Novel Routing Schemes for IC layout
Part I: Two-Layer Channel Routing *

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Abstract

We present new channel routing algorithms and theory that consider the characteristic of net crossings. The routing strategy is based on parallel bubble sorting and river routing techniques. A function named POTENTIAL, can be evaluated to indicate the required channel height for a given channel without actually carrying out the routing steps. Non-Manhattan wires as well as overlapping wires are introduced. Preliminary results show that a class of channel routing problems can be routed in height less than the Manhattan density.

1 Introduction

Channel routing is one of the most important phases of physical design of VLSI chips as well as PC boards. Many breakthroughs [22, 8, 17, 5, 4, 2, 19, 10, 15, 20] in channel routing theory and algorithms have been reported. In this paper, we propose two new routing models, the *mini-swap* model and the *overlap* model are introduced. The routing strategy is very different from the Manhattan approach. In particular, vertical constraints no longer exist. Furthermore, the solution produced by either model consists of a minimal set of net crossings which leads to a small number of vias. To characterize and to evaluate the performance of each model, a function called POTENTIAL is used. Intuitively, the POTENTIAL function measures the degree of difficulty for a given channel routing problem. Based on the performance analysis, we attempt to combine the strength of both models to form a new *hybrid* router.

2 The Problem

A channel is a pair of vectors of nonnegative integers - TOP and BOT - of the same dimension.

TOP = $t(1)$, $t(2)$, ...$t(n)$
BOT = $b(1)$, $b(2)$, ...$b(n)$

We assume that these numbers are the labels of grid points located along the top and bottom edge of a rectangle. Points having the same positive label have to be interconnected, i.e. they define nets. A 100% routing completion is required and the objective is to minimize the channel area and the number of vias.

A channel is dense if every grid point on the top and bottom boundaries is occupied by a terminal. Let $\{1,2,....n\}$ denote the set of nets. Then in a dense 2-terminal net channel, TOP and BOT are permutations of $\{1,2,....n\}$. Without loss of generality, we may assume that nets are arbitrarily ordered on the bottom and are naturally ordered on the top. This is stated as:

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Definition 1 *A dense 2-terminal net (D2TN) channel routing problem is specified by:*
TOP = 1, 2, 3, ... n
BOT = a permutation of { 1, 2, ... n }
We denote the top and bottom terminals of net i by (i, q_i).

3 The Mini-swap Model

3.1 Definition of the Mini-Swap Model

The basic idea of the new model is to swap a pair of neighboring nets by two wires, one in the +45° direction, another in the -45° direction (Figure 1). We call such a swap a "mini-swap".

Routing of a D2TN channel can be viewed as a vertical stack of steps. A step is a unit-high horizontal strip in which a set of mini-swaps is performed. If a net does not change position in a step, it simply propagates to the next track by a unit vertical wire. An example is shown in Figure 2.

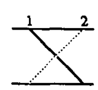


Figure 1. A mini-swap

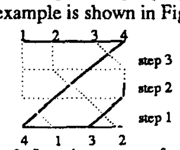


Figure 2. In each step, a set of mini-swaps occur

A solution for the D2TN channel routing problem in this model can be constructed in a bottom-to-top step-by-step fashion . The final channel height is equal to the number of steps required. Clearly, a D2TN channel routing problem can have many possible solutions under the mini-swap model (Figure 3). In search of the optimum solution, the router must determine which pairs of nets to swap in each step so that the final solution is optimum. Theorem 1 states necessary optimality conditions.

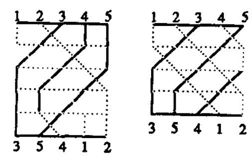


Figure 3. Two realizations of the same channel

Definition 2 *The optimal solution of a two-terminal net channel routing problem under the mini-swap model is one that has*
(a) the minimum channel height and
(b) the shortest total wire length

Definition 3 *A pair of nets is said to be "planar" if it is in the natural order on the bottom. Otherwise, it is said to be "intersecting".*

Theorem 1 *A solution is optimal under the mini-swap model if it has the minimum number of tracks and properties 1 and 2 hold:*
(property 1) planar pairs do not intersect and
(property 2) intersecting pairs intersect only once

Proof: For the sake of contradiction, assume a solution S, has the minimum number of tracks and some planar nets intersect twice. Then there exists a new solution S' (see Figure 4), which has the same number of tracks and no intersecting planar nets.

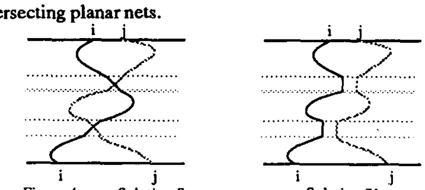


Figure 4. Solution S Solution S'

Clearly S' has shorter total wire length than S, which implies that S is not optimal. The proof for property 2 is similar.

■

In other words, a router never needs to swap a pair of nets that are already in the natural order to obtain an optimum solution. This implies that the set of mini-swaps that leads to the solution corresponds to a minimal set of net crossings.

3.2 The Routing Strategy

We now describe a routing algorithm based on the parallel bubble sort [1]. In the first step, nets located at odd grid points are compared with the net on their right. If the pair is an planar pair, the pair does not switch positions; otherwise, a mini-swap is performed (Figure 5). The second step is identical to the first one, except this time nets located at even grid points are compared with the net on their right. These two steps are repeatedly performed in this order. The algorithm stops when for two consecutive steps, no pairs of nets switch places. Once the algorithm terminates, the nets are ordered in the natural order.

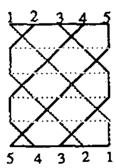


Figure 5.

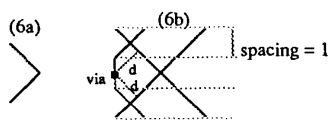


Figure (6a) An impossible situation
(6b) A via is placed at the center of a vertical wire to ensure d > 1

Layer 1 is assigned to wires oriented in the +45° angle; layer 2 is assigned to wires oriented in the -45° angle. Vertical wires can be assigned to any layer since they do not cross any other wires. Vias are introduced for layer changes between a +45° wire and -45° wire. Due to the odd-even transposition procedure, the situation shown in Figure 6a could never happen: the +45° wire of a net always connects to a -45° wire through a vertical segment as shown in Figure 6b. We place a via at the midpoint of the vertical segment. This ensures that the wires satisfy the design rule so that there is no need to magnify either the column spacing or the row spacing by $\sqrt{2}$.

Formally, let us denote the permutation of nets on track t by:

$A_t = (a_t(1), a_t(2), ...a_t(n))$

Then, $A_0 = (a_0(1), a_0(2), ...a_0(n))$, where $a_0(i) = b(i)$, for all i. We also assume there is an infinite number of auxiliary nets, represented by .... a(-1), a(0), and a(n+1), a(n+2),...., where for all $i \leq 0$ and $t > 0, a_t(i) = -\infty$ and for all $i > n$ and $t > 0, a_t(i) = +\infty$. These auxiliary nets do not disturb the routing of regular nets at any time. For each track $t \geq 0$ and each integer i, the net, $a_t(i)$, at the ith grid point at track t is given by:

if (i + t) is even , $a_t(i) = min(a_{(t-1)}(i), a_{(t-1)}(i+1))$

if (i + t) is odd , $a_t(i) = max(a_{(t-1)}(i-1), a_{(t-1)}(i))$

The dynamic behavior of this routing scheme is the same as that of the parallel bubble sorter realized on a two-way infinite linear array. If, for every i, $a_t(i) \leq a_t(i+1)$, then A_t is said to be sorted. The computing time of the parallel bubble system is the smallest t such that A_t is sorted. A router similar to ours has been developed independently by [6]. The main difference is that their method is based on the sequential bubble sort.

3.3 The POTENTIAL function

We are interested in the computing time of a parallel bubble system which corresponds to the required height of the D2TN channel routing problem. A function called POTENTIAL, first introduced by [18], proved to be very useful in evaluating the computing time of the bubble system. This section defines the POTENTIAL function and reviews the theorem proved by [18].

Definition 4 *For each (i,j,t), where $1 \leq i, j \leq n$, and $t \geq 0$, and the set of nets, S, we define:*

- *ORDER(i,j,t,S) is the number of indices $p \in S$ such that $i < p \leq j$ and $a_t(i) \leq a_t(p)$, or such that $j \leq p < i$ and $a_t(p) \leq a_t(i)$*
- *NOTORDER(i,j,t,S) is the number of indices $p \in S$ such that $i < p \leq j$ and $a_t(p) \leq a_t(i)$, or such that $j \leq p < i$ and $a_t(i) \leq a_t(p)$*
- *MAXLT(i,t,S) = max(0 ∪ { ORDER(i,p,t,S) - NOTORDER(i,p,t,S) + 1 | $p \in S, p < i$ and $a_t(i) \leq a_t(p)$ })*
- *MAXGT(i,t,S) = max(0 ∪ { ORDER(i,p,t,S) - NOTORDER(i,p,t,S) + 1 | $p \in S, i < p$ and $a_t(p) \leq a_t(i)$ })*

Definition 5 *For any position indexing $i \leq n$, and any $t \geq 0$, the function POTENTIAL(i,t) is defined as:*

- *When NOTORDER(i,1,t,S) = 0,*
  POTENTIAL(i,t) = NOTORDER(i,n,t,S) + MAXGT(i,t,S)
- *When NOTORDER(i,n,t,S) = 0,*
  POTENTIAL(i,t) = NOTORDER(i,1,t,S) + MAXLT(i,t,S)
- *When NOTORDER(i,1,t,S) ≠ 0 and NOTORDER(i,n,t,S) ≠ 0,*
  POTENTIAL(i,t) = NOTORDER(i,1,t,S) + NOTORDER(i,n,t,S) + max(1, MAXLT(i,t,S), MAXGT(i,t,S))

The POTENTIAL function for the entire bubble system is defined by:
POTENTIAL(t) = max(POTENTIAL(i,t) | $1 \leq i \leq n$)

From the above definition, it is clear that POTENTIAL(i, t) = 0 if and only if NOTORDER($i, 1, t, S$) = NOTORDER(i, n, t, S) = 0. An immediate consequence of this fact is that if POTENTIAL(t) = 0, then A_t is sorted. We now state the main theorem proved by [18].

Theorem 2 *If $t \geq 1$ and A_t is not sorted, then POTENTIAL(t+1) = POTENTIAL(t) - 1*

Corollary 2.1 *The computing time of the bubble system is POTENTIAL(0) or POTENTIAL(0)+1.*

The salient feature of the bubble system is that the POTENTIAL value consistently decreases by 1 per step. In other words, when POTENTIAL(0) = k, the required height of the D2TN channel is k or $k + 1$. The value of POTENTIAL(0) is defined solely by the initial permutation A_0 , without referring to the intermediate configurations A_t for $t > 0$. The POTENTIAL function can be precisely evaluated to compute the number of tracks require to route the D2TN channel without actually carrying out the routing steps. Another feature is that the decision of whether to swap a pair or not is local and does not depend on the locations of the rest of the nets. This feature makes this algorithm attractive in a parallel mode of operation. Also observe that the wiring path for each net is monotonic in the vertical direction.

3.4 The Main Theorem

The parallel bubble sort provides a simple routing strategy to produce a solution in the mini-swap model. Do better algorithms exist for which the POTENTIAL value decreases rapidly , say, by more than 1 per step ? In the following theorem, we prove a sufficient optimality condition for any algorithm under the mini-swap model. See [21] for detail of the proof.

Theorem 3 *Potential value decreases at most by 1 per net per step under the mini-swap model.*

The following theorem gives a lower bound on the POTENTIAL function.

Theorem 4 *For each net i, we define the displacement(i) as the horizontal distance between its two terminals. Then POTENTIAL(0) $\geq$ $max(\{displacement(i) \mid 1 \leq i \leq n\})$.*

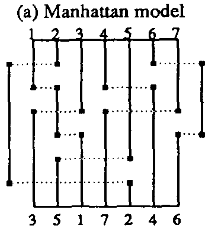
3.5 Results and Comparison

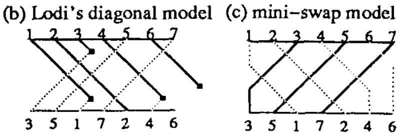
Routing results of two D2TN channels are shown in Figure 8 and 9. Compared to the Manhattan routers, our router has the following advantages:

- There is no need to deal with vertical constraints
- The required channel height, POTENTIAL(0), can be precisely computed
- For D2TN channels that have a POTENTIAL(0) value less than the Manhattan density, our router can out-perform any Manhattan routers
- Extra columns outside the channel's span are never used
- Wirelength is expected to be shorter due to the use of diagonal wires
- Being a minimal crossing solution, we expect only a small number of vias is required. We observed that most nets require zero or one via, whereas Manhattan routing typically requires at least 2 vias per net.
- It is inherently suitable for parallel mode of operations.
- In standard cell design, it is very difficult to optimize the assignment of pins to feedthroughs when the objective function is the density. In our environment, one would want to minimize the maximum displacement of a net. This can be easily done by linear assignment.

Figure 8. A channel is routed three models shown in (a), (b), and (c).

(a) Manhattan model (b) Lodi's diagonal model (c) mini-swap model





(d) Comparison

	Man	Lodi	Mini Swap
height	7	3	3
area	56	42	18
vias	20	4	0
wire length	71	42	28

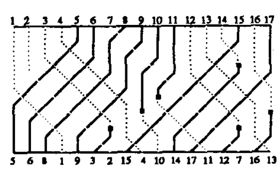


Figure 9. A channel with density = 7 is optimally routed in the mini swap model.

The diagonal model channel router proposed by [14] can complete routing of a D2TN channel in $max(\{displacement(i) \mid 1 \le i \le n\})$ tracks. Although this number is smaller than the channel height required by our router (see theorem 4), the diagonal router [14] magnifies both the column spacing and row spacing by $\sqrt{2}$, which implies that the channel area is doubled. Our router ensures that the wiring paths are design rule correct and does not magnify spacing in either direction.

3.6 Concluding Remarks on the Mini-Swap Model

A D2TN channel can be routed in the miniswap model by parallel bubble sort. The function POTENTIAL can be evaluated to compute the precise number of tracks required to route the channel. The function evaluation of POTENTIAL for a given channel can be obtained without referring to the intermediate routing steps. We have established necessary and sufficient optimality conditions for routing under the "mini-swap" model. The final routing solution has an unambiguous layer assignment and is design-rule correct. Our results show that a class of dense two-terminal net channels can be routed in a height less than the Manhattan density.

4 An Overlap Model

The mini-swap model is good for routing channel whose POTENTIAL value is less than or equal to the Manhattan density. However, we need to overcome its deficency in routing channels with "long" nets (nets spanning a number of columns which is larger than the Manhattan density). Since Theorem 3 confirmed that we could not reduce the required channel height by cleverly selecting pairs to swap, the only alternative that may lead us to a better solution is to relax the constraint of pair-wise transpositions.

4.1 Algorithm for the Overlap Model

The basic idea of the new scheme is to allow long nets to swap with more than one net while maintaining the integrity of net crossings. To improve the results of the mini-swap model, the long nets must be given a better chance in swapping with other nets. At the same time, we want to keep the number of net crossings minimal so that the number of vias will not increase.

The routing algorithm is based on the river routing technique. A dense 2-terminal net channel routing problem is specified in Definition 1. A net is said to be a *right* net if $i > q_i$; otherwise, it is said to be a *left* net (Figure 12). The basic idea of the overlap model is to divide the nets into two disjoint subsets: *S1* = { *right* nets } and *S2* = { *left* nets } and route each set independently. A top level description of the algorithm is given below:

step 1) Divide nets into two sets: S1 = { right nets }; S2 = { left nets }

step 2) Sort S1 in decreasing order of their top terminals. Route S1.

step 3) Sort S2 in decreasing order of their bottom terminals. Route S2.

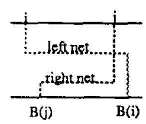


Figure 12.

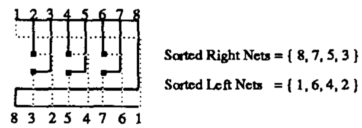


Figure 13. Routing in the overlap model

The solution for the D2TN channel routing problem is constructed in a bottom- to-top net-by-net fashion. Let us demonstrate the algorithm on the example shown in Figure 13. The right nets are sorted and routed sequentially in a river routing fashion. The procedure begins by constructing a rectilinear wiring path for the first right net. The path begins at the bottom terminal and ends at the top terminal. The wiring path for the second right net simply follows the path of the first right net and ends at its top terminal. This process continues until all of the right nets are routed. All of the paths are monotonic in both the horizontal and vertical directions. Step 3 is identical to step 2, except that this time the left nets are routed.

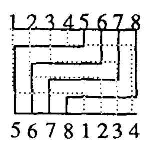
Layer assignment for the wiring paths is trivial. For the right (left) nets, the vertical segment attached to their bottom (top) terminals is assigned layer 2 (1) while the remaining wire segments are assigned layer 1 (2), see Figure 13. A via is introduced for each layer change. Clearly, the routing paths for a pair of planar nets do not intersect and intersecting pairs intersect only once. This implies that the routing solution contains a minimal set of net crossings.

4.2 Performance Analysis

Suppose the D2TN channel routing problem is specified as a pair of vectors: $TOP = \{ 1, 2, ..., n \}$ and $BOT = \{ b(1), b(2), ... b(n) \}$.

Theorem 5 *The lower bound on the channel height under the arbitrary overlap model is* $max\{$ *NOTORDER(1,i,0,S1) + NOTORDER(i,n,0,S2)* $\mid 1 \le i \le n \}$. *(Deailed proof can be found in [21].)*

The above lower bound is derived by considering the permutation of the nets. It is not only a tighter bound than the Manhattan density, d, but also demonstrates that $d/(L-1)$ [13, 11] is not a universal lower bound under the unrestricted overlap models, where L is the number of layers. The example in Figure 14 is routed by the overlap router in a height equal to the lower bound, which is half of the Manhattan density. We prove in the following theorem that, in contrast to the bubble system, the POTENTIAL value will decrease by more than 1 per track (amortizedly) under the overlap model.



density = 8

tracks = 4

vias = 0

Figure 14.

Theorem 6 *The upper bound on the channel height under the arbitrary overlap model is* $POTENTIAL(0) + 2$.

The essential implication of Theorem 3 and Theorem 6 is that when long nets are present in a channel routing problem, the POTENTIAL value of the channel is much larger than the density. This means that the overlap model works better than the mini-swap model. On the other hand, if all nets are "short", i.e. the POTENTIAL value of the channel is equal to or less than the density, the mini-swap model produces a slightly better solution than the overlap router. In section 5 we shall combines the strengths of both models to form a hybrid router.

The wiring path for each net is monotonic in both the horizontal and vertical direction. There are no detours in the final solution. This is a direct consequence of the routing algorithm. The overlap routing procedure has the distinct advantage of generating a small number of vias for D2TN channel routing problems. Given any D2TN channel routing problem, the overlap routing algorithm introduces at most one via per net.

4.3 Concluding Remarks on the Overlap Model

Unlike other overlap models [12, 16, 3] that used two or more vias per net, the routing solution produced by our router can be wired using two interconnect layers so that at most one via is required per net. Compared to the Manhattan model, the overlap model has the following advantages:

- There is no need to deal with vertical constraints
- For channels that have POTENTIAL(0) value less than the Manhattan density, our model can out-perform any Manhattan routers
- Extra columns outside the channel's span are never used
- Because it is a minimal crossing solution, we expect only a small number of vias to be required.

The overlap model is better than the mini-swap model in two respects: (1) It yields a small channel area for channel with long nets. (2) Variable wire widths can be incorporated easily. However, overlapping wires may increase crosstalk between signals and unlike the mini-swap algorithm, the overlap algorithm is not suitable for parallel operations.

5 The Hybrid Router

We combine the advantages of the overlap and the miniswap models to form a new hybrid router. This router pre-routes long nets using the overlap algorithm so that the amount of overlapping wire is limited. The remaining nets, ie. relatively short nets, are routed by the miniswap algorithm. The user can specify the maximum amount of overlap allowed.

5.1 Algorithm

The basic idea is as follows: When the channel is routed solely by the miniswaps, we know the channel height is POTENTIAL(0). Suppose we pick some long nets which would cause the mini-swap model to disgrace itself, and pre-route them using the overlap strategy. If the resulting channel height is better than before, we can choose more long nets and continue the process provided the maximum amount of overlap is not exceeded. A top level description of the algorithm is given below. (See figure 17 for an example.)

For $i \leftarrow 1$ to $n-1$ do

- i) pre-route i longest nets by the overlap model
  - amount of overlap $= i$,
  - calculate current channel height, t(i),
  - $t(i)$ = (tracks for pre-routes) + (POTENTIAL value of remaining channel)
- ii) compare height to last iteration:
  - if ($t(i) < t(i-1)$) and (max overlap is not exceeded)
  - $i = i + 1$
  - continue
  - else stop

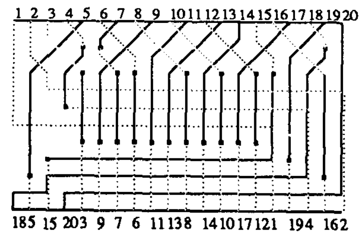


Figure 17. A channel is routed by the hybrid router in density.

5.2 Performance Analysis

Since the hybrid router is a combination of the mini-swap model and the overlap model, it should perform at least as well as either model in the worst case. Given a channel routing problem, if the algorithm terminates in the first (($n-1$)th) iteration, the solution has purely mini-swaps (overlaps). Therefore, the mini-swap model and the overlap model can be considered special cases of the hybrid router. We summarize the performance bounds of the hybrid router in the following theorem.

Theorem 7 *The channel height required by the hybrid router has*

Lower Bound $= max\{NOTORDER(i,1,0,S1) + NOTORDER(i,1,n,S2) \mid 1 \le i \le n\})$

Upper Bound $=$ *POTENTIAL(0)* $+ 1$.

6 Routing Results for 2-terminal Nets

The hybrid router is implemented in the C language on a DEC3100 running Ultrix Worksystem V2.1. Figure 18 shows the routing result of a channel with 48 nets. We have attempted to run this example using other routers available [19, 7, 9] but they either failed to complete routing or produced substantially worse results. Table I lists the channels tested with 100% routing completion in all cases. Several D2TN channels are routed in a height less than the density.

Table 1. Experimental Results

Example	density	final height	CPU
Ex1	8	7	0.1
Ex2	7	8	0.1
Ex3	4	3	0.1
Ex4	18	22	0.1
Ex5	4	5	0.1
Ex6	8	5	0.1

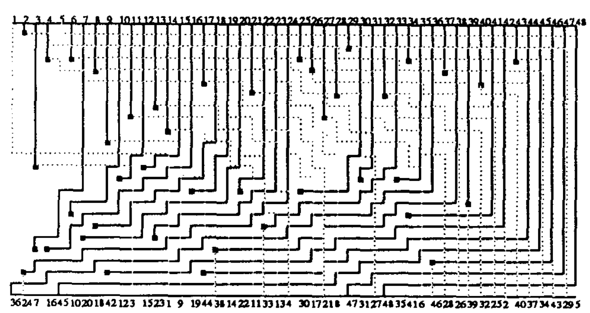


Figure 18. Routing result of Ex4

7 Multi-terminal Net Routing

To extend the routing algorithm to handle multi-terminal nets, we partition each multi-terminal net into 2-terminal subnets and classify each subnet as a *right* net or a *left* net. A 2-terminal subnet with one terminal on TOP and the other on BOTTOM is said to be a *2-sided* subnet; otherwise, it said to be a *1-sided* subnet. The 2-sided subnets can be categorized as left or right easily. However, the classification of 1-sided subnets is ambiguous and may affect the quality of solution. In the following lemmas, we show that not all 1-sided subnets can be classified either way. For detailed proof,see [21].

Definition 6 *Given I, a two-terminal 1-sided subnet:*

- *I is top-sided iff both terminals of I lie on TOP.*
- *I is bottom-sided iff both terminals of I lie on BOTTOM.*

Lemma 1 *Given a 1-sided subnet I,*

i) I is top-sided and I intersects a right net ⇒ I must be a left net.

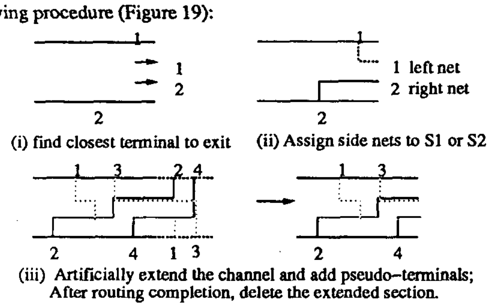
ii) I is bottom-sided and I intersects a left net ⇒ I must be a right net.

iii) I is top-sided and I intersects another top-sided subnet J ⇒ I and J can not both be right nets.

iv) I is bottom-sided and I intersects another bottom-sided subnet J ⇒ I and J can not both be left nets.

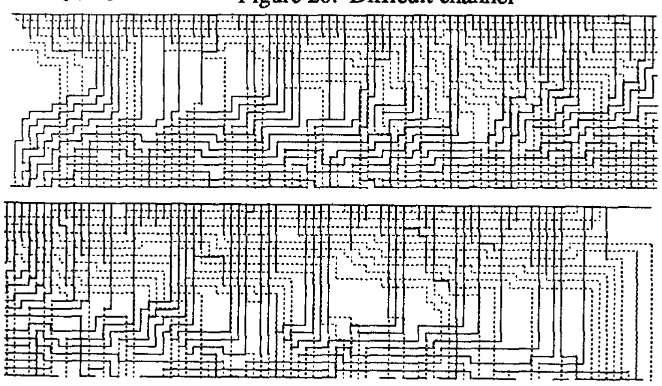
Most 1-sided nets in practical examples are classified as above. The classification of the remaining 1-sided subnets is guided by local congestion analysis. The left subnets and right subnets then are sorted independently by their terminal positions and routed in the rivering routing fashion as discussed in section 5.1. We observe that this method for routing multi-terminal nets is straight-forward but not necessarily optimal. Future work should investigate better strategy for handling multi-terminal nets.

In a channel routing problem, nets may exit the channel on either its left end or its right end. Routing of these so-called side nets is done by the following procedure (Figure 19):



The analysis and theorems introduced for the D2TN channel routing problem in Section 5.2 can be generalized for the multi-terminal nets problem by replacing nets with subnets in the equations. Figure 20 shows that the Deutsch's difficult channel is routed with 23 tracks and has 25% less vias than [5, 19].

Figure 20. Difficult channel



8 Conclusion and Future Work

In this paper, we propose new methods which perform well on practical channels and attempt to provide accurate and formal analysis on the quality of the solutions. Two new routing models, the *mini-swap* model and the *overlap* model are introduced. Non-Manhattan geometry as well as rectilinear wires are used. The routing strategy is based on the parallel sorting and river routing techniques and is very different from the Manhattan approach. In particular, vertical constraints no longer exist. Furthermore, the solution produced by either model consists of a minimal set of net crossings which leads to a small number of vias. To characterize and to evaluate the performance of each model, a function called POTENTIAL is used. Intuitively, the POTENTIAL function measures the degree of difficulty for a given channel routing problem. Based on the performance analysis, we attempt to combine the strength of both models to form a new *hybrid* router. The routing solution produced by our router has unambiguous layer assignment and is design-rule correct. Finally, a straight-forward extension to handle multi-terminal nets and and side-nets is proposed. Preliminary results show that a class of two-terminal net channel routing problems can be routed in height less than the Manhattan density. Future work should optimize the extension to handle multi-terminal nets and multi-layers.

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