ABOUT THE PERFORMANCE OF PRACTICAL DIRTY PAPER CODING SCHEMES IN GAUSSIAN MIMO BROADCAST CHANNELS

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ABSTRACT

This paper describes a way of implementing DPC in a Gaussian MIMO broadcast channel. The outer encoder is based on a vector TCQ designed to possess certain "good properties". Simpler schemes such as the Tomlinson Harashima or Scalar Costa's scheme are also considered by a way of comparison. The inner encoder is implemented through a vector version of the ZF-DPC and the MMSE-DPC. The BER performance of the DPC schemes is evaluated and compared to that of conventional interference cancellers (pre-ZF, pre-MMSE). From simulation results the choices of the inner encoder, the outer encoder (THS/SCS/TCQ) and the interference cancellation technique (conventional or DPC) are discussed.

1. INTRODUCTION

The situation under consideration in this paper is that of a Gaussian downlink channel in which both the transmitter and receivers are equipped with multiple antennas. Each receiver is interested in his own message. We assume that the transmitter knows the channel transfer matrix perfectly. In this context the task of the transmitter is to use channel state information knowledge the best way possible in order to maximize a given performance criterion. In this respect it is known that interference (multiuser interference + spatial interference) plays a critical role. In terms of data rates, the best information-theoretic transmit scheme is known [1]. Although the capacity region for this channel has been completely determined by Weingarten et al [1], who showed the optimality of dirty-paper coding (DPC), there is still some work to be done to know what practical DPC schemes have to be used in order to optimize practical performance criteria such as the bit error rates (BER), coding/decoding complexity, robustness to channel estimation errors, etc.

When we look at the transmitter structure suggested by Information Theory (IT) for Gaussian MIMO broadcast channels (BC) we see that the encoder comprises two stages, one stage implementing DPC (say the outer encoder) followed by a precoding stage (say the inner encoder). One of the main points of this paper is to discuss the benefits of mimicing such a structure for treating interference compared to well-known pre-cancellation schemes (see e.g. [2]). Assuming that we want to mimic the IT-based structure two natural questions arise: How do we implement DPC for a MIMO BC? How do we choose the channel precoder for a MIMO BC? In this paper we propose the following transmitter structure. For the inner encoder we use an extended version of the ZF-DPC introduced by [3] for the Gaussian MIMO BC with single-antenna receivers and the MMSE-DPC of [4], [5]. For the outer encoder we use Trellis Coded Quantization (TCQ), initially used by [6] for data hiding, which we have adapted to the vector case in a DPC context. The authors also note that Yu et al. [7] proposed a more advanced trellis-based solution (with shaping gain) for the single-user channel with Tx side information. Of course, one of the purposes of this paper is precisely to discuss the proposed choice in the light of typical simulation results. But, a priori, the proposed choices are motivated by a search for a good trade-off between performance and complexity.

In fact, the most efficient outer encoding strategy would be to use general nested lattices (NL) [8], [9]. However the construction of such lattices for reasonably high dimensions is not systematic and the underlying coding/decoding complexity can be very high. On the other hand if the complexity issue matters the most, one can use the Tomlinson-Harashima scheme (THS) or scalar Costa's scheme (SCS) introduced by Eggers et al. in [10]. So the TCQ-based outer encoding scheme lies between these two kinds of schemes in terms of performance and implements, in fact, a suboptimum nested lattice. For the inner encoders, the most efficient (in terms of sum rate) precoding schemes have been proposed by [3] (ZF-DPC) and [11], [4], [5] (MMSE-DPC). The MMSE-DPC is sum-capacity achieving while the sum-rate performance of ZF-DPC is generally a little bit lower. But here the key practical issue by using ZF-DPC or MMSE-DPC is to know how well these linear precoding schemes perform in terms of BER. In our context, what is expected from these precoders is that interference is dealt with properly instead of having a high overall (or cell) throughput.

In this context one of the goals of this paper is to assess the performance (BERs) of the proposed transmit scheme (section 5). More specifically we want to compare the considered DPC-based pre-cancellers (ZF-DPC, MMSE-DPC) with their conventional counterparts (pre-ZF, pre-MMSE) and the outer encoders (THS, SCS, TCQ) between themselves for a given inner encoder (ZF-DPC/MMSE-DPC). Technically speaking, to do this, we need to extend (section 3) the ZF-DPC to the case of multi-antenna receivers and generalize the scalar TCQ to a vector TCQ (section 4) in order to use this coding scheme when multi-antenna receivers are present.

2. MIMO BC SIGNAL MODEL

In this paper, the notations s, $\underline{v}$, $\mathbf{M}$ stand for scalar, vector, and matrix, respectively. The superscripts $(.)^T$ and $(.)^H$ denote transpose and transpose conjugate, respectively. We will also use the notation $\mathbf{M} \overset{d}{=} r \times c$ where r, c are the number of rows and columns of $\mathbf{M}$.

For each user $k \in \{1, ..., K\}$ the received signal writes as: $\underline{Y}_k = \mathbf{H}_k \underline{X} + \underline{Z}_k$ where $\underline{X}$ is the transmitted signal, $\mathbf{H}_k$ is the $r_k \times t$ channel matrix for user k, t is the number of transmit anten-

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nas, r_k is the number or receive antennas and $\underline{Z}_k \sim \mathcal{N}(\underline{0}, \mathbf{I})$. In this paper $\mathbf{H}$ is a constant matrix, but most of the results can be extended to block fading channels. Also the transmitted signal is subject to a power constraint $\text{Tr}[E(\underline{X}\,\underline{X}^H)] = \text{Tr}(\mathbf{\Sigma}) \leq P$. By concatenating the received signal vectors we get $\underline{Y} = \mathbf{H}\underline{X} + \underline{Z}$ where $\mathbf{H}$ is the $r \times t$ broadcast channel matrix defined by $\mathbf{H} = [\mathbf{H}_1^T ... \mathbf{H}_K^T]^T$, with $r = \sum_k r_k$. The transmit signal $\underline{X}$ is not built directly from the information messages $\{\forall k \in \{1,...,K\}, W_k \in \{1,...,2^{nR_k}\}\}$ intended for the different users and CSI knowledge (R_k denotes the individual rate associated with user "k".) In fact the assumed encoder structure is as follows (figure 1). The outer coder builds from $W_1,...,W_K$ and $\mathbf{H}$ an intermediate signal denoted by $\tilde{\underline{X}}$. Then the inner encoder builds $\underline{X}$ from $\tilde{\underline{X}}$ and $\mathbf{H}$. Additionally we will assume a linear precoder: $\underline{X} = \mathbf{B}\tilde{\underline{X}} = [\mathbf{B_1}...\mathbf{B}_K][\tilde{\underline{X}}_1^T ... \tilde{\underline{X}}_K^T]^T$ where $\mathbf{B}$ is a $t \times r$ matrix. Under this assumption the transmit signal writes as $\underline{X} = \sum_{k=1}^{K} \mathbf{B}_k \tilde{\underline{X}}_k$.

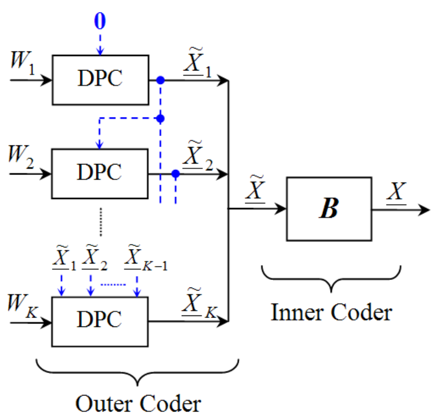

Fig. 1. Successive coding using DPC + linear precoding

3. INNER ENCODING

3.1. ZF-DPC for multi-antenna receivers

Here we present the extended version of [3] for any number of users and receive antennas, and for an arbitrary channel matrix rank (say m). As in [3] we perform a QR-type decomposition $\mathbf{H} = \mathbf{GQ}$. From this decomposition we have:

$$\mathbf{H} = \begin{bmatrix} \mathbf{G}_{1,1} & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \mathbf{0} \\ \mathbf{G}_{K',1} & \dots & \mathbf{G}_{K',K'} \\ \vdots & & \vdots \\ \mathbf{G}_{K,1} & \dots & \mathbf{G}_{K,K'} \end{bmatrix} \mathbf{Q}$$

where $K' \leq K$ is the smallest integer such that $\sum_{k=1}^{K'} r_{i(k)} \geq m$. The indices $i(1),...,i(K')$ are defined by the chosen user ordering. To avoid any ambiguity here is an example illustrating our notations. Let $K' = K = 2$. First we allocate arbitrarily an index to each user, which defines r_1 and r_2. If user $\#2$ is encoded before user $\#1$ then $i(1) = 2, i(2) = 1$ that is $r_{i(1)} = r_2, r_{i(2)} = r_1$.

Regarding matrix dimensions we have:

- $\forall k \in \{1,...,K'-1\}$, $\mathbf{G}_{k,k} \overset{d}{=} r_{i(k)} \times r_{i(k)}$;
- $\mathbf{G}_{K',K'} \overset{d}{=} r_{i(K')} \times \overline{r}_{i(K')}$ with $\overline{r}_{i(K')} = m - \sum_{k=1}^{K'-1} r_{i(k)}$;
- $\forall k \in \{1,...,K'-1\}, \forall \ell \in \{1,...,k\}$, $\mathbf{G}_{k,\ell} \overset{d}{=} r_{i(k)} \times r_{i(\ell)}$;
- $\forall k \in \{K',...,K\}, \forall \ell \in \{1,...,K'-1\}$, $\mathbf{G}_{k,\ell} \overset{d}{=} r_{i(k)} \times r_{i(\ell)}$;
- $\forall k \in \{K',...,K\}$, $\mathbf{G}_{k,K'} \overset{d}{=} r_{i(k)} \times \overline{r}_{i(K')}$.

The matrix $\mathbf{Q}$ is a $m \times t$ matrix with orthonormal rows. Note that K' is merely the maximum number of active users (with non-zero powers). Finally the precoding matrix is given by:

$$\mathbf{B}^{zf-dpc} = \left[\mathbf{Q}^H \; \mathbf{0}_{t\times(r-m)}\right]. \tag{1}$$

By using this precoding matrix the received signals writes for all $k \in \{1,...,K'\}$:

$$Y_k = \mathbf{G}_{k,k}\tilde{\underline{X}}_k + \sum_{\ell<k} \mathbf{G}_{k,\ell}\tilde{\underline{X}}_\ell + \underline{Z}_k \tag{2}$$

where the covariance matrices $\tilde{\mathbf{\Sigma}}_k = E\left[\tilde{\underline{X}}_k \tilde{\underline{X}}_k^H\right]$ are such that $\sum_{k=1}^{K'} \text{Tr}(\tilde{\mathbf{\Sigma}}_k) = \sum_k \tilde{P}_k \leq P$. One notices that for a given user "k" the interference for $\ell > k$ has been totally cancelled out thanks to the ZF-DPC precoding matrix. The other part of the interference ($\ell < k$) will be cancelled out by the outer encoder in the Costa's manner [13]. This point will be detailed in the next section. Now we turn our attention to the power allocation and coding order issues. When it turns to power allocation and user ordering it is necessary to define the performance criterion to be optimized. At least two criteria are worth being considered in general.

Sum-rate optimization for static channels. Thanks to the ZF-DPC precoding matrix and the DPC implementation of the outer encoder, the MIMO BC is equivalent to K' MIMO single-user sub-channels. The sum-rate is then optimized by performing a water-filling over $\{\lambda_{1,1},...,\lambda_{1,r_1},...,\lambda_{K',1},...,\lambda_{K',r_{K'}}\}$, which are the singular values of the matrices $\mathbf{G}_{1,1},...,\mathbf{G}_{K',K'}$.

$$R_{sum}^{zf-dpc} = \sum_{k=1}^{K'} \sum_{\ell=1}^{r_{i(k)}} \max\left\{0, \log(\mu\lambda_{k,\ell})\right\} \tag{3}$$

$$\sum_{k=1}^{K'} \underbrace{\sum_{\ell=1}^{r_{i(k)}} \max\left\{0, \mu - \frac{1}{\lambda_{k,\ell}}\right\}}_{\tilde{P}_k} = P. \tag{4}$$

Of course, the user ordering has also to be optimized in order to maximize the sum-rate. The exhaustive approach consists in testing all the user orderings, performing water-filling for all these cases and keeping the best order.

Multiuser diversity optimization for quasi-static channels. For example, in the case of block Rayleigh fading BCs with single-antenna receivers, Tu and Blum [12] have proposed an algorithm (called the greedy algorithm, based on a subspace decomposition) that is optimal in terms of multiuser diversity. We have checked that this result can be extended to the case of multi-antenna receivers.

The corresponding algorithm is almost optimal in terms of ergodic sum-rate $E_{\mathbf{H}}\left[\sum_k R_k\right]$ but the rate loss due to the use of the greedy algorithm is generally more severe if the channel is fixed, which is the case under investigation in this paper.

In fact, in the simulation part we will use *another performance criterion*: We will compare the DPC-based interference cancellation technique with conventional interference pre-cancellers in terms of transmit power for given target spectral efficiencies and target QoS (qualities of service).

3.2. Review of the MMSE-DPC [4], [5]

Each block $\mathbf{B}_k$ of the MMSE-DPC precoding matrix is simply given by:

$$\mathbf{B}_k^{mmse-dpc} = \left(\mathbf{I} + \sum_{\ell=k+1}^{K} \mathbf{H}_\ell^{\mathbf{H}} \mathbf{\Sigma}_\ell \mathbf{H}_\ell\right)^{-1} \mathbf{H}_k^H \quad (5)$$

where the matrices $\{\mathbf{\Sigma}_\ell\}$ needs to be optimized under the constraints $\mathrm{Tr}[\mathbf{\Sigma}_\ell] = P_\ell$, $\sum_\ell P_\ell \leq P$. Notice that the interference term associated with the users $i < k$ does not appear in the precoding matrix. This is because it is assumed that the outer encoder pre-cancels this interference in the Costa's manner. For the coding ordering issue, one can check that the sum rate achieved by MMSE-DPC does not depend on the user ordering in contrast with the ZF-DPC case. For a given coding order, there will always be a set of powers achieving the sum capacity, which means that the matrices $\mathbf{B}_1^{mmse-dpc}, ..., \mathbf{B}_K^{mmse-dpc}$ can "automatically" adapt to the given order in an optimal way. We insist on the fact that independency regarding the coding order is only true for the sum-rate maximization but not for any point of the capacity region and certainly not for another performance criterion. As for the optimum set of user powers it cannot be found explicitly since we have to maximize the sum rate which is given by $R_{sum}^{mmse-dpc} = \log|\mathbf{I} + \mathbf{H}^H \mathbf{\Sigma} \mathbf{H}|$ with $\mathbf{\Sigma} = \mathrm{Diag}(\mathbf{\Sigma}_1, ..., \mathbf{\Sigma}_K)$. For single-antenna receivers one can use the optimization algorithm derived by [14] which insures convergence to the optimum solution. Although the convergence remains insured in the case of multi-antenna receivers the complexity of the power allocation algorithm becomes considerable and can be critical if it has to be repeated (think of the block-fading case). This is one of the reasons why the ZF-DPC, although suboptimal in terms of sum capacity, can be more suited in certain contexts thanks to a simpler power allocation scheme.

3.3. Review of the pre-ZF and pre-MMSE [2]

Knowing $\mathbf{H}$ the pre-MMSE and pre-ZF are respectively given by $\mathbf{B}^{pre-mmse} = (\mathbf{I} + \mathbf{H}^H \mathrm{Diag}(P_1, ..., P_K)\mathbf{H})^{-1}\mathbf{H}^H$, $\mathbf{B}^{pre-zf} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$. The pre-ZF assumes that $\mathbf{H}\mathbf{H}^H$ is invertible. If it is not case, a generalized matrix inversion can be used by performing an eigenvalue decomposition $\mathbf{H}\mathbf{H}^H = \sum_i \lambda_i \underline{u}_i \underline{u}_i^H$, then $(\mathbf{H}\mathbf{H}^H)^{\#} = \sum_i^{m'} \lambda_i^{-1} \underline{u}_i \underline{u}_i^H$, where $m' = \mathrm{rank}(\mathbf{H}\mathbf{H}^H)$. Note that for a conventional pre-canceller only the inner coding treats the interference while in its DPC counterpart both the inner et outer encoders treat it. This results in a loss in terms of sum capacity for the conventional pre-cancellers.

4. OUTER ENCODING

In the previous section we have seen how to implement the precoding part. In fact the precoder assumes that the outer encoding is implemented by a good dirty paper encoder. So, in this section we describe a way of generating the signal $\underline{\tilde{X}}$ from the information messages $W_1, ..., W_K$ and the knowledge of the channel matrix $\mathbf{H}$. The outer encoding scheme is presented for the ZF-DPC case but the proposed scheme can be easily extended to the MMSE-DPC case.

4.1. Problem statement

In [3] the sum-rate optimization performed by the ZF-DPC assumes a good outer encoder. More precisely, the authors assumes that the inner encoder is implemented by successive coding and a bank of Costa's encoders. Indeed, from equation (2) we see that the ZF-DPC precoding matrix transforms the MIMO channel into several subchannels, each of them being a vector Costa's channel. For k ranging from 1 to K' the received signal has the same form: $\underline{Y}_k = \underline{X}_k + \underline{S}_k + \underline{Z}_k$, where $\underline{X}_k = \mathbf{G}_{k,k}\underline{\tilde{X}}_k$ and $\underline{S}_k = \sum_{\ell<k} \mathbf{G}_{k,\ell}\underline{\tilde{X}}_\ell$ is non-causally known to the transmitter. Yu and Cioffi [11] showed the result of [13] generalizes by using the following DPC scheme:

1. $\underline{X}_k = \underline{U}_k - \mathbf{A}_k \underline{S}_k$, $\underline{U}_k$ is an auxiliary encoding variable and $\mathbf{A}_k = \mathbf{\Sigma}_k (\mathbf{\Sigma}_k + \mathbf{I})^{-1}$ and $\mathbf{\Sigma}_k = E\left(\underline{X}_k \underline{X}_k^H\right)$.
2. $I(\underline{X}_k; \underline{S}_k) = 0$.

By using this theoretical coding scheme, user "k" achieves the following rate: $R_k^{opt} = \max_{\tilde{\mathbf{\Sigma}}_k, \mathrm{Tr}(\tilde{\mathbf{\Sigma}}_k) \leq \tilde{P}_k} \log|\mathbf{I} + \mathbf{G}_k \tilde{\mathbf{\Sigma}}_k \mathbf{G}_k^H|$ with $\mathbf{\Sigma}_k = \mathbf{G}_k \tilde{\mathbf{\Sigma}}_k \mathbf{G}_k^H$. Therefore $\tilde{\mathbf{\Sigma}}_k$ must also have its eigenvectors equal to the right singular vectors of $\mathbf{G}_k$ and its eigenvalues obeying the water-filling power allocation on the singular values of $\mathbf{G}_k$:
$\mathbf{G}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^H \Rightarrow \tilde{\mathbf{\Sigma}}_k = \mathbf{V}_k \tilde{\mathbf{P}}_k^{opt} \mathbf{V}_k^H$ where $\tilde{\mathbf{P}}_k$ is the optimum power matrix obtained through water-filling. This imposes a third condition on the coding scheme:

3. $\mathbf{\Sigma}_k^{opt} = \mathbf{G}_k \mathbf{V}_k \tilde{\mathbf{P}}_k^{opt} \mathbf{V}_k^H \mathbf{G}_k^H$.

The purpose of what follows in this section is precisely to describe a practical way of implementing a dirty paper coding scheme having the aforementioned "good" properties. Note that the approach consisting in imposing information theoretic properties to a real DP coding scheme (namely data hiding in the case under consideration) has been shown to be fruitful for the scalar Costa's channel [10].

4.2. Practical implementation of DPC

The final version of the proposed coding/decoding scheme is represented by figure 2. This subsection aims at explaining and detailing the corresponding choices. As mentioned in the introduction part of this paper, the choice for implementing DPC is to use a vector TCQ, which implements a suboptimum nested lattice. The TCQ works exactly as a trellis coded modulation. The $R_k = \frac{1}{n}\log_2|\mathcal{W}_k|$ ($\mathcal{W}_k$ is the message alphabet of user k) bits associated with user k are split into two parts. One part is encoded by a conventional encoder is used for selecting a subset of quantizers and the other is used to select a quantizer within a given subset. For the modulation we simply used a PAM (pulse amplitude modulation) but more efficient modulation can be used if necessary. We also mentioned in the introduction that a good way to implement each DP encoder would be to use nested lattices (see e.g. [9]) since the corresponding Costa's single-user capacity is achieved. Indeed nested lattices allow for implementing good source-channel codes. The coarse lattice (say Λ_2) implements a good source code (i.e the quantizer or modulo-operator necessary to implement Costa's scheme) while the fine lattice (say Λ_1) implements a good channel code. As far as we are concerned, the main advantages of the vector TCQ in our context is that it allows for a relatively simple construction of the fine lattice (through a good design of the trellis, say of size "n") and the coarse lattice. The coarse lattice implements the vector quantizer needed to deal with side information. In a cellular system for example, the number of receive antennas has to be small (say $2-4$). When this specificity is available, the design of the coarse lattice is quite easy. So, for each user "k", the vector TCQ consists in designing "n" r_k-dimensional quantizers instead of one $n \times r_k-$dimensional nested lattice. In the proposed solution there are essentially three technical points to be detailed: The design of the coarse lattice and the corresponding mapping and the way of insuring the DP encoder to have the properties mentioned above.

Coarse lattice/modulo operator design.
In the most simple case where $\forall k \in \{1, ..., K\}, r_k = 1$ the coarse lattice shape is fixed, it is simply cubic: As we use a scalar $mod - \Delta$ quantizer for trellis transition, the coarse lattice is equivalent to a cubic lattice of dimension n. This cubic lattice has to meet the power constraint $\Delta^2/12 = P$. In the case of multi-antenna receivers, the power constraint is replaced with a covariance matrix constraint, therefore a more complicated design for the coarse lattice is needed. In order to simplify this design a desirable feature of the coarse lattice has to be symmetric. The consequence of this is that the covariance matrix at the output of the vector TCQ is diagonal, which allows us to impose the "information theoretic" properties to the overall DP encoder easily. For example, in the case of $2-$antenna receivers, the well-known hexagonal lattice A_2 is a good candidate since one can easily impose $E[\underline{X}'_k \underline{X}'^H_k] = \mathbf{I}$.

Imposing good properties to the DP encoder.
By using a coarse lattice as we just described we know that the output $\underline{X}'_k$ of the modulo-Λ_2 operator (implementing the coarse lattice) is such that $\mathbf{\Sigma}'_k = \mathbf{I}$. Therefore in order to meet the condition (3.) we must have simultaneously $\underline{X}_k = \mathbf{G}_k \underline{\tilde{X}}_k$ and $\underline{X} = \mathbf{\Sigma}_k^{1/2} \underline{X}'_k$. This is why the modulo operator is followed by the linear operator $\mathbf{G}_k^{-1} \mathbf{\Sigma}_k^{1/2}$ in figure 2. Now the conditions (1.) and (2.) are implemented as follows. The information message W_k intended for user "k" is mapped into a coset $\underline{C}_k$ and the independency between $\underline{X}'_k$ and $\underline{S}_k$ is insured thanks to a dithering [9]. The coset $\underline{C}_k$ comprises all the points of Λ_1 that are located in the fundamental Voronoï cell of Λ_2. One notices that the idea of the auxiliary variable is implemented here through a coset code that is added to the signal $\mathbf{A}'_k \underline{S}_k$ with $\mathbf{A}'_k = \mathbf{\Sigma}'_\mathbf{k} \left(\mathbf{\Sigma}'_\mathbf{k} + \mathbf{\Sigma}_\mathbf{k}^{-1} \right)^{-1} = \left(\mathbf{I} + \mathbf{\Sigma}_k^{-1} \right)^{-1}$.

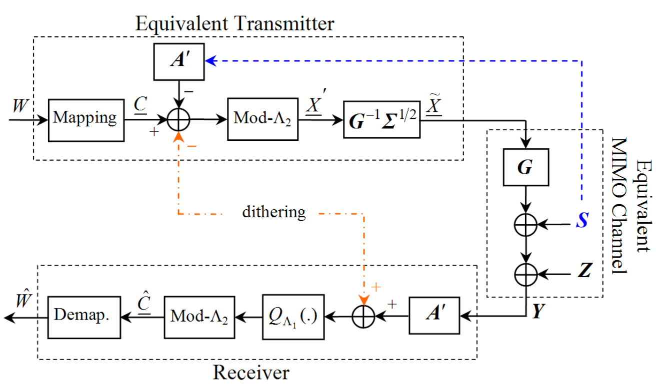

Fig. 2. Proposed coding scheme for implementing a vector DPC

Mapping inside a coarse lattice cell.
In the scalar case ($\forall k \in \{1, ..., K\}, r_k = 1$) the mapping of quantizer representatives is simply done according to Ungerboek's rules just like a trellis coded modulation. If $\forall k \in \{1, ..., K\}, r_k = 2$, an empirical mapping can still be done. However, for higher dimensions no simple and complete solution to this problem, which consists in quantizing a source with Voronoi constellations, is available. For example, suboptimum mapping schemes such as those proposed by Pepin et al. [15] can be used.

5. SIMULATION RESULTS

Chosen performance criterion
The situation under consideration is essentially of practical interest. The spectral efficiencies are fixed (corresponding to certain target data rates): $R_1 = 1$ bpcu, $R_2 = 1$ bpcu. Each of the user wants to reach a minimum QoS, which imposes $P_{e1} \leq QoS_1$, $P_{e2} \leq QoS_2$. The question is then: What is the minimum transmit power needed for satisfying these qualities of service? In what follows this minimal power has been computed for a given coding scheme (say $\mathcal{C}_1$) and compared to another one (say $\mathcal{C}_2$), which defines our performance criterion $G_{dB}(\frac{\mathcal{C}_1}{\mathcal{C}_2}) = 10 \log_{10} \left(\frac{P_{min}(\mathcal{C}_1)}{P_{min}(\mathcal{C}_2)} \right)$. Power allocation scheme: for each tested transmit power, the power is shared between the 2 users in order to reach the target pair of QoS. This procedure is applied to all the possible coding schemes (8 in total).

Simulation setup
Let $K = 2$. Three kind of "parameters" are considered in our setup: 1. The pair of the target qualities of service (QoS_1, QoS_2). Two cases: $(QoS_1, QoS_2) = (10^{-5}, 10^{-3})$ and $(QoS_1, QoS_2) = (10^{-5}, 10^{-5})$. 2. The triple of numbers of antennas (t, r_1, r_2). Three cases: $(t, r_1, r_2) = (2, 1, 1)$, $(t, r_1, r_2) = (4, 1, 1)$, $(t, r_1, r_2) = (4, 2, 2)$. 3. The broadcast channel "asymmetry" which can be defined as $\gamma = ||\mathbf{H}_1||^2/||\mathbf{H}_2||^2$ where $\mathbf{H}_k$ is the submatrix associated with user k. Two cases: $\gamma = 1$ and $\gamma = 6.5$. The following channel matrices have been considered: $\mathbf{H}_a = [1 \;\; 0.4; 0.4 \;\; 1]$, $\mathbf{H}_b = [3 \;\; 2; 1 \;\; 1]$, $\mathbf{H}_c = [1 \;\; 0.2 \; 0.25 \;\; 0.25; 0.2 \;\; 1 \;\; 0.25 \;\; 0.25]$, $\mathbf{H}_d = [2 \;\; 1 \;\; 0.8 \;\; 1.12; 0.2 \;\; 1 \;\; 0.1 \;\; 0.1]$,

$$H_e = \begin{bmatrix} -0.55 & 1.12 & 0.52 & -0.32 \\ 2.79 & -0.03 & -0.92 & -0.60 \\ 0.25 & -0.66 & -1.01 & -2.26 \\ 0.39 & 0.32 & -1.31 & -1.52 \end{bmatrix},$$

$$H_f = \begin{bmatrix} -0.43 & -1.15 & 0.33 & -0.59 \\ -1.67 & 1.19 & 0.17 & 2.18 \\ 0.11 & 1.06 & -0.17 & -0.12 \\ 0.26 & -0.03 & 0.65 & 0.10 \end{bmatrix}.$$

For the lack of space we will only provide here one part of the numerical results corresponding to the aforementioned scenarios.

Comparison of the outer coding schemes
Here the inner coding scheme is fixed (MMSE-DPC) and THS, SCS and TCQ are compared. For all the scenarios that have been tested the gain provided by using SCS instead of THS has been found to be small (typically 0.2 dB). It is known [7] that THS can be obtained by forcing α (or $\mathbf{A}$ in the vector case) to be "1" (or $\mathbf{I}$) in the SCS. This means that the performance of SCS depends on the knowledge of $E(X^2)$ (or $\mathbf{\Sigma}$) at the receiver. In practice, this means that the SCS performance will be dependent on the estimation accuracy of the corresponding parameters, in contrast with THS. As a consequence, using THS seems to be a better choice regarding the issue of robustness to estimation errors. On the other hand the proposed TCQ-based outer coding scheme always provides a significant transmit power gain ranging from 2 dB to 5 dB over the SCS (in fact we should say VCS when $r_k > 1$). More significant gains could be obtained by using a better modulation scheme in the TCQ implementation (e.g. QAM + shaping gain).

$(QoS_1, QoS_2) = (10^{-5}, 10^{-3})$	$(t, r_1, r_2) = (2,1,1)$		$(t, r_1, r_2) = (4,1,1)$		$(t, r_1, r_2) = (4,2,2)$	
	$\mathbf{H}_a$	$\mathbf{H}_b$	$\mathbf{H}_c$	$\mathbf{H}_d$	$\mathbf{H}_e$	$\mathbf{H}_f$
$G_{dB}(\frac{THS}{SCS})$	0.15	0.28	-0.03	0.41	***	***
$G_{dB}(\frac{SCS}{TCQ})$	2.02	2.23	2.26	1.85	4.56	5.31

Comparison of the inner coding schemes

Outer coding scheme ≡ TCQ. Here the ZF-DPC is compared to the MMSE-DPC. In terms of sum-rate, we know from the theory that their performance is generally close. In terms of BERs the results can be different and it turns out that using MMSE-DPC is almost always the better choice. In fact, when the broadcast channel asymmetry is marked (channels $\mathbf{H}_b$, $\mathbf{H}_d$, $\mathbf{H}_f$) the MMSE-DPC can provide a gain of about 2 dB.

$(QoS_1, QoS_2) = (10^{-5}, 10^{-3})$	$(t, r_1, r_2) = (2,1,1)$		$(t, r_1, r_2) = (4,1,1)$		$(t, r_1, r_2) = (4,2,2)$	
	$\mathbf{H}_a$	$\mathbf{H}_b$	$\mathbf{H}_c$	$\mathbf{H}_d$	$\mathbf{H}_e$	$\mathbf{H}_f$
$G_{dB}(\frac{zfdpc}{mmsedpc})$	0.16	2.62	-0.02	0.12	0.58	2.04

Comparison of the interference cancellation technique

Now we want to compare a DPC-based pre-cancellation scheme to its conventional counterpart. We compared the ”TCQ + MMSE-DPC” with the pre-MMSE interference canceller. In the case of single-antenna receivers, the pre-MMSE simply implements a ”pre-equalization” matrix applied to non-coded BPSK symbols ($R_1 = R_2 = 1$ bpcu). In the case of multi-antenna receivers, the pre-MMSE implements a ”pre-equalization” matrix applied to BPSK symbols coming from a $1/2$-rate repetition code ($R_1 = R_2 = 1$ bpcu since $r_1 = r_2 = 2$). Typically the gain provided the DPC approach provides a transmit power gain ranging from 1 dB to 3 dB.

$(QoS_1, QoS_2) = (10^{-5}, 10^{-3})$	$(t, r_1, r_2) = (2,1,1)$		$(t, r_1, r_2) = (4,1,1)$		$(t, r_1, r_2) = (4,2,2)$	
	$\mathbf{H}_a$	$\mathbf{H}_b$	$\mathbf{H}_c$	$\mathbf{H}_d$	$\mathbf{H}_e$	$\mathbf{H}_f$
$G_{dB}(\frac{pre-mmse}{mmse-dpc})$	2.38	3.85	1.27	0.85	3.02	2.95

6. CONCLUDING REMARKS

The performance of the DPC approach is both driven by the inner and outer coding schemes. For the DPC approach: From our analysis it turns out that implementing the outer coding through a TCQ is a good choice in terms of the performance-complexity tradeoff since it always provides a significant gain on the Tx power (typically 2.5 dB). This gain can be improved by using: 1. A QAM instead of a PAM 2. Shaping gain 3. Possibly a turbo-TCQ. For the inner encoder we also note that the Tomlinson Harashima scheme seems to be a more appropriate choice than the SCS (or VCS) taken into account the small gain provided by SCS over the THS and the potential performance degradation due to estimation errors for SCS (or VCS). On the other hand the results for the inner coding (linear precoder) are less significant and more channel-dependent. Even though the ZF-DPC performance is theoretically (in terms of sum-rate) close to that of the MMSE-DPC the BER analysis leads to a difference of 2 dB on the Tx power for asymmetric broadcast channels. Eventually, the gain provided by using the DPC approach over a conventional pre-cancellation technique (pre-MMSE) is not always significant (from 1 dB to 3 dB). It is clear that a better precoding scheme could be designed in order to optimize practical performance criteria such as the maximum number of satisfied users (target data rates, target QoSs) and also to provide a certain robustness to imperfect CSI.

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