Analysis of Space-Time Block Coded Spatial Modulation in Correlated Rayleigh and Rician Fading Channels

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***Abstract*—Space-time block coded spatial modulation (STBC-SM) system is the multiple input multiple output (MIMO) communication system that gives better error performance than space-time block coded (STBC) MIMO system when compared at the same spectral efficiency. It performs better than spatial modulation (SM) MIMO systems. In this paper, we analyze the bit error probability (BEP) of STBC-SM systems over correlated Rayleigh and Rician fading channels. A closed form expression for upper bound on the BEP is derived and evaluated. The analytical results are validated using Monte Carlo simulation results. The performance of STBC-SM system is also compared with STBC systems and SM systems in correlated and uncorrelated fading channels.**

I. Introduction

Multiple input multiple output (MIMO) systems give higher spectral efficiency and better performance than single input single output (SISO) systems without consuming extra bandwidth and power. Recent developments in the MIMO technologies focus on reducing computational and hardware complexity using different transmit and receive diversity schemes [1]–[4]. In [1], S. Alamouti proposed a simple two branch transmit diversity scheme. It was then generalized as space time block codes (STBC) for any number of antennas by Tarokh et. al. in [2]. In [3], R. Mesleh et. al. proposed and analyzed spatial modulation (SM) in which single antenna is active at a time and the antenna index of the active antenna also carries the information resulting in increased spectral efficiency. SM systems are further investigated and a joint detection scheme is proposed to improve the performance in [5]. SM system can be combined with STBC system to get two fold advantage of improved performance and better spectral efficiency. E. Basar et. al. proposed and analyzed space-time block coded spatial modulation (STBC-SM) system in [4]. It is also shown that STBC-SM systems give better bit error rate (BER) performance than SM and Vertical Bell Laboratories Layered Space Time (V-BLAST). Further, the computational complexity of optimal maximum-likelihood (ML) decoder for STBC-SM has been reduced through proposals like hard decision simplified ML detector, hard decision low-complexity near-ML detector and soft-output low-complexity near-ML detector [4], [6], [7]. The spectral efficiency of STBC-SM was improved by using cyclic structure in SM but with slightly degraded error performance [8].

To the best of our knowledge, the analysis of STBC-SM systems reported so far in the literature is done over independent and identically distributed (i.i.d.) Rayleigh channel only, albeit, simulation results for exponentially correlated Rayleigh channels are reported in [4]. But in practical scenario, i.i.d. MIMO channels are very rare due to limited spacing among the antennas. In this paper, we analyze the bit error probability (BEP) of STBC-SM systems over correlated Rayleigh and Rician fading channels. A closed form expression for upper bound on the BEP is derived and evaluated. The analytical results are validated by Monte Carlo simulation results. We also show results for BER performance comparison of SM, STBC and STBC-SM systems over correlated Rayleigh fading channels.

The rest of the paper is organized as follows. In Section II, we describe STBC-SM transmission scheme and system model. The expression for BEP of STBC-SM over correlated Rayleigh fading channels is derived in Section III. Section IV describes the analysis of STBC-SM systems over correlated Rician fading channels. Analytical and simulation results are presented and discussed in Section V and finally the paper is concluded in Section VI.

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II. Space-Time Block Coded Spatial Modulation

In this paper, we have used Alamouti's STBC in which two complex symbols taken from an M-PSK or M-QAM constellations are transmitted from two transmit antennas in two symbol intervals in an orthogonal manner [1]. We consider a MIMO system with n_T transmit antennas and n_R receive antennas. In STBC-SM technique [4], the input data is divided into three streams. Two streams carry the Alamouti's STBC symbols and the third stream carries the transmit antenna indices. As per the bits in the third stream, 2 antennas out of n_T transmitting antennas are selected for transmission. The Alamouti's STBC symbol is transmitted from the selected antennas and the remaining antennas are idle at this moment. In general, for an STBC-SM system with symbol length of N, the received signal matrix can be given as [4]

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mu}}\mathbf{HX} + \mathbf{N} \tag{1}$$

where μ is the normalization factor to ensure that ρ is the average signal to noise ratio (SNR) at each receiver antenna, $\mathbf{Y}$ is the $n_R \times N$ received signal matrix, $\mathbf{N}$ is the $n_R \times N$ zero mean circularly symmetric complex Gaussian (ZMCSCG) distributed noise matrix, $\mathbf{X}$ is the $n_T \times N$ transmitted codeword matrix and $\mathbf{H}$ is the $n_R \times n_T$ channel matrix which is assumed to be quasi-static correlated Rayleigh or Rician fading. For Alamouti's STBC scheme ($N = 2$), the dimensions of $\mathbf{Y}$, $\mathbf{N}$ and $\mathbf{X}$ will reduce to $n_R \times 2$, $n_R \times 2$ and $n_T \times 2$ respectively. The transmitted symbol is detected at the receiver using ML detection algorithm. It does extensive search over all possible transmitted matrices and detects the matrix which is most likely to have been transmitted using the following minimization criteria [4].

$$\hat{\mathbf{X}} = \begin{matrix}\arg\\ \min\\ \mathbf{X} \in \chi\end{matrix} \left\| \mathbf{Y} - \sqrt{\frac{\rho}{\mu}}\mathbf{HX} \right\|^2 \tag{2}$$

where χ is the signal matrix alphabet. Though, the above minimization criteria look like the decision criteria for STBC systems but the set of signal matrix alphabets (χ) for STBC-SM systems is different than that for STBC systems. The details of signal matrix alphabets can be found in Table I and equation (2) of [4].

III. BEP of STBC-SM over Correlated Rayleigh Fading Channels

The conditional pairwise error probability (PEP) of decoding STBC-SM symbol matrix $\mathbf{X}_l$ when STBC-SM symbol matrix $\mathbf{X}_k$ is transmitted can be given by [9]

$$\begin{aligned} P\left(\mathbf{X}_k \to \mathbf{X}_l | \mathbf{H}\right) &= Q\left(\sqrt{\frac{\rho}{\mu}\|\mathbf{H\Delta}\|^2}\right) \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{\rho\|\mathbf{H\Delta}\|^2}{2\mu\sin^2\theta}} d\theta \end{aligned} \tag{3}$$

where $\mathbf{\Delta} = \mathbf{X}_k - \mathbf{X}_l$ is the codeword difference matrix.

Without loss of generality, assuming $\mu = 1$, the unconditional PEP with unit energy symbol transmission, i.e. $E\left\{trace\left(\mathbf{X}^H\mathbf{X}\right)\right\} = 2$, can be given by

$$P\left(\mathbf{X}_k \to \mathbf{X}_l\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \Phi_{\|\mathbf{H\Delta}\|^2}\left(-\frac{\rho}{4\sin^2\theta}\right) d\theta \tag{4}$$

where $\Phi_{\|\mathbf{H\Delta}\|^2}(\cdot)$ is the moment generating function (MGF) of $\|\mathbf{H\Delta}\|^2$. Considering Kronecker MIMO channel model, the channel matrix can be represented as

$$\mathbf{H} = \mathbf{R}_{R_X}^{1/2}\tilde{\mathbf{H}}\left(\mathbf{R}_{T_X}^{1/2}\right)^T \tag{5}$$

where $\tilde{\mathbf{H}}$ is the i.i.d. channel matrix, $\mathbf{R}_{R_X}$ is the receiver side correlation matrix and $\mathbf{R}_{T_X}$ is the transmitter side correlation matrix. Our analysis is applicable to the correlation matrices that can be represented in the following forms

$$\mathbf{R}_{R_X} = \begin{pmatrix} 1 & pr_{12} & \cdots & pr_{1n_R} \\ pr_{21} & 1 & \ddots & pr_{2n_R} \\ \vdots & \vdots & \ddots & \vdots \\ pr_{n_R1} & pr_{n_R2} & \cdots & 1 \end{pmatrix} \tag{6}$$

$$\mathbf{R}_{T_X} = \begin{pmatrix} 1 & pt_{12} & \cdots & pt_{1n_T} \\ pt_{21} & 1 & \ddots & pt_{2n_T} \\ \vdots & \vdots & \ddots & \vdots \\ pt_{n_T1} & pt_{n_T2} & \cdots & 1 \end{pmatrix} \tag{7}$$

where pr_{ij} is the correlation coefficient between the i^{th} and j^{th} receive antennas and pt_{ij} is the correlation coefficient between the i^{th} and j^{th} transmit antennas. Both the correlation matrices are symmetric in nature, i.e. $pr_{ij} = pr_{ji}$ and $pt_{ij} = pt_{ji}$.

The MGF of correlated Rayleigh fading channels can be given by [9]

$$\Phi(s) = \left|\mathbf{I}_{n_Rn_T} - s\mathbf{\Psi}\right| = \prod_{i=1}^{r}\prod_{j=1}^{\hat{r}}\left(1 - s\sigma_i\lambda_j\right)^{-1} \tag{8}$$

where $\mathbf{\Psi} = \left(\mathbf{R}^{1/2}\right)^H\left(\mathbf{I}_{n_R} \otimes \mathbf{\Delta\Delta}^H\right)\left(\mathbf{R}^{1/2}\right)$, $s = -\frac{\rho}{4\sin^2\theta}$, σ_i are the eigenvalues of $\mathbf{\Delta\Delta}^H\mathbf{R}_{T_X}$, λ_j are the eigenvalues of $\mathbf{R}_{R_X}$, $\mathbf{R} = \mathbf{R}_{R_X} \otimes \mathbf{R}_{T_X}$, $r = rank\left(\mathbf{\Delta\Delta}^H\mathbf{R}_{T_X}\right)$ and $\hat{r} = rank\left(\mathbf{R}_{R_X}\right)$

From the average PEP using (4) and (8), the union bound on BEP can be calculated as

$$P_b \le \frac{1}{2^{2m}}\sum_{k=1}^{2^{2m}}\sum_{l=1}^{2^{2m}} n_{k,l}\frac{P\left(\mathbf{X}_k \to \mathbf{X}_l\right)}{2m} \tag{9}$$

where $n_{k,l}$ is the number of bits in error when the codeword matrix $\mathbf{X}_l$ is received when the codeword matrix $\mathbf{X}_k$ is transmitted assuming $2m$ bits are transmitted during two consecutive symbol intervals using one of the 2^{2m} possible STBC-SM symbol matrices.

Approximating the average PEP by Chernoff bound (put $\sin^2\,\theta = 1$ in the integrand of (4)), BEP can be given as follows

$$P_b \leq \frac{1}{2^{2m}} \sum_{k=1}^{2^{2m}} \sum_{l=1}^{2^{2m}} \frac{n_{k,l}}{4m} \prod_{i=1}^{r} \prod_{j=1}^{\hat{r}} \left(1 + \frac{\rho \sigma_i \lambda_j}{4}\right)^{-1} \quad (10)$$

The above expression can be further simplified for constant correlation at receiver, i.e. each $pr_{ij} = pr$. For such a case, all the off diagonal elements in the receiver correlation matrix of (6) bear the same value, pr. For such a matrix, $\hat{r} = n_R$ and there will be n_R eigen values taking any of the two distinct values, $1 + (n_R - 1)pr$ and $1 - pr$. Out of n_R eigen values one eigen value equals $1 + (n_R - 1)pr$ and the remaining $n_R - 1$ equals $1 - pr$. The expression of PEP for such a case can be given as

$$P_b \leq \frac{1}{2^{2m}} \sum_{k=1}^{2^{2m}} \sum_{l=1}^{2^{2m}} \frac{n_{k,l}}{4m} \prod_{i=1}^{r} \left\{ \left(1 + \frac{\rho \sigma_i (1 - pr)}{4}\right)^{1 - n_R} \times \left(1 + \frac{\rho \sigma_i (1 + (n_R - 1)\, pr)}{4}\right)^{-1} \right\} \quad (11)$$

IV. BEP of STBC-SM over Correlated Rician Fading Channels

The BEP analysis of STBC-SM over correlated Rician fading channels can be done in the similar way as in previous section. The MGF of correlated Rician fading channels can be given as [9]

$$\Phi(s) = \frac{\exp\left[s \bar{\mathbf{h}}^H \mathbf{\Psi} \left\{ \mathbf{I}_{n_R n_T} - \frac{s \mathbf{\Psi}}{(K+1)} \right\}^{-1} \bar{\mathbf{h}} \right]}{\left| \mathbf{I}_{n_R n_T} - \frac{s \mathbf{\Psi}}{(K+1)} \right|} \quad (12)$$

where $\bar{\mathbf{h}} = vect\left(\bar{\mathbf{H}}^H\right)$ and $\bar{\mathbf{H}} = \sqrt{\frac{K}{K+1}}\breve{\mathbf{H}}\left(n_R, n_T\right)$ is the mean channel matrix with Rice parameter K.

The average PEP can be calculated as

$$P\left(\mathbf{X}_k \rightarrow \mathbf{X}_l\right) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{e^{\left[-\frac{\rho}{4\sin^2\theta} \bar{\mathbf{h}}^H \mathbf{\Psi} \left\{ \mathbf{I}_{n_R n_T} + \frac{\rho \mathbf{\Psi}}{4\sin^2\theta(K+1)} \right\}^{-1} \bar{\mathbf{h}} \right]}}{\left| \mathbf{I}_{n_R n_T} + \frac{\rho \mathbf{\Psi}}{4\sin^2\theta(K+1)} \right|} d\theta \quad (13)$$

Using $\sin^2\,\theta = 1$ (Chernoff bound) in above equation and applying union bound, the BEP can be evaluated as

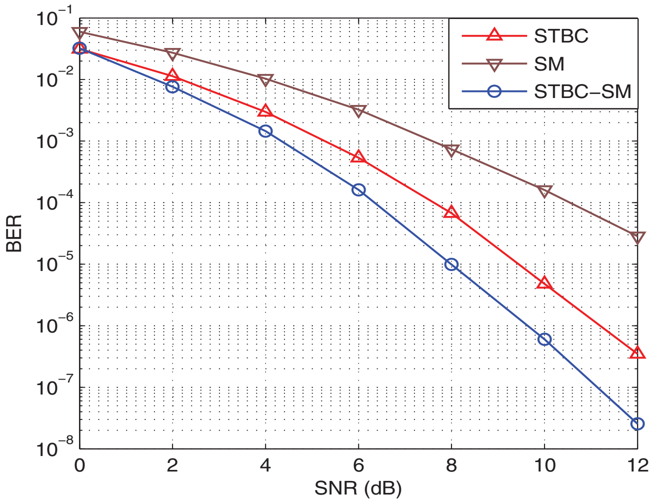

Fig. 1. Performance comparison of SM, STBC and STBC-SM systems over uncorrelated Rayleigh fading channels (2 bits/s/Hz)

$$P_b \leq \frac{1}{2^{2m}} \sum_{k=1}^{2^{2m}} \sum_{l=1}^{2^{2m}} \frac{n_{k,l}}{4m} e^{\left[-\frac{\rho}{4} \bar{\mathbf{h}}^H \mathbf{\Psi} \left\{ \mathbf{I}_{n_R n_T} + \frac{\rho \mathbf{\Psi}}{4(K+1)} \right\}^{-1} \bar{\mathbf{h}} \right]} \prod_{i=1}^{r} \prod_{j=1}^{\hat{r}} \left(1 + \frac{\rho \sigma_i \lambda_j}{4(K+1)}\right)^{-1} \quad (14)$$

Following the explanation in previous section, above expression can be represented as follows for constant correlation at receiver.

$$P_b \leq \frac{1}{2^{2m}} \sum_{k=1}^{2^{2m}} \sum_{l=1}^{2^{2m}} \frac{n_{k,l}}{4m} e^{\left[-\frac{\rho}{4} \bar{\mathbf{h}}^H \mathbf{\Psi} \left\{ \mathbf{I}_{n_R n_T} + \frac{\rho \mathbf{\Psi}}{4(K+1)} \right\}^{-1} \bar{\mathbf{h}} \right]} \prod_{i=1}^{r} \left\{ \left(1 + \frac{\rho \sigma_i (1 - pr)}{4(K+1)}\right)^{1 - n_R} \times \left(1 + \frac{\rho \sigma_i (1 + (n_R - 1)\, pr)}{4(K+1)}\right)^{-1} \right\} \quad (15)$$

It is important to note that expressions for BEP of STBC-SM systems (11) and (15) are similar to those for STBC systems. In our case, we are assuming Alamouti scheme based spatial modulation in which only 2 antennas are selected for transmission at particular time interval. So, the codewords ($\mathbf{X}$) and hence the codeword difference matrices ($\mathbf{\Delta}$) are different from STBC systems.

V. Results and Discussions

The STBC-SM system was simulated assuming Alamouti's STBC transmission scheme for a 4×4 MIMO system. For spatial modulation with STBC, two antennas are selected at transmitter to transmit each Alamouti's STBC symbol matrix. The performance of STBC-SM system is compared with 2×4 SM system and 2×4 STBC system in uncorrelated Rayleigh fading channels. Fig. 1 gives the BER performance of the SM, STBC and STBC-SM systems for $2\ bits/s/Hz$. It can be

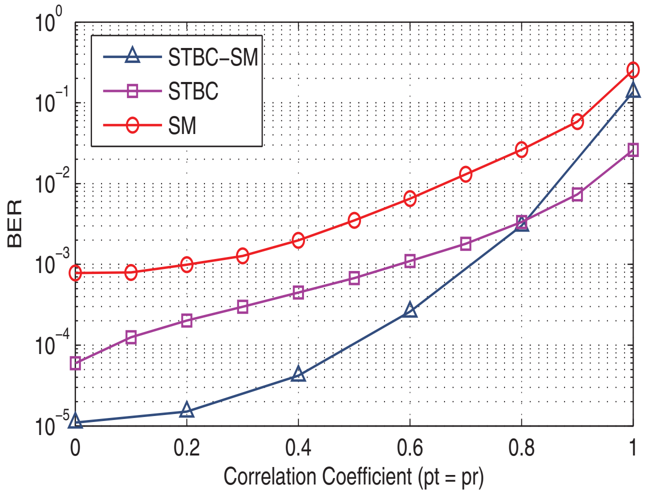

Fig. 2. Performance comparison of SM, STBC and STBC-SM systems over correlated Rayleigh fading channels for SNR = 8 dB (2 bits/s/Hz)

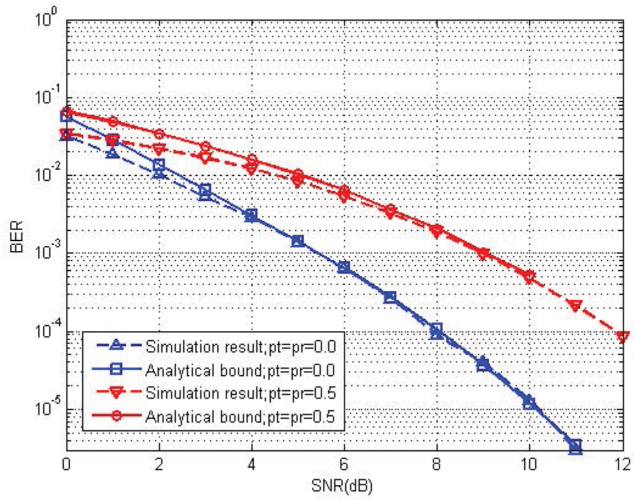

Fig. 4. Average BER of STBC-SM systems over correlated Rician fading channels (2 bits/s/Hz)

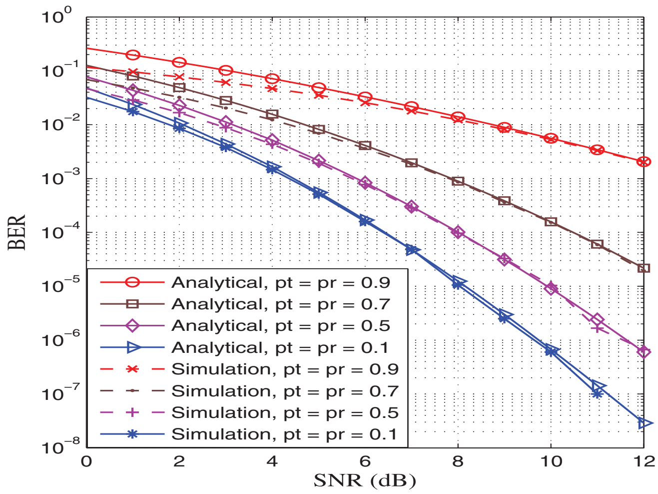

Fig. 3. Average BER of STBC-SM systems over correlated Rayleigh fading channels (2 bits/s/Hz)

observed that, for the same spectral efficiency of 2 $bits/s/Hz$, STBC-SM systems give better error performance as compared to STBC or SM systems. It is clear that STBC systems achieve full diversity order and hence performs better than SM systems which achieve diversity order of n_R. But, while moving from spectral efficiency of 1 $bits/s/Hz$ to 2 $bits/s/Hz$ or higher, STBC systems need to use higher order modulation scheme whereas in STBC-SM systems, higher spectral efficiency can be achieved by deploying more number of transmit antennas without changing the modulation scheme.

The BER performance of SM, STBC and STBC-SM systems is compared in correlated Rayleigh fading channels in Fig. 2. For correlated fading channels, we refer transmitter correlation coefficient by pt and receiver correlation coefficient by pr. It can be observed that STBC-SM systems perform better than SM and STBC systems for the same spectral efficiency in both uncorrelated channels and correlated channels with correlation coefficients $pt = pr < 0.8$. For $pt = pr > 0.8$ the BER performance of STBC is superior to STBC-SM systems when observed at an SNR of 8 dB. The reason behind degradation of BER performance of STBC-SM systems for high correlation coefficients is that highly correlated links becomes similar to each other making antenna index estimation more difficult and erroneous. Hence, it results in degraded overall BER performance.

The upper bound on BEP of STBC-SM systems over correlated Rayleigh fading channels is evaluated using (11) and plotted in Fig. 3 along with the results of Monte Carlo simulations at the spectral efficiency of 2 $bits/s/Hz$. The upper bound on BEP of STBC-SM systems over correlated Rician fading channels is evaluated using (15) and plotted in Fig. 4 along with the results of Monte Carlo simulations of the STBC-SM system at the spectral efficiency of 2 $bits/s/Hz$ and Rice fading parameter, $K = 2$. From Fig. 3 and Fig. 4, it can be observed that the analytical upper bounds calculated using (11) and (15) are tighter for BER$< 0.5 \times 10^{-2}$.

VI. Conclusion

In this paper, we derive closed form expressions for upper bound on BEP of STBC-SM systems over correlated Rayleigh and Rician fading channels. The upper bound on BEP is derived using Chernoff bound and the union bound. The analytical results are validated using the results obtained from Monte Carlo simulations of STBC-SM systems. The analytical results are in agreement with the Monte Carlo simulation results and the upper bound on BEP is highly accurate for high SNR regions (BER$< 0.5 \times 10^{-2}$). The results show that STBC-SM systems give better BER performance than SM and STBC systems in uncorrelated fading channels and in correlated fading channels up to correlation coefficient of 0.8. Under highly correlated fading conditions ($pt = pr > 0.8$), STBC systems outperform STBC-SM systems.

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