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New Results on Synchronization for Complex Dynamical Networks with Time-Varying Coupling Delay and Sampled-Data

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Abstract: This paper is concerned with the synchronization problem for complex dynamical networks with time-varying coupling delay and sampled-data. The sampling period considered here is assumed to be time-varying but bounded. A novel exponential synchronization condition is established based on the Gronwall's inequality, and an explicit expression for a set of sampled-data synchronization controllers is also given in terms of the solution to linear matrix inequalities. A numerical example is introduced to show the effectiveness of the given result.

Keywords: Dynamical networks; Synchronization control; Sampled-data control; Time-Varying sampling

1. INTRODUCTION

In the past decades, complex dynamical networks (CDNs) have received much attention because they have extensive applications in both science and engineering such as Internet, World Wide Web, food webs, electric power grids, cellular and metabolic networks, scientific citation networks, social networks, etc [1]. In the recent years, synchronization problem of CDNs has attracted rapidly increasing attention in the scientific community due to past physics and potential engineering applications, including parallel image processing, pattern storage and retrieval, secure communication, and so on [2–4]. For example, in [5], the global exponential synchronization in arrays of coupled identical delayed neural networks (DNNs) with constant and delayed coupling has been studied by referring to Lyapunov functional method and Kronecker product technique, and some sufficient conditions have been derived for global synchronization of such systems. In [6], a detailed analysis has been presented for the synchronization of CDNs with impulsive coupling based on the concept of average impulsive interval, and a unified synchronization criterion has been derived for directed impulsive dynamical networks, which takes into account two types of impulses simultaneously. In [4], the global synchronization of complex dynamical networks with network failures has been studied based on the framework of switching system. The problem of guaranteed cost synchronization for a complex network has been investigated in [7], where two types of guaranteed cost dynamic feedback controller have been designed.

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On the other hand, sampled-data systems have been studied extensively over the past decades due to the fact that the sampled-data control technology has shown more and more superiority over other control approaches. Thus, many important and interesting results have been proposed [8–12]. Very recently, the sampled-data synchronization control problem has been investigated for a class of general complex networks with time-varying coupling delays in [13], where a sufficient condition has been derived to ensure the exponential stability of the closed-loop error system. Based on the condition, the desired sampled-data feedback controllers have been designed. However, there is room for further investigation. For example, the delay terms $\tau(t)$ and $\tau - \tau(t)$ with $0 \leq \tau(t) \leq \tau$ are enlarged as τ, and the delay terms $d(t)$ and $p - d(t)$ with $0 \leq d(t) \leq p$ are enlarged as p, that is, $\tau = \tau(t) + \tau - \tau(t)$ and $p = p(t) + p - p(t)$ are enlarged as 2τ and $2p$, respectively. It is clear the aforementioned treatment may lead to a conservative result.

In this paper, the problem of exponential synchronization is studied for complex dynamical networks with time-varying coupling delay and variable samplings. In the framework of the input delay approach, an exponential synchronization condition is proposed based on the Gronwall's inequality, and an explicit expression for a set of sampled-data synchronization controllers is also given. A numerical example is given to demonstrate the effectiveness of the proposed method.

Notation: The notations used throughout this paper are fairly standard. $\mathcal{R}^n$ and $\mathcal{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively. The notation $X > Y$ ($X \geq Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive definite (positive semidefinite). I and 0 represent the identity matrix and a zero matrix, respectively. The superscript "T" represents the transpose, and $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $||\cdot||$ denotes the Euclidean norm of a vector and its induced norm of a matrix. For an arbitrary matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where "$*$" denotes the term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following complex dynamical network consisting of N identical coupled nodes:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} G_{ij}Ax_j(t-\tau(t)) + u_i(t) \quad (1)$$

for all $i = 1, 2, \cdots, N$, where $x_i(t)$ and $u_i(t)$ are, respectively, the state variable and the control input of the node i, and $f : \mathcal{R}^n \to \mathcal{R}^n$ is a continuous vector-valued function. The scalar $\tau(t)$ denotes the time-varying delay satisfying

$$0 \leq \tau(t) \leq \mu,\ \dot{\tau}(t) \leq \nu \quad (2)$$

where $\tau > 0$ and $\nu > 0$ are known constants; c is a constant denoting the coupling strength; $A = (a_{ij})_{n\times n} \in \mathcal{R}^{n\times n}$ is a constant inner-coupling matrix of the nodes, and $G = (G_{ij})_{N\times N}$ is the outer-coupling matrix of the network. If there is a connection between node i and node j $(i \neq j)$, then $G_{ij} = 1$, otherwise, $G_{ij} = 0$ $(i \neq j)$. The diagonal elements of matrix G are defined by $G_{ii} = -\sum_{j=1, j\neq i}^{N} G_{ij}$ for all $i = 1, 2, \cdots, N$.

Letting $e_i(t) = x_i(t) - s(t)$ be the synchronization error, where $s(t) \in \mathcal{R}^n$ is the state trajectory of the unforced isolate node $\dot{s}(t) = f(s(t))$. Then, the error dynamics of complex network (1) can be obtained as follows:

$$\dot{e}_i(t) = g(e_i(t)) + c\sum_{j=1}^{N} G_{ij}Ae_j(t-\tau(t)) + u_i(t) \quad (3)$$

for all $i = 1, 2, \cdots, N$, where $g(e_i(t)) = f(x_i(t)) - f(s(t))$.

The control signal is assumed to be generated by using a Zero-Order-Hold (ZOH) function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$. Therefore, the state feedback controller takes the following form:

$$u_i(t) = K_ie_i(t_k),\ t_k \leq t < t_{k+1} \quad (4)$$

for all $i = 1, 2, \cdots, N$, where K_i is sampled-data feedback controller gain matrix to be determined, $e_i(t_k)$ is discrete measurement of $e_i(t)$ at the sampling instant t_k, $\lim_{k\to+\infty} t_k = +\infty$. It is assumed that $t_{k+1} - t_k = h_k \leq p$ for any integer $k \geq 0$, where p is a positive scalar and represents the largest sampling interval.

By substituting (4) into (1), we obtain

$$\begin{aligned}\dot{e}_i(t) &= g(e_i(t)) + c\sum_{j=1}^{N} G_{ij}Ae_j(t-\tau(t)) \\ &\quad + K_ie_i(t-d(t)),\ i = 1, 2, \cdots, N\end{aligned} \quad (5)$$

where $d(t) = t - t_k$. It can be seen that

$$0 \leq d(t) \leq p. \quad (6)$$

It is clear that (5) can be rewritten as:

$$\begin{aligned}\dot{e}(t) &= \bar{g}(e(t)) + c(G\otimes A)e(t-\tau(t)) \\ &\quad + Ke(t-d(t))\end{aligned} \quad (7)$$

where $K = \mathrm{diag}\{K_1,\ K_2\ \cdots, K_N\}$, and

$$e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_N(t) \end{bmatrix},\ \bar{g}(e(t)) = \begin{bmatrix} g(e_1(t)) \\ g(e_2(t)) \\ \vdots \\ g(e_N(t)) \end{bmatrix}.$$

Definition 1: The complex network (1) is said to be exponentially synchronized if the closed-loop error system (7) is exponentially stable, i.e., there exist two constants $\alpha > 0$ and $\beta > 0$ such that

$$||e(t)||^2 \leq \alpha e^{-\beta t} \sup_{-\max\{\mu,p\}\leq\theta\leq 0} ||e(\theta)||^2 \quad (8)$$

Assumption 1 [14]: The nonlinear function $f : \mathcal{R}^n \to \mathcal{R}^n$ satisfies

$$[f(x)-f(y)-U(x-y)]^{\mathrm{T}}[f(x)-f(y)-V(x-y)] \leq 0 \quad (9)$$

for $\forall x, y \in \mathcal{R}^n$.

The aim of this paper is to design a set of controllers with the form (4) to ensure the exponential synchronization of the complex network (1).

3. MAIN RESULTS

Before presenting the main results, for the sake of presentation simplicity, we denote:

$$\bar{U} = \frac{(I_N\otimes U)^{\mathrm{T}}(I_N\otimes V)}{2} + \frac{(I_N\otimes V)^{\mathrm{T}}(I_N\otimes U)}{2}$$

$$\bar{V} = -\frac{(I_N\otimes U)^{\mathrm{T}} + (I_N\otimes V)^{\mathrm{T}}}{2}$$

Theorem 1: The error system (7) is exponentially stable if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, S_1, S_2, and a scalar $\lambda > 0$ such that

$$\Xi_1 = \begin{bmatrix} p^{-1}Z_1 & S_1 \\ * & p^{-1}Z_1 \end{bmatrix} > 0 \quad (10)$$

$$\Xi_2 = \begin{bmatrix} \mu^{-1}(Z_2 + (1-\nu)Z_3) & S_2 \\ * & \mu^{-1}Z_2 \end{bmatrix} > 0 \tag{11}$$

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & S_1 & \Xi_{14} & S_2 & \Xi_{16} & 0 \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & K^{\mathrm{T}}Z \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & \Xi_{47} \\ * & * & * & * & \Xi_{55} & 0 & 0 \\ * & * & * & * & * & -\lambda I & Z \\ * & * & * & * & * & * & Z \end{bmatrix} < 0 \tag{12}$$

where

$$\begin{aligned} \Xi_{11} &= Q_1 + Q_2 + Q_3 - \lambda\bar{U} - p^{-1}Z_1 \\ &\quad - \mu^{-1}(Z_2 + (1-\nu)Z_3) \\ \Xi_{12} &= PK + p^{-1}Z_1 - S_1 \\ \Xi_{14} &= cP(G \otimes A) + \mu^{-1}(Z_2 + (1-\nu)Z_3) - S_2 \\ \Xi_{16} &= P - \lambda\bar{V} \\ \Xi_{22} &= -2p^{-1}Z_1 + S_1 + S_1^{\mathrm{T}} \\ \Xi_{23} &= -S_1 + p^{-1}Z_1 \\ \Xi_{33} &= -Q_1 - p^{-1}Z_1 \\ \Xi_{44} &= -(1-\nu)Q_3 - \mu^{-1}(Z_2 + (1-\nu)Z_3) - \mu^{-1}Z_2 \\ &\quad + S_2 + S_2^{\mathrm{T}} \\ \Xi_{45} &= -S_2 + \mu^{-1}Z_2 \\ \Xi_{47} &= c(G \otimes A)^{\mathrm{T}}Z \\ \Xi_{55} &= -Q_2 - \mu^{-1}Z_2 \\ Z &= pZ_1 + \mu Z_2 + \mu Z_3. \end{aligned}$$

Proof: Consider the following Lyapunov functional for the error system (7):

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{13}$$

where

$$\begin{aligned} V_1(t) &= e(t)^{\mathrm{T}}Pe(t) + \int_{t-p}^{t} e(s)^{\mathrm{T}}Q_1e(s)\,ds \\ &\quad + \int_{t-\mu}^{t} e(s)^{\mathrm{T}}Q_2e(s) + \int_{t-\tau(t)}^{t} e(s)^{\mathrm{T}}Q_3e(s)\,ds \\ V_2(t) &= \int_{-p}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}}Z_1\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\ V_3(t) &= \int_{-\mu}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}}Z_2\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\ &\quad + \int_{-\tau(t)}^{0}\int_{t+\theta}^{t} \dot{e}(s)^{\mathrm{T}}Z_3\dot{e}(s)\,\mathrm{d}s\,\mathrm{d}\theta. \end{aligned}$$

Taking the derivative of (13) along the solution of system (7) yields

$$\begin{aligned} \dot{V}_1(t) &\le 2e(t)^{\mathrm{T}}P\dot{e}(t) + e(t)^{\mathrm{T}}Q_3e(t) \\ &\quad + e(t)^{\mathrm{T}}Q_1e(t) - e(t-p)^{\mathrm{T}}Q_1e(t-p) \\ &\quad + e(t)^{\mathrm{T}}Q_2e(t) - e(t-\mu)^{\mathrm{T}}Q_2e(t-\mu) \\ &\quad - (1-\nu)e(t-\tau(t))^{\mathrm{T}}Q_3e(t-\tau(t)), \end{aligned} \tag{14}$$

$$\dot{V}_2(t) = p\dot{e}(t)^{\mathrm{T}}Z_1\dot{e}(t) - \int_{t-p}^{t} \dot{e}(s)^{\mathrm{T}}Z_1\dot{e}(s)\,\mathrm{d}s, \tag{15}$$

$$\begin{aligned} \dot{V}_3(t) &\le \mu\dot{e}(t)^{\mathrm{T}}Z_2\dot{e}(t) + \mu\dot{e}(t)^{\mathrm{T}}Z_3\dot{e}(t) \\ &\quad - \int_{t-\mu}^{t} \dot{e}(s)^{\mathrm{T}}Z_2\dot{e}(s)\,\mathrm{d}s \\ &\quad - (1-\nu)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}Z_3\dot{e}(s)\,\mathrm{d}s. \end{aligned} \tag{16}$$

On the other hand, by applying (10), (11) and the lower bounds lemma of [15], we have

$$-\int_{t-p}^{t} \dot{e}(s)^{\mathrm{T}}Z_1\dot{e}(s)\,\mathrm{d}s \le -\Omega_1^{\mathrm{T}}\Xi_1\Omega_1 \tag{17}$$

and

$$\begin{aligned} &-\int_{t-\mu}^{t} \dot{e}(s)^{\mathrm{T}}Z_2\dot{e}(s)\,\mathrm{d}s \\ &\quad -(1-\nu)\int_{t-\tau(t)}^{t} \dot{e}(s)^{\mathrm{T}}Z_3\dot{e}(s)\,\mathrm{d}s \le -\Omega_2^{\mathrm{T}}\Xi_2\Omega_2 \end{aligned} \tag{18}$$

where

$$\Omega_1 = \begin{bmatrix} \int_{t-d(t)}^{t} \dot{e}(s)\,\mathrm{d}s \\ \int_{t-p}^{t-d(t)} \dot{e}(s)\,\mathrm{d}s \end{bmatrix}, \ \Omega_2 = \begin{bmatrix} \int_{t-\tau(t)}^{t} \dot{e}(s)\,\mathrm{d}s \\ \int_{t-\mu}^{t-\tau(t)} \dot{e}(s)\,\mathrm{d}s \end{bmatrix}.$$

Furthermore, based on Assumption 1, we have that any $\lambda > 0$

$$y(t) = \lambda \begin{bmatrix} e(t) \\ \bar{g}(e(t)) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \bar{U} & \bar{V} \\ * & I \end{bmatrix} \begin{bmatrix} e(t) \\ \bar{g}(e(t)) \end{bmatrix} \le 0. \tag{19}$$

Thus,

$$\begin{aligned} \dot{V}(t) &\le \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) - y(t) \\ &= \begin{bmatrix} \delta_1(t) \\ \delta_2(t) \end{bmatrix}^{\mathrm{T}} \Delta \begin{bmatrix} \delta_1(t) \\ \delta_2(t) \end{bmatrix} \end{aligned} \tag{20}$$

where

$$\delta_1(t) = \begin{bmatrix} e(t) \\ e(t-d(t)) \\ e(t-p) \end{bmatrix}, \delta_2(t) = \begin{bmatrix} e(t-\tau(t)) \\ e(t-\mu) \\ \bar{g}(e(t)) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} \Xi_{11} & \Xi_{12} & S_1 & \Xi_{14} & S_2 & \Xi_{16} \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 \\ * & * & \Xi_{33} & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 \\ * & * & * & * & \Xi_{55} & 0 \\ * & * & * & * & * & -\lambda I \end{bmatrix} + \begin{bmatrix} 0 \\ K^{\mathrm{T}} \\ 0 \\ c(G\otimes A)^{\mathrm{T}} \\ 0 \\ I \end{bmatrix} Z \begin{bmatrix} 0 \\ K^{\mathrm{T}} \\ 0 \\ c(G\otimes A)^{\mathrm{T}} \\ 0 \\ I \end{bmatrix}^{\mathrm{T}}.$$

By using the Schur complement, we can find from (12) that $\Delta < 0$, and thus there exists a scalar $\beta > 0$ such that

$$\dot{V}(t) \leq -\beta||e(t)||^2. \tag{21}$$

Thus,

$$\lambda_{\min}(P)||e(t)||^2 \leq V(t) \leq V(0) - \beta \int_0^t ||e(s)||^2 \, \mathrm{d}s \tag{22}$$

which implies that

$$||e(t)||^2 \leq \frac{V(0)}{\lambda_{\min}(P)} - \frac{\beta}{\lambda_{\min}(P)} \int_0^t ||e(s)||^2 \, \mathrm{d}s. \tag{23}$$

Applying Gronwall's inequality, we can immediately get that

$$||e(t)||^2 \leq \frac{V(0)}{\lambda_{\min}(P)} e^{-\frac{\beta}{\lambda_{\min}(P)}t} \tag{24}$$

which, from Definition 1, means that the error system (7) is exponentially stable. The proof is now complete.

In the following theorem, we make use of Theorem 1 to design the controller ensuring the dynamical network (1) exponentially synchronized.

Theorem 2: The error system (7) is exponentially stable if there exist matrices $P = \mathrm{diag}\{P_1, P_2, \cdots, P_N\} > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, S_1, S_2, $X = \mathrm{diag}\{X_1, X_2, \cdots, X_N\} > 0$ and a scalar $\lambda > 0$ such that (10), (11) and the following LMI hold

$$\begin{bmatrix} \Xi_{11} & \hat{\Xi}_{12} & S_1 & \Xi_{14} & S_2 & \Xi_{16} & 0 \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & X^{\mathrm{T}} \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & \hat{\Xi}_{47} \\ * & * & * & * & \Xi_{55} & 0 & 0 \\ * & * & * & * & * & -\lambda I & P \\ * & * & * & * & * & * & -2P+Z \end{bmatrix} < 0 \tag{25}$$

where $\hat{\Xi}_{12} = X + p^{-1}Z_1 - S_1$, $\hat{\Xi}_{47} = c(G\otimes A)^{\mathrm{T}}P$, and the other parameters follow the same definitions as those in Theorem 1. Moreover, if the desired controllers gain matrices

$$K_i = P_i^{-1}X_i. \tag{26}$$

Proof: Define matrix $J = \mathrm{diag}\{I, I, I, I, I, I, PZ^{-1}\}$ and $X = PK$. Then, pre- and post-multiplying (12) with J and J^{T}, respectively, we obtain that (12) is equivalent to

$$\begin{bmatrix} \Xi_{11} & \hat{\Xi}_{12} & S_1 & \Xi_{14} & S_2 & \Xi_{16} & 0 \\ * & \Xi_{22} & \Xi_{23} & 0 & 0 & 0 & X^{\mathrm{T}} \\ * & * & \Xi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & 0 & \hat{\Xi}_{47} \\ * & * & * & * & \Xi_{55} & 0 & 0 \\ * & * & * & * & * & -\lambda I & P \\ * & * & * & * & * & * & \Upsilon \end{bmatrix} < 0 \tag{27}$$

where $\Upsilon = -PZ^{-1}P$. Noting $Z > 0$, we have $-PZ^{-1}P \leq -2P+Z$. Thus, it is clear that if (25) holds, then (27) holds, which implies that (12) holds. This completes the proof.

Remark 1: It is noted that when $S_1 = S_2 = 0$, we can find that Theorem 1 and Theorem 2 reduces to those of [13]. Thus, Our results have theoretically less conservatism than the existing ones.

4. NUMERICAL EXAMPLE

In this section, we will demonstrate the effectiveness of the method proposed in this paper via an example, which is borrowed from [13]. Consider CND (1) with three nodes. The outer-coupling matrix is assumed to be $G = (G_{ij})_{N\times N}$ with

$$G = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.$$

The inner-coupling matrix is given as

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $c = 0.5$. The time-varying delay is chosen as $\tau(t) = 0.2 + 0.05\sin(10t)$. A straightforward calculation gives $\mu = 0.25$ and $\nu = 0.5$. The nonlinear function f is taken as

$$f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}.$$

It can be found that f satisfies (9) with

$$U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}.$$

By applying Theorem 2 of [13], the maximum values of sampling period $p = 0.5409$. While using Theorem 2 in our paper, the maximum values of sampling period $p = 0.5791$. Thus, our result has less conservatism than the existing one. Moreover, the gain matrices of the desired controllers can be obtained as follows

$$K_1 = \begin{bmatrix} -0.4826 & -0.1515 \\ -0.0001 & -1.1610 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.4826 & -0.1515 \\ -0.0001 & -1.1610 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -0.0601 & -0.1906 \\ -0.0018 & -1.0784 \end{bmatrix}.$$

In the following, we will show the effectiveness of the aforementioned controller parameters. Let $x_1(0) = \begin{bmatrix} 5 & 2 \end{bmatrix}^{\mathrm{T}}$, $x_2(0) = \begin{bmatrix} -4 & -2 \end{bmatrix}^{\mathrm{T}}$, $x_3(0) = \begin{bmatrix} 2 & -7 \end{bmatrix}^{\mathrm{T}}$, $S(0) = \begin{bmatrix} 2 & -1 \end{bmatrix}^{\mathrm{T}}$. The state trajectories of the error system (7) are given in Figs. 1-3, and the the control input $u(t)$ are shown in Figs. 4-6.

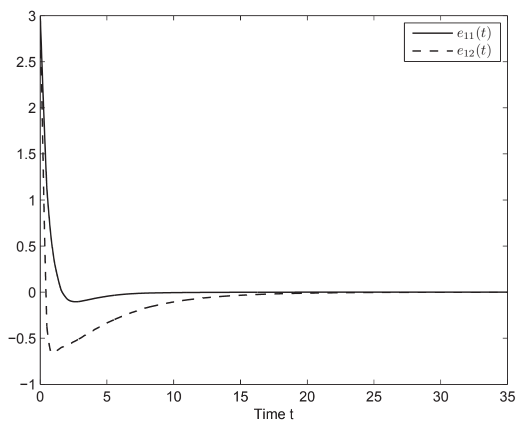

Fig. 1 State trajectory $e_1(t)$.

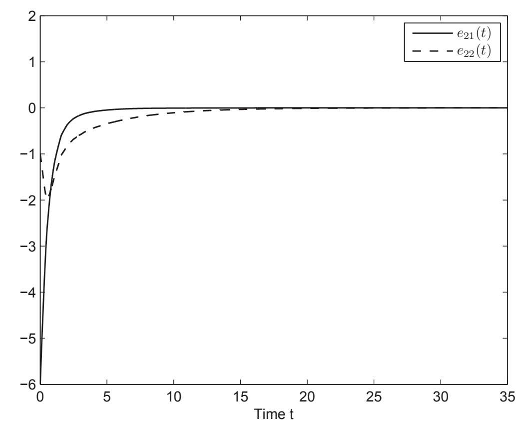

Fig. 2 State trajectory $e_2(t)$.

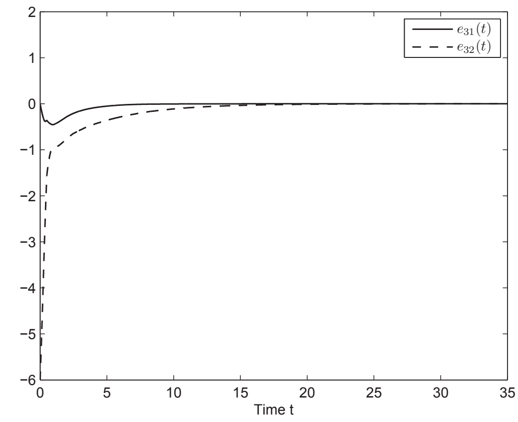

Fig. 3 State trajectory $e_3(t)$.

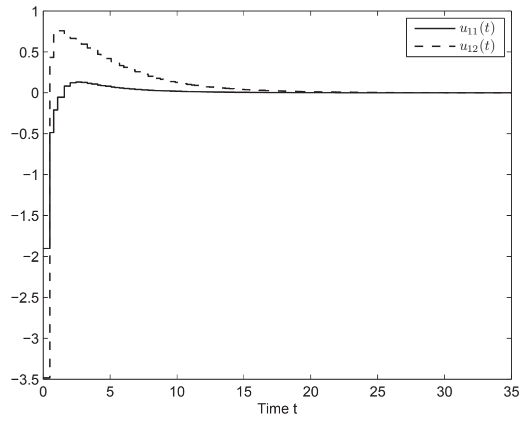

Fig. 4 Responses of the control input $u_1(t)$.

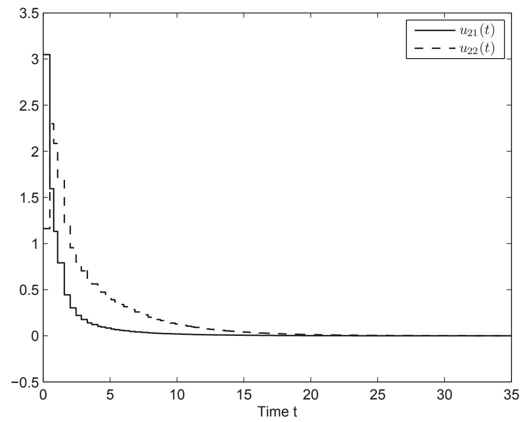

Fig. 5 Responses of the control input $u_2(t)$.

5. CONCLUSIONS

In this paper, the sampled-data synchronization control problem has been investigated for dynamical networks with time-varying coupling delay and sampled-data. Based on the Gronwall's inequality, a new exponential synchronization condition has been proposed, which is formulated by LMIs and thus can be checked easily. The set of sampled-data synchronization controllers has been designed. The given results have theoretically less conservatism than the existing ones. A numerical example has been provided to demonstrate the effectiveness of

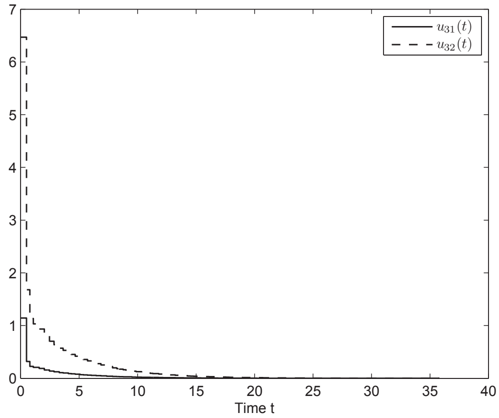

Fig. 6 Responses of the control input $u_3(t)$.

the proposed result.

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