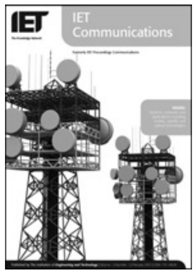
Published in IET Communications
Received on 17th September 2007
Revised on 23rd December 2007
doi: 10.1049/iet-com:20070445



ISSN 1751-8628

Error performance of general order selection in correlated Nakagami fading channels

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Abstract: A procedure for determining the probability distribution of the *r*th order statistic, $G_{r:L}$, $r = 1, 2, \ldots, L$, among a set of L correlated Nakagami diversity branch gains $G_1, G_2, \ldots, G_L$ has been described in David and Nagaraja (2003) and Elkashlan *et al.* (2008). The results are used to evaluate the bit error rate (BER) of general order selection (GOS), a diversity method in which the *r*th order branch is selected for transmission, over correlated Nakagami fading branches. GOS can be used to improve system throughput and provide various levels of services, both of which are highly desirable in high-speed communication systems. Numerical and simulation results are presented and used to illustrate the effects of fading correlation on the BER associated with the *r*th order gain branch.

1 Introduction

To increase system capacity and improve the quality-of-service (QoS) offerings in wireless communication systems, numerous techniques have been proposed to mitigate the deleterious effects of channel fading and to improve the received signal-to-noise ratio (SNR). These techniques include diversity reception, dynamic channel allocation and power control. In this paper, the focus is on diversity and channel allocation methods.

The theory of order statistics underpins the performance analyses of many diversity reception techniques that involve efficient channel allocation and signal processing algorithms for signal detection and estimation. Selection combining (SC) is relatively simple since it involves processing only the signal on the diversity branch with the highest SNR. The performance analysis of SC assuming independent channel fading, has been studied extensively in the literature [1–4]. However, the assumption of independent fading is valid only if the diversity branches are spaced sufficiently far apart, a situation which is not always possible in practice. The performance analysis of SC over correlated fading branches, for dual and triple branch diversity, is treated in [5–9]. As far as higher order diversity is concerned, bit error rate (BER) expressions for the highest SNR with an arbitrary number of branches are given in [10, 11]. In [12], the performance of a hybrid selection/maximal-ratio combining (H-S/MRC) diversity scheme over exchangeable correlated Nakagami fading branches is analysed. A set of random variables $A_1, \ldots, A_L$ are said to be exchangeable if the joint distribution of $A_{\pi_1}, \ldots, A_{\pi_L}$ does not depend on the permutation π.

The distribution of the *r*th order SNR is required for the performance evaluation of general order selection (GOS), in which the *r*th order branch is selected for transmission. This distribution is also often required in the performance analyses of various diversity systems in the presence of incorrect channel estimation and signal detection. For example, it is useful in calculating the performance loss of SC when an error in selecting the highest SNR branch occurs. GOS can be utilised to support applications with different QoS requirements. As an example, it can be employed in IEEE 802.16 OFDM-based WiMAX systems to offer differentiated personal broadband services, such as mobile entertainment [13]. The selective allocation of OFDM subcarriers, bit and power loading to different data streams based on their respective QoS requirements provides a highly scalable and flexible method to support differentiated high-bandwidth and low-latency entertainment applications.

A performance analysis of multibranch GOS over correlated fading channels is not available in the literature. In this paper, the BER of GOS for binary phase shift keying (BPSK) modulation over a set of correlated and not necessarily exchangeable Nakagami fading channels is studied. We formulate the problem in a general framework and derive a solution which is applicable to various multiple access and diversity schemes such as the channel-aware frequency hopping (CAFH) scheme in [14]. In CAFH, a given mobile station (MS) may not be able to transmit in its highest SNR subband because this particular subband may already have been assigned to another MS. The remainder of this paper is organised as follows. In Section 2, the system model is described and a discussion of relevant cumulative distribution functions (cdf's) is provided. In Section 3, an expression for the BER of GOS is derived. Numerical results are presented in Section 4, and in Section 5, some concluding remarks are drawn.

2 Channel and system model

Let $G_1, G_2, \ldots, G_L$ be L arbitrarily correlated random variables (rv's) corresponding to the branch gains in a diversity communication system. The received baseband signal at the ℓth branch is

$$r_\ell(t) = G_\ell s(t) e^{-j\psi_\ell} + n_\ell(t), \quad \ell = 1, 2, \ldots, L \tag{1}$$

where $s(t)$, $t \in (0, T)$ is the transmitted signal and $n_\ell(t)$ is an independent additive white Gaussian noise (AWGN) process. The phase ψ_ℓ is uniformly distributed over the range $[0, 2\pi)$. The corresponding branch SNR's are denoted by $\Gamma_1, \Gamma_2, \ldots, \Gamma_L$, with

$$\Gamma_\ell \triangleq [G_\ell]^2 E/N_0, \quad \ell = 1, 2, \ldots, L \tag{2}$$

where $E = \int_0^T s^2(t)\mathrm{d}t$ is the transmitted bit energy and N_0 is the one-sided noise power spectral density (PSD).

If the rv's $G_1, \ldots, G_L$ are arranged in increasing order of their magnitudes and written as

$$G_{1:L} \le \cdots \le G_{L:L} \tag{3}$$

we refer to $G_{r:L}$ as the rth order statistic. The rv's $G_1, \ldots, G_L$ are statistically dependent and not necessarily exchangeable branch gains. The SNR of the rth order statistic is then

$$\Gamma_{r:L} = [G_{r:L}]^2 E/N_0 \tag{4}$$

From [15, p. 99], and [16] the cdf of the rth order statistic can be obtained as

$$F_{\Gamma_{r:L}}(\gamma) = \sum_{i=r}^{L}\left[(-1)^{i-r}\binom{i-1}{r-1}\sum_{1\le\pi_1<\cdots<\pi_i\le L} F_{\Gamma_{i:i}}^{(\pi_1,\ldots,\pi_i)}(\gamma)\right] \tag{5}$$

In (5), the inner sum is over all possible ways of selecting i out of L rv's and the superscript notation in $F_{\Gamma_{i:i}}^{(\pi_1,\ldots,\pi_i)}$ indicates that only $\Gamma_{\pi_1}, \ldots, \Gamma_{\pi_i}$ are included in the selection. The G_ℓ's are modelled as correlated Nakagami-m rv's with a marginal Nakagami probability density function (pdf) given by [17]

$$f_{G_\ell}(g) = \frac{2}{\Gamma(m_\ell)}\cdot\left(\frac{m_\ell}{\Omega_\ell}\right)^{m_\ell}\cdot g^{2m_\ell-1}\cdot \mathrm{e}^{-(m_\ell/\Omega_\ell)g^2}, \quad g \ge 0, \quad \ell = 1, 2, \ldots, L \tag{6}$$

where $\Gamma(\cdot)$ is the Gamma function, m_ℓ is the fading parameter, and $\Omega_\ell = \boldsymbol{E}[(G_\ell)^2]$. It is well-known that the pdf of Γ_ℓ follows a Gamma distribution, that is

$$f_{\Gamma_\ell}(\gamma) = \frac{m_\ell^{m_\ell}}{\Gamma(m_\ell)}\cdot\frac{\gamma^{m_\ell-1}}{(\overline{\Gamma}_\ell)^{m_\ell}}\cdot \mathrm{e}^{-m_\ell(\gamma/\overline{\Gamma}_\ell)}, \quad \gamma \ge 0, \quad \ell = 1, 2, \ldots, L \tag{7}$$

where the average SNR on the ℓth branch is given by $\overline{\Gamma}_\ell = (E/N_0)\Omega_\ell$. We assume that each branch experiences the same fading parameter value, i.e., $m_\ell = m$ for $\ell = 1, 2, \ldots, L$. Given the fact that the instantaneous SNR's are always non-negative, we can express their joint cdf as [18, p. 140]

$$F_{\Gamma_1,\ldots,\Gamma_L}(\gamma_1, \ldots, \gamma_L) = \frac{1}{(2\pi)^L}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi(t_1, \ldots, t_L)\prod_{\ell=1}^{L}\left(\frac{1-\mathrm{e}^{-\mathrm{j}t_\ell\gamma_\ell}}{\mathrm{j}t_\ell}\right)\mathrm{d}t_1\cdots\mathrm{d}t_L \tag{8}$$

where $\mathrm{j} = \sqrt{-1}$ and $\phi(t_1, \ldots, t_L)$ is the joint characteristic function (CF) of the Γ_ℓ's given in (24). From (8), the cdf of the highest order statistic $\Gamma_{i:i} \triangleq \max\{\Gamma_1, \ldots \Gamma_i\}$ can be written as

$$F_{\Gamma_{i:i}}(\gamma) = \Pr\{\Gamma_1, \ldots, \Gamma_i \le \gamma\} = \frac{1}{(2\pi)^i}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi(t_1, \ldots, t_i)\prod_{\ell=1}^{i}\left(\frac{1-\mathrm{e}^{-\mathrm{j}t_\ell\gamma}}{\mathrm{j}t_\ell}\right)\mathrm{d}t_1\cdots\mathrm{d}t_i \tag{9}$$

3 Error performance of GOS

By differentiating (5), we can write the pdf of $\Gamma_{r:L}$ as

$$f_{\Gamma_{r:L}}(\gamma) = \frac{dF_{\Gamma_{r:L}}(\gamma)}{d\gamma} = \sum_{i=r}^{L}\left[(-1)^{i-r}\binom{i-1}{r-1}\sum_{1\le\pi_1<\cdots<\pi_i\le L}\frac{dF_{\Gamma_{i:i}}^{(\pi_1,\ldots,\pi_i)}(\gamma)}{d\gamma}\right] = \sum_{i=r}^{L}\left[(-1)^{i-r}\binom{i-1}{r-1}\sum_{1\le\pi_1<\cdots<\pi_i\le L}f_{\Gamma_{i:i}}^{(\pi_1,\ldots,\pi_i)}(\gamma)\right] \tag{10}$$

where the pdf of the highest order statistic, $\Gamma_{i:i}$, is given by [11]

$$f_{\Gamma_{i:i}}^{(\pi_1,\ldots,\pi_i)}(\gamma) = \frac{1}{(2\pi)^i}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi(t_{\pi_1},\ldots,t_{\pi_i}) \times h(\gamma,\boldsymbol{t})dt_{\pi_1}\cdots dt_{\pi_i} \tag{11}$$

and

$$h(\gamma,\boldsymbol{t}) = \frac{d}{d\gamma}\left[\prod_{\ell=1}^{i}\left(\frac{1-e^{-jt_{\pi_\ell}\gamma}}{jt_{\pi_\ell}}\right)\right] = \left[\prod_{\ell=1}^{i}(jt_{\pi_\ell})\right]^{-1}\left[\sum_{n=1}^{i}(-1)^{n+1} \times \sum_{b_{\pi_1}+\cdots+b_{\pi_i}=n}\frac{j(b_{\pi_1}t_{\pi_1}+\cdots+b_{\pi_i}t_{\pi_i})}{\exp(j\gamma[b_{\pi_1}t_1+\cdots+b_{\pi_i}t_{\pi_i}])}\right] \tag{12}$$

where $b_{\pi_1},\ldots,b_{\pi_i}$ is a binary sequence whose elements assume the value of zero or one. For BPSK modulation over an AWGN channel with one-sided PSD N_0, the average error probability when transmitting on the rth order branch out of L branches is obtained by averaging the conditional error probability, $P(e|\gamma)$, over the pdf of $\Gamma_{r:L}$, that is

$$P_e^{r:L} = \int_0^{\infty}P(e|\gamma)f_{\Gamma_{r:L}}(\gamma)d\gamma \tag{13}$$

where [19]

$$P(e|\gamma) = Q(\sqrt{2\gamma}) \tag{14}$$

Using (10) in (13) yields

$$P_e^{r:L} = \sum_{i=r}^{L}(-1)^{i-r}\binom{i-1}{r-1}\times\sum_{1\le\pi_1<\cdots<\pi_i\le L}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\phi^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) \times w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i})dt_1\cdots dt_{\pi_i} \tag{15}$$

with

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) = \frac{1}{(2\pi)^i}\int_0^{\infty}P(e|\gamma)h^{(\pi_1,\ldots,\pi_i)}(\gamma,\boldsymbol{t})d\gamma \tag{16}$$

As noted in [11], the integrand in (15) consists of two terms. The first term $\phi(\boldsymbol{t})$ depends only on the channel whereas the second term $w(\boldsymbol{t})$ depends only on the modulation scheme. Substituting (14) into (16) yields

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) = \frac{1}{(2\pi)^i}\int_0^{\infty}Q(\sqrt{2\gamma})h^{(\pi_1,\ldots,\pi_i)}(\gamma,\boldsymbol{t})d\gamma \tag{17}$$

Integration by parts as

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) = \frac{1}{(2\pi)^i}\left[\prod_{\ell=1}^{i}(jt_{\pi_\ell})\right]^{-1}\sum_{n=1}^{i}(-1)^{n+1} \sum_{b_{\pi_1}+\cdots+b_{\pi_i}=n}\int_0^{\infty}\overbrace{Q(\sqrt{2\gamma})}^{u}\underbrace{\frac{j(b_{\pi_1}t_{\pi_1}+\cdots+b_{\pi_i}t_{\pi_i})}{\exp(j\gamma[b_{\pi_1}t_{\pi_1}+\cdots+b_{\pi_i}t_{\pi_i}])}d\gamma}_{ds} \tag{18}$$

yields

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) = \frac{1}{(2\pi)^i}\left[\prod_{\ell=1}^{i}(jt_{\pi_\ell})\right]^{-1}\sum_{n=1}^{i}(-1)^{n+1} \times\sum_{b_{\pi_1}+\cdots+b_{\pi_i}=n}\frac{1}{2}\left[1-\frac{1}{\sqrt{\pi}} \times\int_0^{\infty}\frac{e^{-\gamma(1+j[b_{\pi_1}t_{\pi_1}+\cdots+b_{\pi_i}t_{\pi_i}])}}{\sqrt{\gamma}}d\gamma\right] \tag{19}$$

Letting $x=\sqrt{\gamma}$, we obtain

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1},\ldots,t_{\pi_i}) = \frac{1}{(2\pi)^i}\left[\prod_{\ell=1}^{i}(jt_{\pi_\ell})\right]^{-1}\sum_{n=1}^{i}(-1)^{n+1}\sum_{b_{\pi_1}+\cdots+b_{\pi_i}=n}\left[\frac{1}{2}-\frac{1}{\sqrt{\pi}}\int_0^{\infty}e^{-x^2(1+j[b_{\pi_1}t_{\pi_1}+\cdots+b_{\pi_i}t_{\pi_i}])}dx\right] \tag{20}$$

Using

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_0^{\infty}e^{-x^2/2\sigma^2}dx = \frac{1}{2} \tag{21}$$

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doi: 10.1049/iet-com:20070445

in (20) with $\sigma^2 = 1/(2(1+j[b_{\pi_1}t_{\pi_1} + \cdots + b_{\pi_i}t_{\pi_i}]))$, we have

$$w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1}, \ldots, t_{\pi_i}) = \frac{1}{(2\pi)^i}\left[\prod_{\ell=1}^{i}(jt_{\pi_\ell})\right]^{-1} \sum_{n=1}^{i}(-1)^{n+1} \sum_{b_{\pi_1}+\cdots+b_{\pi_i}=n} \frac{1}{2}\left[1 - \frac{1}{\sqrt{1+j[b_{\pi_1}t_{\pi_1} + \cdots + b_{\pi_i}t_{\pi_i}]}}\right] \quad (22)$$

The error performance can then be obtained by inserting $w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1}, \ldots, t_{\pi_i})$ and $\phi^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1}, \ldots, t_{\pi_i})$ for a given channel into (15).

4 Numerical results and discussion

To illustrate the usefulness of the approach described in Section 3, numerical examples are provided for a commonly used correlation model. We calculate the BER, $P_e^{r:L}$, when transmitting on the rth order SNR branch (subband) out of L available subbands in a Nakagami-m correlated fading environment. We assume that the various subbands are subject to correlated fading, where the degree of correlation depends on, among other factors, the subband frequency separation. The correlation at any instant of time between the fade envelopes, G_k and G_l of the kth and lth subbands, respectively, is assumed to be [20]

$$E\{G_k, G_l\} = \frac{1}{1 + \left(f_k - f_l/B_c\right)^2} \equiv \rho_{k,l} \quad (23)$$

where B_c is the coherence bandwidth of the channel and $\rho_{k,l}$ is the correlation coefficient. Therefore the $L \times L$ covariance matrix $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \bar{\Gamma}\begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,L} \\ \rho_{2,1} & 1 & \cdots & \rho_{2,L} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{L,1} & \rho_{L,2} & \cdots & 1 \end{bmatrix}$$

where $\boldsymbol{R}$ is symmetric and positive definite [21] and $\bar{\Gamma}$ denotes the average SNR. For correlated Nakagami-m fading, the joint CF is given by [22, p. 359]

$$\phi(t_1, \ldots, t_L) = |\boldsymbol{I} - \mathrm{j}\boldsymbol{TS}|^{-m} \quad (24)$$

where $\boldsymbol{I}$ is the identity matrix of size $L \times L$, $|\cdot|$ denotes the determinant, $\boldsymbol{T} = \mathrm{diag}\{t_1, t_2, \ldots, t_L\}$, $m \in [0.5, \infty)$ and $\boldsymbol{S}$ is a symmetric matrix with elements

$$\boldsymbol{S}_{k,l} = \sqrt{\frac{\boldsymbol{R}_{k,l}}{m}} \quad (25)$$

In (25), $\boldsymbol{R}_{k,l}$ is the element in row k and column l of the covariance matrix $\boldsymbol{R}$.

Substituting the component correlation coefficients into (15) and using (22) to calculate $w^{(\pi_1,\ldots,\pi_i)}(t_{\pi_1}, \ldots, t_{\pi_i})$, we can obtain $P_e^{r:L}$ in a Nakagami-m correlated fading environment. The results are plotted in Fig. 6, with $m = 1$, $L = 8$ and $B_n \triangleq B_s/B_c = 0.9$, where B_s is the bandwidth of each subband. In this case, the covariance matrix $\boldsymbol{R}$ for $B_n = 0.9$ can be calculated using (23) as

$$\boldsymbol{R} = \bar{\Gamma}\begin{bmatrix} 1.000 & 0.552 & 0.235 & 0.120 & 0.071 & 0.047 & 0.033 & 0.024 \\ 0.552 & 1.000 & 0.552 & 0.235 & 0.120 & 0.071 & 0.047 & 0.033 \\ 0.235 & 0.552 & 1.000 & 0.552 & 0.235 & 0.120 & 0.071 & 0.047 \\ 0.120 & 0.235 & 0.552 & 1.000 & 0.552 & 0.235 & 0.120 & 0.071 \\ 0.071 & 0.120 & 0.235 & 0.552 & 1.000 & 0.552 & 0.235 & 0.120 \\ 0.047 & 0.071 & 0.120 & 0.235 & 0.552 & 1.000 & 0.552 & 0.235 \\ 0.033 & 0.047 & 0.071 & 0.120 & 0.235 & 0.552 & 1.000 & 0.552 \\ 0.024 & 0.033 & 0.047 & 0.071 & 0.120 & 0.235 & 0.552 & 1.000 \end{bmatrix}$$

Since the off-diagonal elements are not all equal, the subband gains are not exchangeable. The BER curves for BPSK GOS in this fading environment with $r = 1$, 4, 7 and 8 are shown as solid lines. Corresponding curves for an uncorrelated fading environment are shown as dotted lines. Computer simulation results for GOS for the correlated fading case were obtained to ascertain the accuracy of the analysis and are shown as circles. It can be seen that the analytic and simulation results agree closely. For comparison, the BER of BPSK with no diversity is also plotted (dashed line).

From Fig. 1, when the highest SNR subband is selected (i.e. $\Gamma_{8:8}$), the performance is better when the subbands are independently fading. On the other hand, if the lowest SNR subband is selected (i.e. $\Gamma_{1:8}$), correlated fading yields a slightly better performance. This is due to the fact that there is less variability among the different subband gains in a correlated environment. It can also be seen that the

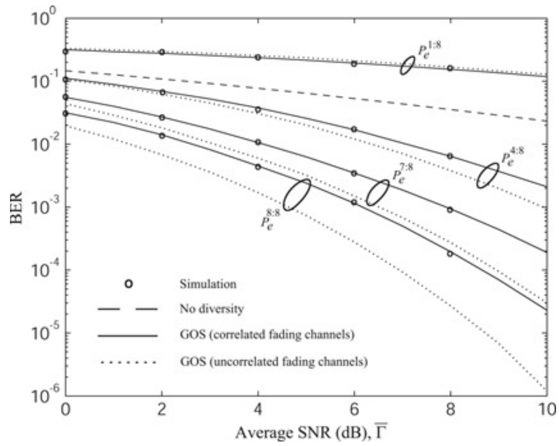

Figure 1 *BPSK BER against average SNR for $m = 1$ and $B_n = 0.9$*

performance difference between the correlated and uncorrelated fading cases increases with r. At a SNR of 10 dB, operation in an uncorrelated fading environment provides roughly a 20-fold, 7-fold and 2-fold reduction in BER compared with a correlated case for $r = 8$, 7 and 4, respectively.

To demonstrate the impact of the fading correlation on the BER, B_n is chosen as 0.5, with $m = 1$ and $L = 8$ in Fig. 2. The covariance matrix for $B_n = 0.5$ is given by

$$\boldsymbol{R} = \overline{\Gamma}\begin{bmatrix} 1.000 & 0.800 & 0.500 & 0.307 & 0.200 & 0.137 & 0.100 & 0.075 \\ 0.800 & 1.000 & 0.800 & 0.500 & 0.307 & 0.200 & 0.137 & 0.100 \\ 0.500 & 0.800 & 1.000 & 0.800 & 0.500 & 0.307 & 0.200 & 0.137 \\ 0.307 & 0.500 & 0.800 & 1.000 & 0.800 & 0.500 & 0.307 & 0.200 \\ 0.200 & 0.307 & 0.500 & 0.800 & 1.000 & 0.800 & 0.500 & 0.307 \\ 0.137 & 0.200 & 0.307 & 0.500 & 0.800 & 1.000 & 0.800 & 0.500 \\ 0.100 & 0.137 & 0.200 & 0.307 & 0.500 & 0.800 & 1.000 & 0.800 \\ 0.075 & 0.100 & 0.137 & 0.200 & 0.307 & 0.500 & 0.800 & 1.000 \end{bmatrix}$$

It is evident from a comparison of Figs. 1 and 2 that for high values of r, the BER significantly increases with increased correlation, that is, increases with lower values of B_n. In Fig. 2, at a SNR of 10 dB, the performance difference between the uncorrelated and correlated fading cases is roughly 100-fold, 30-fold and 4-fold $r = 8$, 7 and 4, respectively. For $r = 1$ the BER slightly decreases with correlation and the performance difference is more noticeable as the correlation increases. The BER curves for $B_n = 0.25$, $m = 1$ and $L = 8$ are shown in Fig. 3. The covariance matrix for $B_n = 0.25$ is given by

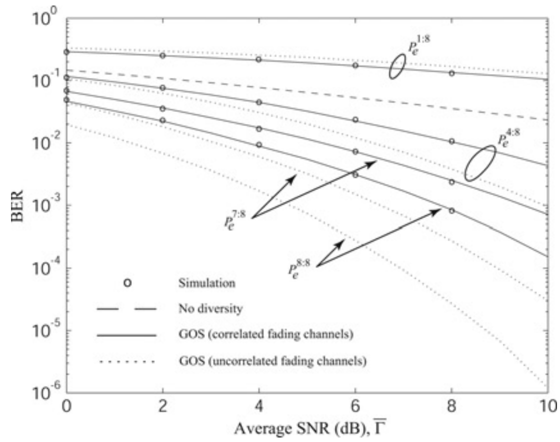

Figure 2 *BPSK BER against average SNR for $m = 1$ and $B_n = 0.5$*

$$\boldsymbol{R} = \overline{\Gamma}\begin{bmatrix} 1.000 & 0.941 & 0.800 & 0.640 & 0.500 & 0.390 & 0.307 & 0.246 \\ 0.941 & 1.000 & 0.941 & 0.800 & 0.640 & 0.500 & 0.390 & 0.307 \\ 0.800 & 0.941 & 1.000 & 0.941 & 0.800 & 0.640 & 0.500 & 0.390 \\ 0.640 & 0.800 & 0.941 & 1.000 & 0.941 & 0.800 & 0.640 & 0.500 \\ 0.500 & 0.640 & 0.800 & 0.941 & 1.000 & 0.941 & 0.800 & 0.640 \\ 0.390 & 0.500 & 0.640 & 0.800 & 0.941 & 1.000 & 0.941 & 0.800 \\ 0.307 & 0.390 & 0.500 & 0.640 & 0.800 & 0.941 & 1.000 & 0.941 \\ 0.246 & 0.307 & 0.390 & 0.500 & 0.640 & 0.800 & 0.941 & 1.000 \end{bmatrix}$$

The results from Figs. 1, 2 and 3 show that the difference in BER between the correlated and uncorrelated cases is small for $r = 1$ with the BER decreasing slightly as the correlation increases. For $r = 8$, the BER degrades rapidly with increased correlation.

Figs. 4–6 show the BER against average SNR for $m = 0.5$, $L = 8$ and $B_n = 0.9$, 0.5 and 0.25, respectively. The curves for $m = 0.5$ are qualitatively similar to the $m = 1$ results. The case $m = 0.5$ corresponds to a one-sided Gaussian distribution. As expected, the BER is generally higher with a smaller value of m since it corresponds to a more severe fading environment.

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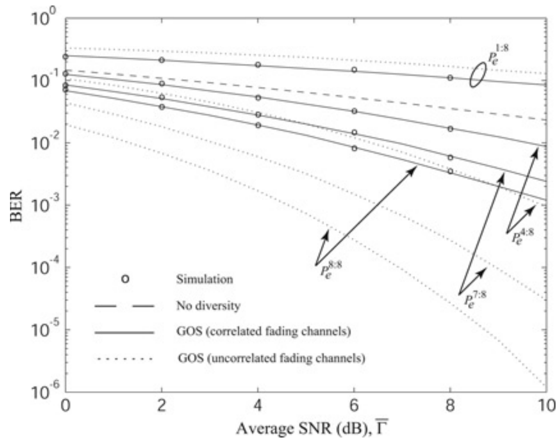

Figure 3 *BPSK BER against average SNR for m = 1 and $B_n = 0.25$*

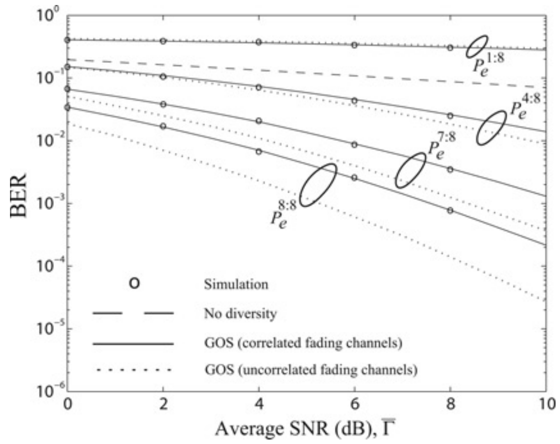

Figure 4 *BPSK BER against average SNR for m = 0.5 and $B_n = 0.9$*

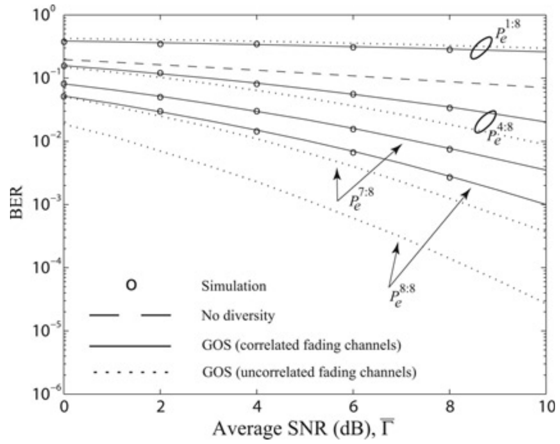

Figure 5 *BPSK BER against average SNR for m = 0.5 and $B_n = 0.5$*

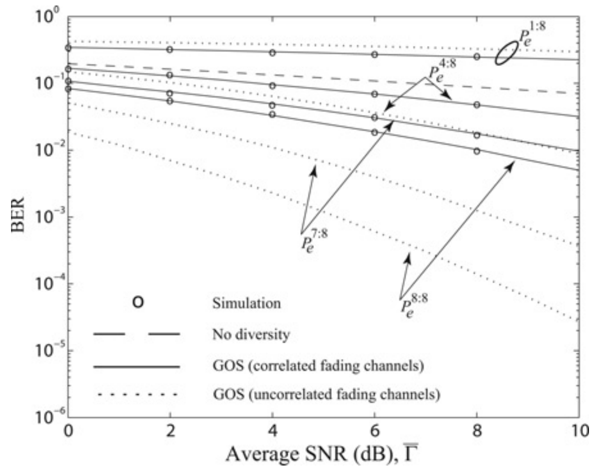

Figure 6 *BPSK BER against average SNR for m = 0.5 and $B_n = 0.25$*

5 Conclusion

An expression for the BER of *r*th order selection involving a set of arbitrarily correlated (not necessarily exchangeable) diversity branch gains was obtained. The proposed approach may be applied in the performance analyses of various diversity systems operating over correlated Nakagami fading channels. It was found that the sensitivity of the BER performance to correlation among the branches depends on the branch order. The strongest branch showed the largest degradation in a correlated environment compared with an uncorrelated environment. On the other hand, the weakest branch showed an improvement in performance in a correlated environment. The performance difference between the correlated and uncorrelated fading cases increases with *r* and correlation. This is due to the fact that there is less variability among the different subband gains as the correlation increases.

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