Robust Distributed Beamforming for Wireless Relay Networks

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***Abstract*—In this paper, we consider a robust distributed beamforming design that minimizes total relay transmit power with signal-to-noise ratio (SNR) constraint for a wireless relay network in the presence of imperfect channel state information (CSI) at the relays. We consider a system with a transmit and a receive node, and a set of relay nodes. We assume there is no direct link between the transmit and receive nodes. Each node is equipped with a single antenna. The relay nodes perform amplify-and-forward (AF) relaying. The distributed beamforming design based on the assumption of perfect CSI at the relays fails to guarantee the SNR when the CSI available at the relay nodes is imperfect. We present a robust design which ensures that the SNR constraint is satisfied in the presence of imperfect CSI. We adopt a worst-case design and formulate the problem as a convex optimization problem that can be solved efficiently. The robustness of the proposed design to imperfections in CSI is illustrated through simulations.**

I. Introduction

Wireless communications systems with cooperation among users have been studied widely [1]–[3]. The major benefit of the cooperation among users is the availability of spatial diversity without need for multiple antennas at each user [3]. For user cooperation in relay networks, the non-regenerative (amplify-and-forward) and regenerative (decode-and-forward) schemes have been studied widely [1], [2]. In the non-regenerative relaying, the relay nodes scale the received signal and transmit it. Whereas, in the regenerative relaying, the relay nodes decode the received signal, re-encode and then transmit it to the receive node. The non-regenerative relaying is of practical interest due to its low complexity and amenability to implementation.

In this paper, we consider distributed beamforming for non-regenerative relaying with imperfect channel state information (CSI) at the relays. The beamformer design is based on the minimization of the total relay transmit power under a signal-to-noise ratio (SNR) constraint. Distributed beamforming with perfect CSI at the relays has been reported in [4]. In practice, the CSI is usually imperfect due to different factors like estimation error, feedback delay, quantization etc. Further, the performance of the designs based on the assumption of perfect CSI degrades in the presence of CSI imperfections. Hence, it is of interest to develop distributed beamforming designs that are robust to errors in CSI. Distributed beamforming with imperfect CSI has been studied in [5], [6]. In [5], robust beamformer designs are based on the second-order statistics of the CSI. This is a stochastically robust design, and does not guarantee QoS for all realizations of the CSI. In [6], the robust designs consider only large-scale fading. Our robust design differs from those in [5] and [6] in that the QoS constraint is satisfied for all realizations of CSI errors belonging to a given uncertainty set, and is based on imperfections in the instantaneous CSI. This is ensured by adopting a worst-case design. We show that this worst-case design can be formulated as convex optimization problem which can be solved efficiently using interior-point methods [7].

The rest of the paper is organized as follows. The system model is presented in Section II. The proposed robust beamformer design are presented in Section III. Simulation results and comparisons are presented in Section IV. Conclusions are presented in Section V.

II. System Model

We consider a wireless relay system with one transmit (source) node, M relay nodes and one receive (destination) node. All the nodes are equipped with single antenna. We assume that there is no direct link between the transmit and the receive nodes. However, the proposed approach is applicable to systems with a direct link as well. We consider non-regenerative relaying scheme with the half-duplex relay mode. In this mode, during the first time slot, the transmit node transmits the symbol[1] $x \in \mathbb{C}$ with $\mathbb{E}\{|x|^2\} = 1$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. The signal received at the relay nodes is given by

$$y_k \;=\; \alpha_k x + \mu_k, \quad 1 \le k \le M, \tag{1}$$

where $\alpha_k \in \mathbb{C}$ is the channel gain between the transmit node and the kth relay node, and $\mu_k \in \mathbb{C}$ is the noise at the kth relay node. During the second time slot, the relay nodes transmit the received signal after multiplying it by a complex weight. The signal received by the destination node is given by

$$z \;=\; \sum_{k=1}^{M} \beta_k b_k y_k + \nu \tag{2}$$

$$\;=\; \sum_{k=1}^{M} \beta_k b_k (\alpha_k x + \mu_k) + \nu, \tag{3}$$

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $\|\cdot\|_F$ denotes the Frobenius norm. $\mathbf{A} \succeq \mathbf{B}$ implies $\mathbf{A}-\mathbf{B}$ is positive semi-definite, and $\mathbf{A} \succ \mathbf{B}$ implies $\mathbf{A}-\mathbf{B}$ is positive definite. $\odot$ denotes the (element-wise) Hadamard product. $\Re(\cdot)$ and $\Im(\cdot)$ denote real part and imaginary part of the argument respectively.

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where $b_k \in \mathbb{C}$ is the scaling factor at the kth relay, $\beta_k \in \mathbb{C}$ is the channel gain between the kth relay node and the receive node, and $\nu \in \mathbb{C}$ is the noise at the receive node.

Let $\boldsymbol{\alpha} = [\alpha_1\ \alpha_2\ \cdots\ \alpha_M]^T$, $\boldsymbol{\beta} = [\beta_1\ \beta_2\ \cdots\ \beta_M]^T$, $\mathbf{b} = [b_1\ b_2\ \cdots\ b_M]^T$, $\boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\beta})$, and $\boldsymbol{\mu} = [\mu_1\ \mu_2\ \cdots\ \mu_M]^T$, where the diag$(\cdot)$ operator constructs a diagonal matrix with the components of the argument vector as the diagonal entries. Based on these definitions, we can represent the received signal as

$$z = \mathbf{b}^T\mathbf{h}x + \mathbf{b}^T\boldsymbol{\Gamma}\boldsymbol{\mu} + \nu, \tag{4}$$

where $\mathbf{h} = \boldsymbol{\alpha} \odot \boldsymbol{\beta}$. We consider CSI uncertainties that can be modeled as

$$\boldsymbol{\alpha} = \widehat{\boldsymbol{\alpha}} + \mathbf{e}_\alpha, \tag{5}$$

$$\boldsymbol{\beta} = \widehat{\boldsymbol{\beta}} + \mathbf{e}_\beta, \tag{6}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the true CSI, $\widehat{\boldsymbol{\alpha}}$ and $\widehat{\boldsymbol{\beta}}$ are the imperfect CSI available at the relay nodes, and $\mathbf{e}_\alpha$ and $\mathbf{e}_\beta$ represent the additive error in the CSI. Further, we assume that $\|\mathbf{e}_\alpha\| \le \delta_\alpha$, and $\|\mathbf{e}_\beta\| \le \delta_\beta$. Equivalently, $\boldsymbol{\alpha}$ belongs to the uncertainty set $\mathcal{R}_\alpha$, and $\boldsymbol{\beta}$ belongs to the uncertainty set $\mathcal{R}_\beta$, where

$$\mathcal{R}_\alpha = \{\zeta \big| \zeta = \widehat{\boldsymbol{\alpha}} + \mathbf{e}_\alpha, \|\mathbf{e}_\alpha\| \le \delta_\alpha\}, \tag{7}$$

and

$$\mathcal{R}_\beta = \{\zeta \big| \zeta = \widehat{\boldsymbol{\beta}} + \mathbf{e}_\beta, \|\mathbf{e}_\beta\| \le \delta_\beta\}. \tag{8}$$

III. Proposed Robust Beamformer design with SNR constraint

In this section, we propose a design of the distributed beamforming vector, $\mathbf{b}$, that minimizes the total relay transmit power with constraints on the SNR at the receive node, when the CSI (both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$) available at the relay nodes is imperfect.

i) Case of Perfect CSI: When perfect CSI is available at the relay nodes, the power minimization problem can be written as

$$\min_{\mathbf{b}} \quad P \qquad \text{subject to} \quad \text{SNR} \ge \eta, \tag{9}$$

where P is the total relay transmit power, and η is the minimum SNR required at the receive node. Based on (4), the total signal power received at the receive node is given by

$$\overline{P}_s = \mathbb{E}\{|\mathbf{b}^T\mathbf{h}x|^2\} = |\mathbf{b}^T\mathbf{h}|^2. \tag{10}$$

The noise power at the receive node is given by

$$\overline{P}_n = \sum_{k=1}^{M} \mathbb{E}\{|b_k\beta_k\mu_k|^2\} + \mathbb{E}\{|\nu|^2\} \tag{11}$$

$$= \sum_{k=1}^{M} |b_k\beta_k|^2\sigma_\mu^2 + \sigma_\nu^2 \tag{12}$$

$$= \|\boldsymbol{\Gamma}\mathbf{b}\|^2\sigma_\mu^2 + \sigma_\nu^2, \tag{13}$$

where we have assumed that $\mu_k,\ 1 \le k \le M$, are independent and identically distributed, and are independent of ν. The total relay transmit power can be expressed as

$$\begin{aligned} P &= \sum_{k=1}^{M} \mathbb{E}\{|b_k y_k|^2 \big| \alpha_k\} \\ &= \sum_{k=1}^{M} |b_k\alpha_k|^2 + \sum_{k=1}^{M} |b_k|^2\sigma_\mu^2 \\ &= \|\boldsymbol{\Lambda}\mathbf{b}\|^2 + \|\mathbf{b}\|^2\sigma_\mu^2, \end{aligned} \tag{14}$$

where $\boldsymbol{\Lambda} = \text{diag}(\boldsymbol{\alpha})$. Based on the above development, and defining SNR $= \overline{P}_s/\overline{P}_n$, we can express the problem in (9) as follows:

$$\min_{\mathbf{b}} \quad \|\boldsymbol{\Lambda}\mathbf{b}\|^2 + \|\mathbf{b}\|^2\sigma_\mu^2 \tag{15a}$$

$$\text{subject to} \quad \frac{|\mathbf{b}^T\mathbf{h}|^2}{\|\boldsymbol{\Gamma}\mathbf{b}\|^2\sigma_\mu^2 + \sigma_\nu^2} \ge \eta \tag{15b}$$

The problem as given in (15) is not a convex optimization program. But, it can be transformed to a convex optimization program as follows. The constraint in (15b) can be written as

$$\|\boldsymbol{\Gamma}\mathbf{b}\|^2\sigma_\mu^2 + \sigma_\nu^2 \le \frac{1}{\eta}|\mathbf{b}^T\mathbf{h}|^2 \tag{16a}$$

$$\Rightarrow \|\boldsymbol{\Gamma}\mathbf{b}\|^2 + \frac{\sigma_\nu^2}{\sigma_\mu^2} \le \frac{1}{\sigma_\mu^2\eta}|\mathbf{b}^T\mathbf{h}|^2 \tag{16b}$$

$$\Rightarrow \left\| \begin{bmatrix} \boldsymbol{\Gamma}\mathbf{b} \\ \frac{\sigma_\nu}{\sigma_\mu} \end{bmatrix} \right\| \le \frac{1}{\sigma_\mu\sqrt{\eta}}\mathbf{b}^T\mathbf{h}, \tag{16c}$$

where, in (16c), we have assumed $\Re(\mathbf{b}^T\mathbf{h}) \ge 0$, and $\Im(\mathbf{b}^T\mathbf{h}) = 0$. We can restrict our search to those $\mathbf{b}$ which satisfy these conditions, as an arbitrary phase rotation of $\mathbf{b}$ does not change the objective or the constraint in (15).

In the rest of this paper, we will consider the optimization problem in (15) in terms of real variables[2]. We will use the same names for the real variables also in order to avoid notational complexity. The optimization problem in (15) can be reformulated as the following convex optimization problem

$$\min_{\mathbf{b}, r_1, r_2} \quad r_1 + \sigma_\mu^2 r_2 \tag{17}$$

$$\text{subject to} \quad \mathbf{b}^T\boldsymbol{\Lambda}^T\boldsymbol{\Lambda}\mathbf{b} \le r_1 \tag{18}$$

$$\|\mathbf{b}\|^2 \le r_2 \tag{19}$$

$$\left\| \begin{bmatrix} \boldsymbol{\Gamma}\mathbf{b} \\ \frac{\sigma_\nu}{\sigma_\mu} \end{bmatrix} \right\| \le \frac{1}{\sigma_\mu\sqrt{\eta}}\mathbf{b}^T\mathbf{h}, \tag{20}$$

where r_1 and r_2 are slack variables. The objective function, and the first and second constraints in the above problem are obviously convex. The third constraint is a second order cone constraint, and hence convex. Hence, the robust power

[2]Any complex matrix $\mathbf{Z}$ is mapped into the corresponding real matrix as $\begin{bmatrix} \Re(\mathbf{Z}) & \Im(\mathbf{Z}) \\ -\Im(\mathbf{Z}) & \Re(\mathbf{Z}) \end{bmatrix}$. The complex vector $\mathbf{h} = \boldsymbol{\alpha} \odot \boldsymbol{\beta}$ is mapped into the corresponding real vector as $[\Re(\mathbf{h})\ \ \Im(\mathbf{h})]^T$. Any other complex vector $\mathbf{z}$ is mapped into the corresponding real vector as $[\Re(\mathbf{x})\ \ -\Im(\mathbf{x})]^T$

minimization problem formulated as above is a convex optimization program which can be solved efficiently by interior point methods [7].

ii) Case of Imperfect CSI: When the CSI available at the relay nodes is imperfect, we adopt a worst-case design approach. The beamforming vector is designed so as to meet the SNR target for all channel vectors in the CSI uncertainty region of given size. Mathematically, this robust beamforming design can be expressed as

$$\min_{\mathbf{b},r_1,r_2} \quad r_1+\sigma_\mu^2 r_2 \tag{21a}$$

$$\text{subject to} \quad \mathbf{b}^T\boldsymbol{\Lambda}^T\boldsymbol{\Lambda}\mathbf{b}\le r_1, \forall \boldsymbol{\alpha}\in\mathcal{R}_\alpha \tag{21b}$$

$$\|\mathbf{b}\|^2\le r_2 \tag{21c}$$

$$\left\|\begin{bmatrix}\boldsymbol{\Gamma}\mathbf{b}\\ \frac{\sigma_\nu}{\sigma_\mu}\end{bmatrix}\right\|\le\frac{1}{\sigma_\mu\sqrt{\eta}}\mathbf{b}^T\mathbf{h}, \forall\boldsymbol{\beta}\in\mathcal{R}_\beta. \tag{21d}$$

Let $\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}+\mathbf{E}_\Gamma$, where $\widehat{\boldsymbol{\Gamma}}=\text{diag}(\widehat{\boldsymbol{\beta}})$, and $\mathbf{E}_\Gamma=\text{diag}(\mathbf{e}_\beta)$, and let $\boldsymbol{\Lambda}=\widehat{\boldsymbol{\Lambda}}+\mathbf{E}_\Lambda$, where $\widehat{\boldsymbol{\Lambda}}=\text{diag}(\widehat{\boldsymbol{\alpha}})$, and $\mathbf{E}_\Lambda=\text{diag}(\mathbf{e}_\alpha)$. Then $\boldsymbol{\beta}\in\mathcal{R}_\beta\Longrightarrow\boldsymbol{\Gamma}\in\mathcal{R}_\Gamma=\{\mathbf{X}=\widehat{\boldsymbol{\Gamma}}+\mathbf{E}_\Gamma \,|\, \|\mathbf{E}_\Gamma\|_F\le\delta_\beta\}$, and $\boldsymbol{\alpha}\in\mathcal{R}_\alpha\Longrightarrow\boldsymbol{\Lambda}\in\mathcal{R}_\Lambda=\{\mathbf{X}=\widehat{\boldsymbol{\Lambda}}+\mathbf{E}_\Lambda \,|\, \|\mathbf{E}_\Lambda\|_F\le\delta_\alpha\}$. Further,

$$\begin{aligned}\mathbf{h} &= (\widehat{\boldsymbol{\alpha}}+\mathbf{e}_\alpha)\odot(\widehat{\boldsymbol{\beta}}+\mathbf{e}_\beta)\\ &= \widehat{\boldsymbol{\alpha}}\odot\widehat{\boldsymbol{\beta}}+\widehat{\boldsymbol{\alpha}}\odot\mathbf{e}_\beta+\mathbf{e}_\alpha\odot\widehat{\boldsymbol{\beta}}+\mathbf{e}_\alpha\odot\mathbf{e}_\beta\\ &= \widehat{\mathbf{h}}+\mathbf{e}_h,\end{aligned} \tag{22}$$

where $\widehat{\mathbf{h}}=\widehat{\boldsymbol{\alpha}}\odot\widehat{\boldsymbol{\beta}}$, and $\mathbf{e}_h=\widehat{\boldsymbol{\alpha}}\odot\mathbf{e}_\beta+\mathbf{e}_\alpha\odot\widehat{\boldsymbol{\beta}}+\mathbf{e}_\alpha\odot\mathbf{e}_\beta$. Then,

$$\boldsymbol{\alpha}\in\mathcal{R}_\alpha,\boldsymbol{\beta}\in\mathcal{R}_\beta\Longrightarrow\mathbf{h}\in\mathcal{R}_h=\{\mathbf{x}=\widehat{\mathbf{h}}+\mathbf{e}_h \,|\, \|\mathbf{e}_h\|\le\delta_h\},$$

where $\delta_h=\delta_\beta\|\widehat{\boldsymbol{\alpha}}\|+\delta_\alpha\|\widehat{\boldsymbol{\beta}}\|+\delta_\alpha\delta_\beta$.

The optimization problem in (21) is a semi-infinite optimization problem, and hence intractable [8]. We apply some results from robust convex optimization theory in order to transform the above problem into a mathematically tractable problem.

Lemma 1 [8]: Consider the uncertain quadratically constrained convex quadratic program (QCQP)

$$\min_{\mathbf{x}} \quad \mathbf{c}^T\mathbf{x} \tag{23}$$

$$\text{subject to} \quad \mathbf{x}^T\mathbf{A}^T\mathbf{A}\mathbf{x}-2\mathbf{p}^T\mathbf{x}\le\gamma \tag{24}$$

$$\forall(\mathbf{A},\mathbf{p},\gamma)\in\mathcal{U}, \tag{25}$$

where $\mathcal{U}=\{(\mathbf{A},\mathbf{b},\gamma)=(\mathbf{A}_0,\mathbf{b}_0,\gamma_0)+\sum_{k=1}^N u_k(\mathbf{A}_k,\mathbf{b}_k,\gamma_k)\big|\|\mathbf{u}\|\le 1\}$. The robust counterpart of this uncertain QCQP is equivalent to the following SDP problem:

$$\min_{\mathbf{x},\lambda} \quad \mathbf{c}^T\mathbf{x} \tag{26}$$

$$\text{subject to} \quad \begin{bmatrix}\gamma_0+2\mathbf{x}^T\mathbf{p}_0-\lambda & \mathbf{N} & \mathbf{A}_0\mathbf{x}^T\\ \mathbf{N}^T & \lambda\mathbf{I} & \mathbf{M}^T\\ \mathbf{A}_0\mathbf{x} & \mathbf{M} & \mathbf{I}\end{bmatrix}\succeq\mathbf{0}, \tag{27}$$

where $\mathbf{M}=[\mathbf{A}_1\mathbf{x}\ \cdots\ \mathbf{A}_N\mathbf{x}]$, and $\mathbf{N}=\left[\gamma_1/2+\mathbf{x}^T\mathbf{p}_1\ \cdots\ \gamma_N/2+\mathbf{x}^T\mathbf{p}_N\right]$. ■

Lemma 2 [8]: Consider the uncertain SOCP

$$\min_{\mathbf{x}} \quad \mathbf{c}^T\mathbf{x} \tag{28}$$

$$\text{subject to} \quad \|\mathbf{A}\mathbf{x}+\mathbf{p}\|\le\mathbf{d}^T\mathbf{x}+\gamma \tag{29}$$

$$\forall(\mathbf{A},\mathbf{p})\in\mathcal{U},\forall(\mathbf{d},\gamma)\in\mathcal{V} \tag{30}$$

where $\mathcal{U}=\{[\mathbf{A};\mathbf{b}]=[\mathbf{A}_0;\mathbf{b}_0]+\sum_{k=1}^N u_k[\mathbf{A}_k;\mathbf{b}_k]\,\big|\,\|\mathbf{u}\|\le 1\}$, $\mathcal{V}=\{(\mathbf{d},\gamma)=(\mathbf{d}_0,\gamma_0)+\sum_{k=1}^N u_k(\mathbf{d}_k,\gamma_k)\,\big|\,\|\mathbf{u}\|\le 1\}$. The robust counterpart of this uncertain SOCP is equivalent to the following SDP:

$$\min_{\mathbf{x},\mu,\lambda} \quad \mathbf{c}^T\mathbf{x} \tag{31}$$

$$\text{subject to} \quad \begin{bmatrix}\lambda-\mu & \mathbf{0} & \mathbf{A}_0\mathbf{x}+\mathbf{p}^T\\ \mathbf{0} & \mu\mathbf{I} & \mathbf{M}^T\\ \mathbf{A}_0\mathbf{x}+\mathbf{p} & \mathbf{M} & \lambda\mathbf{I}\end{bmatrix}\succeq\mathbf{0} \tag{32}$$

$$\begin{bmatrix}\mathbf{d}_0^T\mathbf{x}+\gamma_0-\lambda & \mathbf{N}\\ \mathbf{N}^T & (\mathbf{d}_0^T\mathbf{x}+\gamma_0-\lambda)\mathbf{I}\end{bmatrix}\succeq\mathbf{0}, \tag{33}$$

where $\mathbf{M}=[\mathbf{A}_1\mathbf{x}+\mathbf{p}_1\ \cdots\ \mathbf{A}_N\mathbf{x}+\mathbf{p}_N]$, and $\mathbf{N}=\left[\mathbf{d}_1^T\mathbf{x}+\gamma_1\ \cdots\ \mathbf{d}_N^T\mathbf{x}+\gamma_N\right]$. ■

First, consider the first constraint (21b)

$$\mathbf{b}^T\boldsymbol{\Lambda}^T\boldsymbol{\Lambda}\mathbf{b} \le r_1,\forall\boldsymbol{\alpha}\in\mathcal{R}_\alpha. \tag{34}$$

We can represent the uncertainty in $\boldsymbol{\Lambda}$ by

$$\boldsymbol{\Lambda} = \widehat{\boldsymbol{\Lambda}}+\delta_\alpha\text{diag}(\mathbf{u}),\|\mathbf{u}\|\le 1. \tag{35}$$

Application of *Lemma 1* to (21b) leads to the following linear matrix inequality (LMI):

$$\mathbf{R}\equiv\begin{bmatrix}r_1-\zeta & \mathbf{0} & \widehat{\boldsymbol{\Lambda}}\mathbf{b}^T\\ \mathbf{0} & \zeta\mathbf{I} & \delta_\alpha\mathbf{b}^T\\ \widehat{\boldsymbol{\Lambda}}\mathbf{b} & \delta_\alpha\mathbf{b} & \mathbf{I}\end{bmatrix}\succeq\mathbf{0}. \tag{36}$$

Next, consider the constraint (21d). We can represent the uncertainty in $\boldsymbol{\Gamma}$ by $\boldsymbol{\Gamma}=\widehat{\boldsymbol{\Gamma}}+\delta_\beta\text{diag}(\mathbf{u}),\|\mathbf{u}\|\le 1$, and the uncertainty in $\mathbf{h}$ by $\mathbf{h}=\widehat{\mathbf{h}}+\delta_h\mathbf{u}$, $\|\mathbf{u}\|\le 1$. Application of *Lemma 2* to (21d) leads to the following LMIs:

$$\mathbf{S}\equiv\begin{bmatrix}\lambda-\mu & \mathbf{0} & \begin{bmatrix}\widehat{\boldsymbol{\Gamma}}\mathbf{b}\\ \frac{\sigma_\nu}{\sigma_\mu}\end{bmatrix}^T\\ \mathbf{0} & \mu\mathbf{I} & \left[\delta_\beta\mathbf{b}^T\ \ 0\right]\\ \begin{bmatrix}\widehat{\boldsymbol{\Gamma}}\mathbf{b}\\ \frac{\sigma_\nu}{\sigma_\mu}\end{bmatrix} & \begin{bmatrix}\delta_\beta\mathbf{b}\\ 0\end{bmatrix} & \lambda\mathbf{I}\end{bmatrix}\succeq\mathbf{0}, \tag{37}$$

$$\mathbf{T}\equiv\begin{bmatrix}\frac{1}{\sigma_\mu\sqrt{\eta}}\widehat{\mathbf{h}}^T\mathbf{b}-\lambda & \frac{1}{\sigma_\mu\sqrt{\eta}}\delta_h\mathbf{b}\\ \frac{1}{\sigma_\mu\sqrt{\eta}}\delta_h\mathbf{b}^T & \left(\frac{1}{\sigma_\mu\sqrt{\eta}}\widehat{\mathbf{h}}^T\mathbf{b}-\lambda\right)\mathbf{I}\end{bmatrix}\succeq\mathbf{0}. \tag{38}$$

Based on the above development, the robust design of the relay weight vector $\mathbf{b}$ for minimizing total relay transmit

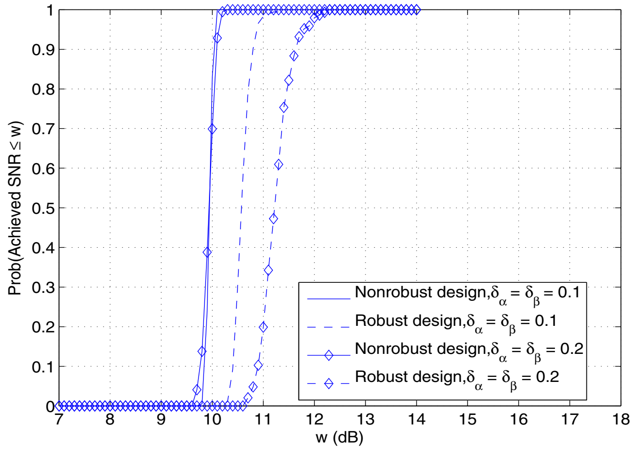

Fig. 1. Cumulative distribution of achieved SNR. Target SNR $\eta = 10$ dB, $\delta_\alpha = \delta_\beta = 0.1,\ 0.2,\ M = 20$.

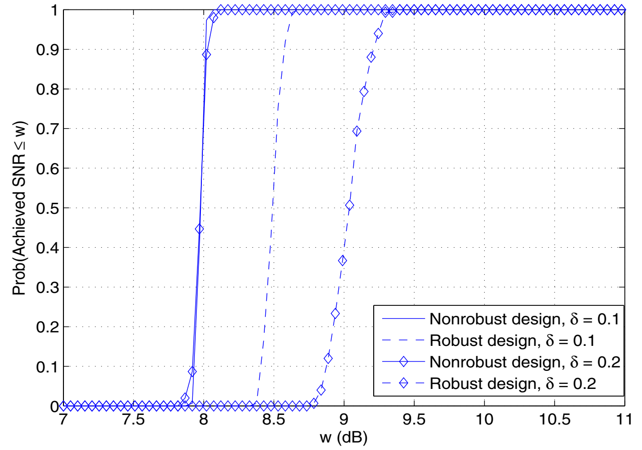

Fig. 2. Cumulative distribution of achieved SNR. Target SNR $\eta = 8$ dB, $\delta = \delta_\alpha = \delta_\beta = 0.1,\ 0.2,\ M = 30$.

power under SNR constraint can be written as

$$\begin{aligned} \min_{\mathbf{b},r_1,r_2,\zeta,\lambda,\mu} \quad & r_1 + \sigma_\mu^2 r_2 && \text{(39a)} \\ \text{subject to} \quad & \mathbf{R} \succeq \mathbf{0} && \text{(39b)} \\ & \mathbf{S} \succeq \mathbf{0} && \text{(39c)} \\ & \mathbf{T} \succeq \mathbf{0}. && \text{(39d)} \end{aligned}$$

The convex optimization problem (39) with the LMI constraints is a semi-definite program (SDP), and it can be solved efficiently by the interior point methods [7], [9]. We note that, we can convert the LMI $\mathbf{T} \succeq \mathbf{0}$ to a second order cone (SOC) constraint. Consider a symmetric matrix $\mathbf{Q}$, which is partitioned as

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{bmatrix}, \tag{40}$$

where $\mathbf{C}$ is symmetric, and $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are of appropriate dimensions. We can show that, if $\mathbf{C} \succ \mathbf{0}$, then $\mathbf{Q} \succeq \mathbf{0}$ if and only if $\mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T \succeq \mathbf{0}$ [7], [10]. The matrix $\mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T$ is called the Schur complement of $\mathbf{C}$ in $\mathbf{Q}$.

Applying this result to (39d), we can reformulate the LMI as the equivalent SOC constraint

$$\delta_h \|\mathbf{b}\| \quad \leq \quad \widehat{\mathbf{h}}^T \mathbf{b} - \frac{1}{\sigma_\mu \sqrt{\eta}} \lambda. \tag{41}$$

Second order cone constraint given above results in a reduced computational complexity compared to LMI.

IV. Simulation Results

In this section, we illustrate the performance of the proposed robust distributed beamforming design, evaluated through simulations. We compare the performance of the proposed robust design with the nonrobust design. The channel fading is modeled as Rayleigh, with the channel gain coefficients α_k, β_k, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each node is

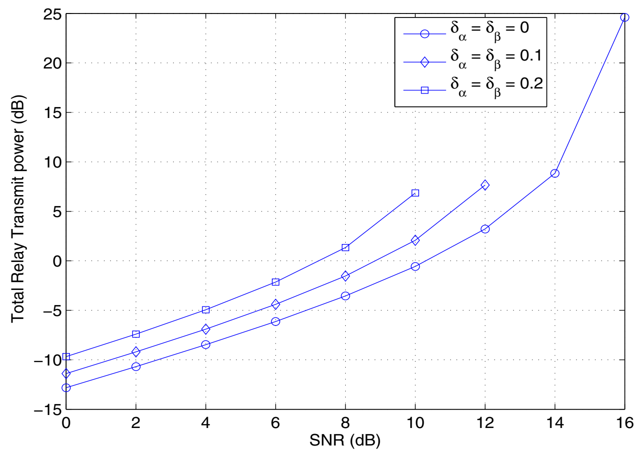

Fig. 3. Total relay transmit power versus SNR threshold. $\delta_\alpha = \delta_\beta = 0,\ 0.1,\ 0.2,\ M = 20$.

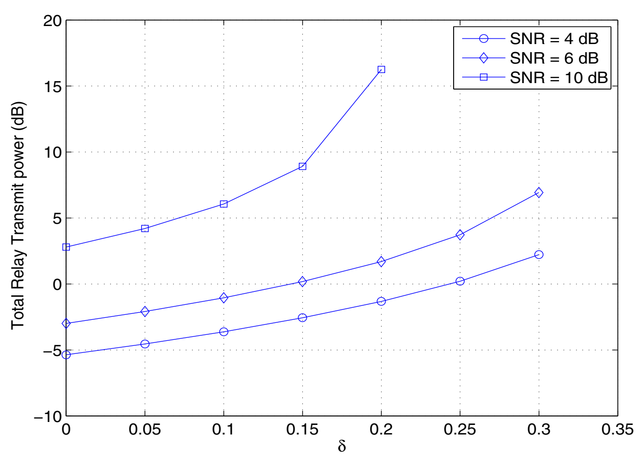

Fig. 4. Total relay transmit power versus CSI error bound $\delta = \delta_\alpha = \delta_\beta$. SNR=4 dB, 6 dB, 10 dB, $M = 10$.

assumed to be zero-mean complex Gaussian random variable. In all the simulations, we have assumed $\delta_\alpha = \delta_\beta$.

First, we compare the performance of the proposed robust design with the nonrobust design in terms of the cumulative distribution of achieved SNR in the presence of CSI errors. In this experiment, we consider a system with one transmit node, one receive node, and $M = 10$ relay nodes. The target SNR is 10 dB. The results are shown in Fig. 1. The nonrobust design fails to achieve the SNR target with higher probabilities for larger values of the CSI error bounds. The robust design is found to achieve the target SNR in the presence of CSI errors. Results for the same experiment with different system parameters are shown in Fig. 2. Here, we consider a system with $M = 30$, and target SNR $\eta = 8$ dB. We can observe that the proposed robust design guarantee the achieved SNR is greater than the target SNR, whereas the nonrobust design fails to meet the target SNR.

Next, we study the performance of the proposed design in terms of total relay transmit power versus SNR target for different values of CSI error bounds. For this purpose, we consider a system with $M = 20$ relay nodes. Total relay transmit power required to achieve different SNR targets with perfect CSI (i.e., $\delta_\alpha = \delta_\beta = 0$), and in the presence of CSI errors is estimated through simulations. The results are shown in Fig. 3. It is found that the total relay transmit power required to achieve the SNR target increases with increase in the CSI error norm bound. The robust beamformer design problem becomes infeasible for target SNR beyond a threshold. From the results, we can observe that this threshold decreases with increase in the error norm bound. Similarly, the performance of the proposed design in terms of total relay transmit power versus CSI error norm bound for different values of SNR targets is studied. For this experiment, we consider a system with $M = 10$ relay nodes. Total relay transmit power required to achieve a particular SIN target in the presence CSI error of various norm bounds are shown in Fig. 4. In this case also, we can observe that the robust design problem becomes infeasible for larger values of the norm error bound.

V. CONCLUSIONS

We presented a robust distributed beamforming design with SNR constraint for wireless relay networks in the presence of imperfect CSI at the relays. The proposed robust beamformer design was based on total relay transmit power minimization under SNR constraint. We showed that the proposed robust design can be formulated as a convex optimization problem that can be solved efficiently. Through simulation results, we illustrated the superior performance of the proposed robust design compared to the non-robust design in the presence of CSI imperfections.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. Part I. system description,” *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[3] Y. W. Hong, W. J. Huang, F. H. Chiu, and C. C. J. Kuo, “Cooperative communications in resource-constrained wireless networks,” *IEEE Signal Process. Mag.*, vol. 24, pp. 47–57, May 2007.

[4] Y. Jing and H. Jafarkhani, “Network beamforming using relays with perfect channel information,” in *Proc. ICASSP’2007*, Apr. 2007, pp. III–473–III–476.

[5] V. H. Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, “Distributed beamforming for relay networks based on second-order statistics of the channel state information,” *IEEE Trans. Signal Process.*, vol. 56, pp. 4306–4306, Sep. 2008.

[6] T. Q. S. Quek and H. S. M. Z. Win, “Robust wireless relay networks: Slow power allocation with guaranteed QoS,” *IEEE J. Sel. Topics in Signal Processing*, vol. 1, pp. 700–713, Dec. 2007.

[7] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[8] A. Ben-Tal and A. Nemirovsky, “Robust convex optimization,” *Mathematics of Operations Research*, vol. 23, no. 4, pp. 769–805, Nov. 1998.

[9] J. Sturm, “Using SeDuMi 1.02, a MATLAB toolbox or optimization over symmetric cones,” *Optimization Methods and Software*, vol. 11, pp. 625–653, 1999.

[10] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.