A Local Minimum Stagnation Avoidance in Design of CSD Coefficient FIR Filters by Adding Gaussian Function

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***Abstract*—In this paper, a method for designing FIR (Finite Impulse Response) filters with CSD (Canonic Signed Digit) coefficient using PSO (Particle Swarm Optimization) is proposed. In such a design problem, there are a large number of local minimums. The updating procedure of normal PSO tends to stagnate around such a local minimum and thus indicates a premature convergence property. Therefore, a new method for avoiding such a situation is proposed, in which the Gaussian function is added as a penalty to the evaluation function. Several design examples are shown to present the effectiveness of the proposed method.**

I. Introduction

Digital filters are widely used in signal processing applications[1]. Especially, FIR filters are able to realize a perfect linear phase characteristic and to assure a stability. In a hardware implementation of FIR filters, the CSD representation is well-known as a promising approach for reducing a scale of filter circuits[2][3]. However, this design problem falls into a NP-hard problem[4]. Thus, several heuristic approaches have been developed. The PSO is one of the heuristic approaches which can solve this problem fast[5].

Essentially, the PSO was applied to the optimization problems having continuous variables. Therefore, the PSO can't be applied to the CSD coefficient FIR filter design problem having discrete variables directly. The simple approach is to round the nearest CSD coefficient every iteration[6]. However, this operation needs an extra procedure for the PSO. In this paper, a constraint for the CSD representation was introduced in the evaluation function as a penalty[7]. As a result, the PSO can work without the extra procedure for the CSD representation.

The searching of the PSO algorithm has a high directivity toward local minimums, and thus the PSO can enumerate local minimum solutions quickly. However, the high directivity causes a local minimum stagnation because the design problem of the CSD coefficient FIR filter has a large number of local minimums. The StPSO (Strengthened PSO) is one of the successful methods for avoiding such a situation[8]. The StPSO algorithm can avoid the local minimum stagnation using a neighborhood search and a reallocation by the pioneer particle. According to our investigation, the StPSO has achieved about 5% error improvement in comparison with the normal PSO.

In this paper, a new method for avoiding a local minimum stagnation is proposed, in which the Gaussian function is added as the penalty to the evaluation function. Then, the evaluation function has a high penalty at a stagnation point. Adding the Gaussian function makes the PSO updating escape from the stagnation point. Several design examples are shown to present the effectiveness of the proposed method.

II. Problem Description of Designing FIR Filters with CSD Coefficient

In this paper, we consider the design problem of linear phase FIR filters. Assuming the filter has the even order N and the even-symmetric impulse response, a magnitude response $H(\omega)$ is described as,

$$H(\omega) = \sum_{n=0}^{M} d_n \cos n\omega, \tag{1}$$

where $M = N/2$, $d_n = 2h_{M-n}$, $d_0 = h_M$ and ω is the angular frequency. The FIR filter design problem in a min-max criterion to a desired response $D(\omega)$ on the approximation band Ω is defined as,

$$\min_{d_0,\cdots,d_M} \max_{\omega\in\Omega} |D(\omega) - H(\omega)|. \tag{2}$$

Then, the error is calculated on the discretized frequency ω_i, $i = 0, 1, \cdots, S-1$. For a reduction of circuit complexity, it is important to reduce the number of nonzero digits included in each filter coefficient. The CSD representation can describe the coefficient by the minimum number of nonzero digits. In the CSD representation, d_n is expressed as,

$$d_n = \sum_{j=1}^{p} c_{n,j} 2^{-j}, \tag{3}$$

where $c_{n,j} \in \{1, 0, -1\}$, $n = 0, 1, \cdots, M$, p is a word length. Then, the maximum number of nonzero digits L_u is fixed for each coefficient to reduce the number of nonzero digits.

For a simplicity of treatment as the integer programming problem, the filter coefficient is scaled as following,

$$a_n = 2^p d_n. \tag{4}$$

Similarly, the desired response $D(\omega)$ is scaled by 2^p.

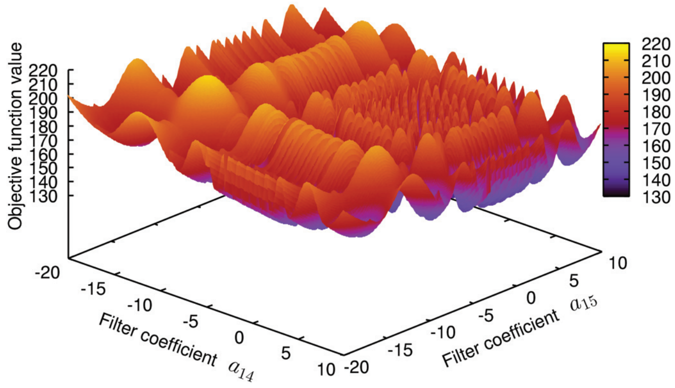

Fig. 1. A surface of evaluation function $F(\boldsymbol{x})$.

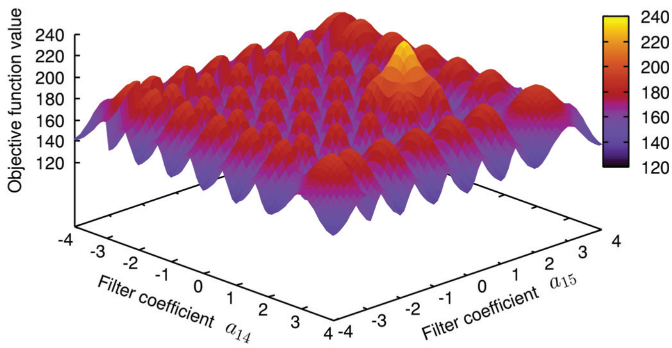

Fig. 2. A surface of evaluation function $F'(\boldsymbol{x})$ after adding the Gaussian function.

III. Particle Swarm Optimization

The PSO is one of the swarm optimization methods and applicable to the nonlinear optimization problems. In the PSO, multiple particles search for the minimum solution of the evaluation function.

The position vector $\boldsymbol{x}_u = [a_0, a_1, \cdots, a_M]^T$ is defined for the particle u and $\boldsymbol{x}_u$ is updated based on both the best solution of its particle $\boldsymbol{p}_u$(p-best) until k-th iteration and the best solution of current swarm $\boldsymbol{p}_g$(g-best) as following,

$$\left\{ \begin{array}{l} \boldsymbol{v}_u^{k+1} = w_k \boldsymbol{v}_u^k + c_1 r_1 (\boldsymbol{p}_u^k - \boldsymbol{x}_u^k) + c_2 r_2 (\boldsymbol{p}_g^k - \boldsymbol{x}_u^k), \\ \boldsymbol{x}_u^{k+1} = \boldsymbol{x}_u^k + \boldsymbol{v}_u^{k+1}, \end{array} \right.$$

where $\boldsymbol{v}_u$ is a speed vector of the particle u, k is the number of iterations, r_1, r_2 is the random number in a interval of $[0, 1]$, c_1, c_2, w is the arbitrary constant.

The evaluation function of the PSO is defined in consideration with the constraint of the CSD representation following,

$$F(\boldsymbol{x}) = W\delta + s\phi(\boldsymbol{x}), \tag{5}$$

where δ is the maximum error. W, s is a weight factor. $\phi(\boldsymbol{x})$ is the penalty function for the CSD representation defined as,

$$\phi(\boldsymbol{x}) = \sum_{n=0}^{M} \frac{1}{2} \left[\sin \left(\frac{2\pi \{ a_n - \frac{1}{4}(b_{n,m+1} + 3b_{n,m}) \}}{b_{n,m+1} - b_{n,m}} \right) + 1 \right], \tag{6}$$

where $b_{n,m}$ is a set of the CSD numbers which can be expressed by the word length p with the number of nonzero digits L_u. Thus, the PSO can work without the extra procedure for the CSD representation.

IV. Local minimum Stagnation Avoidance of PSO

The shape of objective function $F(\boldsymbol{x})$, in which $N = 30$, $p = 8$ and $L_u = 3$ is shown in Fig.1. According to the Fig.1, there are a large number of local minimums in $F(\boldsymbol{x})$.

The updating produce of the normal PSO tends to stagnate around such a local minimum and thus indicates a premature convergence property. A new method for avoiding such a situation is proposed, in which the Gaussian function is added as a penalty to the evaluation function $F(\boldsymbol{x})$. The new evaluation function $F'(\boldsymbol{x})$ is defined as,

$$F'(\boldsymbol{x}) = W\delta + s\phi(\boldsymbol{x}) + \sum_{j=1}^{I} q_j \psi_j(\boldsymbol{x}), \tag{7}$$

where q_j is a weight factor. I is the number of local minimum stagnation. $\psi(\boldsymbol{x})$ is the Gaussian function defined as,

$$\psi_j(\boldsymbol{x}) = \prod_{n=0}^{M} e^{-\left\{ \frac{(a_n - p_{g,n})^2}{2\sigma^2} \right\}}. \tag{8}$$

The shape of the objective function $F'(\boldsymbol{x})$ after adding the Gaussian function at $\boldsymbol{p}_g = \{1, 1\}$ is shown in Fig.2. It is apparent that the shape of objective function $F'(\boldsymbol{x})$ has a high penalty at $\boldsymbol{p}_g$. Thus, the PSO can avoid the local minimum stagnation. Then, a directivity for $\boldsymbol{p}_g$ and $\boldsymbol{p}_u$ tends to restrict the PSO updating to the another local minimum. For this reason, $\boldsymbol{p}_g$ and $\boldsymbol{p}_u$ is initialized after a value of $\boldsymbol{p}_g$ was recorded.

TABLE I
Specification of each design example.

	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
N	30	40	56	76	100	130	150
f_p	0.100	0.185	0.225	0.170	0.220	0.210	0.215
f_s	0.180	0.245	0.265	0.200	0.240	0.228	0.231
p	8	8	8	8	16	16	16
L_u	3	3	3	3	3	3	3
q	100	100	100	100	200	200	200

V. Design Example

Several design examples are shown to verify the effectiveness of the proposed method. The desired response was defined for each example as,

$$D(\omega) = \left\{ \begin{array}{ll} 1, & 0 \leq \omega \leq 2\pi f_p, \\ 0, & 2\pi f_s \leq \omega \leq \pi, \end{array} \right. \tag{9}$$

where f_p is the passband edge frequency and f_s is stopband edge frequency. Then, $S = 10M$. The value of weight factors

TABLE II
PARAMETER OF PSO.

	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
P	20	30	30	60	60	60	70
T	5000	5000	5000	5000	10000	10000	10000
P_P	3	5	5	15	20	30	30
R	100	100	150	200	250	300	320

were chosen as $W = 3$, $s = 1$. The Gaussian function variance was set to $\sigma = 0.09$. The design specification of each example is shown in Table I.

The PSO parameters of each example is listed in Table II. In the Table II, the number of particles was set to P. T denotes the number of iterations. In total, 20 trials were attempted for the each example. From preliminary experiments, the parameters of PSO were chosen as $c_1 = 1.0$, $c_2 = 2.0$, $w = 0.8$. As a comparison, the design using the StPSO was carried out. P_P denotes the number of the pioneer particles, R denotes the number of the neighborhood solutions generated by the pioneer particles using neighborhood search.

Design results of the proposed method and the StPSO are shown in Table III. In the Table III, δ_{mean} denotes the average of the maximum error, δ_{min} denotes the best of maximum error obtained among all trials. The magnitude response of the Ex.4 and its passband response are shown Fig.3 and Fig.4. The magnitude response of the Ex.6 and its passband response are shown Fig.5 and Fig.6.

The results listed in the Table. III showed that the proposed method could reduce the maximum error and the average error in comparison with the StPSO. Updating curve of the evaluation function in the Ex.4 are shown Fig.7. In the Fig.7, the StPSO converged at about the 1500 iteration. On the other hand, the proposed method is able to enumerate the good solution up to the end of iterations.

VI. CONCLUSIONS

In this paper, a method for local minimum stagnation avoidance in designing CSD coefficient FIR filters using the PSO was proposed. In the method, the Gaussian function was added to the evaluation function as the penalty term. As a result, it was shown that the proposed method could reduce the design error in comparison with the StPSO.

REFERENCES

[1] A. V. Oppenheim and R. W. Schafer “Discrete-time Signal Processing,” *Prentise hall,* 1989.

[2] Q. Zhao and Y. Tadokoro “A Simple Design of FIR with Powers–of–Two Coefficients,” *IEEE Trans. CAS,* vol. 35, no.4, 1988, pp.566–570, May

[3] H. Samueli, “An improved Search Algorithm for the Design of Multiplierless FIR Filters with Power-of-two Coefficients,” *IEEE Trans. CAS,* vol. 36, no7, 1989, pp.1044–1047, Jul

[4] N. Takahashi and K. Suyama, “Design of CSD coefficient FIR filters based on branch and bound method,” *Proc. IEEE ISCIT2010,* 2010, pp.570–573.

[5] J. Keneddy and R. D. Eberhart, “Particle Swarm Optimization,” *Proc. IEEE Int. Conf. Neural Network* vol. 4, 1995, pp.1942–1948.

[6] Seyyed Ali Hasheni and Behrouz Nowrouzian, “A Novel Finite-Wordlength Particle Swarm Optimization Technique for FRM IIR Digital Filter,” *Proc. IEEE ISCAS 2011,* 2011, pp.2745–2748. May

[7] D. K. Shin, Z. Gurdal, and O. H. Griffin, “A Penalty Approach for Nonlinear Optimization with Discreate Design Variables,” *Enginnering Optimization,* vol. 16, 1990, pp.29–42.

[8] Zulal Sevkli and Erdgan Sevilgen, “StPSO : Strengthened particle swarm optimization,” *Turkish Journal Of Electrical Engineering & Computer Sciences,* vol.18, no.3, 2010, pp.1095–1114.

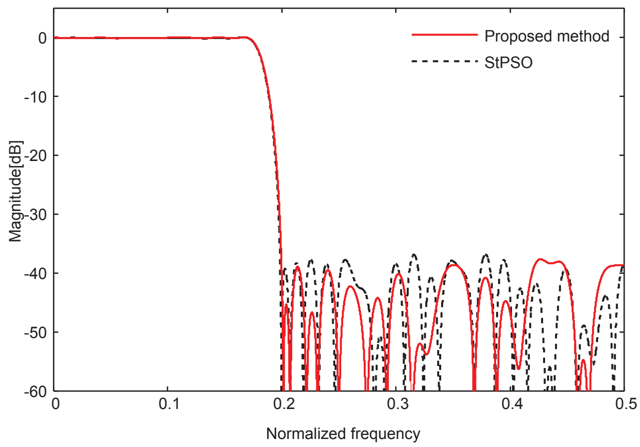

Fig. 3. Magnitude response of the Ex.4.

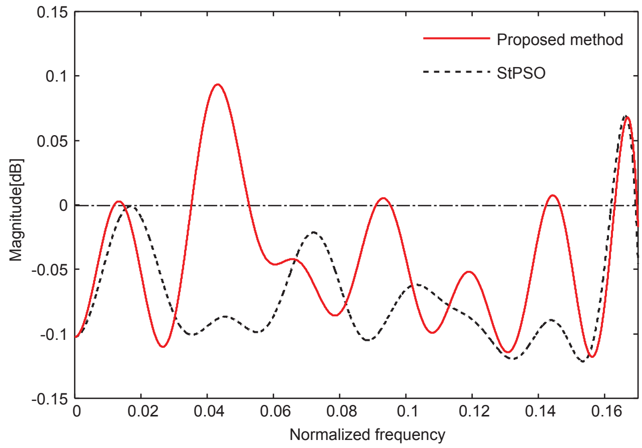

Fig. 4. Passband magnitude response of the Ex.4.

TABLE III
EXPERIMENTAL RESULTS.

		Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
StPSO	δ_{min}	0.886	0.846	1.313	1.456	1.093	0.782	0.678
	δ_{mean}	0.965	0.974	1.561	1.640	1.183	0.990	0.799
	δ_{worst}	1.192	1.216	1.906	1.979	1.287	1.135	1.025
Proposed method	δ_{min}	0.886	0.827	1.194	1.326	1.079	0.747	0.654
	δ_{mean}	0.928	0.916	1.416	1.610	1.164	0.868	0.735
	δ_{worst}	1.112	0.979	1.571	1.957	1.229	0.937	0.779

δ_{min}, δ_{mean}, δ_{worst}:$\times 10^{-2}$

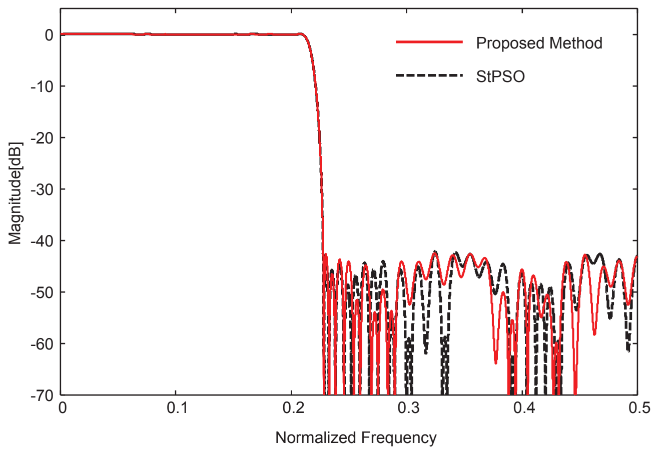

Fig. 5. Magnitude response of the Ex.6.

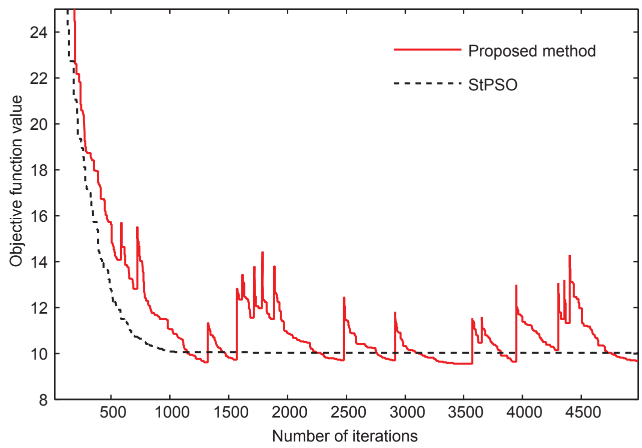

Fig. 7. Updating curve of the evaluation function in the Ex.4.

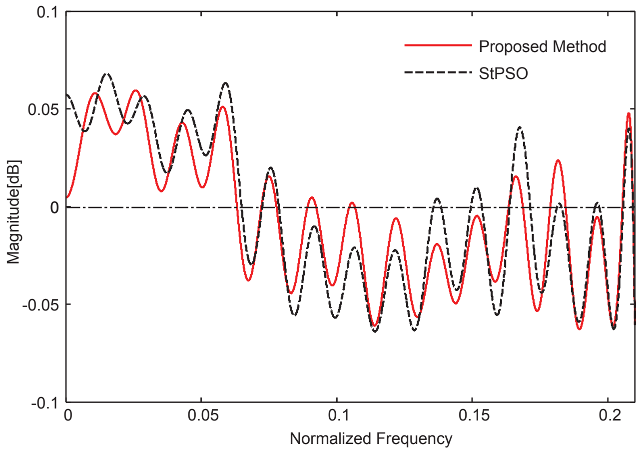

Fig. 6. Passband magnitude response of the Ex.6.