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A Scheduling Algorithm Based on Petri Nets and Simulated Annealing

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Abstract: This study aims at presenting a hybrid Flexible Manufacturing System "HFMS" short-term scheduling problem. Based on the art state of general scheduling algorithms, we present the meta-heuristic, we have decided to apply for a given example of HFMS. That was the study of Simulated Annealing Algorithm SA. The HFMS model based on hierarchical Petri nets, was used to represent static and dynamic behavior of the HFMS and design scheduling solutions. Hierarchical Petri nets model was regarded as being made up a set of single timed colored Petri nets models. Each single model represents one process which was composed of many operations and tasks. The complex scheduling problem was decomposed in simple sub-problems. Scheduling algorithm was applied on each sub model in order to resolve conflicts on shared production resources.

Key word: Scheduling, meta-heuristics, modeling, hybrid petri nets, continuous petri nets, simulated annealing SA, hybrid flexible manufacturing systems "H.F.M.S"

INTRODUCTION

Meta-heuristics have totally changed heuristics elaboration. Whether we begin by asking questions on characteristics and particularities of the problem to solve, before programming a specific method, meta-heuristics have inversed the presses because the main algorithm of the method is given by the metaphore which has inspired the meta heuristic. Given a first heuristic, we search its amelioration by observing its weakness, according to the problem we have to solve. In general, flexible manufacturing systems ''FMS'' are considered as discrete event dynamic systems ''DEDS''. However, some characteristic variables are continuous, like tasks, jobs, evolution of temperature, of pressure and so forth. So, resulting FMS are called hybrid FMS because they can be seen as a set of some continuous phenomena depending on discrete events. Their description is too difficult and complex. We have used hybrid hierarchical Petri Nets, because of their capacity to depict continuous evolution on timed places and to represent discrete events on transitions.

The main problem in an FMS, is how to realize all the presses in an optimal time, with a minimal cost. The objective of scheduling system is to organize in time, realization of interdependent tasks considering constraints on time, cost and resources. Scheduling method must generally reconcile a static aspect related to the production planning, with a dynamic one, related to real time decisions considering the efficient state of tasks evolution. Several methods have been developed. They can be classified in three important classes :

Exact optimization methods like graph theory, branch and bound method they have proved their limitation because of the important computing time they need to reach the optimal solution.

Heuristic methods that the inconvenient consists in their particular and individual elaboration for a specific problem.

Meta-heuristic methods which are iterative research process. They combine different intelligent concepts of known and general algorithms like genetic algorithms, Taboo search, priority rules method, stochastic descent method, neuronal networks, simulated annealing and so forth We have developed two scheduling algorithms based on Taboo search[1,2] and genetic algorithms[3]. In this paper, we developed a scheduling algorithm based on the ''Simulated annealing '' Meta heuristic. The objective of the algorithm is to solve assignment problem of different tasks and jobs for each valuable machine. The optimization criterion is the ''MAKESPAN'' minimization (realization time of complete process) under ''maximization of productivity' constraint.

H.F.M.S modeling by hybrid hierarchical petri nets: A large review of the literature has shown that hybrid

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flexible manufacturing systems has not been well studied, either in modeling or scheduling problems. For these reasons, we have proposed some flexible models which can depict both discrete and continuous variables in an HFMS. Timed Petri nets and continuous Petri nets seem to be a complete modeling tool for HFMS. Hierarchical or well formed Petri nets permit the decomposition of complex schedule into simple and elementary schedules according to different part types manufactured in the HFMS[4].

Continuous Petri nets[5]: They results from the timed Petri nets definition. In timed Petri nets, the marking $m_i(t)$ of a place P_i is integer, however in continuous Petri nets, $m_i(t)$ is real. In timed Petri nets, a transition T_i is fired instantly. In continuous Petri nets, it is fired continuously with a certain speed $V_j(t)$. A continuous Petri nets is characterized by a set of maximal speeds of its transitions.

$$Vmax=(Vmaxj)_{j=1,..,m} \in \Re^m \quad (1)$$

Each transition Tj is fired with real speed less or equal to its maximal firing speed.

$$V(t)=(V_j(t))_{j=1,..,m} \in \Re^m \quad (2)$$

Is the set of firing speeds at time t. The marking evolution in continuous Petri nets is defined by differential equation as follows:

$$\frac{dM(t)}{dt} = W.V(t) \quad \text{W is the global incidence matrix of the P.N} \quad (3)$$

HFMS modeling by hybrid hierarchical PN's[5]: Hybrid systems are characterized by a set of discrete events and a continuous evolution according to these events. Two monitoring structures are proposed depending of the signals which circulate between the system and the monitor.

In hybrid Petri nets modeling, discrete state variables are described by places of HPN. Discreet events are directed on transitions. The system continuous evolution is defined by the marking continuous evolution. The first model is the most close one to the HFMS structure.

Simulated annealing algorithm

Introduction[6]: This method is inspired from Metallurgy science. Slow Quenching of a material gives it a crystalline structure with minimum energy. The same concept can be applied to optimization problems.

Definition[6]: Simulated annealing is an improved version of the method called: iterative improvement. It has been proposed at 1983 by Kirkpatrick and Al. for VLSI placement problem resolution. The method has been proposed under shape of the algorithm called " Metropolis algorithm" that simulates energy modification of a material. The main idea consists in the analogy observed between a complex system optimization and a physical system behavior description. This method is applied in several domains: operational research, imagery, production tools scheduling, and so forth. In this algorithm, movements in research space are based on Boltzman distribution. This one, measure the probability of a system to be in a configuration Ci with an energy E(Ci), in a given temperature T in the space of configurations, which are defined by :

$$\pi_{i \in U} = \frac{\exp \frac{-E(c_i)}{kT}}{\sum_{j \in U} \exp \frac{-E(c_i)}{kT}} \quad \text{K is the Boltzman constant } K=1,3805.10^{-23} j/k \quad (4)$$

In this expression, KT shows that when the temperature is high, all states have the same probability. It means that most of configurations are reachable. When temperature is low, states with high energy become less probable than those with low energy. If we apply this concept to cost minimization problems, research process can be assimilated to the annealing process in metallurgy. When a material is got very hot (temperature is very high), the material becomes liquid and can get any configuration we need. So, when it becomes cold, we say that it has quenched and we must get it hot again if we need to modify its actual configuration. Kirkpatrick algorithm stimuli this process by combining Quenching and annealing.

The annealing concept (increase of temperature) allows the research algorithm to leave local optimum. Temperature have not equivalent in optimization problems. It is only a control parameter that indicates the system configuration. Energy represents the evaluation function or the fitness. The metropolis criterion is used to decide if the new configuration gives an acceptable variation of the fitness value. It permits to leave local optimum if stopping criterion is not reached yet.

Metropolis criterion[6]: After every challenge from a configuration U to an other configuration V, we compute the cost function variation $\Delta g= g(v)- g(u)$. The transformation is accepted with a probability:

$$P(u,v) = \exp(-\Delta g/T) \quad (5)$$

When $\Delta g \le 0$, then $\exp(-\Delta g/T) \ge 1$ and the new configuration is accepted with a probability P(v)=1.

If $\Delta g>0$, we compare P(u,v) with a random number $r \in [0,1[$:

* If r < P(u,v) then the configuration is accepted.
* Else the configuration V is rejected.

In this case, the system try to find another configuration. If it is impossible, the last configuration is accepted and the research is stopped when the stopping criterion is reached.

Algorithm characteristics[6]

Temperature: It is a control parameter that is sufficiently high, in order to skip high level gates of energy. And sufficiently low to be attracted in the most deep minimum. The slow diminution of temperature, allows the research of "attraction basins" that preference is always given to the minimal cost one. The variation law of temperature is also important in order to test a maximum number of configurations, to find global minimum.

Temperature initial value T_0: In Kirkpatrick and Al, T_0 must be chosen so that the acceptance probability of the worst configuration would be equal to Pr=80%. After, the maximal increase of cost function $\Delta g+$ is fixed by the user. T_0 is obtained by the following expression:

$T_0 = \Delta g+ / \ln(Pr)$ (6)

Siarry and Dreyfus suggest that T_0 is computed us follows:

$T_0 = r^*. \max \Delta g$ such us $r \gg 1$, $r \cong 10$ (7)

In these two expressions, it is too difficult to compute or estimate the value of $\Delta g+$ and max Δg for a real big dimension problem.

Van Larhoven and Aarts propose that T_0 would be determined so that system transitions are all accepted at the beginning of the research algorithm. That means:

$\exp(-\Delta g/T_0) \cong 1$ (8)

Stopping criterion: The temperature decrease is stopped:

* By fixing the T_k variations number for which the algorithm is run. Generally, we take this number between 6 and 50.
* When two consecutive configurations are identical.

When temperature T_k is less than a given fraction of T_0:

$T_k < T_{ratio} . T_0$ with $T_{ratio} = 10^{-6}$ for example (9)

Temperature decrease: The temperature challenge from Tk to T_{k+1} is determined by the statistic stability detection. The research is realized by iterating the Markov chain, which is generally configurations number tested at T_k . The most usual decrease functions are given in this table.

Table 1: Usual decrease functions of temperature

	Function	Parameters
Linear	$T_{k+1} = \alpha T_k$	$\alpha \le 1$
Discret	$T_{k+1} = T_k - \Delta T$	$\Delta T>0$ (quenching) $\Delta T<0$ (annealing)
Exponential	$T_{k+1} = T_k \exp(-\lambda^* T_k/ \sigma_k)$ $0 \le T_{k+1} \le 1$	λ: fixed by user

We generally use linear function which permit a mean decrease, not very slow (exponential) and not very fast (discrete).

Markov chain: The Markov chain is the set of finite random states composed of the probabilities set associated to every configuration visited at a temperature T_k. When T_k is constant, the probability is homogeneous. If the transition number tends towards the infinite, the most probable state appears very often, then we obtain the statistic stability at this temperature.

Conclusion: Like in other meta-heuristics, simulated annealing algorithm convergence and efficiently depend on:

* A good choice of the neighborhood function.
* Method tram diversity.
* A good choice of algorithm parameters.
* The research space size.

Simulated annealing adaptation for optimization of the "MAKESPAN":

Introduction: The system studied is a flexible manufacturing cell composed of many numerical machines with unitary capacity (at one time, a machine can realize one operation) and different operating times. The cell receives materials from infinite capacity input buffer. Resulting products are sent to infinite capacity output buffer. There are some precedence constraints in the same product, but different products can be realizing simultaneously, by different machines. Our objective is to resolve conflicts on shared resources when several operations require the same machine, so that we minimize the Global execution time "Makespan".

Optimization criteria are:

* Passed time of current cycle.
* Asking time of conflicting resource.
* Operating time of unrealized operations.
* Operating time of conflicting operation.

The evaluation function is based on the characteristic equation of hybrid stochastic P.N's[4].The algorithm, we are going to present in the following section, can be used for other, kind of products, number of machines and operations and configuration of the flexible manufacturing cell. Simulating results are compared to those obtained with an exact iterative algorithm[7], two specific heuristics[4], Taboo search algorithm[2] and a genetic algorithm method[3].

Problem position: The problem is to make a decision when a shared resource is required to realize simultaneously two or more different operations, so that the Makespan is minimized. Our approach is to resolve the make-decision problem with an optimization algorithm using Simulated Annealing meta-heuristic and Hybrid Stochastic P.N's model.

Simulated annealing association with peti nets model: PN's model is used to describe the flexible manufacturing cell structure and dynamics. We obtain a flexible structure that allows exploitation of evaluation function based on evolution equation of Peti Nets:

$M' = M + C(p,t) * D$ (10)

* When C(p,t) is the incidence matrix and D the set of transitions fired by M.
* Research domain is the set of operations that asking for a same shared resource.

* The neighborhood function is given by the algorithm described in 4.4.
* Initial value of temperature T_0 is defined according to a probability P_0 , such us

$P_0= \exp(-\Delta g/T_0) \cong 1$ (11)

* Temperature decrease is linear : $T_{k+1} = \alpha T_k$ $\alpha \leq 1$ such us $\alpha \in [0.8, 0.95]$
* The final value of Temperature is computed in two different kinds:

Such us a probability $P_{fin}= \exp(-\Delta g/T_{fin}) \cong 0$ (12)

$T_{fin} = T_{ratio} . T_0$ with $T_{ratio} \in [10^{-6}, 10^{-1}]$ (13)

Neighborhood function algorithm

Simulation results: In order to simulate a real time evolution of the cell, as good as possible, we have tried two models:

* In the first model, we use the full colored model based on stochastic well formed colored PN's[4],
* In the second one, the model is horizontally discolored (each type of product is modeled by a stochastic colored PN's)[7].

Computation of different algorithm parameters: Prod: Mean productivity= mean number of realized products/ mean executing time

Table 2: P_0=0.5,0.6,0.7

P_0	0.5	0.5	0.5	0.6	0.6	0.6	0.7	0.7	0.7
P_{fin}	0.2	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1
α	0.8	0.9	0.95	0.8	0.9	0.95	0.8	0.9	0.95
Prod	1.04	1.01	1.06	1.14	1.14	1.19	1.17	1.19	1.22

Table 3: P_0=0.8,0.9,0.99

P_0	0.8	0.8	0.9	0.9	0.99
P_{fin}	0.1	0.1	0.1	0.1	0.1
α	0.90	0.95	0.90	0.95	0.95
Prod	1.21	1.23	1.22	1.24	1.30

Table 4: Choice of α and P_{fin}

P_0	0.5	0.99
P_{fin}	0.3	0.01
α	0.8	0.95
Prod	0.965	1.466

Table 5: Choice of α and T_{ratio}

P_0	0.99	0.99
α	0.80	0.95
T_{ratio}	0.01	0.001
Prod	1.295	1.506

Comments

* Productivity is better when P_{fin} is less.
* Productivity is better when P_0 is close to 1.
* Productivity is better when $\alpha = 0.95$.
* Results computed with T_{ratio} are better than those computed with $P_{fin.}$

Table 6: Machines number =3

N cycles	Prod D	Prod S	Prod D	Prod S
5	1.124	1.430	1.281	1.487
10	1.313	1.398	1.402	1.402
25	1.212	1.550	1.365	1.290
Machines type	MS		MP	

Table 7: Polyvalent machines

N cycles	Prod D	Prod S	Prod D	Prod S
5	1.442	1.597	1.294	0.965
10	1.315	1.390	1.354	1.038
25	1.291	1.449	0.953	1.023
Machines Number	2		1	

Productivity results obtained with the first model

MP: Polyvalent machines. All the three machines of the cell are full flexible; they can execute any operation on any product manufactured in the cell.

MS: Specialized machines. All the three machines of the cell are half flexible Each machine can realize a predefined set of operations , but can be used for any type of product manufactured in the cell.

D: Deterministic functioning.

S: Stochastic functioning.

Comments: According to these results, we can do some remarks, such us:

* Best results are obtained with stochastic functioning of the cell. That can be explained by the fact that the research space is more large and variable, that permit a best exploration and allow a fast convergence to the global optimum.
* With polyvalent machines, best results are obtained with 2 machines. That is because waiting times for machines availability are reduced and conflicts appear less frequently.
* With specialized machines, conflicts number is less than that with polyvalent machines. That explains best productivities obtained in this case (SM).

Productivity results obtained with the second model

Table 8: Polyvalent machines

Ncycles	5	10	25
Prod	1.43	1.61	1.65

Comments

* We can easily remark that best results are given by the second model (horizontally discolored when

each type of product is modeled by a stochastic colored PN's). The global optimum is computed more easily because each algorithm is executed separately for each type of product.

* Productivity results obtained when a failure appear on a machine

Table 9: With failure and repair

Productivity / cycles Nbr	Defected resource R	M1	M2	M3
10	1.52	1.11	1.29	1.16
15	1.30	0.62	1.23	1.59
25	0.66	0.67	1.44	0.71

R,M1,M2,M3 : The four shared resources in the cell (robot, machines 1,2,3)

Comments

* We can see that the algorithm is adaptive because it can recover the lost time spending in repair and reach the maximal productivity.

CONCLUSION

With the simulated annealing meta-heuristic, the parameters have an important effect. Productivity is best when all the parameters are well chosen. The inconvenient of this method is the great probability to fall in a local optimum.The role of temperature parameter is to show the presses evolution. More the presses is advanced; less we accept a solution with a high cost. But, at the beginning, the acceptation of a bad solution permits us to completely explore the solution space in order to reach the global optimum. In this paper, we have attempted to realize a flexible scheduling algorithm, which takes account all types of events, variables, machines Different kinds of evolution, parallel evolution, deterministic, stochastic, full flexible, half flexible, with failure, without failure, with failure and repair

REFERENCES

1. Ghoul, R., 2002. Application de la recherche tabou à l'ordonnancement de production dans une cellule de production flexible. Annaba, Algérie.
2. Ghoul, R. and H. Tebbikh, 2002. Production management in a flexible Manufacturing cell with a petri net model And meta-Heuristic methods. Third assiut university international conference on mechanical engineering advanced technology for industrial production, Assiut, Dec. 24-26.
3. Ghoul, R., H. Tebbikh and S. Kechida, 2004. A genetic algorithm method to solve machine assignment problem in a flexible manufacturing cell modeled with hierarchical petri nets. Intl. C. on Telecommuting and Information Technology/ ICTIT 2004, Applied Science Private University. Sep. 22-24, Amman-Jordan.
4. Ghoul, R., H. Tebbikh and M. Djeghaba, 2002. Conduite d'un système de production flexible par les SCWN . APII. JESA, 36 : 1399-1411.
5. Lefebvre, D., 2000. Contribution à la modélisation des systèmes dynamiques à événements discrets pour la commande et la surveillance. Habilitations à diriger des recherches. Belfort, France.
6. Taillard, E., 1999. Méta-heuristiques et outils nouveaux en Recherche opérationnelle. Thèse doctorat de l'I.I, Suisse occidentale.
7. Ghoul, R., 2003. Modélisation et conduite des systèmes de production flexible par les réseaux de pétri. Thèse de doctorat d'état de l'université de Annaba, Algérie.