# Outage Exponents of Block-Fading Channels With Power Allocation

Khoa D. Nguyen, Member, IEEE, Albert Guillén i Fàbregas, Senior Member, IEEE, and Lars K. Rasmussen, Senior Member, IEEE

***Abstract*—Power allocation is studied for fixed-rate transmission over block-fading channels with arbitrary continuous fading distributions and perfect transmitter and receiver channel state information. Both short- and long-term power constraints for arbitrary input distributions are considered. Optimal power allocation schemes are shown to be direct applications of previous results in the literature. It is shown that the short- and long-term outage exponents for arbitrary input distributions are related through a simple formula. The formula is useful to predict when the delay-limited capacity is positive. Furthermore, this characterization is useful for the design of efficient coding schemes for this relevant channel model.**

***Index Terms*—Block-fading, coded modulation, delay-limited capacity, outage diversity, outage probability, power allocation.**

## I. Introduction

The block-fading channel [1], [2] is a useful model for slowly varying fading in time and/or frequency. The channel consists of a finite number of flat fading blocks, whose fading gains are drawn from system dependent statistics. Transmission schemes such as orthogonal frequency division multiplexing (OFDM) and frequency-hopping, as encountered in the Global System for Mobile Communication (GSM) and the Enhanced Data GSM Environment (EDGE), over frequency-selective channels can be conveniently modeled as block-fading channels.

A codeword transmitted over a block-fading channel spans only a finite number of fading blocks. Therefore, the channel is nonergodic and not information stable [3], [4]. The Shannon capacity under most common fading statistics is zero, since there is an irreducible probability, denoted as the *outage probability*, that the channel is unable to support the target data rate [1], [2]. For sufficiently long codes, the outage probability is the natural fundamental limit of the channel [17]. In some cases zero outage probability can be achieved at nonzero rates and finite signal-to-noise ratio (SNR). The maximum rate with zero outage is commonly referred to as the *delay-limited capacity* [5].

Channel state information (CSI), namely the degree of knowledge that either the transmitter, the receiver, or both, have about the channel gains, greatly influences system design and performance [2]. At the receiver side, channel parameters can often be accurately estimated [6]. Thus, perfect CSI at the receiver (CSIR) is a common assumption. Conversely, CSI at the transmitter (CSIT) depends on the specific system architecture. In a system with time-division duplex (TDD), the same channel estimate can be used for both transmission and reception, provided that the channel varies slowly [7]. In other system architectures, CSIT is provided through direct feedback from the receiver. We consider an OFDM-inspired scenario, for which the channel coefficients of the parallel multi-carriers are perfectly known to the transmitter. When no CSIT is available, transmit power is commonly allocated uniformly over the blocks. In contrast, when CSIT is available, the transmitter can adapt the transmission mode (transmission power, data rate, modulation and coding) to the instantaneous channel characteristics, leading to significant improvements [2]. In this paper, we will consider power adaptation for fixed-rate transmission over delay-limited nonergodic block-fading channels.

The optimal (minimum outage) transmission strategy, subject to a short-term power constraint, was shown in [4] to consist of a random code with independently, identically distributed Gaussian code symbols, followed by optimal power allocation. Systems with short-term (per codeword) and long-term (average over many codewords) power constraints were considered, showing that significant gains in outage performance are possible by allowing for long-term power constraints. In some cases, the optimal power-allocation scheme can even eliminate outages, leading to a strictly positive delay-limited capacity [4], [8]. A criterion for positive delay-limited capacity was obtained for Gaussian inputs and Rayleigh fading in [8].

In this paper, we study power allocation rules that minimize the outage probability of fixed-rate schemes with arbitrary input distribution under long-term power constraints over block-fading channels with a general fading distribution. For channels with arbitrary inputs, the short-term power allocation scheme was developed in [9] using the relationship between mutual information and minimum mean-squared error (MMSE) obtained in [10]. Here we show that the optimal long-term power allocation scheme is a generalization of the

Manuscript received May 17, 2008; revised July 30, 2009. Current version published April 21, 2010. This work was supported by the Australian Research Council under ARC grants RN0459498, DP0558861, and DP0881160. The material in this paper was presented in part at the International Symposium on Information Theory, Toronto, Canada, July 2008.

K. D. Nguyen is with the Institute for Telecommunications Research, University of South Australia, Mawson Lakes 5095, South Australia, Australia (e-mail: khoa.nguyen@unisa.edu.au).

A. Guillén i Fàbregas is with the Department of Engineering, University of Cambridge, Cambridge CB2 1PZ, U.K. (e-mail: guillen@ieee.org).

L. K. Rasmussen was with the Institute for Telecommunications Research, University of South Australia. He is now with the Communication Theory Laboratory, School of Electrical Engineering and the ACCESS Linnaeus Center, Royal Institute of Technology, 100 44 Stockholm, Sweden (e-mail: lars.rasmussen@ieee.org).

Communicated by L. Zheng, Associate Editor for Communications.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TIT.2010.2043768

results in [4]. We study the corresponding outage SNR exponents for arbitrary input distributions and show that a simple formula relates short- and long-term exponents. We show that zero outage can be achieved provided that the corresponding short-term outage exponent is strictly greater than one, implying a positive delay-limited capacity. In some cases, we show that fixed discrete signal constellations like PSK or QAM, pay a small penalty with respect to optimal Gaussian inputs.

The paper is organized as follows. The system model and preliminaries are given in Sections II and III, respectively. Sections IV and V discuss the power allocation schemes for systems with peak and average power constraints, and their corresponding outage exponents, respectively. Examples are given in Section VI. Concluding remarks are given in Section VII. Proofs are included in the Appendices.

*Notation:* Scalar and vector variables are denoted by lowercase and boldfaced lowercase letters, respectively. Expectation of a function $\phi$ of random variables $\Phi_1, \dots, \Phi_n$ is denoted by $\mathbb{E}[\phi(\Phi_1, \dots, \Phi_n)]$, and expectation with respect to a random variable $\Phi$ with the constraint $\Phi \in \mathcal{R}$ is denoted by $\mathbb{E}_{\Phi \in \mathcal{R}}[\cdot]$. We define $\langle \boldsymbol{x} \rangle \triangleq \frac{1}{B}\sum_{i=1}^{B} x_i$ as the arithmetic mean of $\boldsymbol{x} = (x_1, \dots, x_B)$. Exponential equality $f(\xi) \doteq K\xi^{-d}$ indicates that $\lim_{\xi\to\infty} f(\xi)\xi^d = K$, with exponential inequalities $\dot{\leq}, \dot{\geq}$ similarly defined. $\lceil \xi \rceil$ ($\lfloor \xi \rfloor$) denotes the smallest (largest) integer greater (smaller) than $\xi$. Component-wise vector inequalities are denoted by $\succeq$ and $\preceq$.

## II. System Model

Consider transmission over a block-fading channel with $B$ blocks, where each is affected by a flat fading coefficient and additive noise. Assume that the fading coefficients are available at both the transmitter and the receiver, and that the transmitter allocates power to the blocks according to the rule $\boldsymbol{p}(\boldsymbol{\gamma}) = (p_1(\boldsymbol{\gamma}), \dots, p_B(\boldsymbol{\gamma}))$ where $\boldsymbol{\gamma} = \operatorname{diag}(\boldsymbol{h}\boldsymbol{h}^\dagger) \in \mathbb{R}^B$ is the power fading gain vector, with $\boldsymbol{h} \in \mathbb{C}^B$ being the fading vector, i.e., $\gamma_b = |h_b|^2, b = 1, \dots, B$. The equivalent baseband model is given by

$$\boldsymbol{y}_b = \sqrt{p_b(\boldsymbol{\gamma})} h_b \boldsymbol{x}_b + \boldsymbol{n}_b, \quad b = 1, \dots, B \tag{1}$$

where $\boldsymbol{x}_b \in \mathbb{C}^L$ and $\boldsymbol{y}_b \in \mathbb{C}^L$ are correspondingly the portion of the codeword transmitted and received in block $b$. Assume that $\boldsymbol{n}_b \in \mathbb{C}^L$ is a white Gaussian noise vector with entries drawn independently from a unit variance circularly symmetric Gaussian distribution $\mathcal{N}_{\mathbb{C}}(0,1)$, and the transmit symbols $x$ are drawn with input distribution $Q(x)$ from a unit-energy constellation $\mathcal{X}$, i.e., $\mathbb{E}[|X|^2] = 1$, where $X$ denotes the random variable corresponding to the transmitted symbols. Then, the instantaneous received SNR at block $b$ is given by $p_b(\boldsymbol{\gamma})\gamma_b$.

We assume that the complex fading vectors $\boldsymbol{h}$ are independently identically distributed from codeword to codeword, and that $\boldsymbol{h}$ follows a continuous power density distribution (pdf) $f_{\boldsymbol{h}}(\boldsymbol{h})$ with $\mathbb{E}[\|\boldsymbol{h}\|^2] = B$. The power fading gains then have a continuous pdf $f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})$ with normalized power $\mathbb{E}[\langle \boldsymbol{\gamma} \rangle] = 1$. We consider systems with the following power constraints:

$$\text{Short term:} \quad \langle \boldsymbol{p}(\boldsymbol{\gamma}) \rangle \leq P_{\text{st}}$$
$$\text{Long term:} \quad \mathbb{E}[\langle \boldsymbol{p}(\boldsymbol{\gamma}) \rangle] \leq P_{\text{lt}}.$$

Note that short- and long-term power constraints induce peak and average power restrictions.

## III. Preliminaries

The channel model described in (1) corresponds to a parallel channel model, where each subchannel is used a fraction $\frac{1}{B}$ of the total number of channel uses per codeword. Therefore, for any given power fading gain realization $\boldsymbol{\gamma}$ and power allocation scheme $\boldsymbol{p}(\boldsymbol{\gamma})$, the instantaneous input-output mutual information of the channel is given by [11]

$$I_B(\boldsymbol{p}(\boldsymbol{\gamma}), \boldsymbol{\gamma}) = \frac{1}{B}\sum_{b=1}^{B} I(p_b\gamma_b) \tag{2}$$

where $I(\rho)$ is the input-output mutual information of an AWGN channel with input constellation $\mathcal{X}$ and received SNR $\rho$. When the channel inputs are Gaussian, i.e., $\mathcal{X} = \mathbb{C}$ and $X$ is drawn from the unit variance complex Gaussian distribution $\mathcal{N}_{\mathbb{C}}(0,1)$, we have that $I(\rho) = \log_2(1+\rho)$ [11]. On the other hand, when coded modulation over a signal constellation $\mathcal{X}$ is used with probability assignment $Q(X)$, we obtain

$$I(\rho) = \mathbb{E}\left[\log_2 \frac{e^{-|Y-\sqrt{\rho}X|^2}}{\sum_{x'\in\mathcal{X}} Q(x')\, e^{-|Y-\sqrt{\rho}x'|^2}}\right]. \tag{3}$$

A fundamental relationship between the MMSE and the mutual information (in bits) in additive Gaussian channels is introduced in [10] showing that for any input distribution and constellation

$$\frac{d}{d\rho} I(\rho) = \frac{1}{\log 2}\text{MMSE}(\rho) \tag{4}$$

where $\text{MMSE}(\rho)$ is the MMSE in estimating the input symbol transmitted over an AWGN channel with SNR $\rho$. For Gaussian inputs $\text{MMSE}(\rho) = \frac{1}{1+\rho}$, while for coded modulation [9]

$$\text{MMSE}(\rho) = \mathbb{E}[|X|^2] - \frac{1}{\pi}\int_{\mathbb{C}} \frac{\left|\sum_{x\in\mathcal{X}} Q(x)\, x\, e^{-|y-\sqrt{\rho}x|^2}\right|^2}{\sum_{x\in\mathcal{X}} Q(x)\, e^{-|y-\sqrt{\rho}x|^2}} dy. \tag{5}$$

Results in this paper can also be applied to systems with bit-interleaved coded modulation (BICM) [12], where the relationship in (4) is replaced by the corresponding results in [13].

Finally, we define the transmission to be in outage when the instantaneous input-output mutual information is less than the target *fixed* transmission rate $R$. For a given power allocation scheme $\boldsymbol{p}(\boldsymbol{\gamma})$ with power constraint $P$, the outage probability at transmission rate $R$ is given by [1], [2]

$$P_{\text{out}}(\boldsymbol{p}(\boldsymbol{\gamma}), P, R) \triangleq \Pr(I_B(\boldsymbol{p}(\boldsymbol{\gamma}), \boldsymbol{\gamma}) < R) = \Pr\left(\frac{1}{B}\sum_{b=1}^{B} I(p_b\gamma_b) < R\right). \tag{6}$$

## IV. Power Allocation Schemes

In this section, we briefly present the short- and long-term power allocation optimization problems and their corresponding solutions for arbitrary inputs.

### A. Short-Term Power Constraints

Formally, the power allocation scheme $\boldsymbol{p}^{\rm opt}(\boldsymbol{\gamma})$ that minimizes outage probability with short-term power constraint $P_{\rm st}$ is given by

$$\boldsymbol{p}_{\rm st}^{\rm opt}(\boldsymbol{\gamma}) = \arg \min_{\substack{\boldsymbol{p}\succeq \boldsymbol{0}\\ \langle \boldsymbol{p}\rangle \leq P_{\rm st}}} P_{\rm out}(\boldsymbol{p}, P_{\rm st}, R). \tag{7}$$

Following [4], a solution of the problem is given by

$$\boldsymbol{p}_{\rm st}^{\rm opt}(\boldsymbol{\gamma}) = \arg \max_{\substack{\boldsymbol{p}\succeq 0\\ \langle \boldsymbol{p}\rangle \leq P_{\rm st}}} \sum_{b=1}^{B} I(p_b\gamma_b). \tag{8}$$

The problem given in (8) is convex; therefore, applying the Karush–Kuhn–Tucker (KKT) conditions, and noting the relationship in (4), we have that [9]

$$p_b^{\rm opt} = \frac{1}{\gamma_b}\,\mathrm{MMSE}^{-1}\left(\min\left\{\mathrm{MMSE}(0), \frac{\eta}{\gamma_b}\right\}\right) \tag{9}$$

where $\eta$ is chosen such that the peak power constraint is active

$$\frac{1}{B}\sum_{b=1}^{B}\frac{1}{\gamma_b}\,\mathrm{MMSE}^{-1}\left(\min\left\{\mathrm{MMSE}(0), \frac{\eta}{\gamma_b}\right\}\right) = P_{\rm st}. \tag{10}$$

The optimal outage probability is then given by $P_{\rm out}(\boldsymbol{p}_{\rm st}^{\rm opt}(\boldsymbol{\gamma}), P, R)$. The power adaptive allocation rule in (9) is referred to as mercury/water-filling, which turns into the classical water-filling scheme [11] when the input constellation is Gaussian. The relationship between water-filling and mercury/water-filling has been discussed in [9], while further insight can be obtained from [14].

Alternatively, consider the following power allocation rule

$$\boldsymbol{p}^{\star}(\boldsymbol{\gamma}) = \begin{cases} \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), & \langle \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})\rangle \leq P_{\rm st}\\ 0, & \text{otherwise}\end{cases} \tag{11}$$

where $\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})$ is the power allocation rule that minimizes the power required for transmission at rate $R$. In particular, $\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})$ solves the following problem:

$$\begin{cases} \text{Minimize} & \langle \boldsymbol{\wp}(\boldsymbol{\gamma})\rangle\\ \text{Subject to} & \frac{1}{B}\sum_{b=1}^{B} I(\wp_b\gamma_b) \geq R\\ & \wp_b \geq 0, \quad b = 1,\ldots,B.\end{cases} \tag{12}$$

Since the problem in (12) is convex, applying the KKT conditions, a solution for the problem is given by

$$\wp_b^{\rm opt} = \frac{1}{\gamma_b}\,\mathrm{MMSE}^{-1}\left(\min\left\{\mathrm{MMSE}(0), \frac{\eta}{\gamma_b}\right\}\right) \tag{13}$$

where $\eta$ is chosen such that the rate constraint is met with equality, i.e.,

$$\sum_{b=1}^{B} I\left(\mathrm{MMSE}^{-1}\left(\min\left\{\mathrm{MMSE}(0), \frac{\eta}{\gamma_b}\right\}\right)\right) = BR. \tag{14}$$

The outage probability of the power allocation rule $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$ is given by

$$P_{\rm out}(\boldsymbol{p}^{\star}(\boldsymbol{\gamma}), P_{\rm st}, R) = \Pr(\langle \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})\rangle > P_{\rm st}). \tag{15}$$

The power allocation rule is optimal in terms of outage, as given by the following duality result. The result was observed in [4], but is proved rigorously here for the sake of completeness.

*Proposition 1:* Consider transmission at rate $R$ over the block-fading channel given in (1) with input constellation $\mathcal{X}$ and power allocation rules $\boldsymbol{p}^{\rm opt}(\boldsymbol{\gamma})$ and $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$, respectively. For any fading distribution, we have that

$$P_{\rm out}(\boldsymbol{p}^{\star}(\boldsymbol{\gamma}), P_{\rm st}, R) = P_{\rm out}(\boldsymbol{p}^{\rm opt}(\boldsymbol{\gamma}), P_{\rm st}, R). \tag{16}$$

*Proof:* See Appendix A. ■

Compared to the power allocation rule $\boldsymbol{p}^{\rm opt}(\boldsymbol{\gamma})$ given in (9), the scheme $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$ given in (11) is computationally more demanding and less practical for transmission with short-term power constraints. However, as we show in the next section, $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$ has the same structure as the power allocation rule for systems with long-term power constraint, and will therefore prove more useful in the subsequent analysis.

### B. Long-Term Power Constraints

For a system with a long-term power constraint $P_{\rm lt}$, the optimal power allocation scheme $\boldsymbol{p}_{\rm lt}^{\rm opt}$ is given by

$$\boldsymbol{p}_{\rm lt}^{\rm opt}(\boldsymbol{\gamma}) = \arg \min_{\substack{\boldsymbol{p}\succeq 0\\ \mathbb{E}[\langle \boldsymbol{p}\rangle] \leq P_{\rm lt}}} P_{\rm out}(\boldsymbol{p}, P, R). \tag{17}$$

The optimal solution to the above optimization problem was obtained in [4] for Gaussian inputs only. This solution can be trivially generalized to systems with arbitrary inputs using the relationship in (4). For fading distributions with continuous pdf, the solution is given by [15], [16]

$$\boldsymbol{p}_{\rm lt}^{\rm opt}(\boldsymbol{\gamma}) = \begin{cases} \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), & \langle \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})\rangle \leq s^{\star}\\ 0, & \text{otherwise}\end{cases} \tag{18}$$

where $\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})$ is given in (13) and the threshold $s^{\star}$ is such that

$$\begin{cases} s^{\star} = \infty, & \lim_{s\to\infty} \mathcal{P}(\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), s) \leq P_{\rm lt}\\ \mathcal{P}(\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), s^{\star}) = P_{\rm lt}, & \text{otherwise}\end{cases} \tag{19}$$

with

$$\begin{aligned}\mathcal{P}(\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), s) &\triangleq \mathbb{E}_{\{\boldsymbol{\gamma}:\langle \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})\rangle \leq s\}}[\langle \boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma})\rangle]\\ &= \mathbb{E}\left[\langle \boldsymbol{p}_{\rm lt}^{\rm opt}(\boldsymbol{\gamma})\rangle\right]\end{aligned} \tag{20}$$

being the long-term power consumed given a short-term power threshold $s$. The resulting outage probability is therefore $P_{\rm out}(\boldsymbol{\wp}^{\rm opt}(\boldsymbol{\gamma}), s^{\star}, R)$. Note that this is exactly the same as the outage probability achieved by a system with short-term power constraint $s^{\star}$. This duality between short- and long-term power constraints, together with the short-term outage exponents, will be used to characterize the outage exponents of long-term power-constrained systems.

## V. Outage Exponents

In this section, we present the large-SNR analysis of the outage probability with short- and long-term power constraints. In particular, we examine the outage exponents, i.e., the asymptotic slope of the outage probability curve with respect to SNR in log-log scale [17]–[19]

$$d_{\rm st}(R) \triangleq \lim_{P_{\rm st}\to\infty} \frac{-\log(P_{\rm out}(\boldsymbol{p}(\boldsymbol{\gamma}), P_{\rm st}, R))}{\log(P_{\rm st})}. \tag{21}$$

Similarly, the outage exponent of systems with long-term power constraints is defined as

$$d_{\rm lt}(R) \triangleq \lim_{s\to\infty} \frac{-\log(P_{\rm out}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s, R))}{\log(\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s))} \tag{22}$$

namely, the exponent of the outage probability as a function of the average power consumed.

*Proposition 2:* Consider transmission at rate $R$ over the block-fading channel given in (1). The largest outage diversity that short-term power allocation solutions can have is the same as the outage diversity of the uniform power allocation $\boldsymbol{p}^{\rm uni}(P_{\rm st}) = (P_{\rm st}, \ldots, P_{\rm st})$. Furthermore, the optimal short-term power allocation solution achieves this diversity.

*Proof:* See Appendix B. ■

The outage diversity obtained by systems with short-term power constraints $P_{\rm st}$ and uniform power allocation $\boldsymbol{p}^{\rm uni}(P_{\rm st})$ is a well-studied quantity, and has been obtained in various works for multiple fading distributions [17]–[20].

We now investigate the outage behavior of power allocation schemes with long-term power constraints based on the corresponding short-term outage diversity and obtain a criterion for zero outage (i.e., positive delay-limited capacity), since the criterion given in [8] is not applicable to systems with arbitrary inputs. In particular, for an arbitrary short-term power allocation rule $\boldsymbol{\wp}(\boldsymbol{\gamma})$ satisfying $I_B(\boldsymbol{\wp}(\boldsymbol{\gamma}), \boldsymbol{\gamma}) \geq R$, we consider the power allocation scheme

$$\boldsymbol{p}_{\rm lt}(\boldsymbol{\gamma}) = \begin{cases} \boldsymbol{\wp}(\boldsymbol{\gamma}), & \langle \boldsymbol{\wp}(\boldsymbol{\gamma})\rangle \leq \hat{s} \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

where $\hat{s}$ satisfies

$$\begin{cases} \hat{s} = \infty, & \lim_{s\to\infty} \mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s) \leq P_{\rm lt} \\ \mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), \hat{s}) = P_{\rm lt}, & \text{otherwise.} \end{cases} \tag{24}$$

Assume that the power allocation rule $\boldsymbol{\wp}(\boldsymbol{\gamma})$ achieves an outage diversity $d_{\rm st}(R)$ for with short-term power constraint $s$, i.e.

$$P_{\rm out}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s, R) \doteq \mathcal{K}\, s^{-d_{\rm st}(R)}. \tag{25}$$

We then have the following characterization of the average power function $\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s)$.

*Proposition 3:* Consider transmission at rate $R$ over the block-fading channel given in (1). Assume that the fading coefficients have a continuous pdf and that power is allocated according to (23), where $\boldsymbol{\wp}(\boldsymbol{\gamma})$ satisfies (25). Then, for large $s$, we have that

$$\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s) \doteq \mathcal{K}\, d_{\rm st}(R) s^{-d_{\rm st}(R)}. \tag{26}$$

*Proof:* See Appendix C. ■

From the previous proposition, we have the following characterization of the outage probability of systems with long-term power constraints.

*Theorem 1:* Consider transmission at rate $R$ over the block-fading channel given in (1). Assume that the fading coefficients have a continuous pdf and that power is allocated according to (23), where $\boldsymbol{\wp}(\boldsymbol{\gamma})$ satisfies (25). Then, we have the following.

- If $d_{\rm st}(R) > 1$, then $\lim_{s\to\infty} \mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s) \triangleq P_{\rm th} < \infty$ and $d_{\rm lt} = \infty$;
- if $d_{\rm st}(R) = 1$, then $\lim_{s\to\infty} \mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s) = \infty$ and $d_{\rm lt}(R) = \infty$; and
- if $d_{\rm st}(R) < 1$, then

$$d_{\rm lt}(R) = \frac{d_{\rm st}(R)}{1 - d_{\rm st}(R)}. \tag{27}$$

*Proof:* See Appendix D. ■

The above theorem gives a simple relationship between the outage diversity of systems with short- and long-term power constraints. Given a system with power allocation rule $\boldsymbol{\wp}(\boldsymbol{\gamma})$ that achieves a short-term outage diversity $d_{\rm st}(R)$, the long-term outage diversity is readily obtained from the theorem. If $d_{\rm st}(R) \leq 1$, reliable transmission in the strict Shannon sense is not possible for any finite long-term power constraint. Further, when $d_{\rm st}(R) < 1$, the outage diversity of systems with long-term power constraints is given as a function of $d_{\rm st}(R)$ according to (27). However, when $d_{\rm st}(R) > 1$, reliable transmission is possible for long-term power constraints $P_{\rm lt} \geq P_{\rm th}$. Equivalently, the delay-limited capacity [5] of systems with power allocation rule $\boldsymbol{p}_{\rm lt}(\boldsymbol{\gamma})$ and long-term power constraint $P_{\rm th}$ is $R$.

Note that no assumptions regarding the underlying channel model or fading distribution are required in proving Proposition 3 and Theorem 1. The relationship between short-term and long-term outage diversity is derived based solely on the power allocation structure in (18). The result in Theorem 1 can therefore be used to analyze the performance of various systems with long-term power constraint. Examples of particular interest where Proposition 3 and Theorem 1 hold include MIMO block-fading channels (where the corresponding power allocation problem is not convex [21]), or hybrid radio-frequency and free-space optical where the fading distributions can be exponential, lognormal, gamma-gamma, lognormal-Rice or IK [22].

The above result is also key for efficient code design. In particular, if we want to approach the outage probability with powerful codes, we must embed sufficient structure in the code, so that it achieves the optimal short-term diversity exponent. It is well known that the short-term exponent of good codes over the block-fading channel is related to the block-diversity [17]–[19], [23], [20], which is the minimum (over the codebook) number of

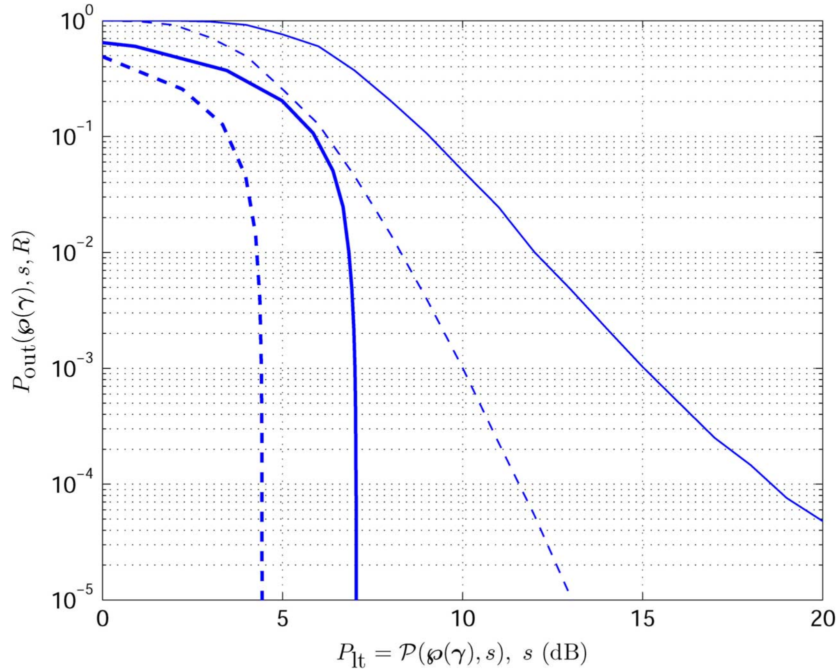


Fig. 1. Outage probability of transmission with optimal power allocation schemes using Gaussian and uniform QPSK inputs over a Nakagami-$m$ block-fading channel with $B = 4, R = 1.7, m = 2$. The solid lines and dashed lines correspondingly represent the outage performance of systems with uniform QPSK and Gaussian inputs. The thin lines are plots of outage probability versus the short-term power $s$ and thick lines are plots of outage probability versus the long-term power $P_{\rm lt} = \mathcal{P}(\wp(\gamma), s)$.

blocks in which two codewords differ, i.e., the blockwise Hamming distance. Hence, the above result immediately provides the optimal diversity design criterion to design codes for this relevant setup, which has been open since the first results of [4].

## VI. Examples

In this section, we show some examples of the above general results for a specific input and fading distributions over the block-fading channel described by (1). In particular, we consider Gaussian and QPSK inputs over block-fading channels with Nakagami-$m$ distributed fading. The power fading gains $\gamma_b, b = 1, \ldots, B$ are then independently identically distributed with the following pdf :

$$f_{\gamma_b}(\xi) = \frac{m^m \xi^{m-1}}{\Gamma(m)} e^{-m\xi}, \quad \xi \geq 0 \tag{28}$$

where $\Gamma(m) = \int_0^\infty t^{m-1} e^{-t} dt$ is the Gamma function [24].

The optimal short-term outage diversity $d_{\rm st}^{\rm opt}(R)$ of systems with fixed discrete input constellation $\mathcal{X}$ of size $2^M$ and uniform power allocation has been studied in [18], [17], and [19] for Rayleigh fading channels ($m = 1$) and in [20] for Nakagami-$m$ fading channels with general $m$. The outage diversity for communication at rate $R$ is given by the Singleton bound

$$d_{\rm st}^{\rm opt}(R) = m\left(1 + \left\lfloor B\left(1 - \frac{R}{M}\right)\right\rfloor\right). \tag{29}$$

On the other hand, the outage diversity for Gaussian inputs is $d_{\rm st}^{\rm opt}(R) = mB$ for any $R > 0$. Theorem 1 allows for characterizing the outage diversity achieved by the optimal long-term power allocation rule with arbitrary input distributions. In particular, if $d_{\rm st}^{\rm opt}(R) > 1$, there exists $P_{\rm th}$ such that for all $P_{\rm av} > P_{\rm th}$ reliable transmission is possible, whereas if $d_{\rm st}^{\rm opt}(R) < 1$, the outage diversity is given by $d_{\rm st}^{\rm opt}(R)/(d_{\rm st}^{\rm opt}(R) - 1)$.

In Figs. 1 and 2, we illustrate the outage probabilities for transmission with uniform QPSK and Gaussian inputs at rate $R = 1.7$ over block-fading channels with $B = 4$, $m = 0.5$ and $m = 2$, respectively. With $R = 1.7$, we obtain that $d_{\rm st}^{\rm opt}(R) = m$ for QPSK, while $d_{\rm st}^{\rm opt}(R) = 4\ m$ for Gaussian inputs. For $m = 2$, both QPSK and Gaussian inputs achieve zero outage with long-term power constraints, as predicted by Theorem 1. In this case, we observe that uniform QPSK pays a small penalty in average SNR with respect to Gaussian inputs.

On the other hand, for $m = 0.5$, Fig. 2 shows, in agreement with Theorem 1, that with long-term power constraints, the outage diversity for QPSK is $d_{\rm lt}(R) = \frac{m}{1-m} = 1$; while zero outage can still be achieved with Gaussian inputs. Due to the Singleton bound, QPSK would still achieve zero outage with lower rates. In fact, the Singleton bound characterizes the minimum rate that can be transmitted with zero outage for any fixed discrete signal constellation.

Note that, while [4] only observed the no-outage phenomenon in certain cases, and [8] provided a condition for Gaussian inputs only, Theorem 1 rigorously characterizes this zero-outage behavior for general input and fading distributions.

## VII. Conclusion

We have studied the high-power behavior of power allocation with short- and long-term power constraints in block-fading channels with arbitrary input and fading distributions. We have shown a duality property between short- and long-term power constrained systems, which enables a simple expression of the long-term outage diversity as a function of its short-term

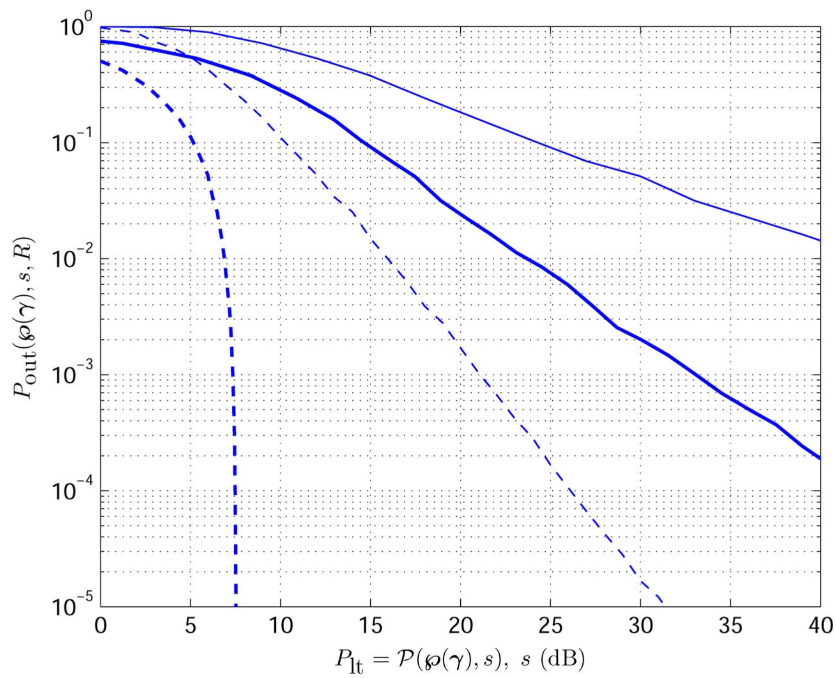


Fig. 2. Outage probability of transmission with optimal power allocation schemes using Gaussian and uniform QPSK inputs over a Nakagami-$m$ block-fading channel with $B = 4, R = 1.7, m = 0.5$. The solid and dashed lines correspondingly represent the outage performance of systems with uniform QPSK and Gaussian inputs. The thin lines are plots of outage probability versus the short-term power $s$ and thick lines are plots of outage probability versus the long-term power $P_{\mathrm{lt}} = \mathcal{P}(\wp(\boldsymbol{\gamma}), s)$.

counterpart. We prove that when the short-term outage diversity is strictly larger than one, reliable communication in the strict Shannon sense is possible above a certain threshold. Otherwise, the long-term outage diversity can be obtain via a simple function of the short-term counterpart. This result generalizes previous observations and results from [4] and [8], where Gaussian inputs on Rayleigh fading channels were studied. In turn, the result provides a key design criterion for efficient design of outage-approaching codes, a problem that has been open since the early results of [4].

## Appendix A
## Proof of Proposition 1

We prove that an outage event with power allocation rule $\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma})$ yields an outage event with power allocation rule $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$, and vice versa.

Consider a system with power allocation rule $\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma})$. Assume that a fading realization $\boldsymbol{\gamma}$ results in an outage, i.e., we have that $I_B(\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma}), \boldsymbol{\gamma}) < R$. Since $\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma})$ is a solution of (8), $\langle \boldsymbol{p}(\boldsymbol{\gamma}) \rangle \geq P_{\mathrm{st}}$ for all power allocation rules such that $I_B(\boldsymbol{p}(\boldsymbol{\gamma}), \boldsymbol{\gamma}) \geq R$. Therefore, $\langle \boldsymbol{\wp}^{\mathrm{opt}}(\boldsymbol{\gamma}) \rangle > P_{\mathrm{st}}$, and $\boldsymbol{p}^{\star}(\boldsymbol{\gamma}) = 0$, which results in an outage for the system with power allocation rule $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$.

By using similar argument, an outage event in with power allocation rule $\boldsymbol{p}^{\star}(\boldsymbol{\gamma})$ results in outage with power allocation rule $\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma})$.

## Appendix B
## Proof of Proposition 2

Consider an arbitrary power allocation rule $\boldsymbol{p}(\boldsymbol{\gamma})$ satisfying short-term power constraint $P_{\mathrm{st}}$, we have that $\boldsymbol{p}(\boldsymbol{\gamma}) \preceq \boldsymbol{p}^{\mathrm{uni}}(BP_{\mathrm{st}})$. Therefore

$$P_{\mathrm{out}}(\boldsymbol{p}(\boldsymbol{\gamma}), P_{\mathrm{st}}, R) \geq P_{\mathrm{out}}(\boldsymbol{p}^{\mathrm{uni}}(BP_{\mathrm{st}}), BP_{\mathrm{st}}, R). \quad (30)$$

Consequently, the outage diversity of systems with power allocation rule $\boldsymbol{p}(\boldsymbol{\gamma})$ satisfies

$$\begin{aligned} d_{\mathrm{st}}(R) &= \lim_{P_{\mathrm{st}}\to\infty} \frac{-\log(P_{\mathrm{out}}(\boldsymbol{p}(\boldsymbol{\gamma}), P_{\mathrm{st}}, R))}{\log(P_{\mathrm{st}})} \\ &\leq \lim_{P_{\mathrm{st}}\to\infty} \frac{-\log(P_{\mathrm{out}}(\boldsymbol{p}^{\mathrm{uni}}(BP_{\mathrm{st}}), BP_{\mathrm{st}}, R))}{\log(P_{\mathrm{st}})} \\ &= \lim_{P_{\mathrm{st}}\to\infty} \frac{-\log(P_{\mathrm{out}}(\boldsymbol{p}^{\mathrm{uni}}(BP_{\mathrm{st}}), BP_{\mathrm{st}}, R))}{\log(BP_{\mathrm{st}})} \\ &= d_{\mathrm{st}}^{\mathrm{uni}}(R). \end{aligned} \quad (31)$$

Thus, $d_{\mathrm{st}}(R) \leq d_{\mathrm{st}}^{\mathrm{uni}}(R)$ is the largest outage diversity with short-term power constraints. Furthermore, since the optimal power allocation scheme is such that

$$P_{\mathrm{out}}(\boldsymbol{p}^{\mathrm{opt}}(\boldsymbol{\gamma}), P_{\mathrm{st}}, R) \leq P_{\mathrm{out}}(\boldsymbol{p}^{\mathrm{uni}}(P_{\mathrm{st}}), P_{\mathrm{st}}, R) \quad (32)$$

it follows that $d_{\mathrm{st}}^{\mathrm{opt}}(R) \geq d_{\mathrm{st}}^{\mathrm{uni}}(R)$, hence, proving that the optimal short-term solution has the same diversity as uniform power allocation.

## APPENDIX C
## PROOF OF PROPOSITION 3

From the definition of differentiation, we have that

$$\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)=\lim_{a\downarrow 1}\frac{\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),as)-\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}{as-s}. \quad (33)$$

Let $\mathcal{R}(s)=\{\boldsymbol{\gamma}:\langle\boldsymbol{\wp}(\boldsymbol{\gamma})\rangle\leq s\}$, we have $\mathcal{R}(s)\subset\mathcal{R}(as)$. Therefore

$$\begin{aligned}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),as)&-\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\\&=\int_{\boldsymbol{\gamma}\in\mathcal{R}(as)}\langle\boldsymbol{\wp}(\boldsymbol{\gamma})\rangle f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})d\boldsymbol{\gamma}-\int_{\boldsymbol{\gamma}\in\mathcal{R}(s)}\langle\boldsymbol{\wp}(\boldsymbol{\gamma})\rangle f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})d\boldsymbol{\gamma}\\&=\int_{\boldsymbol{\gamma}\in\mathcal{R}(as)\setminus\mathcal{R}(s)}\langle\boldsymbol{\wp}(\boldsymbol{\gamma})\rangle f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})d\boldsymbol{\gamma}. \quad (34)\end{aligned}$$

Noting that $\forall\boldsymbol{\gamma}\in\mathcal{R}(as)\setminus\mathcal{R}(s), s\leq\langle\boldsymbol{\wp}(\boldsymbol{\gamma})\rangle\leq as$, we have

$$\begin{aligned}s\int_{\boldsymbol{\gamma}\in\mathcal{R}(as)\setminus\mathcal{R}(s)}f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})d\boldsymbol{\gamma}&\leq\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)-\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),as)\\&\leq as\int_{\boldsymbol{\gamma}\in\mathcal{R}(as)\setminus\mathcal{R}(s)}f_{\boldsymbol{\gamma}}(\boldsymbol{\gamma})d\boldsymbol{\gamma} \quad (35)\\ s\mathcal{F}(as,s)&\leq\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)-\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),as)\\&\leq as\mathcal{F}(as,s) \quad (36)\end{aligned}$$

where $\mathcal{F}(as,s)\triangleq\Pr(\boldsymbol{\gamma}\in\mathcal{R}(as))-\Pr(\boldsymbol{\gamma}\in\mathcal{R}(s))$. Since $\Pr(\boldsymbol{\gamma}\in\mathcal{R}(\xi))=1-P_{\rm out}(\boldsymbol{\wp}(\boldsymbol{\gamma}),\xi,R)\doteq 1-\mathcal{K}\xi^{-d_{\rm st}(R)}$, we have

$$\mathcal{F}(as,s)\doteq\left(1-a^{-d_{\rm st}(R)}\right)\mathcal{K}s^{-d_{\rm st}(R)}. \quad (37)$$

Therefore, (36) gives

$$\begin{aligned}\left(1-a^{-d_{\rm st}(R)}\right)\mathcal{K}s^{1-d_{\rm st}(R)}&\dot{\leq}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),as)-\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\\&\dot{\leq}a\left(1-a^{-d_{\rm st}(R)}\right)\mathcal{K}s^{1-d_{\rm st}(R)}. \quad (38)\end{aligned}$$

Inserting (38) into (33) and let $a\downarrow 1$, we have

$$\frac{d}{ds}P(s)\doteq\mathcal{K}d_{\rm st}(R)s^{-d_{\rm st}(R)} \quad (39)$$

as required.

## APPENDIX D
## PROOF OF THEOREM 1

From Proposition 3, we have that

$$\lim_{s\to\infty}\left(\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\right)s^{d_{\rm st}(R)}=\mathcal{K}d_{\rm st}(R). \quad (40)$$

Therefore, for any $\epsilon>0$, there exists a finite $s_1$ such that for all $s>s_1$,

$$\begin{aligned}\mathcal{K}d_{\rm st}(R)-\epsilon&<\left(\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\right)s^{d_{\rm st}(R)}\\&<\mathcal{K}d_{\rm st}(R)+\epsilon\\(\mathcal{K}d_{\rm st}(R)-\epsilon)s^{-d_{\rm st}(R)}&<\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\\&<(\mathcal{K}d_{\rm st}(R)+\epsilon)s^{-d_{\rm st}(R)}. \quad (41)\end{aligned}$$

Therefore, if $d_{\rm st}(R)>1$, we have that

$$\begin{aligned}\lim_{s\to\infty}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)&=\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s_1)\\&\quad+\lim_{s\to\infty}\int_{s_1}^{s}\frac{d}{dt}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),t)dt\\&<\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s_1)\\&\quad+\lim_{s\to\infty}\int_{s_1}^{s}(\mathcal{K}d_{\rm st}(R)+\epsilon)t^{-d_{\rm st}(R)}dt \quad (42)\\&=\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s_1)\\&\quad+\frac{(\mathcal{K}d_{\rm st}(R)+\epsilon)s_1^{1-d_{\rm st}(R)}}{d_{\rm st}(R)-1}\triangleq P_{\rm th}<\infty \quad (43)\end{aligned}$$

and

$$\begin{aligned}d_{\rm lt}(R)&=\lim_{s\to\infty}\frac{-\log P_{\rm out}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s,R)}{\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}\\&\geq\lim_{s\to\infty}\frac{-\log\left(\mathcal{K}s^{-d_{\rm st}(R)}\right)}{P_{\rm th}}=\infty \quad (44)\end{aligned}$$

as required.

Meanwhile, if $d_{\rm st}(R)\leq 1$, from (41) and (42), we have

$$\begin{aligned}&\lim_{s\to\infty}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)\\&\quad>P(s_1)+\lim_{s\to\infty}\int_{s_1}^{s}(\mathcal{K}d_{\rm st}(R)-\epsilon)t^{-d_{\rm st}(R)}dt\\&\quad=\infty. \quad (45)\end{aligned}$$

Now, the outage diversity is given by

$$\begin{aligned}d_{\rm lt}(R)&=\lim_{s\to\infty}\frac{-\log P_{\rm out}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s,R)}{\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}\\&=\lim_{s\to\infty}\frac{-\log\left(\mathcal{K}s^{-d_{\rm st}(R)}\right)}{\log\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}\\&=\lim_{s\to\infty}\frac{d_{\rm st}(R)\log(s)}{\log\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}.\end{aligned}$$

Since $\lim_{s\to\infty}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)=\infty$, applying L'Hôpital's rule, we have

$$d_{\rm lt}(R)=\lim_{s\to\infty}\frac{d_{\rm st}(R)\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}{s\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}. \quad (46)$$

Applying Proposition 3, we can further write

$$d_{\mathrm{lt}}(R) = \lim_{s\to\infty} \frac{\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s)}{\mathcal{K}s^{1-d_{\mathrm{st}}(R)}}. \quad (47)$$

Therefore, if $d_{\mathrm{st}}(R) = 1$, we have $d_{\mathrm{lt}}(R) = \lim_{s\to\infty} \frac{\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}),s)}{\mathcal{K}} = \infty$, while if $d_{\mathrm{st}}(R) < 1$, further applying L'hôpital's rule and Proposition 3, we obtain

$$d_{\mathrm{lt}}(R) = \lim_{s\to\infty} \frac{\frac{d}{ds}\mathcal{P}(\boldsymbol{\wp}(\boldsymbol{\gamma}), s)}{\mathcal{K}(1-d_{\mathrm{st}}(R))s^{-d_{\mathrm{st}}(R)}} = \frac{d_{\mathrm{st}}(R)}{1-d_{\mathrm{st}}(R)} \quad (48)$$

which completes the proof.

## References


[1] L. H. Ozarow, S. Shamai, and A. D. Wyner, "Information theoretic considerations for cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. 43, no. 2, pp. 359–378, May 1994.

[2] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2619–2692, Oct. 1998.

[3] S. Verdú and T. S. Han, "A general formula for Shannon capacity," *IEEE Trans. Inf. Theory*, vol. 40, no. 4, pp. 1147–1157, July 1994.

[4] G. Caire, G. Taricco, and E. Biglieri, "Optimal power control over fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1468–1489, July 1999.

[5] S. V. Hanly and D. N. C. Tse, "Multiaccess fading channels-Part II: Delay-limited capacities," *IEEE Trans. Inf. Theory*, vol. 44, no. 7, pp. 2816–2831, Nov. 1998.

[6] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[7] R. Knopp and G. Caire, "Power control and beamforming for systems with multiple transmit and receive antennas," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 638–648, Oct. 2002.

[8] E. Biglieri, G. Caire, and G. Taricco, "Limiting performance of block-fading channels with multiple antenna," *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1273–1289, May 2001.

[9] A. Lozano, A. M. Tulino, and S. Verdú, "Optimum power allocation for parallel Gaussian channels with arbitrary input distributions," *IEEE Trans. Inf. Theory*, vol. 52, no. 7, pp. 3033–3051, Jul. 2006.

[10] D. Guo, S. Shamai, and S. Verdú, "Mutual information and minimum mean-square error in Gaussian channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 4, pp. 1261–1282, Apr. 2005.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York: Wiley, 2006.

[12] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inf. Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[13] A. Guillén i Fàbregas, A. Martinez, and G. Caire, "Bit-interleaved coded modulation," *Foundations and Trends in Commuications and Information Theory*, vol. 5, no. 1–2, pp. 1–153, 2008.

[14] K. Nguyen, A. Guillén i Fàbregas, and L. Rasmussen, "Power allocation for block-fading channels with arbitrary input constellations," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, May 2009.

[15] G. Caire and K. R. Kumar, "Information-theoretic foundations of adaptive coded modulation," *Proc. IEEE*, vol. 95, no. 12, pp. 2274–2298, Dec. 2007.

[16] K. D. Nguyen, A. Guillén i Fàbregas, and L. K. Rasmussen, "Power allocation for discrete-input non-ergodic block-fading channels," in *Proc. IEEE Inf. Theory Workshop*, Lake Tahoe, CA, Sept. 2007.

[17] E. Malkamäki and H. Leib, "Coded diversity on block-fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 771–781, Mar. 1999.

[18] R. Knopp and P. A. Humblet, "On coding for block fading channels," *IEEE Trans. Inf. Theory*, vol. 46, no. 1, pp. 189–205, Jan. 2000.

[19] A. Guillén i Fàbregas and G. Caire, "Coded modulation in the block-fading channel: Coding theorems and code construction," *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 91–114, Jan. 2006.

[20] K. D. Nguyen, A. Guillén i Fàbregas, and L. K. Rasmussen, "A tight lower bound to the outage probability of block-fading channels," *IEEE Trans. Inf. Theory*, vol. 53, no. 11, pp. 4314–4322, Nov. 2007.

[21] F. Pérez-Cruz, M. Rodrigues, and S. Verdú, "MIMO Gausian channels with arbitrary inputs: Optimal precoding and power allocation," *IEEE Trans. Inf. Theory*, vol. 56, no. 3, pp. 1070–1084, Mar. 2010.

[22] N. Letzepis, K. Nguyen, A. Guillén i Fàbregas, and W. Cowley, "Outage analysis of the hybrid free-space optical and radio-frequency channel," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 9, p. 1, 2009.

[23] A. Guillén i Fàbregas, "Coding in the block-erasure channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5116–5121, 2006.

[24] M. Abramowitz and I. A. Stegun*, Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1964.



**Khoa D. Nguyen** (S'06-M'10) was born in An Giang, Vietnam, in 1982. He received the B.Eng. degree in electrical and electronics engineering from the University of Melbourne, Australia, in 2005, and the Ph.D. degree in telecommunications from the University of South Australia in 2010.

He was a summer research scholar with the Australian University in 2004 and was a visiting researcher with the University of Cambridge, Cambridge, U.K., in 2007. He is currently a Research Fellow with the Institute of Telecommunications Research, University of South Australia. His research interests are in information theory and adaptive transmission for wireless systems.

**Albert Guillén i Fàbregas** (S'01–M'05–SM'09) was born in Barcelona, Catalunya, Spain, in 1974. He received the Telecommunication Engineering degree and the Electronics Engineering degree from the Universitat Politècnica de Catalunya, Barcelona, Catalunya, Spain, and the Politecnico di Torino, Torino, Italy, respectively, in 1999, and the Ph.D. degree in communication systems from Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland, in 2004.

From August 1998 to March 1999, he conducted his Final Research Project at the New Jersey Institute of Technology, Newark. He was with Telecom Italia Laboratories, Italy, from November 1999 to June 2000 and with the European Space Agency (ESA), Noordwijk, The Netherlands, from September 2000 to May 2001. During his doctoral studies, from 2001 to 2004, he was a Research and Teaching Assistant with the Institut Eurecom, Sophia-Antipolis, France. From June 2003 to July 2004, he was a Visiting Scholar at EPFL. From September 2004 to November 2006, he was a Research Fellow with the Institute for Telecommunications Research, University of South Australia, Mawson Lakes. Since 2007, he has been a Lecturer with the Department of Engineering, University of Cambridge, Cambridge, U.K., where he is also a Fellow of Trinity Hall. He has held visiting appointments at Ecole Nationale Supérieure des Télécommunications, Paris, France, Universitat Pompeu Fabra, Barcelona, Spain, and the University of South Australia. His research interests are in communication theory, information theory, coding theory, digital modulation, and signal processing techniques with wireless applications.

Dr. Guillén i Fàbregas is currently an Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS. He received a predoctoral Research Fellowship of the Spanish Ministry of Education to join ESA. He received the Young Authors Award of the 2004 European Signal Processing Conference EUSIPCO 2004, Vienna, Austria, and the 2004 Nokia Best Doctoral Thesis Award in Mobile Internet and 3rd Generation Mobile Solutions from the Spanish Institution of Telecommunications Engineers. He is also a member of the ARC Communications Research Network (ACoRN) and a Junior Member of the Isaac Newton Institute for Mathematical Sciences.

**Lars K. Rasmussen** (S'92–M'93–SM'01) was born on March 8, 1965, in Copenhagen, Denmark. He received the M.Eng. degree in 1989 from the Technical University of Denmark, Lyngby, and the Ph.D. degree from the Georgia Institute of Technology, Atlanta, in 1993.

From 1993 to 1995, he was a Research Fellow with the Institute for Telecommunications Research (ITR), University of South Australia, Adelaide. From 1995 to 1998, he was a Senior Member of Technical Staff with the Centre for Wireless Communications, National University of Singapore. From 1999 to 2002, he was an Associate Professor with Chalmers University of Technology, Göteborg, Sweden, where he maintained a part-time appointment until 2005. From 2002 to 2008, he was a Research Professor with ITR, University of South Australia, where he was the leader of the Communications Signal Processing research group, the Convenor of the Australian Research Council (ARC) Communications Research Network (ACoRN), and a cofounder of Cohda Wireless Pty, Ltd., Adelaide, Australia. He has held visiting positions with the University of Pretoria, Pretoria, South Africa, Southern Poro Communications, Sydney, Australia, and Aalborg University, Aalborg, Denmark. He now holds a position as Professor in Communications Theory, School of Electrical Engineering, and the ACCESS Linnaeus Center, Royal Institute of Technology, Stockholm, Sweden. His research interests include adaptive coding and modulation, multiuser communications, coding for fading channels, cognitive communications and cooperative communications for wireless networks, and anytime coding for controls.

Dr. Rasmussen is a member of the IEEE Information Theory and Communications Societies and served as Chairman for the Australian Chapter of the IEEE Information Theory Society 2004–2005, and has been a board member of the joint IEEE Communications Society and IEEE Vehicular Technology Chapter in Sweden since 2010. He is an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, and was a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2007. He was also a member of the organizing committees for the IEEE 2004 International Symposium on Spread Spectrum Systems and Applications, Sydney, and the IEEE 2005 International Symposium on Information Theory, Adelaide, as well as the co-chair of the Communications Theory Symposium at the IEEE Global Communications Conference (Globecom) 2009.