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Research Article

Optimally Joint Subcarrier Matching and Power Allocation in OFDM Multihop System

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Orthogonal frequency division multiplexing (OFDM) multihop system is a promising way to increase capacity and coverage. In this paper, we propose an optimally joint subcarrier matching and power allocation scheme to further maximize the total channel capacity with the constrained total system power. First, the problem is formulated as a mixed binary integer programming problem, which is prohibitive to find the global optimum in terms of complexity. Second, by making use of the equivalent channel power gain for any matched subcarrier pair, a low-complexity scheme is proposed. The optimal subcarrier matching is to match subcarriers by the order of the channel power gains. The optimal power allocation among the matched subcarrier pairs is water-filling. An analytical argument is given to prove that the two steps achieve the optimally joint subcarrier matching and power allocation. The simulation results show that the proposed scheme achieves the largest total channel capacity as compared to the other schemes, where there is no subcarrier matching or power allocation.

1. INTRODUCTION

Multihop networks have gained recently a lot of interests in the research community. By introducing relay that forwards the signal from the source to far distant destination, channel capacity can be improved and coverage area can be extended. Two main relay strategies have been identified to be usable in such scenarios: amplify-and-forward (AF) and decode-and-forward (DF). AF means that the received signal is multiplied by a parameter and then retransmitted by the relay without performing any decoding. In contrast to this, the signal is decoded at the relay and re-encoded for retransmission in the DF strategy. This has the main advantage that the transmission can be optimized for both links, separately. Furthermore, the signal is regenerated at the relay, which will not amplify the noise including the received signal. In this paper, the relay strategy is DF.

Orthogonal frequency division multiplexing (OFDM) is a mature technique to mitigate the problems of frequency of selectivity and intersymbol interference. Therefore, for the wide bandwidth multihop system, the combination of multihop system and OFDM modulation is an even more promising way to increase capacity and coverage. However, as the fading gains of different channels are mutually independent, the subcarriers which experience deep fading over the source-relay channel may not be in deep fading over the relay-destination channel. Thus, the channel capacity of a matched subcarrier pair is limited by the worse subcarrier, which will reduce the total channel capacity if the subcarriers are not matched correctly. Here, the matched subcarrier pair means that the bits transmitted on a subcarrier over the source-relay channel will be retransmitted on the other subcarrier over relay-destination channel. This motivates us to consider an adaptive subcarrier matching and power allocation scheme, where the bits transmitted on a subcarrier over the source-relay channel are possibly reallocated to another different subcarrier over the relay-destination channel.

There exist already a large number of publications on different aspects of multihop system. A fundamental analysis of cooperative relay systems was done by Kramer et al. [1], who gave channel capacities of several schemes. The system performance analysis in terms of diversity gain was done by Laneman et al. [2]. Also Sendonaris et al. [3, 4] considered the advantages in code division multiplexing access (CDMA) system using relay. Other issues that were investigated in the past were distributed space-time coding [5], selective cooperative diversity system [6], cooperative diversity in sensor network [7, 8], and the references therein.

Relaying for OFDM systems was considered theoretically in [9]. In [10], the power allocation problem for nonregenerative OFDM relay links was investigated; in this work, the instantaneous rate is maximized for given source and relay power constraints. Multiuser OFDM relay networks were studied by Han et al. [11]. Relay selectivity in OFDM multihop system was considered by Dai et al. [12]. Bit loading algorithms for cooperative OFDM systems to minimize the system power were considered by Gui et al., where the greedy algorithm and suboptimal algorithm were proposed [13]. Kaneko et al. considered resource allocation for OFDMA system [14]. Adaptive relaying scheme for OFDM that taking channel state information at the relay node into account has been proposed in [15], where subcarrier matching was considered for OFDM amplify-and-forward scheme and power allocation was not considered. To the best of our knowledge, the optimally joint subcarrier and power allocation scheme in OFDM multihop system has not been proposed.

In this paper, we formulate the optimally joint subcarrier matching and power allocation problem as a mixed binary integer programming problem, which is NP-hard and very difficult to find global optimum. Then, by making use of the equivalent channel power gain for any matched subcarrier pair, we propose a low-complexity and optimally joint subcarrier matching and power allocation scheme, where the subcarrier matching is to match the subcarriers by the order of the channel power gains and power allocation among the matched subcarrier pairs is water-filling.

The rest of the paper is organized as follows. Section 2 presents system model used throughout the paper and formulates the problem as a mixed binary integer programming problem. Section 3 provides the optimally joint subcarrier matching and power allocation scheme for the system including only two subcarriers. The scheme is extended to the system including unlimited number of subcarriers in Section 4. Section 5 compares the capacity of the proposed scheme with those of several other schemes by simulations. Section 6 concludes the paper.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1. System model

An OFDM multihop system is considered where the source communicates with the destination using a single relay. The relay strategy is decode-and-forward. All nodes hold one antenna. It is assumed that the destination can receive signal from the relay but not from the source because of distance or obstacle. A two-stage transmission protocol is adopted. This means that the communication between the source and the destination covers two equal time slots. The source transmits an OFDM symbol over the source-relay channel during the first time slot. At the same time, the relay receives and decodes the symbol. During the second time slot, the relay re-encodes the signal with the same codebook as the one used at the source, and transmits it towards the destination over the relay-destination channel. The destination decodes the signal based on the received signal only from the relay. The system architecture researched in this paper is shown as Figure 1. Full channel state information (CSI) is assumed. The source transmits the signal to the relay with power allocation among the subcarriers based on the algorithm of joint subcarrier matching and power allocation. The relay receives the signal and decodes the signal. Then, the relay reorders the subcarrier to match subcarrier, and allocates power among the subcarriers according to the algorithm of joint subcarrier matching and power allocation. At last, the destination decodes the signal by using the CSI over the relay-destination channel.

Throughout this paper, we assume that the different channels experience independent fading. The system consists of N subcarriers with total system power constraint. The power spectral densities of additive white Gaussian noise (AWGN) are equal at the source and the relay. The channel capacity of the subcarrier i over the source-relay channel is given:

$$R_{s,i}(P_{s,i}) = \frac{B}{2N}\log_2\left(1 + \frac{P_{s,i}h_{s,i}}{N_0 B/N}\right), \tag{1}$$

where $P_{s,i}$ is the power allocated to the subcarrier i $(1 \le i \le N)$ at the source, $h_{s,i}$ is the corresponding channel power gain, N_0 is the power spectral density of AWGN, B is the total available bandwidth. Similarly, the channel capacity of the subcarrier j over the relay-destination channel is given:

$$R_{r,j}(P_{r,j}) = \frac{B}{2N}\log_2\left(1 + \frac{P_{r,j}h_{r,j}}{N_0 B/N}\right), \tag{2}$$

where $P_{r,j}$ is the power allocated to the subcarrier j $(1 \le j \le N)$ at the relay, $h_{r,j}$ is the corresponding channel power gain.

When the subcarrier i over the source-relay channel is matched to the subcarrier j over the relay-destination channel, the channel capacity of this subcarrier pair is given:

$$R_{ij} = \min\left\{R_{s,i}(P_{s,i}), R_{r,j}(P_{r,j})\right\}. \tag{3}$$

2.2. Problem formulation

Theoretically, the bits transmitted at the source can be reallocated to the subcarriers at the relay in arbitrary way. But for simplification in this paper, an additional constraint is that the bits transported on a subcarrier over the source-relay channel can be reallocated to only one subcarrier over the relay-destination channel, that is, only one-to-one subcarrier matching is permitted. This means that the bits on different subcarriers over the source-relay channel will not be reallocated to the same subcarrier at the relay.

For the optimally joint subcarrier matching and power allocation problem, we can formulate it as an optimization problem. The optimization problem is given as

$$\max_{P_{s,i},P_{r,j},\rho_{ij}} \sum_{i=1}^{N} \min\left\{R_{s,i}(P_{s,i}), \sum_{j=1}^{N}\rho_{ij}R_{r,j}(P_{r,j})\right\}$$
$$\text{subject to} \sum_{j=1}^{N}P_{s,i}+\sum_{j=1}^{N}P_{r,j} \le P_{\text{tot}},$$
$$P_{s,i},P_{r,j} \ge 0 \ \forall\, i,j, \tag{4}$$
$$\rho_{ij} = \{0,1\} \ \forall\, i,j,$$
$$\sum_{j=1}^{N}\rho_{ij} = 1,$$

where P_{tot} is the total system power constraint, ρ_{ij} can only be either 1 or 0, indicating whether the bits transmitted on the subcarrier i at the source are retransmitted on the subcarrier j at the relay. The last constraint shows that only one-to-one subcarrier matching is permitted. By introducing the parameter C_i, the optimization problem can be transformed into

$$\max_{P_{s,i},P_{r,j},\rho_{ij},C_i} \sum_{i=1}^{N} C_i \quad \text{subject to } R_{s,i}(P_{s,i}) \ge C_i,$$
$$\sum_{j=1}^{N}\rho_{ij}R_{r,j}(P_{r,j}) \ge C_i,$$
$$\sum_{j=1}^{N}P_{s,i}+\sum_{j=1}^{N}P_{r,j} \le P_{\text{tot}}, \tag{5}$$
$$P_{s,i},P_{r,j} \ge 0 \ \forall\, i,j,$$
$$\rho_{ij} = \{0,1\} \ \forall\, i,j,$$
$$\sum_{j=1}^{N}\rho_{ij} = 1.$$

That is, the original maximization problem is transformed into a mixed binary integer programming problem. It is prohibitive to find the global optimum in terms of computational complexity. However, when ρ_{ij} is given, the objective function and all constraint functions are convex, so the optimization problem is a convex optimization problem. Then the optimal power allocation can be achieved by interior-point algorithm. Therefore, the optimally joint subcarrier matching and power allocation can be found by finding the largest objective function among all subcarrier matching possibilities, and the corresponding subcarrier matching and power allocation are jointly optimal. But, it has been proved to be NP-hard and is fundamentally difficult [16]. In next section, with analytical argument, a low-complexity and optimally joint subcarrier matching and power allocation scheme is given, where the optimal subcarrier matching is to match subcarriers by the order of the channel power gains and the optimal power allocation among the subcarrier pairs is water-filling.

3. OPTIMALLY JOINT SUBCARRIER MATCHING AND POWER ALLOCATION FOR THE SYSTEM INCLUDING TWO SUBCARRIERS

Supposing that the system includes only two subcarriers ($N = 2$): the channel power gains over the source-relay channel are $h_{s,1}$ and $h_{s,2}$, and the channel power gains over the relay-destination channel are $h_{r,1}$ and $h_{r,2}$. Without loss of generality, we assume that $h_{s,1} \le h_{s,2}$ and $h_{r,1} \le h_{r,2}$. The total system power constraint is also P_{tot}. From Section 2, the optimally joint subcarrier matching and power allocation can be found by two steps: (1) for every matching possibility (i.e., ρ_{ij} is given), find the optimal power allocation and the total channel capacity; (2) compare all the total channel capacities, the largest one is the largest total channel capacity whose subcarrier matching and power allocation are joint optimally. But this process is prohibitive in terms of complexity. In this section, an analytical argument is given to prove that the optimal subcarrier is to match subcarrier by the order of the channel power gains and the optimal power allocation between the matched subcarrier pairs is water-filling. More important is that they are joint optimally.

Before giving the scheme, the equivalent channel power gain is given for any matched subcarrier pair. For any given matched subcarrier pair, with the total power constraint, an equivalent channel power gain can be given by the following proposition, whose channel capacity is equivalent to the channel capacity of this subcarrier pair.

Proposition 1. *For any given matched subcarrier pair, with total power constraint, an equivalent subcarrier channel power gain (e.g., h_i') can be given, which is related to the channel power gains (e.g., $h_{s,i}$ and $h_{r,j}$) of the subcarrier pair as follows:*

$$\frac{1}{h_i'} = \frac{1}{h_{s,i}} + \frac{1}{h_{r,j}}. \tag{6}$$

Proof. With the total power constraint P_i', the channel capacity of this subcarrier pair is

$$R_i' = \max_{P_{s,i}} \min\left\{\frac{B}{4}\log_2\left(1+\frac{P_{s,i}h_{s,i}}{N_0B/2}\right), \frac{B}{4}\log_2\left(1+\frac{(P_i'-P_{s,i})h_{r,i}}{N_0B/2}\right)\right\}, \tag{7}$$

where $P_{s,i}$ is the power allocated to the subcarrier i at the source, $P_i' - P_{s,i}$ is the remainder power allocated to the subcarrier j at the relay.

The first term is a monotonically increasing function of $P_{s,i}$ and the second term is a monotonically decreasing function of $P_{s,i}$. Therefore, the optimal power allocation between the corresponding subcarriers can be gotten easily so that

$$\frac{B}{4}\log_2\left(1+\frac{P_{s,i}h_{s,i}}{N_0B/2}\right) = \frac{B}{4}\log_2\left(1+\frac{(P_i'-P_{s,i})h_{r,j}}{N_0B/2}\right), \tag{8}$$

which means that $h_{s,i}P_{s,i} = h_{r,j}(P_i' - P_{s,i})$. As a result, the channel capacity of this subcarrier pair is

$$R_i' = \frac{B}{4}\log_2\left(1+\frac{h_{s,i}h_{r,j}P_i'}{(h_{s,i}+h_{r,j})N_0B/2}\right). \tag{9}$$

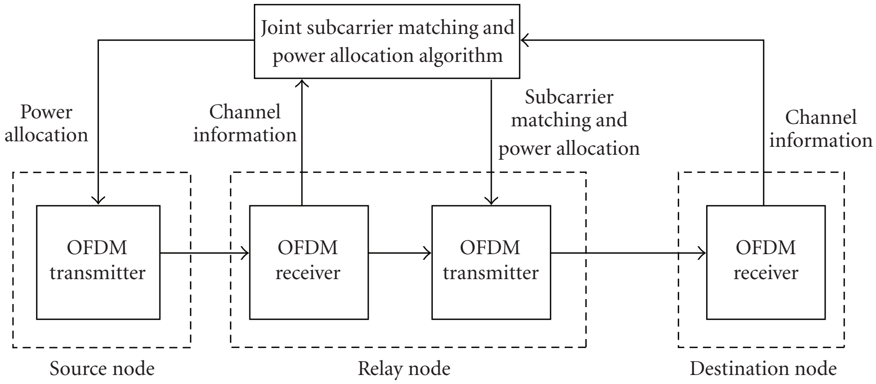

Figure 1: Block diagram of joint subcarrier matching and power allocation.

It can be seen that, by the expression of the channel capacity, the subcarrier pair can be equivalent to a single subcarrier channel with the same total power constraint. The equivalent channel power gain h_i' can be expressed:

$$h_i' = \frac{h_{s,i}h_{r,j}}{h_{s,i}+h_{r,j}}, \quad (10)$$

which can be expressed in another way:

$$\frac{1}{h_i'} = \frac{1}{h_{s,i}} + \frac{1}{h_{r,j}}. \quad (11)$$

□

Here, there are two ways to match the subcarriers: (i) the subcarrier 1 over the source-relay channel is matched to the subcarrier 1 over the relay-destination channel, and the subcarrier 2 over the source-relay channel is matched to the subcarrier 2 over the relay-destination channel (i.e., $h_{s,1}\sim h_{r,1}$ and $h_{s,2}\sim h_{r,2}$); (ii) the subcarrier 1 over the source-relay channel is matched to the subcarrier 2 over the relay-destination channel, and the subcarrier 2 over the source-relay channel is matched to the subcarrier 1 over the relay-destination channel (i.e., $h_{s,1}\sim h_{r,2}$ and $h_{s,2}\sim h_{r,1}$).

For the two ways of matching subcarriers, the equivalent channel power gains are denoted as $h_{k,i}'$ which can be gotten easily based on Proposition 1. Here, the k implies the method of matching subcarrier and the i is the index of the equivalent subcarrier. Then, the power allocation between the subcarrier pairs can be reformulation as follows:

$$\max_{P_i'}\sum_{i=1}^{2}\frac{B}{4}\log_2\left(1+\frac{h_{k,i}'P_i'}{N_0B/2}\right) \quad \text{subject to} \sum_{i=1}^{2}P_i' \leq P_{\text{tot}}, \quad (12)$$

where P_i' is the power allocated to the equivalent subcarrier i.

It is clear that the optimal power allocation is water-filling [17]. Therefore, once the subcarrier matching is provided, the optimal power allocation is given. The remainder task is to decide which way of subcarrier matching is better. The better method can be found by getting the channel capacities of the two ways and comparing them. But, here, we give an analytical argument to prove that the optimal subcarrier matching way is the first way.

Before giving the optimal subcarrier matching way, based on Proposition 1, we can get following lemma.

Lemma 1. *For the two ways of matching subcarrier, the relationship between the equivalent channel power gains can be expressed:*

$$\frac{1}{h_{1,1}'} + \frac{1}{h_{1,2}'} = \frac{1}{h_{2,1}'} + \frac{1}{h_{2,2}'}. \quad (13)$$

Proof. Based on Proposition 1, the equivalent channel power gains of the two ways can be expressed $1/h_{1,1}' = 1/h_{s,1}+1/h_{r,1}$, $1/h_{1,2}' = 1/h_{s,2} + 1/h_{r,2}$ and $1/h_{2,1}' = 1/h_{s,1} + 1/h_{r,2}$, $1/h_{2,2}' = 1/h_{s,2} + 1/h_{r,1}$. By summing up the corresponding terms, it is clear that the relationship can be gotten. □

By making use of Lemma 1, the following proposition can be proved, which states the optimal subcarrier matching way.

Proposition 2. *For the system including two subcarriers, the optimal subcarrier matching is to match the subcarriers by the order of the channel power gains. Together with the optimal power allocation for this subcarrier matching, they are optimally joint subcarrier matching and power allocation. In this system, the optimal subcarrier matching is as* $h_{s,1}\sim h_{r,1}$ *and* $h_{s,2}\sim h_{r,2}$.

Proof. For the two ways of matching subcarrier, based on Lemma 1, the equivalent channel power gains satisfy the following constraint: $1/h_{k,1}' + 1/h_{k,2}' = H$ ($H \geq 0$), where the parameter H is a constant. For the first way, we can get $1/h_{1,1}' - 1/h_{1,2}' = x_1$ ($H \geq x_1 \geq 0$). For the second way, without loss of generality, it is assumed that $1/h_{2,1}' - 1/h_{2,2}' = x_2$ ($H \geq x_2 \geq 0$). Therefore, the $h_{k,i}'$ can be expressed as $h_{k,1}' = 2/(H + x_k)$ and $h_{k,2}' = 2/(H - x_k)$. The corresponding total channel capacity is

$$R_{\text{tot},k}(P_1', P_2') = \frac{B}{4}\log_2\left(1+\frac{P_1'}{(H+x_k)N_0B/2}\right) + \frac{B}{4}\log_2\left(1+\frac{P_2'}{(H-x_k)N_0B/2}\right). \quad (14)$$

For denotation simplicity, we denote $N_0B/2$ as σ_2^2. The partial derivative of the channel capacity with respect to x_k can be gotten by making use of $P_2' = P_{\text{tot}} - P_1'$:

$$\frac{\partial R_{\text{tot},k}(P_1', P_2')}{\partial x_k} = \frac{B}{4\ln 2} \times \frac{(H^2\sigma_2^2 + 2H^2\sigma_2^2 x_k)(P_{\text{tot}} - P_1') + 2\sigma_2^2(P_{\text{tot}}H - P_1'x_k) + P_{\text{tot}}x_k^2\sigma_2^2}{(H^2 - x_k^2)[(H + x_k)\sigma_2^2 + P_1'][(H - x_k)\sigma_2^2 + (P_{\text{tot}} - P_1')]}. \tag{15}$$

It is clear that $\partial R_{\text{tot},k}/\partial x_k$ is greater than 0. Therefore, the total channel capacity is a monotonically increasing function of x_k for the given power allocation. This means that, for the given power allocation, the larger the difference between the equivalent channel power gains, the larger the total channel capacity. At the same time, it is clear that the difference between the equivalent channel power gains of the first way is larger than the one of the second way. Therefore, for the same power allocation, the relationship of the total channel capacities of the two ways can be expressed:

$$R_{\text{tot},2}(P_1', P_2') \le R_{\text{tot},1}(P_1', P_2'). \tag{16}$$

Therefore, we can get the following relationship:

$$\begin{aligned} \max_{P_i'} R_{\text{tot},2}(P_1', P_2') &= R_{\text{tot},2}(\overline{P}_1', \overline{P}_2') \le R_{\text{tot},1}(\overline{P}_1', \overline{P}_2') \\ &\le \max_{P_i'} R_{\text{tot},1}(P_1', P_2'), \end{aligned} \tag{17}$$

where $\overline{P}_1'$ and $\overline{P}_2'$ are the optimal power allocation for the first term. Note that the first term is the total channel capacity of the first way and the last term is the one of the second way. It proves that the first way, whose difference between the equivalent channel power gains is larger, is optimal subcarrier matching way. The more important is that, as the total channel capacity of the fisrt way is the largest one, this subcarrier matching and the corresponding power allocation are the optimally joint subcarrier matching and power allocation. Specially, the optimal subcarrier matching is to match subcarriers by the order of the channel power gains. □

The optimally joint subcarrier matching and power allocation scheme have been given by now. Specially, the optimal subcarrier matching is to match the subcarriers by the order of the channel power gains and the optimal power allocation between the matched subcarrier pairs is according to the water-filling. The power allocation between the matched subcarrier pair is to make the channel capacities of the two subcarriers equivalent.

4. EXTEND TO THE SYSTEM INCLUDING UNLIMITED NUMBER OF SUBCARRIERS

This section extends the method in Section 2 to the system including unlimited number of the subcarriers. The number of the subcarriers is finite, where the subcarrier channel power gains are $h_{s,i}$ $(i \ge 2)$ and $h_{r,j}$ $(j \ge 2)$. First, the optimal power allocation among the matched subcarrier pair is proposed for given subcarrier matching. Second, we prove that the subcarrier matching by the order of the channel power gains is optimal.

When the subcarrier matching is given, the equivalent channel gains of the subcarrier pairs can be gotten based on Proposition 1, for example, h_i' $(1 \le i \le N)$. The power allocation can be formulated as

$$\max_{P_i'} \sum_{i=1}^{N} \frac{B}{2N} \log_2\left(1 + \frac{h_i' P_i'}{\sigma_N^2}\right) \quad \text{subject to} \sum_{i=1}^{N} h_i' \le P_{\text{tot}}, \tag{18}$$

where $\sigma_N^2 = N_0B/N$. It is clear that the power allocation is also water-filling. Therefore, the optimal power allocation among the matched subcarrier pairs is according to the water-filling.

Here, without loss of generality, the channel power gains are assumed $h_{s,i} \le h_{s,i+1}$ and $h_{r,j} \le h_{r,j+1}$. The following proposition gives the optimal subcarrier matching.

Proposition 3. *For the system including unlimited number of the subcarriers, the optimal subcarrier matching is*

$$h_{s,i} \sim h_{r,i}. \tag{19}$$

Together with the optimal power allocation for this subcarrier matching, they are optimally joint subcarrier matching and power allocation

Proof. This proposition will be proved in the contrapositive form. Assuming that there is a subcarrier matching method whose matching result includes two matched subcarrier pairs $h_{s,i} \sim h_{r,i+n}$ and $h_{s,i+n} \sim h_{r,i}$ $(n > 0)$, which means that $h_{s,i} \le h_{s,i+n}, h_{r,i} \le h_{r,i+n}$, and the total capacity is larger than that of the matching method in Proposition 3.

When the power allocated to other subcarrier pairs and the other subcarrier matching are constant, the total channel capacity of this two-subcarrier pair can be improve based on Proposition 2, which implies that the channel capacity can be improved by rematching the subcarriers to $h_{s,i} \sim h_{r,i}$ and $h_{s,i+n} \sim h_{r,i+n}$. It is contrary to the assumption. Therefore, there is no subcarrier matching way better than the way in Proposition 3. At the same time, as the total capacity of this subcarrier matching and the corresponding optimal power allocation scheme is the largest, this subcarrier matching together with the corresponding optimal power allocation are the optimally joint subcarrier matching and power allocation. □

For the system including unlimited number of the subcarriers, the optimally joint subcarrier matching and power allocation scheme has been given by now. Here, the steps are summarized as follow

Step 1. Sort the subcarriers at the source and the relay in ascending order by the permutations π and π', respectively. The process is according to the channel power gains, that is, $h_{s,\pi(i)} \le h_{s,\pi(i+1)}$, $h_{r,\pi'(i)} \le h_{r,\pi'(i+1)}$.

Step 2. Match the subcarriers into pairs by the order of the channel power gains (i.e., $h_{s,\pi(i)} \sim h_{r,\pi'(i)}$), which means that the bits transported on the subcarrier $\pi(i)$ over the source-relay channel will be retransmitted on the subcarrier $\pi'(i)$ over the relay-destination channel.

Step 3. Based on Proposition 1, get the equivalent channel power gain $h'_{\pi(i)}$ according to the matched subcarrier pair, that is, $h'_{\pi(i)} = (h_{s,\pi(i)} h_{r,\pi'(i)})/(h_{s,\pi(i)} + h_{r,\pi'(i)})$.

Step 4. For the equivalent channel power gains, the power allocation is water-filling as follows:

$$P'_{\pi(i)} = \left(\frac{h'_{\pi(i)} B}{2N\lambda \ln 2} - \frac{h'_{\pi(i)}}{\sigma_N^2} \right)^{+}, \tag{20}$$

where $(a)^+ = \max(a, 0)$ and λ can be found by the following equation:

$$\sum_{i=1}^{N} P'_{\pi(i)} = P_{\text{tot}}. \tag{21}$$

The power allocation between the subcarriers in the given matched subcarrier pair is as follows:

$$\begin{aligned} P_{s,\pi(i)} &= \frac{h_{r,\pi'(i)} P'_{\pi(i)}}{h_{s,\pi(i)} + h_{r,\pi'(i)}}, \\ P_{r,\pi'(i)} &= \frac{h_{s,\pi(i)} P'_{\pi(i)}}{h_{s,\pi(i)} + h_{r,\pi'(i)}}. \end{aligned} \tag{22}$$

Step 5. The total system channel capacity is

$$R_{\text{tot}} = \frac{B}{2N} \sum_{i=1}^{N} \log_2 \left(1 + \frac{h'_{\pi(i)} P'_{\pi(i)}}{\sigma_N^2} \right). \tag{23}$$

5. NUMERICAL RESULTS

In this section, we compare the total channel capacity of the proposed scheme with those of several other schemes by computer simulations.

These schemes include:

(i) no subcarrier matching and no power allocation: the bits transmitted on the subcarrier i at the source will be retransmitted on the subcarrier i at the relay; the power is allocated equally over all subcarriers at the source and the relay (i.e., $P_{s,i} = P_{r,j} = P_{\text{tot}}/2N$);

(ii) optimal power allocation and no subcarrier matching: the bits transmitted on the subcarrier i at the source will be retransmitted on the subcarrier i at the relay; the power allocation is according to water-filling among the subcarrier pairs;

(iii) subcarrier matching and no power allocation: the bits transmitted on the subcarrier $\pi(i)$ at the source will be retransmitted on the subcarrier $\pi'(i)$ at the relay; the power is allocated equally over all subcarriers at the source and the relay (i.e., $P_{s,i} = P_{r,j} = P_{\text{tot}}/2N$).

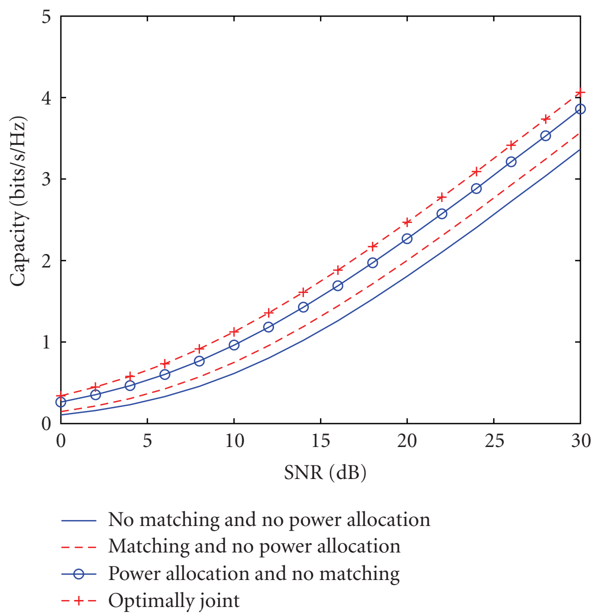

FIGURE 2: Channel capacity versus SNR ($N = 32$).

Here, the subcarrier matching is the same as in Steps 1 and 2 in Section 4 and the power allocation means that the water-filling algorithm is performed among the subcarrier pairs. In the computer simulations, we assume that each subcarrier undergoes Rayleigh fading independently. The total bandwidth is $B = 1$ MHz. The SNR is defined as $\text{SNR} = P_{\text{tot}}/N_0 B$. To obtain the average data rate, we have simulated 10 000 independent trials.

Figure 2 shows the total channel capacity versus SNR, where the number of the subcarriers is constant (e.g., $N = 32$) and the average channel power gains are assumed to be one, that is, $E(h_{s,i}) = 1$ and $E(h_{r,j}) = 1$ for all i and j. The capacity of the scheme (i), where there is no subcarrier matching and no power allocation, is the least one compared with that of the other schemes. The capacity of the optimally joint subcarrier and power allocation scheme is the largest one than those of all other schemes. If other conditions remain unchanged, both subcarrier matching and power allocation can improve the total channel capacity. Specially, subcarrier matching can improve the capacity when comparing the capacity of the scheme (i) to that of the scheme (iii). The system capacity can be improved by power allocation when comparing the capacity of the scheme (i) to that of scheme (ii). Another important result is that power allocation is more effective than subcarrier matching, when only one of the two ways can be applied.

The relationship between the total channel capacity and the number of the subcarriers is shown in Figure 3, where the SNR is constant, for example, $\text{SNR} = 20$ dB. The average channel power gains are also assumed to be one, that is, $E(h_{s,i}) = 1$ and $E(h_{r,j}) = 1$ for all i and j. Almost the same conclusions about the comparison among all the schemes can be gotten from Figure 3 as those from Figure 2. It is noted that the total channel capacity is almost constant with the

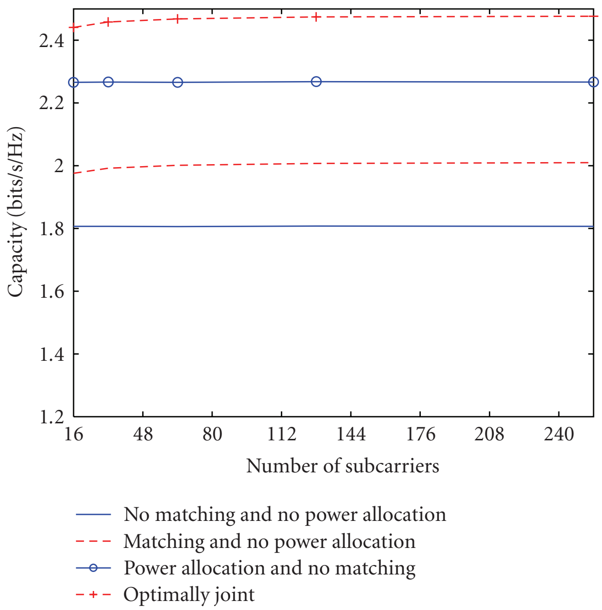

Figure 3: Channel capacity versus the number of the subcarriers (SNR = 20 dB).

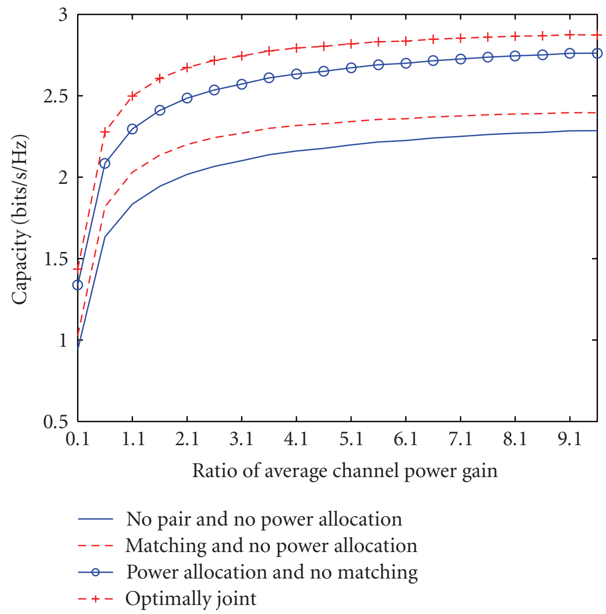

Figure 4: Channel capacity versus the ratio of the channel power gains ($E(h_{s,i}) = 1$, SNR = 20 dB).

growth of the number of the subcarriers because the total power constraint is constant.

Figure 4 shows the total channel capacity versus the ratio of the average channel power gains over the relay-destination channel to the ones over the source-relay channel, that is, $E(h_{r,i})/E(h_{s,i})$, where the average channel power gains over the source-relay channel are assumed to be one, that is, $E(h_{s,i}) = 1$. Again the same conclusions about the comparison among all the schemes can be gotten from Figure 4 as those from Figure 2. The total channel capacities increase very quickly with the ratio increasing from 0.1 to 1, this is because of the total channel capacities are limited by the channel capacities over the relay-destination channels in this interval. The total channel capacities increase slowly with the ratio increasing from 1 to 10 because the total channel capacities are limited by the channel capacities over the source-relay channels in this interval.

6. CONCLUSION

For the OFDM multihop system, as the fading gains of different channels are independent, subcarrier matching is a promising way to further improve capacity. Here, subcarrier matching means that the bits on a subcarrier over the source-relay channel are possibly reallocated to another different subcarrier over the relay-destination channel. In this paper, we propose an optimally joint subcarrier matching and power allocation scheme to maximize channel capacity, where the relay is based on the decode-and-forward and the total system power is constrained. Though the problem can be formulated as a mixed binary integer programming problem, it is NP-hard and prohibitive to find the global optimum. A low-complexity scheme is proposed, which is still jointly optimal. First, for any matched subcarrier pair, an equivalent channel power gain is proposed. Then, for the system including only two subcarriers, the optimally joint subcarrier matching and power allocation can be gotten by matching the subcarriers by the order of the channel power gains and allocating power according to water-filling between the two subcarrier pairs. Second, the scheme of optimally joint subcarrier matching and power allocation is extended to the system including unlimited number of the subcarriers. The analytical argument proves that the scheme also gives optimally joint subcarrier matching and power allocation. The simulation results prove that the proposed scheme achieves the largest total channel capacity as compared to the other schemes, where there is no subcarrier or no power allocation.

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