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Min–max mean squared error-based linear transceiver design for multiple-input–multiple-output interference relay channel

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Abstract: In this study, the authors consider the min–max mean squared error (MSE)-based linear transceiver–relay design for multiple-input–multiple-output (MIMO) interference relay channel, where a finite number of half-duplex MIMO amplify and forward relays assist the communication between multiple source–destination pairs. The problem is formulated as minimising the maximum MSE among all data streams of all users subject to individual transmit power constraints at each source and relay node. Since the optimisation problem is non-convex, globally optimal solution cannot be guaranteed. They propose a suboptimal solution based on alternating minimisation, where the beamforming matrices at all the source, relay and destination nodes are jointly computed in an iterative manner. Numerical simulations show that the proposed algorithm not only ensures fairness among all users' data streams, but also achieves good sum-rate and bit error rate performance.

1 Introduction

An interference channel [1] consists of multiple source nodes simultaneously communicating with multiple destination nodes via a common channel. There is one-to-one correspondence between source nodes and destination nodes. Many wireless communication systems can be modelled using an interference channel, for example, interfering base-stations in a cellular network, wireless local area networks, interfering secondary users in a cognitive radio and so forth. When the quality of the direct link between source and destination nodes is severally degraded because of high path loss and shadowing, relays can be used to assist the communication. Interference relay channel models the scenario where a finite number of relay nodes assist the communication between multiple source–destination pairs. Relay-assisted cooperative communication can offer significant benefits such as throughput enhancement, coverage extension and increased reliability in wireless communication systems [2]. Therefore it is being actively considered as a promising technique for the next generation wireless standards, such as long-term evolution (LTE)-advanced and worldwide interoperability for microwave access [3]. Multiple antennas in a multi-user system not only provide diversity and multiplexing gain, but also help in reducing the inter user interference [4]. The advantages of multiple-input–multiple-output (MIMO) system can be achieved in an interference relay channel by accommodating multiple antennas at source, destination and relay nodes.

Relays can operate in various relaying protocols such as decode and forward (DF), amplify and forward (AF), compressed and forward and so forth depending on their signal processing capability [5]. Compared with other relaying protocols, AF relays are the simplest as they do not need to decode the received signal. AF relay simply amplifies the received signal and forwards the linearly processed signal towards the destination node. Half-duplex relay operation is the most practical in which transmission occurs over two hops in two different time slots.

A number of works on one-way interference relay channel have been reported in the literature [6–16]. In [6], Cadambe and Jafar showed that cooperation through relays does not increase the degrees of freedom (DoFs) of a K user fully connected interference network with time-varying/frequency selective channel coefficients. The sum DoF of a network is the first-order approximation of its sum capacity at high signal-to-noise ratio (SNR). However the relays in an interference channel have potential benefits in terms of reduced channel-state-information (CSI) requirements at source nodes [8, 9] and less number of independent symbol extensions [7, 8] for relay aided interference alignment at the destination nodes. Prior work in [10] develop transmit–receive beamformer at relay nodes using zero-forcing or linear minimum mean square error criterion such that inter user interference is completely suppressed at destination nodes. A cooperative technique, where source and relay nodes cooperate to perform zero forcing at the destination nodes, is considered in [11]. All the above works require assumptions that there are enough number of relays or enough antennas at a MIMO relay to remove all interference at destination nodes.

Relay beamforming design in a general interference relay channel using total relay power minimisation and minimum signal-to-interference-plus noise ratio (SINR) maximisation are considered in [12, 13] for single and multiple antenna relays, respectively. Prior works in [14–16] develop iterative algorithms to jointly optimise source precoders, relay processing matrices and destination filters with multiple antennas at all source, destination and relay nodes, respectively. Different objective functions including total interference plus noise leakage minimisation [16], sum mean squared error (MSE) minimisation [15], weighted sum-MSE minimisation [16], sum power minimisation with quality of service (QoS) constraints at each destination [14] have been explored. Total leakage minimisation algorithm seeks perfect alignment of interference and relay-enhanced noise signals at all destination nodes. This is not optimal at low to moderate SNR since it does not maximise the signal power in the desired signal subspace. Sum-MSE-based algorithms result in unfairness, that is, some users have much smaller data rates than others. Sum power minimisation with QoS constraints at each destination achieves fairness among users, but it is not suitable when there are strict power constraints at source and relay nodes. Moreover, it also assumes only one transmit stream for each source node.

In this paper, we consider a MIMO interference relay channel where multiple source nodes communicate with their intended destination nodes via a finite number of half-duplex AF relays. All source, destination and relay nodes are equipped with multiple antennas. Moreover, there are no restrictions on number of data streams transmitted by each source node. We focus on a novel transceiver–relay design based on min–max fairness criterion. We formulate the joint transceiver–relay design problem aiming at minimising the maximum per-stream MSE among all users subject to individual transmit power constraints at each source and relay node. This problem does not lead to closed-form solutions and is non-convex. Therefore a globally optimal solution is not easily tractable. We propose an iterative algorithm to decouple the joint transceiver–relay design problem into three sub-problems which can be solved efficiently. In particular destination filters, transmit precoders, relay processing matrices are updated in an alternating manner, each update being a convex sub-problem. Since each sub-problem can be solved optimally, maximum per-stream MSE is non-increasing in each iteration. This guarantees the convergence of the iterative algorithm. We use Monte Carlo simulation to evaluate the average sum-rate, fairness and bit error rate (BER) performance of the proposed algorithm.

The remainder of this paper is organised as follows. In Section 2, we provide the system model. In Section 3, we formulate the min–max fairness-based transceiver–relay design problem. Next, in Section 4, we propose an iterative algorithm and prove its convergence. In Section 5, we evaluate the convergence and performance of our algorithm using numerical simulation. Section 6 concludes this paper with suggestions for future work.

Notations: We use lowercase letters for scalars, lowercase bold font for vectors and uppercase bold font for matrices. We use $R^{M\times N}$ and $C^{M\times N}$ to denote set of real and complex $M\times N$ matrices. $E\{\cdot\}$ is the statistical expectation operator, $\mathrm{tr}\{\cdot\}$ represents the trace operator and $\otimes$ stands for Kronecker product. We use $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ to denote transpose and conjugate transpose. $|a|$ denotes the absolute value of a, $\|\boldsymbol{a}\|$ denotes the Euclidean norm of $\boldsymbol{a}$. $\boldsymbol{I}_N$ represents the $N\times N$ identity matrix and $\boldsymbol{0}_{M\times N}$ represents the $M\times N$ zero matrix. The notation $\mathrm{vec}(\boldsymbol{X})$ denotes the vec operator to vectorise an $M\times N$ matrix $\boldsymbol{X}$ into an $MN\times 1$ column vector $\boldsymbol{x}$ by stacking the columns of the matrix $\boldsymbol{X}$ on top of one another and $\mathrm{vec}^{-1}(\boldsymbol{x})$ denotes the inverse vec operator to convert vector $\boldsymbol{x}$ into matrix $\boldsymbol{X}$. Lastly, $\mathcal{K}\triangleq\{1, 2, \ldots, K\}$ is the index set of K source–destination pairs, $\mathcal{D}_k\triangleq\{1, 2, \ldots, d_k\}$ is the index set of d_k streams for source k and $\mathcal{M}\triangleq\{1, 2, \ldots, M\}$ is the index set of M relays.

2 System model

Consider a MIMO interference relay channel with K users, comprised of K source–destination pairs, as illustrated in Fig. 1. Each source communicates with only its corresponding destination with the aid of M half-duplex AF relays. The kth source and destination nodes are equipped with $N_{s,k}$ and $N_{d,k}$ antennas, respectively, and the mth relay is equipped with $N_{r,m}$ antennas. Since all the relays operate in half-duplex mode, the transmission between source and destination nodes is completed in two time slots. In the first slot, the source nodes transmit data to the relays. In the second slot, relays multiply the received signals by amplifying matrices and forwards the linearly processed signals to the destination nodes. The direct signals between source and destination nodes are ignored by the destination nodes because of high path loss and shadowing. We assume quasi-static Rayleigh flat fading channel between source nodes and relay nodes as well as between relay nodes and destination nodes. We assume perfect and global CSI is available at a central unit, which computes the beamforming matrices for all source, destination and relay nodes. We assume that transmission is fully synchronous on each hop, that is, all source nodes and all relay nodes transmit simultaneously in the first and second hops, respectively. This is the same model as in the previous work for transceiver–relay design for MIMO interference relay channel [14–16].

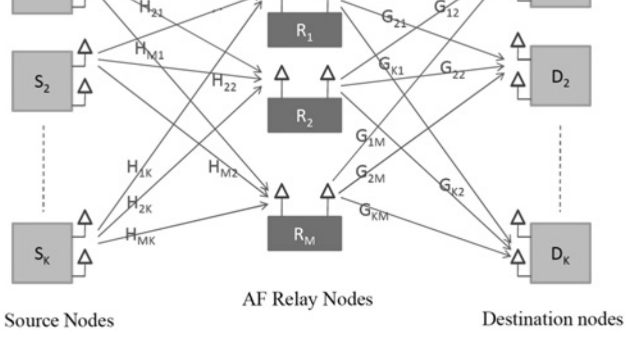

Fig. 1 *MIMO interference relay channel with K source–destination pairs and M half-duplex AF relays*

Source k wishes to transmit d_k data streams $\boldsymbol{s}_k = [s_k^{(1)}, \ldots, s_k^{(d_k)}]^{\mathrm{T}} \in C^{d_k\times 1}$ to destination k where $d_k \le \min\{N_{s,k}, N_{d,k}\}$. The transmit streams are assumed independent and identically distributed such that $E(\boldsymbol{s}_k\boldsymbol{s}_k^{\mathrm{H}}) = \boldsymbol{I}_{d_k}$. All source nodes send independent transmit signals such that $E(\boldsymbol{s}_k\boldsymbol{s}_j^{\mathrm{H}}) = \boldsymbol{0}$ for $j \ne k$ and all transmit signals are statistically independent from the noise vector at all relay and destination nodes. Source k precodes the data streams using a linear transmit precoder $\boldsymbol{V}_k = [\boldsymbol{v}_k^{(1)}, \ldots, \boldsymbol{v}_k^{(d_k)}] \in C^{N_{s,k}\times d_k}$. The transmit signal vector at the kth source node is given as

$$\boldsymbol{x}_{s,k} = \boldsymbol{V}_k\boldsymbol{s}_k = \sum_{l=1}^{d_k}\boldsymbol{v}_k^{(l)}s_k^{(l)} \tag{1}$$

The received signal vector at the mth relay node is given by

$$\boldsymbol{y}_{r,m} = \sum_{k=1}^{K}\boldsymbol{H}_{m,k}\boldsymbol{V}_k\boldsymbol{s}_k + \boldsymbol{z}_{r,m} \tag{2}$$

where $\boldsymbol{H}_{m,k} \in C^{N_{r,m}\times N_{s,k}}$ is the channel matrix from source k to relay m and $\boldsymbol{z}_{r,m} \in C^{N_{r,m}\times 1}$ is the zero-mean additive white Gaussian noise vector at relay m with covariance matrix $E(\boldsymbol{z}_{r,m}\boldsymbol{z}_{r,m}^{\mathrm{H}}) = \sigma_r^2\boldsymbol{I}$. The mth relay node linearly processes its received signal $\boldsymbol{y}_{r,m}$ by an amplifying matrix $\boldsymbol{U}_m \in C^{N_{r,m}\times N_{r,m}}$ and forwards $\boldsymbol{x}_{r,m}$ to the destination nodes, where

$$\boldsymbol{x}_{r,m} = \boldsymbol{U}_m\boldsymbol{y}_{r,m} = \sum_{k=1}^{K}\boldsymbol{U}_m\boldsymbol{H}_{m,k}\boldsymbol{V}_k\boldsymbol{s}_k + \boldsymbol{U}_m\boldsymbol{z}_{r,m} \tag{3}$$

The received signal vector at destination k is given by

$$\begin{aligned}\boldsymbol{y}_{d,k} &= \sum_{m=1}^{M}\boldsymbol{G}_{k,m}\boldsymbol{x}_{r,m} + \boldsymbol{z}_{d,k}\\ &= \sum_{m=1}^{M}\boldsymbol{G}_{k,m}\boldsymbol{U}_m\left(\sum_{j=1}^{K}\boldsymbol{H}_{m,j}\boldsymbol{V}_j\boldsymbol{s}_j + \boldsymbol{z}_{r,m}\right) + \boldsymbol{z}_{d,k}\end{aligned} \tag{4}$$

where $\boldsymbol{G}_{k,m} \in C^{N_{d,k}\times N_{r,m}}$ is the channel matrix from relay m to destination k and $\boldsymbol{z}_{d,k} \in C^{N_{d,k}\times 1}$ is the zero-mean additive white Gaussian noise vector at destination k with covariance matrix $E(\boldsymbol{z}_{d,k}\boldsymbol{z}_{d,k}^{\mathrm{H}}) = \sigma_d^2\boldsymbol{I}$. We now introduce the following definitions

$$\begin{aligned}\tilde{\boldsymbol{H}}_k &\triangleq [\boldsymbol{H}_{1,k}^{\mathrm{T}}, \ldots, \boldsymbol{H}_{M,k}^{\mathrm{T}}]^{\mathrm{T}} \in C^{N_R\times N_{s,k}}\\ \tilde{\boldsymbol{G}}_k &\triangleq [\boldsymbol{G}_{k,1}, \ldots, \boldsymbol{G}_{k,M}] \in C^{N_{d,k}\times N_R}\\ \tilde{\boldsymbol{U}} &\triangleq \mathrm{blkdiag}(\boldsymbol{U}_1, \ldots, \boldsymbol{U}_M) \in C^{N_R\times N_R}\\ \tilde{\boldsymbol{z}}_r &\triangleq [\boldsymbol{z}_{r,1}^{\mathrm{T}}, \ldots, \boldsymbol{z}_{r,M}^{\mathrm{T}}]^{\mathrm{T}} \in C^{N_R\times 1}\end{aligned} \tag{5}$$

where $\boldsymbol{U}$ is a block-diagonal matrix and $N_R \triangleq \sum_{m=1}^{M} N_{r,m}$. Using the

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definitions of (5), (4) can be rewritten as

$$\begin{aligned} \boldsymbol{y}_{\mathrm{d},k} &= \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\left(\sum_{j=1}^{K}\tilde{\boldsymbol{H}}_j\boldsymbol{V}_j\boldsymbol{s}_j + \tilde{\boldsymbol{z}}_{\mathrm{r}}\right) + \boldsymbol{z}_{\mathrm{d},k} \\ &= \sum_{j=1}^{K}\tilde{\boldsymbol{G}}_k\boldsymbol{U}\tilde{\boldsymbol{H}}_j\boldsymbol{V}_j\boldsymbol{s}_j + \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_{\mathrm{r}} + \boldsymbol{z}_{\mathrm{d},k} \\ &= \sum_{j=1}^{K}\boldsymbol{T}_{k,j}\boldsymbol{V}_j\boldsymbol{s}_j + \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_{\mathrm{r}} + \boldsymbol{z}_{\mathrm{d},k} \end{aligned} \tag{6}$$

where $\boldsymbol{T}_{k,j} = \tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{H}}_j$ is the equivalent channel matrix between jth source and the kth destination. Each destination applies a linear receiver and let $\boldsymbol{W}_k = [\boldsymbol{w}_k^{(1)}, \ldots, \boldsymbol{w}_k^{(d_k)}] \in C^{N_{\mathrm{d},k}\times d_k}$ be the receiver filter of destination k. Then the linearly processed received signal at destination k is given by

$$\begin{aligned} \tilde{s}_k &= \boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{y}_{\mathrm{d},k} \\ &= \underbrace{\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{V}_k\boldsymbol{s}_k}_{\text{desired signal}} + \underbrace{\sum_{\substack{j=1\\ j\neq k}}^{K}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{T}_{k,j}\boldsymbol{V}_j\boldsymbol{s}_j}_{\text{interference}} + \underbrace{\boldsymbol{W}_k^{\mathrm{H}}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_r}_{\text{relay enhanced noise}} + \underbrace{\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{z}_{\mathrm{d},k}}_{\text{local noise}} \end{aligned} \tag{7}$$

Similarly, the estimate of lth data stream of the kth destination is given by

$$\begin{aligned} \tilde{s}_k^{(l)} &= (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{y}_{\mathrm{d},k} \\ &= \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)}s_k^{(l)}}_{\text{desired signal}} + \underbrace{\sum_{\substack{p=1\\ p\neq l}}^{d_k}(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(p)}s_k^{(p)}}_{\text{inter stream interference}} \\ &+ \underbrace{\sum_{\substack{j=1\\ j\neq k}}^{K}\sum_{p=1}^{d_j}(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,j}\boldsymbol{v}_j^{(p)}s_j^{(p)}}_{\text{inter user interference}} \\ &+ \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{z}}_{\mathrm{r}}}_{\text{relay enhanced noise}} + \underbrace{(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{z}_{\mathrm{d},k}}_{\text{local noise}} \end{aligned} \tag{8}$$

From (1), the transmit power of the kth source is given as

$$p_{\mathrm{s},k} = \mathrm{tr}(E\{\boldsymbol{x}_{\mathrm{s},k}\boldsymbol{x}_{\mathrm{s},k}^{\mathrm{H}}\}) = \mathrm{tr}(\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}) = \sum_{l=1}^{d_k}(\boldsymbol{v}_k^{(l)})^{\mathrm{H}}\boldsymbol{v}_k^{(l)} \tag{9}$$

and using (3), the transmit power of the mth relay is given as

$$\begin{aligned} p_{\mathrm{r},m} &= \mathrm{tr}(E\{\boldsymbol{x}_{\mathrm{r},m}\boldsymbol{x}_{\mathrm{r},m}^{\mathrm{H}}\}) \\ &= \sum_{k=1}^{K}\mathrm{tr}(\boldsymbol{U}_m\boldsymbol{H}_{m,k}\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{H}}\boldsymbol{H}_{m,k}^{\mathrm{H}}\boldsymbol{U}_m^{\mathrm{H}}) + \sigma_{\mathrm{r}}^2\mathrm{tr}(\boldsymbol{U}_m\boldsymbol{U}_m^{\mathrm{H}}) \end{aligned} \tag{10}$$

The achievable end-to-end data rate for the kth user is given by

$$R_k = \frac{1}{2}\log\det(\boldsymbol{I}_{\mathrm{d}_k} + (\boldsymbol{W}_k^{\mathrm{H}}\mathrm{R}_k\boldsymbol{W}_k)^{-1}\boldsymbol{W}_k^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{V}_k\boldsymbol{V}_k\boldsymbol{T}_{k,k}^{\mathrm{H}}\boldsymbol{W}_k) \tag{11}$$

where the 1/2 factor accounts for two time slots needed for transmission because of half-duplex relays and

$$\boldsymbol{R}_k = \textstyle\sum_{\substack{j=1\\ j\neq k}}^{K}\boldsymbol{T}_{k,j}\boldsymbol{V}_j(\boldsymbol{T}_{k,j}\boldsymbol{V}_j)^{\mathrm{H}} + \sigma_{\mathrm{r}}^2\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{U}}^{\mathrm{H}}\tilde{\boldsymbol{G}}_k^{\mathrm{H}} + \sigma_{\mathrm{d}}^2\boldsymbol{I}_{N_{\mathrm{d},k}}$$

is the covariance matrix of the pre-processing interference plus noise at the kth destination. The sum-rate and minimum user rate of the system is given by

$$\begin{aligned} R_{\mathrm{sum}} &= \sum_{k=1}^{K}R_k \\ R_{\min} &= \min_{k\in\mathcal{K}}R_k \end{aligned} \tag{12}$$

3 Problem formulation

In this section, we formulate a min–max fair transceiver–relay design problem for MIMO interference AF relay channel to minimise the maximum per-stream MSE among all user's data streams, subject to individual transmit power constraints at source and relay nodes. The MSE of the data stream estimate $\tilde{s}_k^{(l)}$ can be written as

$$\begin{aligned} \mathrm{MSE}_{k,l} &= E\{(\tilde{s}_k^{(l)} - s_k^{(l)})(\tilde{s}_k^{(l)} - s_k^{(l)})^{\mathrm{H}}\} \\ &= (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\left(\sum_{j=1}^{K}\sum_{p=1}^{d_j}\boldsymbol{T}_{k,j}\boldsymbol{v}_j^{(p)}(\boldsymbol{v}_j^{(p)})^{\mathrm{H}}\boldsymbol{T}_{k,j}^{\mathrm{H}}\right)\boldsymbol{w}_k^{(l)} \\ &- (\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)} - (\boldsymbol{T}_{k,k}\boldsymbol{v}_k^{(l)})^{\mathrm{H}}\boldsymbol{w}_k^{(l)} \\ &+ \sigma_{\mathrm{r}}^2(\boldsymbol{w}_k^{(l)})^{\mathrm{H}}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{U}}^{\mathrm{H}}\tilde{\boldsymbol{G}}_k^{\mathrm{H}}\boldsymbol{w}_k^{(l)} + \sigma_d^2\left\|(\boldsymbol{w}_k^{(l)})\right\|^2 + 1 \end{aligned} \tag{13}$$

We formulate the min–max MSE-based transceiver–relay design problem as follows

$$\begin{aligned} &\min_{\substack{\{\boldsymbol{V}_k\}_{k=1}^K,\{\boldsymbol{U}_m\}_{m=1}^M\\ \{\boldsymbol{W}_k\}_{k=1}^K}} \quad \max_{\substack{l\in\mathcal{D}_k\\ k\in\mathcal{K}}} \quad \mathrm{MSE}_{k,l} \\ &\quad\text{subject to} \quad p_{\mathrm{s},k} \le P_{\mathrm{s},k}, \quad \forall k\in\mathcal{K} \\ &\qquad\qquad\qquad p_{\mathrm{r},m} \le P_{\mathrm{r},m}, \quad \forall m\in\mathcal{M} \end{aligned} \tag{14}$$

where $P_{\mathrm{s},k}$ is the maximum transmit power at source k and $P_{\mathrm{r,m}}$ is the maximum transmit power at relay m. Using an auxiliary variable MSE, the optimisation problem (14) can be rewritten as

$$\begin{aligned} &\min_{\substack{\{\boldsymbol{V}_k\}_{k=1}^K,\{\boldsymbol{U}_m\}_{m=1}^M\\ \{\boldsymbol{W}_k\}_{k=1}^K}} \quad \mathrm{MSE} \\ &\text{subject to} \quad \sqrt{\mathrm{MSE}_{k,l}} \le \mathrm{MSE}, \quad \forall l\in\mathcal{D}_k,\ \forall k\in\mathcal{K} \\ &\qquad p_{\mathrm{s},k} \le P_{\mathrm{s},k}, \quad \forall k\in\mathcal{K} \\ &\qquad p_{\mathrm{r},m} \le P_{\mathrm{r},m}, \quad \forall m\in\mathcal{M} \end{aligned} \tag{15}$$

Remark: In wireless networks, where some data streams have different QoS requirements, for example, streams with different priorities or applications, we can replace each $\mathrm{MSE}_{k,\ l}$ in the optimisation problem with $\mathrm{MSE}_{k,l}/\rho_{k,l}$, where $\rho_{k,l}$ are constant weights that depend on the importance of the data streams. This maximum weighted MSE minimisation problem will ensure weighted fairness in the network.

4 Proposed solution

The transceiver–relay design problem formulated in (14) and (15) is non-convex, and hence cannot be efficiently solved for globally optimal solution. We propose an iterative algorithm based on alternating minimisation for finding a suboptimal solution for the problem. We decouple the joint design problem into three sub-problems and solve each of them in an alternating manner. The three design sub-problems and their solution for destination filters design, source precoders design and relay processing matrices design are presented in Sections 4.1–4.3, respectively.

4.1 Destination filter design

In this section, we focus on designing destination filter matrices for all users $\boldsymbol{W}_k$; $\forall k \in \mathcal{K}$ with fixed transmit precoders and relay processing matrices by solving the optimisation problem

$$\{\boldsymbol{W}_k^*\}_{k=1}^K := \underset{\{\boldsymbol{W}_k\}_{k=1}^K}{\operatorname{argmin}} \max_{\substack{l \in \mathcal{D}_k \\ k \in \mathcal{K}}} \mathrm{MSE}_{k,l} \tag{16}$$

The $\mathrm{MSE}_{k,\ l}$ depends only on $\boldsymbol{w}_k^{(l)}$. Therefore the destination beamforming vectors for each data stream $\boldsymbol{w}_k^{(l)}$; $\forall l \in \mathcal{D}_k,\ k \in \mathcal{K}$ can be determined separately by solving the following optimisation problem such that it minimises the MSE of that data stream

$$\boldsymbol{w}_k^{(l)^*} = \underset{\boldsymbol{w}_k^{(l)} \in C^{N_{\mathrm{d},k} \times 1}}{\operatorname{argmin}} \mathrm{MSE}_{k,l} \tag{17}$$

The objective function is convex with respect to $\boldsymbol{w}_k^{(l)}$. The optimal receive beamformer can be obtained by setting the first derivative of $\mathrm{MSE}_{k,\ l}$ over $\boldsymbol{w}_k^{(l)}$ to zero. This gives the linear MMSE receiver

$$\boldsymbol{w}_k^{(l)^*} = \left(\sum_{j=1}^{K} \sum_{p=1}^{d_j} \boldsymbol{T}_{k,j} \boldsymbol{v}_j^{(p)} (\boldsymbol{v}_j^{(p)})^{\mathrm{H}} \boldsymbol{T}_{k,j}^{\mathrm{H}} + \tilde{\boldsymbol{R}}_k \right)^{-1} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)} \tag{18}$$

where $\tilde{\boldsymbol{R}}_k = \sigma_{\mathrm{r}}^2 \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \tilde{\boldsymbol{U}}^{\mathrm{H}} \tilde{\boldsymbol{G}}_k^{\mathrm{H}} + \sigma_d^2 \boldsymbol{I}_{N_{\mathrm{d},k}}$ is the covariance matrix of the equivalent noise vector $\tilde{\boldsymbol{z}}_k = \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \tilde{\boldsymbol{z}}_{\mathrm{r}} + \boldsymbol{z}_{\mathrm{d},k}$ at the kth destination

4.2 Source precoder design

In this section, we focus on designing source precoder matrices for all users $\boldsymbol{V}_k$; $\forall k \in \mathcal{K}$ with fixed destination filters and relay processing matrices by solving the following optimisation problem

$$\begin{aligned} \{\boldsymbol{V}_k^*\}_{k=1}^K := \ & \underset{\{\boldsymbol{V}_k\}_{k=1}^K}{\operatorname{argmin}} \quad \mathrm{MSE} \\ \text{s.t.} \ & \sqrt{\mathrm{MSE}_{k,l}} \le \mathrm{MSE}, \quad \forall l \in \mathcal{D}_k,\ \forall k \in \mathcal{K} \\ & p_{\mathrm{s},k} \le P_{\mathrm{s},k}, \quad \forall k \in \mathcal{K} \\ & p_{\mathrm{r},m} \le P_{\mathrm{r},m}, \quad \forall m \in \mathcal{M} \end{aligned} \tag{19}$$

From (13), the MSE of the lth stream estimate of the kth user can be written as

$$\begin{aligned} \mathrm{MSE}_{k,l} = & \left| 1 - (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(l)} \right|^2 + \sum_{\substack{j=1 \\ j \ne k}}^{K} (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,j} \boldsymbol{V}_j \boldsymbol{V}_j^{\mathrm{H}} \boldsymbol{T}_{k,j}^{\mathrm{H}} \boldsymbol{w}_k^{(l)} \\ & + \sum_{\substack{p=1 \\ p \ne l}}^{d_k} (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{T}_{k,k} \boldsymbol{v}_k^{(p)} (\boldsymbol{v}_k^{(p)})^{\mathrm{H}} \boldsymbol{T}_{k,k}^{\mathrm{H}} \boldsymbol{w}_k^{(l)} \\ & + \sigma_{\mathrm{r}}^2 (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \tilde{\boldsymbol{U}}^{\mathrm{H}} \tilde{\boldsymbol{G}}_k^{\mathrm{H}} \boldsymbol{w}_k^{(l)} + \sigma_d^2 \left\| (\boldsymbol{w}_k^{(l)}) \right\|^2 \end{aligned} \tag{20}$$

We define the following matrices $\forall l \in \mathcal{D}_k$ and $\forall k \in \mathcal{K}$

$$\begin{aligned} \boldsymbol{T} & \triangleq \begin{bmatrix} \boldsymbol{T}_{1,1} & \boldsymbol{T}_{1,2} & \cdots & \boldsymbol{T}_{1,K} \\ \boldsymbol{T}_{2,1} & \boldsymbol{T}_{2,2} & \cdots & \boldsymbol{T}_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{T}_{K,1} & \boldsymbol{T}_{K,2} & \cdots & \boldsymbol{T}_{K,K} \end{bmatrix} \\ \boldsymbol{V} & \triangleq \mathrm{blkdiag}(\boldsymbol{V}_1,\ \boldsymbol{V}_2,\ \ldots,\ \boldsymbol{V}_K) \\ \boldsymbol{A}_k & \triangleq \left[\boldsymbol{0}_{N_{\mathrm{d},k} \times \sum_{j=1}^{k-1} N_{\mathrm{d},j}},\ \boldsymbol{I}_{N_{\mathrm{d},k}},\ \boldsymbol{0}_{N_{\mathrm{d},k} \times \sum_{j=k+1}^{K} N_{\mathrm{d},j}} \right] \\ \boldsymbol{B}_k & \triangleq \left[\boldsymbol{0}_{N_{\mathrm{s},k} \times \sum_{j=1}^{k-1} N_{\mathrm{s},j}},\ \boldsymbol{I}_{N_{\mathrm{s},k}},\ \boldsymbol{0}_{N_{\mathrm{s},k} \times \sum_{j=k+1}^{K} N_{\mathrm{s},j}} \right] \\ C_k & \triangleq \left[\boldsymbol{0}_{d_k \times \sum_{j=1}^{k-1} d_j},\ \boldsymbol{I}_{d_k},\ \boldsymbol{0}_{d_k \times \sum_{j=k+1}^{K} d_j} \right] \\ \mathrm{e}_{k,l} & \triangleq [0_{l-1 \times 1}^T,\ 1,\ 0_{d_{k-l \times 1}}^T]^{\mathrm{T}} \end{aligned} \tag{21}$$

Using the definitions of (21), (20) can be rewritten as

$$\begin{aligned} \mathrm{MSE}_{k,l} = & \left| 1 - (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k \boldsymbol{V} C_k^{\mathrm{T}} \boldsymbol{e}_{k,l} \right|^2 \\ & + \sum_{\substack{j=1 \\ j \ne k}}^{K} \mathrm{tr}(C_j \boldsymbol{V}^{\mathrm{H}} \tilde{\boldsymbol{B}}_j \boldsymbol{T}^{\mathrm{H}} \boldsymbol{A}_k^{\mathrm{H}} \boldsymbol{w}_k^{(l)} (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_j \boldsymbol{V} C_j^{\mathrm{T}}) \\ & + \sum_{\substack{p=1 \\ p \ne l}}^{d_k} \left| (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k \boldsymbol{V} C_k^{\mathrm{T}} \boldsymbol{e}_{k,p} \right|^2 + \sigma_r^2 \left\| (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \right\|^2 \\ & + \sigma_d^2 \left\| (\boldsymbol{w}_k^{(l)}) \right\|^2 \end{aligned} \tag{22}$$

where $\tilde{\boldsymbol{B}}_k = \boldsymbol{B}_k^{\mathrm{T}} \boldsymbol{B}_k,\ \forall\, k \in \mathcal{K}$. Using the following matrix equality $\mathrm{vec}(\boldsymbol{ABC}) = (C^{\mathrm{T}} \otimes \boldsymbol{A})\mathrm{vec}(\boldsymbol{B})$, we further rewrite (22) as

$$\begin{aligned} \mathrm{MSE}_{k,l} = & \left| 1 - [\boldsymbol{e}_{k,l}^{\mathrm{T}} C_k \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k] \mathrm{vec}(\boldsymbol{V}) \right|^2 \\ & + \sum_{\substack{j=1 \\ j \ne k}}^{K} \left\| [C_j \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_j] \mathrm{vec}(\boldsymbol{V}) \right\|^2 \\ & + \sum_{\substack{p=1 \\ p \ne l}}^{d_k} \left| [\boldsymbol{e}_{k,p}^{\mathrm{T}} C_k \otimes (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \boldsymbol{A}_k \boldsymbol{T} \tilde{\boldsymbol{B}}_k] \mathrm{vec}(\boldsymbol{V}) \right|^2 \\ & + \sigma_{\mathrm{r}}^2 \left\| (\boldsymbol{w}_k^{(l)})^{\mathrm{H}} \tilde{\boldsymbol{G}}_k \tilde{\boldsymbol{U}} \right\|^2 \\ & + \sigma_{\mathrm{d}}^2 \left\| (\boldsymbol{w}_k^{(l)}) \right\|^2 \end{aligned} \tag{23}$$

The transmit power of the mth relay is written as

$$\begin{aligned} p_{\mathrm{r},m} & = \sum_{k=1}^{K} \mathrm{tr}(\boldsymbol{U}_m \boldsymbol{H}_{m,k} \boldsymbol{V}_k \boldsymbol{V}_k^{\mathrm{H}} \boldsymbol{H}_{m,k}^{\mathrm{H}} \boldsymbol{U}_m^{\mathrm{H}}) + \sigma_{\mathrm{r}}^2 \mathrm{tr}(\boldsymbol{U}_m \boldsymbol{U}_m^{\mathrm{H}}) \\ & = \sum_{k=1}^{K} \left\| \mathrm{vec}(\boldsymbol{U}_m \boldsymbol{H}_{m,k} \boldsymbol{V}_k) \right\|^2 + \sigma_{\mathrm{r}}^2 \mathrm{tr}(\boldsymbol{U}_m \boldsymbol{U}_m^{\mathrm{H}}) \\ & = \sum_{k=1}^{K} \left\| [\boldsymbol{I}_{\mathrm{d}_k} \otimes \boldsymbol{U}_m \boldsymbol{H}_{m,k}] \mathrm{vec}(\boldsymbol{V}_k) \right\|^2 + \sigma_{\mathrm{r}}^2 \mathrm{tr}(\boldsymbol{U}_m \boldsymbol{U}_m^H) \end{aligned} \tag{24}$$

The optimisation problem (19) is equivalently written as

$$\underset{\{V_k\}_{k=1}^K}{\text{minimise}} \quad \text{MSE}$$

$$\text{s.t.} \left\| \begin{matrix} \sigma_\text{d}||(\boldsymbol{w}_k^{(l)})^\text{H}|| \\ \sigma_\text{r}||(\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}|| \\ 1-[\boldsymbol{e}_{k,l}^\text{T}C_k \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_k]\text{vec}(\boldsymbol{V}) \\ [\text{C}_1 \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_1]\text{vec}(\boldsymbol{V}) \\ \vdots \\ [C_{k-1} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_{k-1}]\text{vec}(\boldsymbol{V}) \\ [C_{k+1} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_{k+1}]\text{vec}(\boldsymbol{V}) \\ \vdots \\ [\text{C}_K \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_K]\text{vec}(\boldsymbol{V}) \\ \boldsymbol{e}_{k,1}^\text{T}\text{C}_k \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_k]\text{vec}(\boldsymbol{V}) \\ \vdots \\ [(\boldsymbol{e}_{k,l-1}^\text{T}C_k) \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_k]\text{vec}(\boldsymbol{V}) \\ [(\boldsymbol{e}_{k,l+1}^\text{T}C_k) \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_k]\text{vec}(\boldsymbol{V}) \\ \vdots \\ [(\boldsymbol{e}_{k,d_k}^\text{T}\text{C}_k) \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\boldsymbol{A}_k\boldsymbol{T}\tilde{\boldsymbol{B}}_k]\text{vec}(\boldsymbol{V}) \end{matrix} \right\| \le \text{MSE}$$

$$\forall l \in \mathcal{D}_k \quad \text{and} \quad \forall k \in \mathcal{K}$$

$$\|\text{vec}(\boldsymbol{V}_k)\| \le \sqrt{P_{\text{s},k}}, \quad \forall k \in \mathcal{K}$$

$$\left\| \begin{matrix} \sigma_\text{r}\sqrt{\text{tr}(\boldsymbol{U}_m\boldsymbol{U}_m^\text{H})} \\ [\boldsymbol{I}_{d_1} \otimes \boldsymbol{U}_m H_{m,1}]\text{vec}(\boldsymbol{V}_1) \\ \cdots \\ [\boldsymbol{I}_{\text{d}_K} \otimes \boldsymbol{U}_m H_{m,K}]\text{vec}(\boldsymbol{V}_K) \end{matrix} \right\| \le \sqrt{P_{\text{r},m}} \tag{25}$$

$$\forall m \in \mathcal{M}$$

Since the objective function is linear and constraints are second-order convex cones, the optimisation problem (25) is a second-order-cone-programming (SOCP) problem [17] and can be efficiently solved using interior point algorithms [18]. There are readily available software tools for efficiently solving the convex optimisation problems.

4.3 Relay processing matrix design

In this section, we focus on designing relay processing matrices for all relays $\boldsymbol{U}_m$, $\forall m \in \mathcal{M}$ with fixed source precoders and destination filters by solving the following optimisation problem

$$\begin{aligned} \{\{\boldsymbol{U}_m^*\}_{m=1}^M\} := \underset{\{\boldsymbol{U}_m\}_{m=1}^M}{\text{argmin}} \quad & \text{MSE} \\ \text{s.t.} \sqrt{\text{MSE}_{k,l}} \le \text{MSE}, \ & \forall l \in \mathcal{D}_k, \ \forall k \in \mathcal{K} \\ p_{\text{r},m} \le P_{\text{r},m}, \quad & \forall m \in \mathcal{M} \end{aligned} \tag{26}$$

From (13), the MSE of the lth stream estimate of the kth user can be written as

$$\begin{aligned} \text{MSE}_{k,l} = & \left|1-(\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(l)}\right|^2 \\ & + \sum_{\substack{j=1 \\ j\ne k}}^K (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{H}}_j\boldsymbol{V}_j\boldsymbol{V}_j^\text{H}\tilde{\boldsymbol{H}}_j^\text{H}\tilde{\boldsymbol{U}}^\text{H}\tilde{\boldsymbol{G}}_k^\text{H}\boldsymbol{w}_k^{(l)} \\ & + \sum_{\substack{p=1 \\ p\ne l}}^{d_k} (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(p)}(\boldsymbol{v}_k^{(p)})^\text{H}\tilde{\boldsymbol{H}}_k^\text{H}\tilde{\boldsymbol{U}}^\text{H}\tilde{\boldsymbol{G}}_k^\text{H}\boldsymbol{w}_k^{(l)} \\ & + \sigma_\text{r}^2(\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k\tilde{\boldsymbol{U}}\tilde{\boldsymbol{U}}^\text{H}\tilde{\boldsymbol{G}}_k^\text{H}\boldsymbol{w}_k^{(l)} + \sigma_\text{d}^2\left\|\boldsymbol{w}_k^{(l)}\right\|^2 \end{aligned} \tag{27}$$

Using the matrix equality $\text{vec}(\boldsymbol{ABC}) = (C^\text{T} \otimes \boldsymbol{A})\text{vec}(\boldsymbol{B})$, we equivalently rewrite (27) as

$$\begin{aligned} \text{MSE}_{k,l} = & \left|1-[(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(l)})^\text{T} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k]\text{vec}(\tilde{\boldsymbol{U}})\right|^2 \\ & + \sum_{\substack{j=1 \\ j\ne k}}^K \left\|[(\tilde{\boldsymbol{H}}_j\boldsymbol{V}_j)^\text{T} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k]\text{vec}(\tilde{\boldsymbol{U}})\right\|^2 \\ & + \sum_{\substack{p=1 \\ p\ne l}}^{d_k} \left|[(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(p)})^\text{T} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k]\text{vec}(\tilde{\boldsymbol{U}})\right|^2 \\ & + \sigma_\text{r}^2\left\|[\boldsymbol{I}_{N_\text{R}} \otimes (\boldsymbol{w}_k^{(l)})^\text{H}\tilde{\boldsymbol{G}}_k]\text{vec}(\tilde{\boldsymbol{U}})\right\|^2 \\ & + \sigma_\text{d}^2\left\|\boldsymbol{w}_k^{(l)}\right\|^2 \end{aligned} \tag{28}$$

The transmit power of the mth relay is written as

$$\begin{aligned} p_{\text{r},m} & = \sum_{k=1}^K \left\|\text{vec}(\boldsymbol{U}_m\boldsymbol{H}_{m,k}\boldsymbol{V}_k)\right\|^2 + \sigma_\text{r}^2||\text{vec}(\boldsymbol{U}_m)||^2 \\ & = \sum_{k=1}^K \left\|[\boldsymbol{V}_k^\text{T}\boldsymbol{H}_{m,k}^\text{T} \otimes \boldsymbol{I}_{N_{\text{r},m}}]\text{vec}(\boldsymbol{U}_m)\right\|^2 + \sigma_\text{r}^2||\text{vec}(\boldsymbol{U}_m)||^2 \end{aligned} \tag{29}$$

Since, we have different variables in MSE expression (28) and relay power expression (29), lets define a new variable $\boldsymbol{u} \in C^{\tilde{N}_\text{R}\times 1}$, where $\tilde{N}_\text{R} \triangleq \sum_{m=1}^M N_{\text{r},m}^2$ and

$$\boldsymbol{u} = \begin{bmatrix} \text{vec}(\boldsymbol{U}_1) \\ \vdots \\ \text{vec}(\boldsymbol{U}_M) \end{bmatrix}$$

The relation between vectors $\boldsymbol{u}$ and $\text{vec}(\tilde{\boldsymbol{U}})$ is given by the transformation $\text{vec}(\tilde{\boldsymbol{U}}) = \boldsymbol{A}\boldsymbol{u}$ where $\boldsymbol{A} \in R^{N_\text{R}^2\times\tilde{N}_\text{R}}$ is the matrix of ones and zeros formed by observing the non-zero entries of $\text{vec}(\tilde{\boldsymbol{U}})$. Similarly, the relation between vectors $\boldsymbol{u}$ and $\text{vec}(\boldsymbol{U}_m)$ is given by the transformation $\text{vec}(\boldsymbol{U}_m) = \boldsymbol{B}_m\boldsymbol{u}$, $\forall m \in \mathcal{M}$ where $\boldsymbol{B}_m \in R^{N_{\text{r},m}^2\times\tilde{N}_\text{R}}$ defined as $\boldsymbol{B}_m = [\boldsymbol{B}_{m,1}, \ \ldots, \ \boldsymbol{B}_{m,M}]$ where $\boldsymbol{B}_{m,m} = \boldsymbol{I}_{N_{\text{r},m}^2\times N_{\text{r},m}^2}$ and $\boldsymbol{B}_{m,n} = \boldsymbol{0}_{N_{\text{r},m}^2\times N_{\text{r},n}^2}$, $\forall n \in \mathcal{M}$, $n \ne m$. Using these transformations, the optimisation problem (26) is equivalently written as

$$\begin{aligned}
&\underset{\boldsymbol{u}\in\mathbb{C}^{\tilde{N}_R\times 1}}{\text{minimise}} \quad \text{MSE}\\
&\text{s.t.} \left\| \begin{matrix} \sigma_d\|(\boldsymbol{w}_k^{(l)})\| \\ 1-[(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(l)})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ [(\tilde{\boldsymbol{H}}_1\boldsymbol{V}_1)^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ \vdots \\ [(\tilde{\boldsymbol{H}}_{k-1}\boldsymbol{V}_{k-1})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ [(\tilde{\boldsymbol{H}}_{k+1}\boldsymbol{V}_{k+1})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ \vdots \\ [(\tilde{\boldsymbol{H}}_K\boldsymbol{V}_K)^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ [(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(1)})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ \vdots \\ [(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(l-1)})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ [(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(l+1)})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ \vdots \\ [(\tilde{\boldsymbol{H}}_k\boldsymbol{v}_k^{(d_k)})^T\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \\ \sigma_r[\boldsymbol{I}_{N_R}\otimes(\boldsymbol{w}_k^{(l)})^H\tilde{\boldsymbol{G}}_k]\boldsymbol{A}\boldsymbol{u} \end{matrix} \right\| \le \text{MSE}\\
&\qquad \forall l\in\mathcal{D}_k \quad \text{and} \quad \forall k\in\mathcal{K}\\
&\left\| \begin{matrix} \sigma_r\boldsymbol{B}_m\boldsymbol{u} \\ [(\boldsymbol{H}_{m,1}\boldsymbol{V}_1)^T\otimes\boldsymbol{I}_{N_{r,m}}]\boldsymbol{B}_m\boldsymbol{u} \\ \vdots \\ [(\boldsymbol{H}_{m,K}\boldsymbol{V}_K)^T\otimes\boldsymbol{I}_{N_{r,m}}]\boldsymbol{B}_m\boldsymbol{u} \end{matrix} \right\| \le \sqrt{P_{r,m}} \qquad (30)\\
&\qquad \forall m\in\mathcal{M}
\end{aligned}$$

Similarly, the optimisation problem (30) is an SOCP problem, which can be efficient using interior point algorithms [18].

Algorithm 1: Proposed min–max-MSE-based algorithm

Step 1 Initialisation: Initialise $\{\boldsymbol{V}_k^0\}_{k=1}^K$, $\{\boldsymbol{U}_m^0\}_{m=1}^M$ with randomly generated matrices such that transmit power constraints of each source and relay nodes are satisfied. Set $n=0$.
Step 2 Computation: Compute the linear minimum mean squared error (LMMSE) destination filter matrices $\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K$ with fixed source precoding matrices and relay processing matrices as given in (18).
Step 3 Computation: Compute the transmit precoder matrices $\{\boldsymbol{V}_k^{n+1}\}_{k=1}^K$ with fixed relay processing matrices and destination filter matrices by solving the SOCP optimisation problem (25).
Step 4 Computation: Compute the relay processing matrices $\{\boldsymbol{U}_m^{n+1}\}_{m=1}^M$ with fixed transmit precoding matrices and destination filter matrices by solving the SOCP optimisation problem (30).
Step 5 Termination:

– if converge (or predefined number of iterations reached) terminate the algorithm.
– else set $n=n+1$ and go to **Step 2**.

4.4 Convergence and complexity analysis

In this section, we discuss the convergence and complexity analysis of the proposed algorithm. The proposed algorithm iterates over three design sub-problems namely destination filters design, source precoders design and relay processing matrices design in an alternating manner. The proposed algorithm is summarised in Algorithm 1. We now prove that the proposed min–max MSE algorithm is convergent.

Proof: Let $\text{MSE}=\max_{l\in\mathcal{D}_k, k\in\mathcal{K}} \text{MSE}_{k,l}$, denotes the maximum per-stream MSE of the system. In step 1, for the given $\{\boldsymbol{V}_k^n\}_{k=1}^K$ and $\{\boldsymbol{U}_m^n\}_{m=1}^M$, the optimal solution of the optimisation problem (16) is given by (18).
Hence

$$\begin{aligned}&\text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^n\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\\ &\le \text{MSE}(\{\boldsymbol{W}_k^n\}_{k=1}^K, \{\boldsymbol{V}_k^n\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\end{aligned}$$

In step 2, for the given $\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K$ and $\{\boldsymbol{U}_m^n\}_{m=1}^M$ the optimal solution of the optimisation problem (19) is given by (25). Hence we obtain

$$\begin{aligned}&\text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\\ &\le \text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^n\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\end{aligned}$$

Similarly in step 3, for the given $\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K$ and $\{\boldsymbol{V}_k^{n+1}\}_{k=1}^K$ the optimal solution of the optimisation problem (26) is given by (30). Hence we obtain

$$\begin{aligned}&\text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{U}_m^{n+1}\}_{m=1}^M)\\ &\le \text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\end{aligned}$$

Therefore we conclude that

$$\begin{aligned}&\text{MSE}(\{\boldsymbol{W}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{V}_k^{n+1}\}_{k=1}^K, \{\boldsymbol{U}_m^{n+1}\}_{m=1}^M)\\ &\le \text{MSE}(\{\boldsymbol{W}_k^n\}_{k=1}^K, \{\boldsymbol{V}_k^n\}_{k=1}^K, \{\boldsymbol{U}_m^n\}_{m=1}^M)\end{aligned}$$

Thus we see that the proposed algorithm is convergent. □

Although each sub-problem in an iteration can be solved to global optimality in polynomial time, yet the proposed algorithm is not guaranteed to converge to the global optimum of the optimisation problem (13). However, simulation results in Section 5 show that the proposed algorithm can quickly converge with good performance in terms of average sum-rate and average minimum user rate. The quality of the proposed solution is sensitive to the choice of initial points. However, finding the optimal initial points is a difficult problem, so an opportunistic approach can be used by having multiple initial points and then choosing the one which gives the best performance at the cost of increased computational time.

The computational complexity of each iteration of the proposed algorithm is mainly from the computation of solving two SOCP problems, that is, (25) and (30). In [17], it is shown that the worst-case complexity of solving SOCP problem using interior point methods is polynomial in the problem size and the number of constraints. Specifically, solving an SOCP problem by interior point methods is an iterative procedure with the number of iterations bounded above by $\mathcal{O}(\sqrt{N})$ and the amount of work per iteration $\mathcal{O}(n^2\sum_{i=1}^N n_i)$, where N is the number of second-order-cone inequality constraints, n_i is the dimension of the ith second-order-cone constraint and n is the dimension of the optimisation variable. To provide a complexity analysis of the proposed algorithm, we base our discussion on the following simplification and focus on one iteration of the proposed algorithm. We assume all the nodes have N antennas, that is, $N_{s,k}=N_{d,k}=N$, $\forall k\in\mathcal{K}$ and $N_{r,m}=N$, $\forall m\in\mathcal{M}$ and all source nodes transmit d data streams, that is, $d_k=d$, $\forall k\in\mathcal{K}$. According to (25), the number of real optimisation variables of the SOCP-based transmit precoders design problem is $2K^2Nd+1$. More specifically, there are Kd constraints of real dimension $2Kd+3$, K constraints of real dimension $2Nd+1$ and M

constraints of real dimension $2kdN+2$. Combining all these, the worst case per iteration complexity of SOCP problem comes out to be approximately $\mathcal{O}(K^6N^2d^4+K^5N^3Md^3)$. Moreover, the number of iterations to obtain a numerically acceptable value is bounded above by $\mathcal{O}(\sqrt{Kd+K+M})$. Therefore the complexity of solving SOCP-based transmit precoders design problem (25) is $\mathcal{O}(\sqrt{(Kd+K+M)}(K^6N^2d^4+K^5N^3Md^3)\log(1/\epsilon))$, where ε denotes the precision of the numerical algorithm. Similarly, the SOCP-based relay processing matrices design problem (30) has $2MN^2+1$ real variables. There are Kd constraints of real dimension $2Kd+2MN$ and M constraints of real dimension $2kdN+1$. The per iteration complexity of the SOCP algorithm is approximately $\mathcal{O}(K^2N^4M^2d^2+KN^5M^3d)$ and number of iterations to obtain a numerically acceptable value is upper bounded by $\mathcal{O}(\sqrt{Kd+M})$. Therefore the complexity of solving SOCP-based relay processing matrices design problem (30) is $\mathcal{O}(\sqrt{(Kd+M)}(K^2N^4M^2d^2+KN^5M^3d)\log(1/\epsilon))$.

5 Simulation results

In this section, we evaluate the performance of our proposed algorithm via Monte Carlo simulations. As in [16], we consider only symmetric systems denoted as $(N_\text{d}\times N_\text{s},\ d)^K+N_\text{r}^M$ where $N_{\text{s},k}=N_\text{s}$, $N_{\text{d},k}=N_\text{d}\ \forall k\in\mathcal{K}$ and $N_{\text{r},m}=N_\text{r}\ \forall m\in\mathcal{M}$. Noise power at all the relay nodes and all the destination nodes are normalised to unity, that is, $\sigma_\text{r}=\sigma_\text{d}=\sigma=1$. All source and relay nodes have identical power constraints $P_{\text{s},k}=P$ for $k\in\mathcal{K}$ and $P_{\text{r},m}=P$ for $m\in\mathcal{M}$. Here, we define SNR as $\text{SNR}=P/\sigma^2$. All channel coefficients are drawn from independent identically distributed zero-mean unit-variance complex Gaussian distribution. All plots are obtained by averaging over 200 independent channel realisation except for Fig. 2. For each channel realisation, initial source transmit precoders and relay processing matrices are randomly generated such that they satisfy their transmit power constraints. To solve SOCP problems we used CVX, a package for specifying and solving convex programmes [19].

5.1 Convergence

Fig. 2 illustrates the convergence behaviour of the proposed algorithm for a random channel realisation of the $(2\times 4,\ 1)^4+2^4$ system. It is observed that the maximum per-stream MSE is non-increasing over iterations as expected. Although the convergence speed of the proposed algorithm depends on the system parameters such as number of users and number of transmit streams per user, most of the reduction in the maximum per-stream MSE is achieved in the first few iterations.

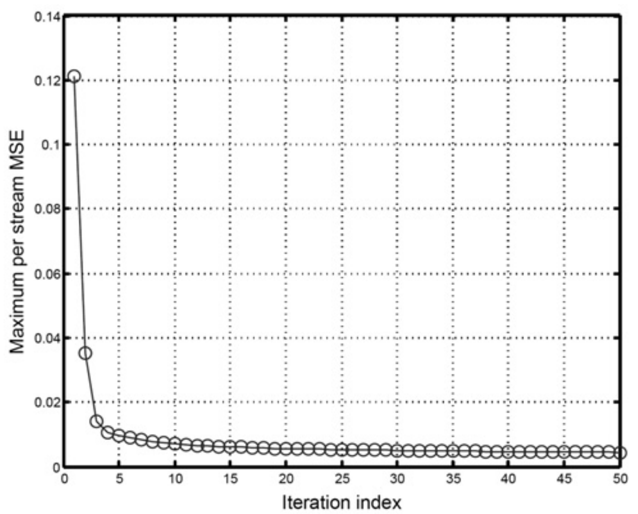

Fig. 2 *Convergence behaviour of the proposed algorithm for a random channel realisation for a $(2\times 4,\ 1)^4+2^4$ system*

5.2 Sum-rate performance

In first experiment, we consider the DF relay interference channel for comparison where a dedicated DF relay assists the communication between each source–destination pair. We simulate four strategies for DF relay case in which single-hop MIMO interference channel strategies are independently applied on source–relay hop and relay–destination hop. The single-hop strategies used are (i) interference alignment based on interference leakage minimisation [20], (ii) max-SINR algorithm [20], (iii) max-MSE minimisation [21] and (iv) iteratively weighted sum-MSE minimisation [22]. The achievable rate of a source–destination pair is defined as half of the minimum between the achievable rate from the source node to DF relay node and that from the DF relay node to the destination node. We also simulate time division multiple access (TDMA)-based distributed beamforming for AF relay case, where all the AF relays assist only one source–destination pair at a time. Source precoder, destination filter and relay processing matrices are jointly designed using MMSE criterion. Fig. 3 shows the average sum-rate against maximum transmit power at source and relay nodes of the $(2\times 2,\ 1)^4+2^4$ system. It is observed that the proposed min–max MSE-based joint transceiver–relay design solution outperforms all the other strategies in all regions. Interference leakage minimisation and max-SINR strategies perform worst because interference alignment is not feasible for the given system configuration on both the hops [23]. Weighted MSE minimisation scheme achieves higher sum-rate because it may turn off some data streams to make interference alignment feasible. Although AF-TDMA distributed beamforming effectively eliminates multi-user interference, it leads to inefficient use of communication resources. This shows that joint design of beamforming matrices at source, relay and destination nodes can achieve much higher sum-rate than the single-hop strategies applied independently on each hop.

In the following experiments, we compare the performance of the proposed algorithm with the following existing transceiver–relay designs for MIMO interference AF relay channel [16]

(1) Total interference plus noise leakage minimisation (min-leakage).
(2) Weighted sum-MSE minimisation with equality power constraints (WMMSE-NoPC).

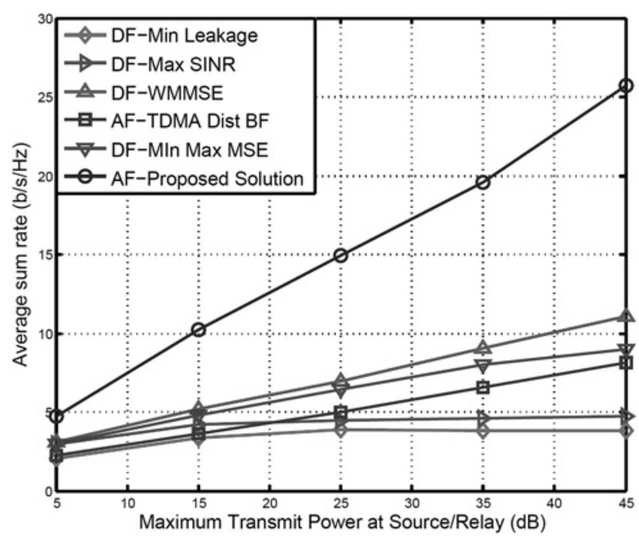

Fig. 3 *Average sum-rate against maximum transmit power for $(2\times 2,\ 1)^4+2^4$ system*

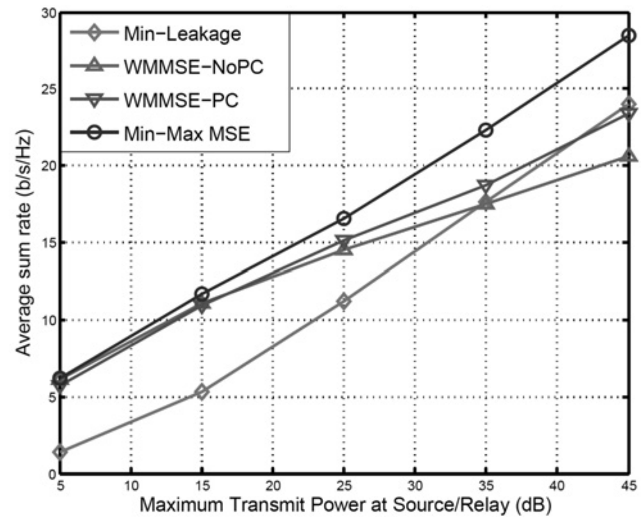

Fig. 4 *Average sum-rate against maximum transmit power for* $(2 \times 4,\ 1)^4 + 2^4$ *system*

(3) Weighted sum-MSE minimisation with inequality power constraints (WMMSE-PC).

Figs. 4 and 5 show the average sum-rate against maximum transmit power at source and relay nodes of a $(2 \times 4,\ 1)^4 + 2^4$ and $(4 \times 4,\ 2)^3 + 4^3$ system, respectively. The proposed min–max MSE-based solution achieves almost the same average sum-rate as the WMMSE-PC and WMMSE-NoPC schemes at low-to-medium SNR values and outperforms WMMSE-based schemes at high SNR values for both system configurations. The proposed min–max MSE-based solution achieves much higher average sum-rate than min-leakage-based solution at low-to-medium SNR values for $(2 \times 4,\ 1)^4 + 2^4$ system. However, the gap between average sum-rate performance of the two solutions reduces at high SNR. This is because min-leakage-based solution seeks perfect alignment of interference and relay-enhanced noise to create an interference free desired signal subspace at the destination nodes. However, it does not attempt to maximise the signal power in desired signal subspace. It is observed that min-leakage-based solution performs worst for the $(4 \times 4,\ 2)^3 + 4^3$ system. It is because the perfect alignment of interference and relay-enhanced noise signals may not be feasible for $(4 \times 4,\ 2)^3 + 4^3$ system.

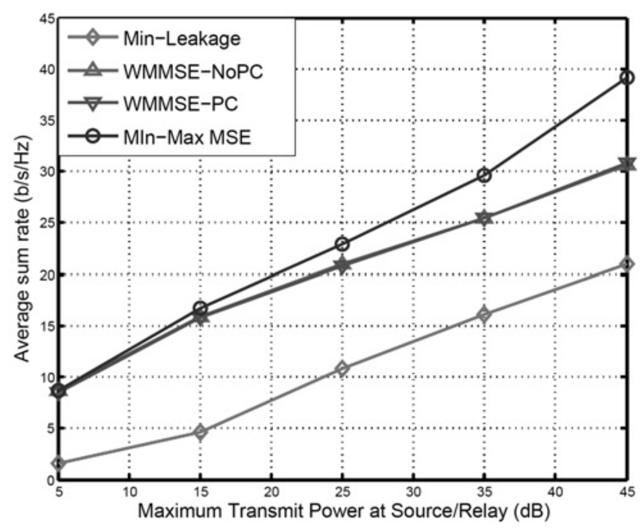

Fig. 5 *Average sum-rate against maximum transmit power for* $(4 \times 4,\ 2)^3 + 4^3$ *system*

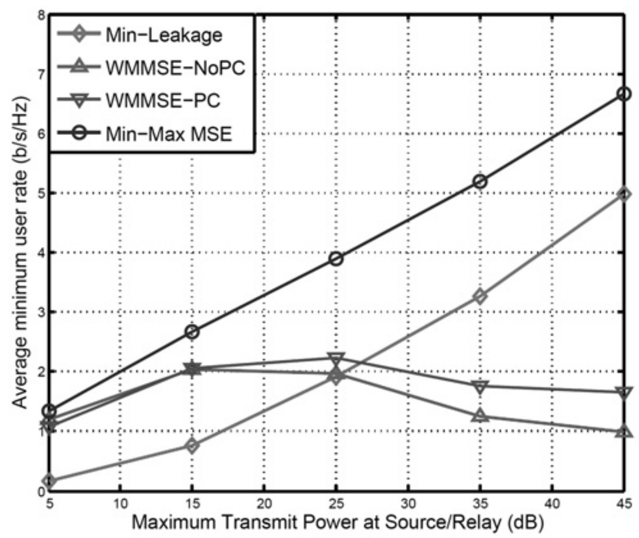

Fig. 6 *Average minimum user data rate against maximum transmit power for* $(2 \times 4,\ 1)^4 + 2^4$ *system*

5.3 Fairness performance

In this experiment, we evaluate the fairness performance of the proposed solution. Figs. 6 and 7 show the average minimum user rate against maximum transmit power at source and relay nodes for a $(2 \times 4,\ 1)^4 + 2^4$ system and $(4 \times 4,\ 2)^3 + 4^3$ system, respectively. We observe that the proposed min–max MSE-based solution outperforms all the other strategies in terms of average minimum user rate performance in all regions. This is because the proposed algorithm improves the fairness among all users' data streams in the sense of almost same per-stream MSE. We observe that for WMMSE-NoPC and WMMSE-PC algorithm, some users have much smaller rates than others because of unequal per-user and per-stream MSE.

In the next experiment, we investigate the performance of the proposed algorithm in a system where number of antennas at each node is small, whereas the number of users in the system can be quite large. This motivates to study the performance of the min–max MSE-based solution for increasing number of users/relays, but with number of antennas at each node fixed. Fig. 8 shows the average minimum user rate for increasing values of K of the $(2 \times 2,\ 1)^K + 2^K$ system. We observe that the proposed min–max

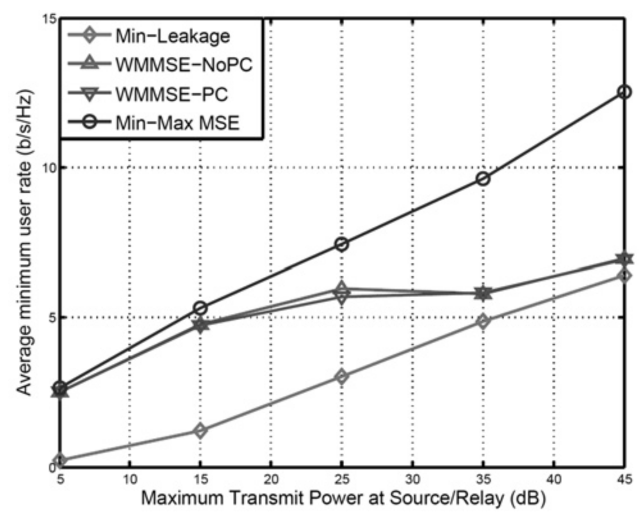

Fig. 7 *Average minimum user data rate against maximum transmit power for* $(4 \times 4,\ 2)^3 + 4^3$ *system*

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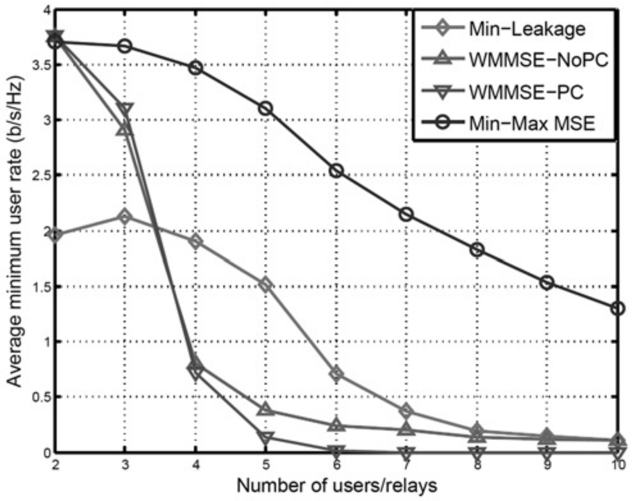

Fig. 8 *Average minimum user rate against the number of users/relays with SNR = 25 dB for* $(2\times 2,\ 1)^K + 2^K$ *system*

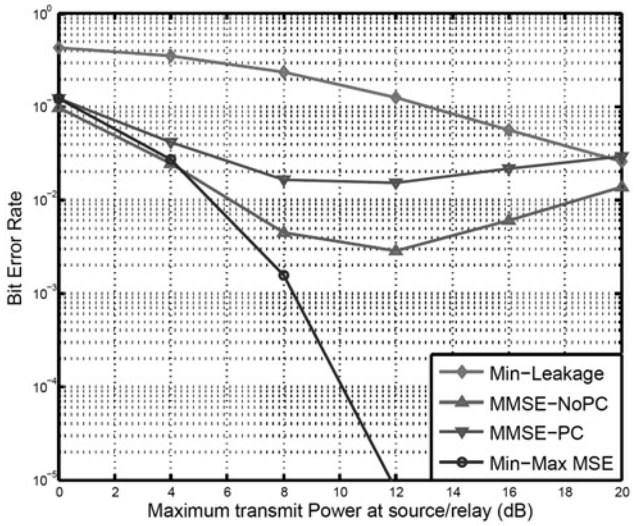

Fig. 9 *Average BER against SNR for a* $(2\times 4,\ 1)^4 + 2^4$ *system*

MSE-based solution outperforms all the other strategies in terms of average minimum user rate performance regardless the number of users in the network.

5.4 BER performance

In this experiment, we evaluate the average BER performance of the proposed algorithm. Fig. 9 shows the average BER against SNR of the $(2\times 4,\ 1)^4 + 2^4$ system for quadrature phase shift keying modulation. It is observed that the proposed min–max MSE-based solution outperforms all the other strategies in terms of average BER performance in all regions. Min-leakage-based solution has the worst BER performance as it provides no diversity gain [24]. We observe that for WMMSE-NoPC and WMMSE-PC algorithms, at high SNR, some users have much higher BER than others because of unequal per-user MSE.

6 Conclusion

We developed a min–max MSE-based linear transceiver–relay design for MIMO interference AF relay channel. The algorithm aims to minimise the maximum per-stream MSE among all users subject to transmit power constraints at each source and relay nodes. An iterative solution is developed to jointly design the beamforming matrices at all the source, relay and destination nodes. Numerical simulation results show that the proposed algorithm ensures fairness among all users' data streams and provide good sum-rate and BER performance.

Although this work and prior works for MIMO interference AF relay channel shows the potential benefits of adding relays in a constant MIMO interference channel, several challenges must be overcome before these schemes translate into practice. One key assumption is that the global and perfect CSI is available at a central unit which computes the beamforming matrices at all source, relay and destination nodes. Distributed algorithms which require less CSI overhead and robust algorithms which make more practical CSI assumptions are topics for future work.

7 References

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