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A. Proposed Scheme for QOSTBC with One Bit Feedback

After multiplying the entries of S_J by four phase factors, we present our proposed scheme as below:

$$S_P = \begin{bmatrix} s_1 e^{j\alpha} & s_2 e^{j\beta} & s_3 e^{j\gamma} & s_4 e^{j\theta} \\ -s_2^* e^{-j\alpha} & s_1^* e^{-j\theta} & -s_4^* e^{-j\gamma} & s_3^* e^{-j\beta} \\ -s_3^* e^{-j\alpha} & -s_4^* e^{-j\beta} & s_1^* e^{-j\gamma} & s_2^* e^{-j\theta} \\ s_4 e^{j\alpha} & -s_3 e^{j\theta} & -s_2 e^{j\gamma} & s_1 e^{j\beta} \end{bmatrix}. \quad (3)$$

The received signals are given as:

$$\begin{bmatrix} r_1 \\ r_2^* \\ r_3^* \\ r_4 \end{bmatrix} = \begin{bmatrix} h_1 e^{j\alpha} & h_2 e^{j\beta} & h_3 e^{j\gamma} & h_4 e^{j\theta} \\ h_2^* e^{j\theta} & -h_1^* e^{j\alpha} & h_4^* e^{j\beta} & -h_3^* e^{j\gamma} \\ h_3^* e^{j\gamma} & h_4^* e^{j\theta} & -h_1^* e^{j\alpha} & -h_2^* e^{j\beta} \\ h_4 e^{j\beta} & -h_3 e^{j\gamma} & -h_2 e^{j\theta} & h_1 e^{j\alpha} \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \\ n_3^* \\ n_4 \end{bmatrix} \quad (4)$$
$$= H_P S + N,$$

where the relevant channel matrix $H_P = H_J . * C_4$, and C_4 is circulant matrix which can be expressed as:

$$C_4 = \begin{bmatrix} e^{j\alpha} & e^{j\beta} & e^{j\gamma} & e^{j\theta} \\ e^{j\theta} & e^{j\alpha} & e^{j\beta} & e^{j\gamma} \\ e^{j\gamma} & e^{j\theta} & e^{j\alpha} & e^{j\beta} \\ e^{j\beta} & e^{j\gamma} & e^{j\theta} & e^{j\alpha} \end{bmatrix}, \quad (5)$$

where $e^{j\alpha}, e^{j\beta}, e^{j\gamma}$ and $e^{j\theta}$ are introduced phase factors. With the conditions $e^{j(\beta-\alpha)} = e^{j(\alpha-\theta)}, e^{j(\theta-\gamma)} = e^{j(\gamma-\beta)}, e^{j(\alpha-\gamma)} = e^{j(\gamma-\alpha)}$ and $e^{j(\beta-\theta)} = e^{j(\theta-\beta)}$. The Grammian matrix which can be calculated by left-multiplying the matched filtering H_P^H with H_P is given as:

$$G_P = H_P^H H_P = \underbrace{h^2 \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix}}_{U_P} + \underbrace{\omega \begin{bmatrix} 0 & J_2 \\ -J_2 & 0 \end{bmatrix}}_{V_P}, \quad (6)$$

where $h^2 = \sum_{i=1}^{4} |h_i|^2$ indicates the total channel gain for the four transmit antennas. ω can be interpreted as the channel dependent interference parameter, and given by

$$\omega = e^{j(\alpha-\beta)} \cdot 2Re(h_1^* h_4) - e^{j(\gamma-\theta)} \cdot 2Re(h_2 h_3^*), \quad (7)$$

I_2 is identity matrix and $J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

As presented in (6), the Grammian matrix G_P , can be divided into two components, which are the channel gain matrix U_P and the interference matrix V_P . i.e.,$G_P = U_P + V_P$. It is well known that the presence of the channel dependent interference ω in V_P can cause the performance degradation in contrast with the optimal orthogonal design. Hence, in order to achieve the ideal 4-path diversity, G_P should approach U_P as close as possible, in other words, the absolute value of ω in V_P should be minimized. The effect of ω in V_P is explained in [4]. From (7), on the premise of knowing the partial channel information, we can achieve the minimal absolute value of ω by adjusting the value of the two factors $e^{j(\alpha-\beta)}$ and $e^{j(\gamma-\theta)}$, namely: when $Re(h_1^* h_4) \cdot Re(h_2 h_3^*) \geq 0$, we set $e^{j(\alpha-\beta)} \cdot e^{j(\gamma-\theta)} = 1$; when $Re(h_1^* h_4) \cdot Re(h_2 h_3^*) < 0$, we set $e^{j(\alpha-\beta)} \cdot e^{j(\gamma-\theta)} = -1$. This issue can be interpreted as follows:

Assuming we know the channel information at the receiver, and adopt one bit $k = 0$ or 1 to indicate $Re(h_1^* h_4) \cdot Re(h_2 h_3^*) \geq 0$ or $Re(h_1^* h_4) \cdot Re(h_2 h_3^*) < 0$ respectively.

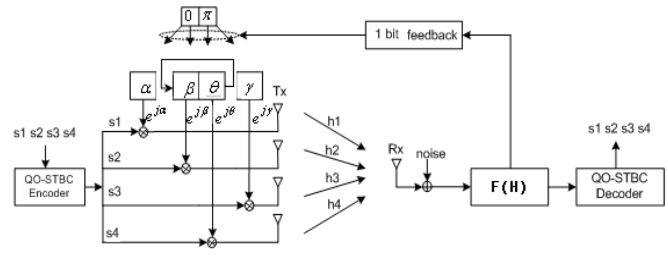

Fig. 1. The proposed closed-loop scheme for S_J.

Then this one bit information will be fed back to the transmitter. Supposing the system channel is quasi-static flat fading channel, at the transmitter we first judge the value of k , if $k = 0$, we set $\alpha = \gamma = \pi$ and $\beta = \theta = 0$, which ensure $e^{j(\alpha-\beta)} \cdot e^{j(\gamma-\theta)} = 1$, if $k = 1$, we set $\alpha = \beta = \theta = 0$ and $\gamma = \pi$ which ensure $e^{j(\alpha-\beta)} \cdot e^{j(\gamma-\theta)} = -1$. Hence, we give the solution for this closed-loop scheme as follows:

$$\begin{cases} if Re(h_1^* h_4) \cdot Re(h_2 h_3^*) \geq 0 \; feedback k=0 \; setting \alpha=\gamma=\pi, \beta=\theta=0 \\ if Re(h_1^* h_4) \cdot Re(h_2 h_3^*) < 0 \; feedback k=1 \; setting \alpha=\beta=\theta=0, \gamma=\pi. \end{cases} \quad (8)$$

The block diagram of the proposed closed-loop QOSTBC scheme extended from S_J with four transmit antennas and one receive antenna is depicted in Fig.1, where $F(H) = Re(h_1^* h_4) \cdot Re(h_2 h_3^*)$. Since α, β, θ and γ only equal to 0 or π, we have $e^{-j\alpha} = e^{j\alpha}, e^{-j\beta} = e^{j\beta}, e^{-j\theta} = e^{j\theta}$, and $e^{-j\gamma} = e^{j\gamma}$. Moreover, β and θ will take a right circulation during every time slot.

It is worth pointing out that this proposed scheme in Fig.1 for S_J is also fit for all of the existing QOSTBC. However, in terms of various code words, the introduced angles at the transmitter and the function $F(H)$ at the receiver will be a little different.

B. Proposed Scheme for QOSTBC with Optimal Rotation

In [5], in order to provide a full-diversity, a rotated QOSTBC based on S_J is introduced as below:

$$S_{RJ} = \begin{bmatrix} s_1 & s_2 & \mu s_3 & \mu s_4 \\ -s_2^* & s_1^* & -(\mu s_4)^* & (\mu s_3)^* \\ -(\mu s_3)^* & -(\mu s_4)^* & s_1^* & s_2^* \\ \mu s_4 & -\mu s_3 & -s_2 & s_1 \end{bmatrix}, \quad (9)$$

where $\mu = e^{j\phi}$ is the rotated factor. It has been proved that when $\phi = \frac{\pi}{4}$, it is optimal for QPSK constellation.

Based on this optimal rotated QOSTBC, we present our scheme as:

$$S_{PR} = \begin{bmatrix} s_1 e^{j\alpha} & s_2 e^{j\beta} & \mu s_3 e^{j\gamma} & \mu s_4 e^{j\theta} \\ -s_2^* e^{-j\alpha} & s_1^* e^{-j\theta} & -(\mu s_4)^* e^{-j\gamma} & (\mu s_3)^* e^{-j\beta} \\ -(\mu s_3)^* e^{-j\alpha} & -(\mu s_4)^* e^{-j\beta} & s_1^* e^{-j\gamma} & s_2^* e^{-j\theta} \\ \mu s_4 e^{j\alpha} & -\mu s_3 e^{j\theta} & -s_2 e^{j\gamma} & s_1 e^{j\beta} \end{bmatrix}, \quad (10)$$

and the expression of the Grammian matrix is:

$$G_{PR} = H_{PR}^H H_{PR} = \underbrace{h^2 \begin{bmatrix} I_2 & 0 \\ 0 & I_2 \end{bmatrix}}_{U_{PR}} + \underbrace{\begin{bmatrix} \omega_1 & \omega_2 \end{bmatrix} \begin{bmatrix} 0 & J_2 \\ -J_2 & 0 \end{bmatrix}}_{V_{PR}}, \quad (11)$$

where $\omega_1 = \omega e^{\frac{\pi j}{4}}$ and $\omega_2 = \omega e^{\frac{-\pi j}{4}}$. Distinctly, both of ω_1 and ω_2 are depended on ω . Hence the solution (8) is also fit for the optimal rotated QOSTBC.

III. Closed-Loop Scheme for Optimal Rotated QOSTBC with Multiple Receive Antennas

In last section, we described the proposed closed-loop scheme based on one receive antenna. However, our scheme can also be applied for a system with M receive antennas. Assuming a quasi-static flat fading channel with four transmit and M receive antennas, using the proposed feedback scheme for QOSTBC with optimal rotation, the received signals during four successive time slots can be expressed as:

$$\begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_M \end{bmatrix} \cdot S + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_M \end{bmatrix} = HS + N, \quad (12)$$

where the channel matrix H_j for the $jth\,(1 \le j \le M)$ receive antenna is

$$H_j = \begin{bmatrix} h_{j1}e^{j\alpha} & h_{j2}e^{j\beta} & \mu h_{j3}e^{j\gamma} & \mu h_{j4}e^{j\theta} \\ h_{j2}^*e^{j\theta} & -h_{j1}^*e^{j\alpha} & \mu h_{j4}^*e^{j\beta} & -\mu h_{j3}^*e^{j\gamma} \\ h_{j3}^*e^{j\gamma} & h_{j4}^*e^{j\theta} & -\mu h_{j1}^*e^{j\alpha} & -\mu h_{j2}^*e^{j\beta} \\ h_{j4}e^{j\beta} & -h_{j3}e^{j\gamma} & -\mu h_{j2}e^{j\theta} & \mu h_{j1}e^{j\alpha} \end{bmatrix}, \quad (13)$$

the received signal vector R_j and the noise vector N_j on the jth receive antenna are

$$R_j = \begin{bmatrix} r_{j1} & r_{j2}^* & r_{j3}^* & r_{j4} \end{bmatrix}^T, \quad (14)$$

$$N_j = \begin{bmatrix} n_{j1} & n_{j2}^* & n_{j3}^* & n_{j4} \end{bmatrix}^T, \quad (15)$$

and the transmitted symbol vector:

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \end{bmatrix}^T. \quad (16)$$

Since the new channel matrix H is a $4M \times 4$ matrix, the expression of the (4×4) Grammian matrix takes the same form of (11) but with h and ω replaced by the following:

$$h^2 = \sum_{j=1}^{M}\sum_{i=1}^{4} |h_{ji}|^2, \quad (17)$$

$$\omega = e^{j(\alpha-\beta)} \cdot 2\sum_{j=1}^{M} Re(h_{j1}^* h_{j4}) - e^{j(\gamma-\theta)} \cdot 2\sum_{j=1}^{M} Re(h_{j2}h_{j3}^*). \quad (18)$$

Similar as (8), the channel dependent interference parameter can be minimized with the aid of one bit feedback information which is determined by the value of the new $F(H)$, i.e., $F(H) = \sum_{j=1}^{M} Re(h_{j1}^* h_{j4}) \cdot \sum_{j=1}^{M} Re(h_{j2}h_{j3}^*)$.

IV. Simulation Results

In this section, we evaluate the error performance of the proposed scheme. In all simulations, we consider only one receiving antenna for simplicity. It is assumed that the channel is quasi-static flat fading channel, and the fading is constant within a frame and changes independently from frame to frame. Besides, the receiver has perfect channel state information.

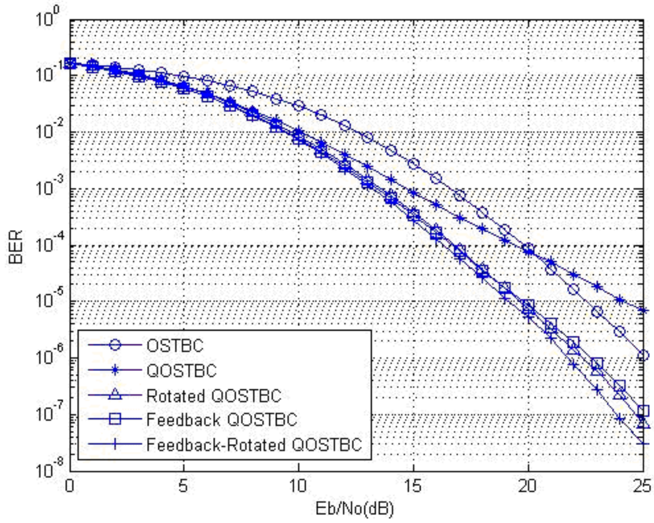

Fig. 2. Bit error probability versus Eb/No for different STBC at 2 bits/(s Hz).

In Fig.2, we show the performances of the proposed closed-loop QOSTBC schemes with ML receiver. The performance of the orthogonal STBC with rate 1/2, the Jafarkhani's QOSTBC, and its optimal rotated scheme are also shown in Fig.2 for comparison. We adopt 16-QAM for the orthogonal STBC and QPSK for other QOSTBC schemes. Comparing the BER of QOSTBC and its corresponding closed-loop scheme, namely, the feedback QOSTBC, we find their curves are nearly the same at low SNR. However, with the increase of SNR, the performance of the closed-loop scheme is much better than the open-loop QOSTBC. Simulation results also show that the performance of feedback QOSTBC is similar with the performance of optimal rotated QOSTBC, nevertheless, the combination of one bit feedback and the optimal constellation rotation can achieve the best performance. For example, when the BER is 10^{-4} , the proposed feedback-rotated QOSTBC gets about 3.12 dB and 0.5 dB over QOSTBC and optimal rotated QOSTBC, respectively.

V. Conclusion

A simple closed-loop QOSTBC based on circulant matrix is proposed in this letter. For systems with four transmit antennas and multiple receive antennas, it can enhance the performance with the feedback information as few as 1 bit. In particular, the presented closed-loop scheme can be applied in any existing QOSTBC without increasing the design complexity. Moreover, one important advantage of the proposed scheme is that it needn't to sacrifice the optimal rotated phase for the feedback variable. Simulation results show that the optimal rotated phase in our proposed closed-loop scheme also makes a great contribution to the system performance.

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