TABLE III
DETECTION AND FALSE ALARM (BRACKETED) RATES IN PERCENT FOR LINEAR CORRELATION-BASED DETECTORS FOR SIGNALS IN SαS INTERFERENCE, WITH $\alpha = 1.6$ AND AN ERROR IN PARAMETER ESTIMATION, $\hat{\alpha}$

	LO & LSO detectors						LOR & LSOR detectors		
$\hat{\alpha}$	LO	MF	hp	triangular	power	Cauchy	LOR	Wilcoxon	triangular
1.4	74.9	3.9	76.5	82.0	52.9	68.5	70.4	73.7	71.6
	(5.0)	(1.5)	(10.3)	(8.3)	(1.3)	(4.4)	(6.1)	(5.2)	(4.1)
1.2	76.1	0.8	86.5	89.7	41.9	71.4	75.8	73.1	74.1
	(6.2)	(0.6)	(19.0)	(16.0)	(0.7)	(6.9)	(5.9)	(5.2)	(5.8)
1	65.4	0.1	93.4	96.6	12.9	70.3	70.9	68.1	73.6
	(4.1)	(0.1)	(28.1)	(26.3)	(0.0)	(6.6)	(3.7)	(4.1)	(6.3)

IV. CONCLUSIONS

The use of correlation detectors for the detection of known signals in impulsive interference modeled by an SαS process has been investigated. A number of nonlinear score functions for both correlation (LSO) detectors, as well as rank correlation detectors (LSOR) have been developed and their performance compared with the LO, LOR, matched filter, and Cauchy detectors. Although the linear and Cauchy detectors are optimal when $\alpha = 2$ and 1, respectively, their performance deteriorated for other values of α.

The LOR detector has been seen to achieve similar performance to the LO detector. It also has inherent advantages for on-line detection. Additionally, the LSOR detector using the triangular rank score function LSOR-tr has achieved high detection rates: close to those of the LO and LOR detectors across all values of α tested. This has been achieved while maintaining computational simplicity and its ability to maintain a constant false alarm rate and high detection rates when parameter estimation errors occur.

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Asymptotic Performance Analysis of DOA Finding Algorithms with Temporally Correlated Narrowband Signals

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***Abstract*—This correspondence focuses on the asymptotic performance analysis of general direction-of-arrival (DOA) finding algorithms under the stochastic model assumption in which source and noise signals are possibly non-Gaussian and possibly temporally correlated. We prove, in particular, that all the covariance-based DOA estimators are sensitive to the temporal correlation of the sources when the noise is temporally correlated; otherwise, most of them are insensitive to the temporal correlation of the sources, except for the Toeplitzation and the augmentation techniques.**

***Index Terms*—Asymptotic performance analysis, augmentation technique, covariance-based DOA estimator, DOA finding algorithms, Toeplitzation technique.**

I. INTRODUCTION

Motivated by the popularity of the second-order algorithms in DOA estimation, many contributions have appeared that aim at establishing the asymptotic statistical performance of DOA estimators in the context of narrowband array processing. These studies rest on different signal models. The deterministic and the stochastic model are the main models that have appeared in the literature. The deterministic model assumes the source signals fixed in all realizations and the noise to be a temporally uncorrelated Gaussian random process. In the stochastic model, the source and noise signals are generally assumed to be temporally uncorrelated Gaussian random processes. Many authors (see [1]–[4] and the reference therein) compared the asymptotic performance of DOA algorithms with these two models and connected their performance to the Cramér–Rao bound. In fact, most DOA estimators have the same asymptotic statistical performance under these two models [3], [4] and with any distribution of the source signals in the stochastic model [5]. However, all these contributions rely on the independence assumption of the successive snapshots. Consequently, performance analyzes of these algorithms under mild assumptions remain of current interest.

It is the aim of this correspondence to investigate the performance of DOA estimators under the general stochastic model assumption in which both the source and noise signals are possibly temporally correlated and possibly non-Gaussian random processes. Ordinarily, the performance analysis of these second-order algorithms relies on the distribution of the empirical spatial covariance matrix $\boldsymbol{R}_x(n) \stackrel{\text{def}}{=} (1/n)\sum_{t=1}^{n} \boldsymbol{x}_t \boldsymbol{x}_t^H$. These studies use two approaches. The first one is based on perturbation calculus induced by its complex Wishart distribution when the snapshots $\boldsymbol{x}_t$ are Gaussian. The second is based on a continuity theorem (e.g., [6, th., p. 122]), which transfers the asymptotic normality issued from its complex asymptotic Gaussian distribution derived from the classical central limit theorem to any regular function of this covariance. When the snapshots $\boldsymbol{x}_t$ are not independent, the distribution of $\boldsymbol{R}_x(n)$ is not complex Wishart in the Gaussian case for the first approach, and the classical central limit theorem cannot be applied for the second approach. We adopt, in this correspondence, the general functional method of [7], in which the Gaussian asymptotic distribution of the covariance-based DOA

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estimates is derived from the Gaussian asymptotic distribution of the empirical covariance matrix. This allows us to give closed-form expressions for the asymptotic covariance matrices of DOA estimates and to specify the conditions for which these expressions are sensitive to the distribution and the temporal correlation of the sources.

This correspondence is organized as follows. The model of dependent snapshots $\boldsymbol{x}_t$ is defined in Section II. Then, in Section III, the asymptotic normality of $\boldsymbol{R}_x(n)$ is established for these models of dependence where a central limit theorem is given. A general functional approach providing a common unifying framework for asymptotic DOA estimation performance analysis is presented in Section IV. The case of uniform linear or rectangular arrays with the Toeplitzation and augmentation techniques are addressed. Finally, in Section V, some simulations are presented.

II. Signal Model

Let an arbitrary array composed of M sensors receive K narrowband waves. These narrowband signals are assumed to have a common center frequency and are in the same bandwidth B. Let $u_{t,k}$, $\boldsymbol{\Theta}_k$, and $\boldsymbol{e}(\boldsymbol{\Theta}_k)$ denote, respectively, for the source k, the complex envelope of the emitted signal by this source at time t, the unknown spatial parameters that are referred to as the DOA, and the so-called steering vector of this source. The M vector of the observed complex envelopes of the sensor outputs is typically modeled by

$$\boldsymbol{x}_t = \sum_{k=1}^{K} u_{t,k}\boldsymbol{e}(\boldsymbol{\Theta}_k) + \boldsymbol{v}_t.$$

Here, $\boldsymbol{v}_t$ represents the M-vector of observed complex envelope of sensor output additive noise at time t. Ordinarily, several *independent* measurements $\boldsymbol{x}_t$ are made by sampling the complex envelopes at times t such that $(u_{t,k}, \boldsymbol{v}_t)_{t=1,\cdots,n}$ are independent. We suppose, in this corrrespondence, that the complex envelopes of the sensor outputs are uniformly sampled at a frequency greater than or equal to B. As a consequence, the observations $(\boldsymbol{x}_t)_{t=1,\cdots,n}$ are no longer independent. $\boldsymbol{v}_t$ and $(u_{t,k})_{t=1,\cdots,n}$ are modeled as zero-mean with finite fourth-order moments that are not necessarily Gaussian stationary random processes. $\boldsymbol{v}_t$ is supposed independent of $(u_{t,k})_{t=1,\cdots,n}$. The spatial covariance matrix $\boldsymbol{R}_x \stackrel{\text{def}}{=} E(\boldsymbol{x}_t\boldsymbol{x}_t^H)$ reads

$$\boldsymbol{R}_x = \boldsymbol{E}(\boldsymbol{\Theta})\boldsymbol{R}_u\boldsymbol{E}^H(\boldsymbol{\Theta}) + \sigma_v^2\boldsymbol{C}$$

with $\boldsymbol{E}(\boldsymbol{\Theta}) \stackrel{\text{def}}{=} [\boldsymbol{e}(\boldsymbol{\Theta}_1), \cdots, \boldsymbol{e}(\boldsymbol{\Theta}_K)]$, $\boldsymbol{R}_u \stackrel{\text{def}}{=} E(\boldsymbol{u}_t\boldsymbol{u}_t^H)$, where $\boldsymbol{u}_t \stackrel{\text{def}}{=} (u_{t,1}, \cdots, u_{t,K})^T$, and $E(\boldsymbol{v}_t\boldsymbol{v}_t^H) = \boldsymbol{R}_v = \sigma_v^2\boldsymbol{C}$, where $\boldsymbol{C}$ is a known positive definite matrix. In order to consider the asymptotic distribution of the estimated spatial covariance matrix $\boldsymbol{R}_x(n)$, we consider for simplicity that $u_{t,k}$ are either harmonic random processes $[u_{t,k} = \sum_{l=1}^{L_k} a_{k,l}e^{i\alpha_{k,l}}e^{i2\pi f_{k,l}t}$, where $(f_{k,l})_{k=1,\cdots,K,\, l=1,\cdots,L_k}$ are fixed *distinct* positive real numbers in $]-1/2, +1/2[$, $a_{k,l}$ are fixed positive real numbers, and $\alpha_{k,l}$ are random variables uniformly distributed in $[0, 2\pi]$ and mutually independent] or complex ARMA processes with power σ_k^2, power spectral density $S_{u_k}(f)$, and power cross-spectral density $K \times K$-matrix $\boldsymbol{S}_u(f)$. If the fourth-order polyspectrum of the K sources $u_{t,k}$ for $k_1, k_2, k_3, k_4 = 1, \cdots, K$ is defined as

$$\begin{aligned}\rho_{k_1,k_2,k_3,k_4}(f, f', f'') &\stackrel{\text{def}}{=} \sum_{\tau,\tau',\tau''} \text{Cum}(u_{0,k_1}, u_{\tau,k_2}^*, u_{\tau',k_3}, u_{\tau'',k_4}^*)\\ &\quad \cdot e^{i2\pi(f\tau + f'\tau' + f''\tau'')}, \quad \text{then}\end{aligned}$$

$$[\boldsymbol{Q}_u]_{K(j-1)+i,\,K(l-1)+k} = \int_{-1/2}^{+1/2}\int_{-1/2}^{+1/2} \rho_{i,j,l,k}(f, f', -f')\, df\, df'$$

denotes the $K^2 \times K^2$ fourth-order cumulant matrix. The same ARMA assumption and notations are adopted to the M-variate $\boldsymbol{v}_t$.

Usually, in the context of narrowband waves, the observations $(\boldsymbol{x}_t)_{t=1,\cdots,n}$ are assumed zero-mean circular Gaussian and independent. Therefore, the ensemble average $\boldsymbol{R}_x(n)$ is a sufficient statistic, and consequently, all direction-finding algorithms are based on $\boldsymbol{R}_x(n)$. In this context, all the asymptotic performance analyzes are based on the distribution of $\sum_{t=1}^{n}\boldsymbol{x}_t\boldsymbol{x}_t^H$, i.e., a central complex Wishart distribution. In this correspondence, we go on using second-order direction-finding algorithms, but as in our signal model, the observations $(\boldsymbol{x}_t)_{t=1,\cdots,n}$ are no longer independent, and all the results based on this Wishart distribution are not usable. We need to know the asymptotic distribution of $\boldsymbol{R}_x(n)$.

III. Spatial Covariance Matrix

Covariance-based DOA estimators will turn out to be asymptotically normal as the number n of observations goes to infinity. In this section, we focus on a central limit theorem to be used for establishing DOA asymptotic normality in the next section. We show that the asymptotic distribution of the spatial covariance matrix $\boldsymbol{R}_x(n)$ is very sensitive to the model of dependence between snapshots. We prove the following theorem.

Theorem 1: $\sqrt{n}(\text{Vec}(\boldsymbol{R}_x(n)) - \text{Vec}(\boldsymbol{R}_x))$ converges in distribution to the zero-mean complex Gaussian distribution of covariance $\boldsymbol{C}_{R_x}$

$$\sqrt{n}(\text{Vec}(\boldsymbol{R}_x(n)) - \text{Vec}(\boldsymbol{R}_x)) \xrightarrow{\mathcal{L}} \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}_{R_x}, \boldsymbol{C}_{R_x}\boldsymbol{K}). \tag{3.1}$$

Furthermore, $E(\boldsymbol{R}_x(n)) = \boldsymbol{R}_x$ and[1]

$$\lim_{n\to\infty} n\text{Cov}(\text{Vec}(\boldsymbol{R}_x(n))) = \boldsymbol{C}_{R_x} \tag{3.2}$$

where $\boldsymbol{C}_{R_x}$ reads

$$\begin{aligned}\boldsymbol{C}_{R_x} = &(\boldsymbol{E}(\boldsymbol{\Theta}) \otimes_c \boldsymbol{E}(\boldsymbol{\Theta}))C_{R_u}(\boldsymbol{E}^H(\boldsymbol{\Theta}) \otimes_c \boldsymbol{E}^H(\boldsymbol{\Theta})) + \boldsymbol{C}_{R_v}\\ &+ (\boldsymbol{E}(\boldsymbol{\Theta}) \otimes_c \boldsymbol{I}_M)\boldsymbol{C}_{R_{u,v}}(\boldsymbol{E}^H(\boldsymbol{\Theta}) \otimes_c \boldsymbol{I}_M)\\ &+ (\boldsymbol{I}_M \otimes_c \boldsymbol{E}(\boldsymbol{\Theta}))C_{R_{v,u}}(\boldsymbol{I}_M \otimes_c \boldsymbol{E}^H(\boldsymbol{\Theta}))\end{aligned} \tag{3.3}$$

with $\boldsymbol{C}_{R_u} = \int_{-1/2}^{+1/2} \boldsymbol{S}_u(f) \otimes_c \boldsymbol{S}_u(f)\, df + \boldsymbol{Q}_u$ in the ARMA case and $\boldsymbol{C}_{R_u} = \boldsymbol{O}$ in the harmonic case, $\boldsymbol{C}_{R_v} = \int_{-1/2}^{+1/2} \boldsymbol{S}_v(f) \otimes_c \boldsymbol{S}_v(f)\, df + \boldsymbol{Q}_v$, $\boldsymbol{C}_{R_{u,v}} = \sum_{n=-\infty}^{+\infty} \boldsymbol{R}_u^n \otimes_c \boldsymbol{R}_v^n$, $\boldsymbol{C}_{R_{v,u}} = \sum_{n=-\infty}^{+\infty} \boldsymbol{R}_v^n \otimes_c \boldsymbol{R}_u^n$, where $\boldsymbol{R}_u^n \stackrel{\text{def}}{=} E(\boldsymbol{u}_t\boldsymbol{u}_{t-n}^H)$ and $\boldsymbol{R}_v^n \stackrel{\text{def}}{=} E(\boldsymbol{v}_t\boldsymbol{v}_{t-n}^H)$. $\boldsymbol{A} \otimes_c \boldsymbol{B}$ denotes the block matrix, the (i, j) block element of which is $b_{i,j}^*\boldsymbol{A}$, and $\boldsymbol{K}$ is the vec-permutation matrix defined by $\text{Vec}(\boldsymbol{A}^T) = \boldsymbol{K}\text{Vec}(\boldsymbol{A})$ for any square matrix $\boldsymbol{A}$.

In (3.1) a complex random $p \times 1$ vector $\boldsymbol{y}$ has a zero-mean complex Gaussian distribution specified by a $p \times p$ positive definite matrix $\boldsymbol{\Sigma}_1$ and $p \times p$ symmetric matrix $\boldsymbol{\Sigma}_2$ and denoted $\mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2)$ if the $2p$-joint distribution of the real and imaginary part of $\boldsymbol{y}$ is $2p$-zero-mean Gaussian, i.e., for any complex $p \times 1$ vector $\boldsymbol{w}$; the real scalar $\boldsymbol{w}^H\boldsymbol{y} + (\boldsymbol{w}^H\boldsymbol{y})^H$ has a zero-mean Gaussian distribution with variance $2\boldsymbol{w}^H\boldsymbol{\Sigma}_1\boldsymbol{w} + \boldsymbol{w}^H\boldsymbol{\Sigma}_2\boldsymbol{w}^* + \boldsymbol{w}^T\boldsymbol{\Sigma}_2^*\boldsymbol{w}$, where $E\boldsymbol{y}\boldsymbol{y}^H) = \boldsymbol{\Sigma}_1$, and $E\boldsymbol{y}\boldsymbol{y}^T) = \boldsymbol{\Sigma}_2$.

Equation (3.2) is proved after straightforward but tedious algebric manipulations. Then, to prove (3.1), we adapt the steps of ([8, Sec. 7.3]) to our problem.

[1] $\text{Cov}(\text{Vec}(\boldsymbol{R}_x(n)))$ denotes $E(\text{Vec}(\boldsymbol{R}_x(n) - \boldsymbol{R}_x)\text{Vec}^H(\boldsymbol{R}_x(n) - \boldsymbol{R}_x))$. We note that $\text{Vec}^T(\boldsymbol{R}_x(n) - \boldsymbol{R}_x) = \text{Vec}^H(\boldsymbol{R}_x(n) - \boldsymbol{R}_x)\boldsymbol{K}$, and therefore, $E(\text{Vec}(\boldsymbol{R}_x(n) - \boldsymbol{R}_x)\text{Vec}^T(\boldsymbol{R}_x(n) - \boldsymbol{R}_x)) = E(\text{Vec}(\boldsymbol{R}_x(n) - \boldsymbol{R}_x)\text{Vec}^H(\boldsymbol{R}_x(n) - \boldsymbol{R}_x))\boldsymbol{K}$. Therefore, the noncircular complex Gaussian asymptotic distribution of $\boldsymbol{R}_x(n)$ is characterized by $\boldsymbol{C}_{R_x}$ only.

IV. Asymptotic Distribution of DOA Estimates

A. Functional Approach

To consider the asymptotic performance of a covariance-based DOA algorithm, we adopt a functional analysis[2] that consists in recognizing that the whole process of constructing an estimate $\boldsymbol{\Theta}(n)$ of $\boldsymbol{\Theta}$ is equivalent to defining a functional relation linking this estimate $\boldsymbol{\Theta}(n)$ to the statistics $\boldsymbol{R}_x(n)$ from which it is inferred. This functional dependence is denoted $\boldsymbol{\Theta}(n) = \text{alg}(\boldsymbol{R}_x(n))$. Clearly, $\boldsymbol{\Theta} = \text{alg}(\boldsymbol{R}_x)$; therefore, the different algorithms $\text{alg}(\cdot)$ constitute distinct extensions of the mapping $\boldsymbol{R}_x \rightarrow \boldsymbol{\Theta}$ generated by any unstructured Hermitian matrix $\boldsymbol{R}_x(n)$. In the following, we consider "regular" algorithms. More specifically, we assume the conditions given in [5].

1) The function $\text{alg}(\cdot)$ is differentiable in a neighborhood of $\boldsymbol{R}_x$, i.e., if $\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}$ denotes the $K \times M^2$ matrix of this differential evaluated at point $\boldsymbol{R}_x$

$$\text{alg}(\boldsymbol{R}_x + \delta\boldsymbol{R}) = \boldsymbol{\Theta} + \boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}\text{Vec}(\delta\boldsymbol{R}) + o(\delta\boldsymbol{R}). \tag{4.1}$$

2) For any $\boldsymbol{\Theta}$ and any positive definite source correlation matrix $\boldsymbol{R}_u$ (condition 2a) or for any $\boldsymbol{\Theta}$ and any positive definite diagonal source correlation matrix $\boldsymbol{R}_u$ (condition 2b)

$$\text{alg}(\boldsymbol{E}(\boldsymbol{\Theta})\boldsymbol{R}_u\boldsymbol{E}^H(\boldsymbol{\Theta}) + \sigma_v^2\boldsymbol{C}) = \boldsymbol{\Theta}. \tag{4.2}$$

Under appropriate hypotheses on the array manifold, requirements 1 and 2a are met by most of the second-order DOA estimators. Requirements 1 and 2b are met by the second-order DOA estimators that suppose the sources spatially uncorrelated. The following lemma (proved under conditions 1 and 2a in [5]) is used in next section to prove the invariance of the asymptotic distribution of the DOA's with respect to the distribution and the temporal correlation of the sources.

Lemma 1: Under conditions 1 and 2a [resp. conditions 1 and 2b], one has the constraints on $\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}$

$$\begin{aligned}&\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}(\boldsymbol{E}(\boldsymbol{\Theta}) \otimes_c \boldsymbol{E}(\boldsymbol{\Theta})) = \boldsymbol{O}\\ &[\text{resp.}, \boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}(\boldsymbol{e}(\boldsymbol{\Theta}_k) \otimes_c \boldsymbol{e}(\boldsymbol{\Theta}_k)) = \boldsymbol{0}\\ &k = 1, \cdots, K].\end{aligned} \tag{4.3}$$

B. Standard Algorithms

By the regularity condition (4.1), the asymptotic behaviors of $\boldsymbol{\Theta}(n)$ and $\boldsymbol{R}_x(n)$ are directly related. The standard result on regular functions of asymptotically normal statistics (see, e.g., [6, p. 122]) applies:

$$\sqrt{n}(\boldsymbol{\Theta}(n) - \boldsymbol{\Theta}) \xrightarrow{\mathcal{L}} \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}_\Theta) \tag{4.4}$$

with

$$\begin{aligned}\boldsymbol{C}_\Theta &= \lim_{n\to\infty} nE((\boldsymbol{\Theta}(n) - \boldsymbol{\Theta})(\boldsymbol{\Theta}(n) - \boldsymbol{\Theta})^T)\\ &= \boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}\boldsymbol{C}_{R_x}(\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x})^H.\end{aligned} \tag{4.5}$$

We can now state our main result.

Theorem 2: For Gaussian or non-Gaussian, ARMA, or harmonic source signals, the asymptotic covariance of any covariance-based DOA estimators that do not require the sources spatially uncorrelated have the common closed-form expression when the noise $\boldsymbol{v}_t$ *is temporally uncorrelated*:

$$\begin{aligned}\boldsymbol{C}_\Theta = \boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}(&\boldsymbol{E}(\boldsymbol{\Theta})\boldsymbol{R}_u\boldsymbol{E}^H(\boldsymbol{\Theta}) \otimes_c \sigma_v^2\boldsymbol{C}\\ &+ \sigma_v^2\boldsymbol{C} \otimes_c \boldsymbol{E}(\boldsymbol{\Theta})\boldsymbol{R}_u\boldsymbol{E}^H(\boldsymbol{\Theta})\\ &+ \sigma_v^4\boldsymbol{C} \otimes_c \boldsymbol{C} + \boldsymbol{Q}_v)(\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x})^H.\end{aligned} \tag{4.6}$$

Proof: In this situation, $\boldsymbol{R}_v^n = \delta_{0,n}\sigma_v^2\boldsymbol{C}$. The closed-form expression follows by application of (3.3), (4.5), and the first part of (4.3). ■

This result extends the result in [5]. We note that if the noise is temporally correlated, the terms $\boldsymbol{C}_{R_{u,v}}$ and $\boldsymbol{C}_{R_{v,u}}$ of (3.3) do not reduce to the spatial terms $\boldsymbol{R}_u \otimes_c \boldsymbol{R}_v$ and $\boldsymbol{R}_v \otimes_c \boldsymbol{R}_u$, and therefore, the performance of all the covariance-based DOA algorithms are sensitive to the temporal correlation of the sources when the noise is temporally correlated. In the next subsections, we show that the asymptotic performance of DOA algorithms that require the sources spatially uncorrelated are sensitive to the distribution and the coloration of the spectrum of the sources, even when the noise is temporally uncorrelated.

C. Toeplitzation Techniques

For M-uniform linear array (ULA) [resp., $M_1 \times M_2$-uniform rectangular array (URA)] spatially uncorrelated sources and spatially white noise, $\boldsymbol{R}_x$ exhibits a Toeplitz [resp., Toeplitz, block-Toeplitz] structure. The estimated spatial covariance matrix $\boldsymbol{R}_x(n)$ accuracy is significantly improved by averaging along its diagonals [resp., its subblock diagonals]. The resulting estimate $\boldsymbol{R}_x^{\text{to}}(n)$ is referred to as the "Toeplitzed" estimated spatial covariance matrix. Because this "Toeplitzation," which is also known as redundancy averaging [10], operates a linear transform on $\boldsymbol{R}_x(n)$ thanks to the "Toeplitzation" projection matrix $\boldsymbol{A}_{\text{to}}$ $(\text{Vec}(\boldsymbol{R}_x^{\text{to}}(n)) = \boldsymbol{A}_{\text{to}}\text{Vec}(\boldsymbol{R}_x(n)))$, Theorem 1 is extending to $\boldsymbol{R}_x^{\text{to}}(n)$ with the asymptotic covariance matrix $\boldsymbol{C}_{R_x}^{\text{to}} = \boldsymbol{A}_{\text{to}}\boldsymbol{C}_{R_x}\boldsymbol{A}_{\text{to}}$. By the regularity condition (4.1) of Section IV-A, the estimated DOA's are asymptotically normal with asymptotic covariance:

$$\boldsymbol{C}_\Theta^{\text{to}} = \boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}\boldsymbol{A}_{\text{to}}\boldsymbol{C}_{R_x}\boldsymbol{A}_{\text{to}}(\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x})^H. \tag{4.7}$$

In contrast to the classical covariance-based DOA algorithms, we show in the following that the Toeplitzed covariance-based DOA algorithms are sensitive to the spectral shape of the spectrum of the sources. More precisely, the following properties are proved.[3]

1) The Toeplitzation is not sensitive to the distribution of the ARMA sources if the sources are not only spatially uncorrelated but are independent.

   Proof: If the sources are independent, the only nonzero terms of the fourth-order cumulant matrix $\boldsymbol{Q}_u$ are the terms $[\boldsymbol{Q}_u]_{K(k-1)+k,\,K(k-1)+k} = c_k \stackrel{\text{def}}{=} \int_{-1/2}^{+1/2}\int_{-1/2}^{+1/2} \rho_{k,k,k,k}(f, f', -f')\,df\,df'$, and the fourth-order cumulant term of (3.3) boils down to $\sum_{k=1}^K c_k(\boldsymbol{e}_M(\theta_k) \otimes_c \boldsymbol{e}_M(\theta_k))(\boldsymbol{e}_M^H(\theta_k) \otimes_c \boldsymbol{e}_M^H(\theta_k))$, where $\boldsymbol{e}_M(\theta_k) \stackrel{\text{def}}{=} (1, e^{i\theta_k}, e^{2i\theta_k}, \cdots, e^{(M-1)i\theta_k})^T$. The DOA algorithms $\text{alg}(\cdot)$ applied to $\boldsymbol{R}_x^{\text{to}}(n)$ define the mapping $\text{alg}^{\text{to}}(\cdot)$. This mapping satisfies condition 2b of Lemma 1. Thus, the second constraint on $\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}$ of Lemma 1 establishes the result. ■

2) In the single source case, the Toeplitzation is not sensitive to the temporal correlation of the sources.

   Proof: In this case, condition 2a of Lemma1 reduces to condition 2b. The expressions of $\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}\boldsymbol{A}_{\text{to}}\boldsymbol{C}_{R_x}\boldsymbol{A}_{\text{to}}\boldsymbol{D}^{\text{alg}\,H}_{\boldsymbol{\Theta},\boldsymbol{R}_x}$ coincide for the ARMA and harmonic sources with $\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x}\boldsymbol{A}_{\text{to}}(\sigma_1^2\boldsymbol{e}_M(\theta_1)\boldsymbol{e}_M^H(\theta_1) \otimes_c \sigma_v^2\boldsymbol{I}_M + \sigma_v^2\boldsymbol{I}_M \otimes_c \sigma_1^2\boldsymbol{e}_M(\theta_1)\boldsymbol{e}_M^H(\theta_1) + \sigma_v^4\boldsymbol{I}_{M^2})\boldsymbol{A}_{\text{to}}(\boldsymbol{D}^{\text{alg}}_{\boldsymbol{\Theta},\boldsymbol{R}_x})^H$. ■

3) In the case of several sources, the Toeplitzation is sensitive to the temporal correlation of the sources.

   Proof: Because $\boldsymbol{e}_M(\theta_k)\boldsymbol{e}_M^H(\theta_l)$, $k \neq l$ does not have a Toeplitz structure, and the column space of $\boldsymbol{A}_{\text{to}}(\boldsymbol{E}(\boldsymbol{\Theta}) \otimes_c \boldsymbol{E}(\boldsymbol{\Theta}))$ does not belong to the column space of $\boldsymbol{E}(\boldsymbol{\Theta}) \otimes_c \boldsymbol{E}(\boldsymbol{\Theta})$. Condition 2a of Lemma 1 is generally not

[2]A similar approach based on the implicit function theorem was introduced by Xu and Kaveh [9].

[3]The following properties are proved for ULA; the proofs can be extended to URA along the same lines.

satisfied. Therefore, the associated terms in (4.7) do not vanish, contrary to the single source case. Therefore, the extra term

$$\boldsymbol{A}_{\rm to}\left(\sum_{1\le l\ne k\le K}\left(\int_{-1/2}^{+1/2} S_{u_k}(f)S_{u_l}(f)\,df\right)\cdot \boldsymbol{e}_M(\theta_k)\boldsymbol{e}_M^H(\theta_k)\otimes_c \boldsymbol{e}_M(\theta_l)\boldsymbol{e}_M^H(\theta_l)\right)\boldsymbol{A}_{\rm to} \qquad (4.8)$$

that appears for ARMA sources does not vanish in $\boldsymbol{C}_\Theta^{\rm to}$. We note that the performance of the ARMA sources case and the harmonic source case coincides when the spectrums of the ARMA sources tend to be disjoint. At low SNR, $\boldsymbol{C}_{R_x}$ can be approximated by $\sigma_v^4\boldsymbol{I}$, and the Toeplitzation becomes insensitive to the temporal correlation of the sources. Furthermore, the closed-form expressions given for two closely spaced sources at low SNR in ([11, rel. 9.118 and 9.119]) and in [12] remains valid for colored sources. At high SNR, the term (4.8) becomes dominant in $\boldsymbol{C}_{R_x}$, and the Toeplitzation becomes very sensitive to the temporal correlation of the sources, which will be confirmed in Section V. ∎

D. Augmention Techniques

The linear or planar sparse arrays attract considerable attention as they lead to significantly improved performance [13] for spatially uncorrelated and white sources.[4] To show that these techniques are sensitive to the temporal correlation of the sources, we consider only the standard method utilizing the direct augmentation approach [15]. To fix notations, consider a planar grid (a half wavelengh of the incident radiation equispaced). Let Γ be the array characteristic function

$$\Gamma(x,\,y)=\begin{cases}1, & \text{if a sensor is in position } (x,\,y)\\ 0, & \text{elsewhere.}\end{cases}$$

Let $\Lambda(dx,\,dy)$ be the autocorrelation function of the array characteristic function Γ [$\Lambda(dx,\,dy)$ represents the number of times the lag $(dx,\,dy)$ is present in the sparse array.] $\boldsymbol{R}_x(n)$ and $\boldsymbol{R}_x^{\rm au}(n)$ denote the spatial covariance matrices associated with the fictitious $M_1'\times M_2'$-URA and the $M_1\times M_2$ augmented array, respectively.[5] Then, the direct augmentation approach [15] operates a linear transform on $\boldsymbol{R}_x(n)$[6] thanks to the augmentation operator $M_1^2M_2^2\times M_1'^2M_2'^2$ matrix $\boldsymbol{A}_{\rm au}$: (Vec($\boldsymbol{R}_x^{\rm au}(n)$) $=\boldsymbol{A}_{\rm au}$Vec($\boldsymbol{R}_x(n)$)).[7]

$$[\boldsymbol{A}_{\rm au}]_{(h-1)M_1^2M_2+(j-1)M_1M_2+(g-1)M_1+i,\;(f-1)M_1'^2M_2'+(l-1)M_1'M_2'+(e-1)M_1'+k} = \begin{cases}\dfrac{\Gamma(k-1,\,e-1)\Gamma(l-1,\,f-1)}{\Lambda(l-k,\,f-e)}, & \text{if } \begin{cases} l-k=j-i\\ f-e=h-g\end{cases}\\ 0, & \text{elsewhere.}\end{cases} \qquad (4.9)$$

Theorem 1 extends to $\boldsymbol{R}_x^{\rm au}(n)$ with the asymptotic covariance matrix $\boldsymbol{C}_{R_x}^{\rm au}=\boldsymbol{A}_{\rm au}\boldsymbol{C}_{R_x}\boldsymbol{A}_{\rm au}^T$. By the regularity condition (4.1) of Section IV-A, the estimated DOA's are asymptotically normal with asymptotic covariance

$$\boldsymbol{C}_\Theta^{\rm au}=\boldsymbol{D}_{\Theta,\boldsymbol{R}_x}^{\rm alg}\boldsymbol{A}_{\rm au}\boldsymbol{C}_{R_x}\boldsymbol{A}_{\rm au}^T(\boldsymbol{D}_{\Theta,\boldsymbol{R}_x}^{\rm alg})^H. \qquad (4.10)$$

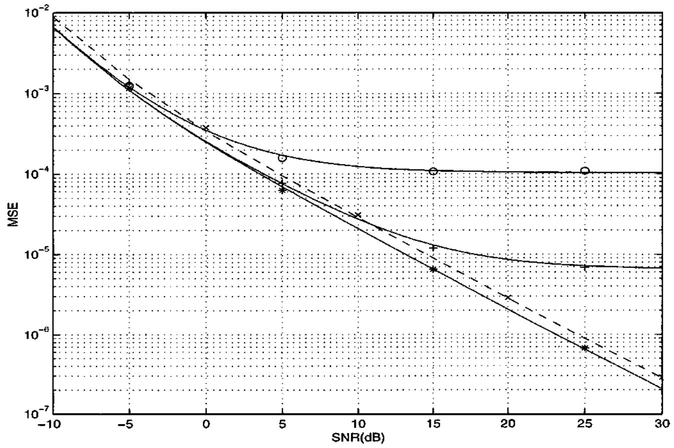

Fig. 1. Theoretical and estimated MSE of $\theta_k(n)$ versus the SNR for, respectively, white (o), colored (+), and harmonic $(*)$ signals for a 10–ULA array, $n=100$ after Toeplitzation (—) and without Toeplitzation (- - -).

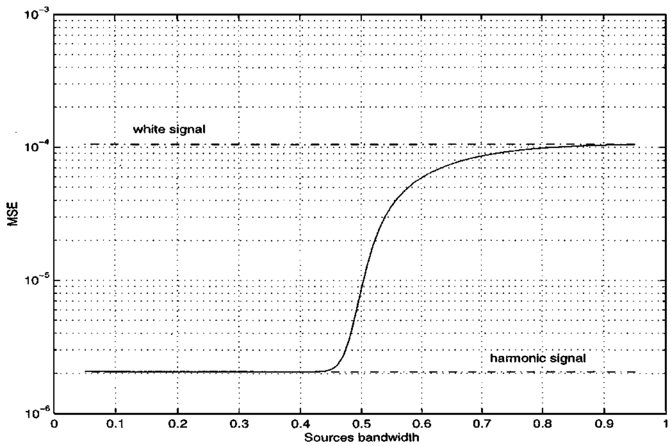

Fig. 2. Theoretical MSE of $\theta_k(n)$ versus the sources bandwidth for a 10–ULA array, SNR=20 dB after Toeplitzation.

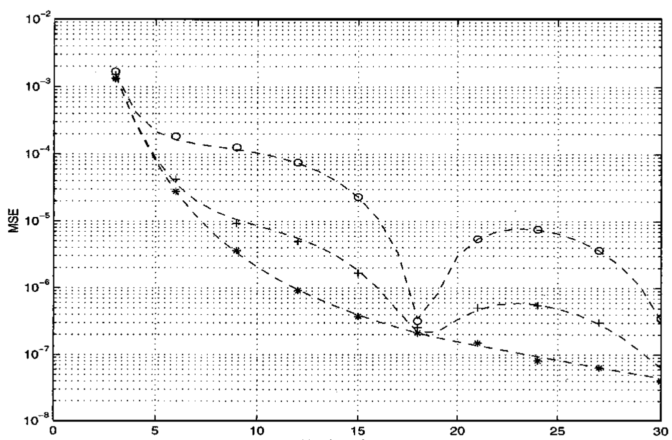

Fig. 3. Theoretical and estimated MSE of $\theta_k(n)$ versus the number of sensors of an ULA array, SNR=20 dB, $n=100$ for white (o), colored (+) and harmonic $(*)$ signals after Toeplitzation.

[4]An improvement of the performance of DOA algorithms when the sources are spatially correlated was proposed by using the redundancy averaging techniques. Under these conditions, these techniques lead to asymptotical inconsistent and biased estimates [14].

[5]$M_1=M_1'$ and $M_2=M_2'$ for restricted redundancy and $M_1<M_1'$ and $M_2<M_2'$ for unrestricted redundancy [13].

[6]We note that this linear transform is defined only for analysis purpose as $\boldsymbol{R}_x(n)$ is not observed.

[7]In the case of linear arrays, $e=f=g=h=1$, and (4.9) reads

$$[\boldsymbol{A}_{\rm au}]_{(j-1)M+i,\,(l-1)M'+k}=\begin{cases}\dfrac{\Gamma(k-1)\Gamma(l-1)}{\Lambda(l-k)}, & \text{if } l-k=j-i\\ 0, & \text{elsewhere.}\end{cases}$$

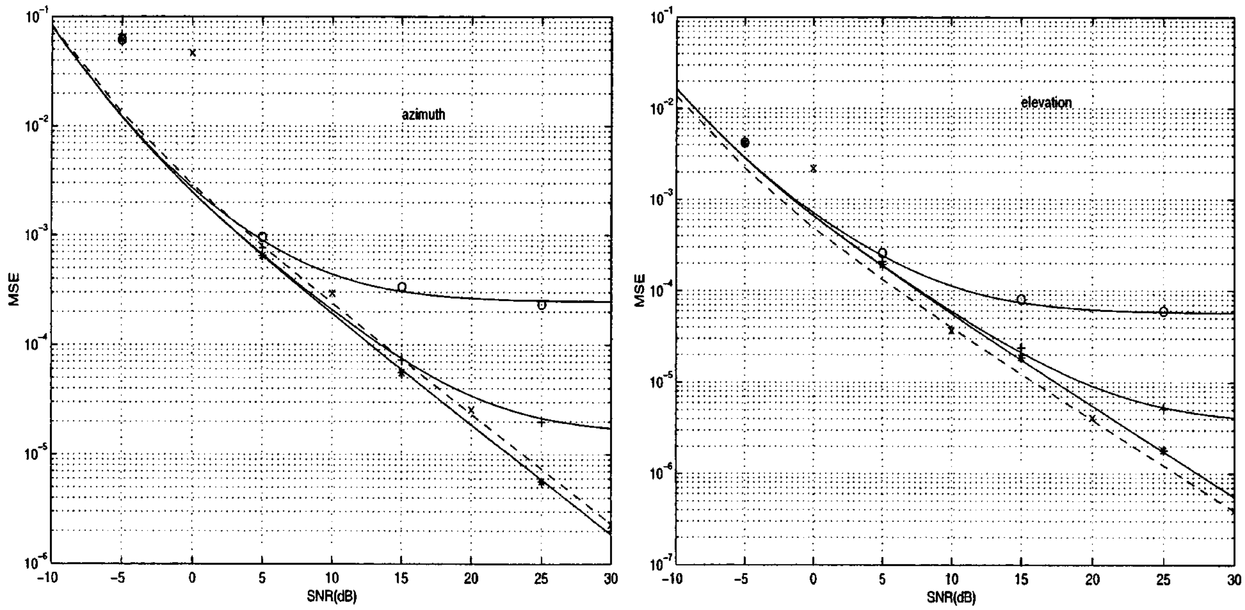

Fig. 4. Theoretical and estimated MSE of $\theta_k(n)$ and $\phi_k(n)$ versus the SNR for, respectively, white (either Gaussian or discrete) (o), colored (+), and harmonic $(*)$ signals for a 12-Greene and Wood array, $n = 100$ after standard augmentation technique (—) and without augmentation (- - -).

We note that Lemma 1 does not apply to this situation because the mapping $\text{alg}^{\text{au}}(\cdot)$ is not defined on $\boldsymbol{R}_x(n)$ but on some terms of $\boldsymbol{R}_x(n)$ only. Consequently, the insensitivity of the augmentation techniques to the distribution and the temporal correlation of the sources is not assured.

V. Simulations

We consider throughout this section two sources of equal power $(\sigma_k^2)_{k=1,2}$. The SNR is defined as the ratio $((\sigma_1^2+\sigma_2^2)/\sigma_v^2)$. The DOA's are estimated by the standard MUSIC algorithm, and the number of Monte Carlo runs is 500. These sources are issued from the DOA's $\theta_k = \pi \sin\theta'_k$ with $\theta'_1 = 30^\circ$, $\theta'_2 = 20^\circ$ for the ULA and from the DOA's $\theta_k = \pi\sin\theta'_k \sin\phi'_k$, $\phi_k = \pi\cos\theta'_k\sin\phi'_k$ with $\theta'_1 = 30^\circ$ $\theta'_2 = 20^\circ$ and $\phi'_1 = 10^\circ$ $\phi'_2 = 40^\circ$ for the URA.

The first experiment presents the case of two sources that are both spatially uncorrelated, white Gaussian, ARMA Gaussian [generated by a (10,10) Butterworth filter driven by a white Gaussian noise], or harmonic. The centered frequencies of the ARMA and the frequencies of the harmonics are -0.25 and 0.25. Fig. 1 plots the theoretical MSE of θ_k $(1/n)[\boldsymbol{C}_\Theta]_{k,k}$ and the estimated MSE $E\|\theta_k(n) - \theta_k\|^2_{\text{Fro}}$ as a function of the SNR for an 10–ULA (the bandwidth is fixed to 0.5 for ARMA signals) after Toeplitzation. We observe that these estimated MSE's are in good agreement with the theoretical MSE's but are very sensitive to the temporal correlation of the sources. Fig. 2 plots the theoretical MSE of θ_k $(1/n)[\boldsymbol{C}_\Theta]_{k,k}$ for the ARMA Gaussian spatially uncorrelated sources as a function of the sources bandwidth for a 10–ULA. We observe that these theoretical MSE's increase with this bandwidth and begin increasing from the bandwidth 0.45, which is associated with the overlapping of the spectrum of the two sources. These MSE's increase from the value associated with two harmonic sources to the value associated with two white sources. The "saturation" phenomena observed in [10] disappears when the spectrum of the two sources are not overlapping. Note that the common expression of the MSE's obtained without Toeplitzation is close to the MSE's obtained after Toeplitzation for nonoverlapping spectra. Fig. 3 plots the theoretical MSE of θ_k $(1/n)[\boldsymbol{C}_\Theta]_{k,k}$ and the estimated MSE $E\|\theta_k(n) - \theta_k\|^2_{\text{Fro}}$ as a function of the number of sensors for an ULA for SNR=20 dB. We observe that these estimated MSE's are in good agreement with the theoretical MSE, but these MSE's are very sensitive to the temporal correlation of the sources (the bandwidth is fixed to 0.5 for ARMA signals). These MSE's are decreasing with the number of sensors except for the case of overlapping spectrums where the "saturation" phenomena can lead to a degradation of the MSE.

The second experiment presents the case of two sources with the same temporal parameters (for white source signals, the distribution is either Gaussian or discrete $\{-1, +1\}$) as in the first experiment but are impinging on a 12–Greene and Wood array [16] utilizing the direct augmentation approach [15]. Fig. 4 plots the theoretical MSE and the estimated MSE of the angles θ_k and ϕ_k as a function of the SNR. The behavior of these MSE's is similar to those of the MSE's obtained for the ULA Toeplitzation situation.

VI. Conclusion

In this correspondence, we have presented an asymptotic performance analysis of DOA finding algorithms using the stochastic model assumption in which both source and noise signals are possibly non-Gaussian and possibly temporally correlated. We have shown that the asymptotic statistical performance of the second-order DOA finding algorithms generally depend on the temporal correlation of the source and noise signals, but when the noise is supposed temporally uncorrelated, it is proved that the covariance-based DOA estimators that do not require the sources to be spatially uncorrelated are insensitive to the distribution and the temporal correlation of the source signals, unlike the Toeplitzation and the augmentation techniques that are very sensitive.

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Blind Identification of Multichannel FIR Systems Based on Linear Prediction

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***Abstract*—In this correspondence, a blind identification algorithm for multichannel FIR systems is proposed. In the approach, the identification of each channel is decoupled, and channel responses are estimated separately without having to solve for the augmented channel responses. The algorithm can be implemented using linear prediction techniques. It is computationally efficient and suitable for real-time applications. Computer simulations are used to demonstrate the effectiveness of the proposed algorithm.**

***Index Terms*—Communication channels, FIR digital filter, identification, least squares method, parameter estimation, Z transforms.**

I. Introduction

The problem of blind channel identification has been extensively studied by researchers since the pioneering work by Sato [1]. It has received considerable attention in communications and signal processing society [2]. Blind identification refers to the process in which the channel responses are estimated based on the channel outputs without using training sequences. Most of the earlier approaches to blind identification are based on the use of higher order statistics, which are known to suffer from many drawbacks. They usually require a large number of data samples and a heavy computational burden, making them unattractive for practical applications. A recent major progress is made by Tong *et al.* [3] in which they explored the cyclostationary properties of an oversampled communications signal and proposed an approach for estimating the channel responses based on the second-order statistics of the channel outputs. Since then, many techniques have been developed including the eigenstructure-based methods [4], [5] and the least squares (LS) method [6]. The LS method has the advantage of not requiring the explicit statistical knowledge of the channel input. This is important for many practical applications where such information is not available. The LS method uses a cross relationship between each pair of sensor outputs to estimate the augmented channel responses, i.e., the channel responses formed by appending one channel response vector to another. In applications where the channel orders are is relatively small, the computation cost of the LS method may be affordable. However, for applications such as speech dereverberation and echo cancellation, where each channel may have an order at the level of several hundreds, the computational burden of the LS method may increase significantly due to the increased dimension of the augmented channel responses, making it difficult for real-time implementations.

In this correspondence, an efficient blind identification algorithm for multichannel FIR systems is proposed based on a deterministic modeling for the channel inputs. The approach uses a relationship between a pair of channel outputs in the z-plane. In the z domain, uniformly distributed sampling points on the unit circle are used to interpolate the z-transforms of the channel impulse responses. In the time domain, we show that each channel response vector forms a linear prediction relationship among the data sequence obtained from the inverse Fourier transforms of a ratio function between the z-transforms of the pair of channel outputs at interpolation points on the z-plane. The channel responses are then estimated by solving a set of linear prediction equations. The proposed algorithm decouples the estimation of each channel impulse responses. Unlike the LS method, which requires a solution for an augmented channel impulse response vector of an increased dimension, the proposed algorithm estimates each individual channel responses separately. The decoupling of the estimation process can reduce the computational complexity of the algorithm. In addition, since the proposed algorithm is a linear prediction process, many existing fast algorithms can be applied, making it practical for real-time applications. This correspondence is organized as follows. In Section II, we formulate the multiple FIR channel data model. Section III is devoted to the development of the new blind channel identification algorithm. In Section IV, we discuss the implementations and computational complexity of the algorithm. Finally, in Section V, computer simulations are used to demonstrate the effectiveness of the proposed algorithm.

II. Problem Formulation

Consider the single-input/multiple-output (SIMO) system shown in Fig. 1. Each channel is assumed to be a unknown finite impulse responses (FIR) system. The multiple sensor model can be found to be useful for many practical applications. For example, in mobile communications systems and multisensor data fusion, multiple sensors are usually deployed to achieve optimum performance and to reduce the data uncertainties. It can also be used for single receiver systems where virtual receivers are formulated by temporally oversampling

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