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Application of Neural Network Models in Modelling Economic Time Series with Non-constant Volatility

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Abstract

In this paper, we investigate the volatility dynamics of EUR/GBP currency using statistical as well as the neural network approach which is an alternative way for time series modelling and forecasting in economics. The goal of this paper is to provide an alternative and reasonable way in modelling dynamic economic time series. We suggest an alternative approach for forecasting time series with non-constant volatility – we suggest and implement several neural network prediction models; we also use a large number of statistical models as well as different optimization techniques for artificial neural network. After discussing the basics of statistical volatility modelling and the basis of artificial neural networks we perform the experiment on real financial data. We quantify several ARCH and GARCH models; we also implement various RBF neural network prediction models. The comparative analysis of out-of-sample forecasts evaluated using MSE evaluation measures is performed. Finally, we state that suggested neural network models performed almost as good as the standard statistical models and are therefore reasonable and acceptable in economic modelling.

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1. Introduction

The most common way in expressing the risk is the volatility. Therefore, volatility is an extremely important factor for risk management for many economical subjects in the world. It plays an important role in an investor's decision making process. Volatility is not only of great concern for investors but also policy makers and regulators

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who are interested in the effect of volatility on the stability of financial markets in particular and the whole economy in general. Finally, companies and economic subjects (mainly financial) use risk management due to many reasons: minimizing potential losses, quantify the most probable development etc. Volatility estimation is an essential input in many VaR (Value at Risk) models, as well as for a number of applications in a firms market risk management practices. Also, a large part of risk management is measuring the potential future losses of a portfolio of assets (volatility modelling provides a simple approach to calculating VaR of a financial position in risk management), and in order to measure these potential losses, estimates must be made of future volatilities and correlations.

Volatility modelling is also important for asset allocation where the Markowitz approach of minimizing risk for a given level of expected returns (Markowitz, 1952) has become a standard approach. Perhaps the most challenging application of volatility forecasting, however, is to use it for developing a volatility trading strategy. Option traders often develop their own forecast of volatility, and based on this forecast they compare their estimate for the value of an option with the market price of that option.

Due to many reasons stated above, it is no surprise that modelling volatility is the must in risk management. However, it is not always easy to predict it. In some cases, mainly in dynamic economic markets like stock market or forex market volatility has some very unique features and has to be modelled in a special way. In this case, taking into account a constant, non-varying volatility is not a right way to go. Various approaches to non-constant volatility modelling have been suggested in the econometric literature. Gooijer and Hyndman (2006) proved that artificial neural networks had the biggest potential in the domain of time series forecasting. Therefore, various types of neural networks have been used for forecasting future values of high frequency financial data such as (Zhang, 2003) or (Marcek, 2009).

To goal of this paper is to provide an alternative way in modelling various economic variables. We take the principle of artificial neural network, which is an extremely helpful tool in various areas and we suggest its application in economics. More specifically, we create and implement the artificial neural network of the feedforward type and we adapt it to be able to forecast economic time series such as GDP, unemployment etc. The correctness of our approach is then verified on real economic data.

2. Material and Methods

2.1. Statistical volatility modelling

The major breakthrough in the history of statistical modelling came with publishing a study from Box & Jenkins (Box, Jenkins, 1976). In this study by Box and Jenkins (1976) authors integrated all the knowledge including autoregressive and moving average models into one book. From that time the ARIMA (AutoRegressive Integrated Moving Averages) models have been very popular in time series modelling for a long time as O'Donovan (1987) showed that these models provided better results than other models used in that time. However, in 1982, Engle showed that this assumption is not always correct. He founded out that in some time series such as financial time series and other very dynamic economical time series, the volatility is not always constant and this is due to its special features. First of all, it is its stochastic character. Moreover, financial time series exhibit a characteristic known as volatility clustering in which large changes tend to follow large changes, and small changes tend to follow small changes. Volatility is hence clustered in time and therefore it has persistence character. Resulting from this, actual variance is dependent on the previous variances and the time series is characterized by the time-variant conditional variance, also called clustering of variances. Another feature of non-constant (financial) volatility is mean reversion. Volatility is often persistent and so has a long memory. Also, it has been experimentally proved that the distribution of many high frequency financial time series usually have fatter tails than a Gaussian distribution.

The weakness of ARIMA models in modeling financial time series is the inability to model stochastic non-constant volatility having the features we described above. In 1982 Engle suggested (Engle, 1982) the solution by creating so called ARCH (Autoregressive Conditional Heteroskedastic) models which assume heteroskedastic variance of ε_t . The conditional variance in the ARCH(p) model is a function of the past squares of random variable e_t . The ARCH model is able to model the basic properties of financial volatility such as volatility clustering, stochastic properties of volatility, mean reversion, fat tails etc. Bollershev (1986) suggested the generalized form of ARCH model called GARCH (Generalized Autoregressive Heteroskedastic Models) where conditional variance of

h_t depends on the previous conditional variances. Later, as time went on, many extensions of the GARCH model have been introduced in the literature since: e.g. GARCH-in-mean (GARCH-M) models (Engle, Lilien, Robins, 1987), EGARCH models (Nelson, 1991), Threshold ARCH (TARCH) and Threshold GARCH (TGARCH) (Glosten, Jagannathan, Runkle, 1993) and Power Arch (PARCH) models (Ding, Granger, Engle, 1993) just to name a few. A number of studies have focused on optimal model specification and the performance of various GARCH models in financial markets providing no clear-cut results (Hansen and Lunde, 2005).

2.2. Statistical vs Neural Network Approach: Real Data Application

This paper focuses on modelling time series with non-constant conditional volatility; we used daily close prices of EUR/GBP exchange rate. The data cover the historical period from October 31, 2003 to October 31, 2013 ($n = 2610$ daily observations).[†] The graphical characteristics of the series is illustrated in Figure 1.

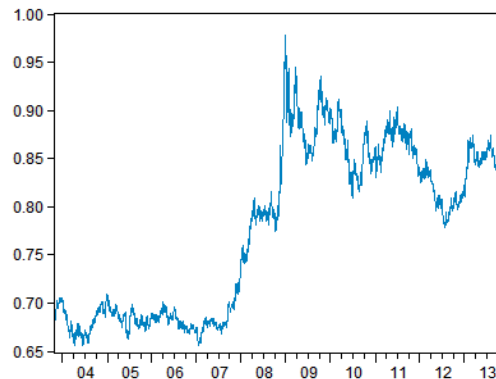


Fig. 1. Time Series of daily close prices of EUR/GBP currency (October, 2003 – October, 2013).

Due to validation, data were divided into two parts. The first part included 1306 observations (from 10/31/2008 to 10/31/2008) and was used for quantification. The second part (11/1/2008 to 10/31/2013), counting 1304 observations, was used for validation by making one-day-ahead ex-post forecast. These observations included new data which had not been incorporated into model estimation. The reason for validation was to find out the real prediction power of the models. In order to evaluate the quantified model as well as to compare the real forecasting performance of our proposed models, the numerical characteristic called Mean Squared Error (MSE) was used.

2.3. Statistical Modelling

The empirical statistical analysis, which was performed according to Box-Jenkins (1976), focused on the original and differentiated series of daily observations of EUR/GBP currency pair covering a historical period from October 31, 2003 to October 31, 2008. Statistical modelling was performed in the Eviews software.

Unit root tests results (see Dickey, Fuller, 1979; Elliott, Rothenberg, Stock, 1996; Kwiatkowski, Phillips, Schmidt, Shin, 1992; Phillips, Perron, 1988) showed that this series was not stationary. In order to stationarize the series, it was differentiated. After that, unit root tests confirmed that the differentiated series became stationary which had been a necessary condition in Box-Jenkins modelling. By analyzing autocorrelation (ACF) and partial autocorrelation functions (PACF) of the differentiated series of EUR/GBP, there were no significant correlation coefficients (on $\alpha = 0.05$). Due to that we supposed that first differences of the original series formed a white noise process. In that case, the original series would have formed random walk process (RWP) as RWP is $I(1)$

[†] The data was downloaded from the website <http://www.global-view.com/forex-trading-tools/forex-history>.

process. Assuming the differences of the original series formed a white noise process, we selected AR(0) as the basic Box-Jenkins model. Ljung-Box Q-statistics confirmed this assumption and the applicability of AR(0) process as the correlations were statistically not significant. However, the assumption of normality of residuals of AR(0) was rejected at 0.05 significance level. The observed asymmetry might have indicated the presence of nonlinearities in the evolution process of residuals. This nonlinearity was also confirmed by graphical quantiles comparison a scatter plot of the series which did not appear to be in the form of a regular ellipsoid (see Figure 2). BDS test also rejected the random walk hypothesis as the BDS statistic was greater than critical value at 0.05.

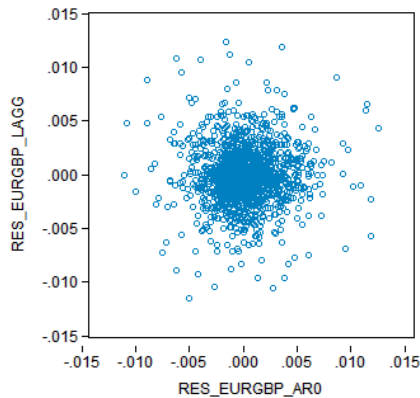


Fig. 2. Scatter plot of EUR/GBP residuals variations.

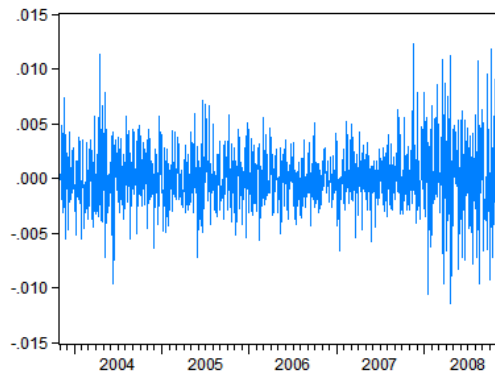


Fig. 3. Evolution of residuals of AR(0) model

Therefore, other tests had to be performed in order to correctly model this series. We noted that the residuals of AR(0) (see Figure 3) were not characterized by a Gaussian distribution. The asymmetry might have indicated nonlinearities in the residuals. When looking at the graph of residuals (see Figure 3), one could observe the variability of these residuals could have been caused by the non-constant variance. Residual with small value followed another residuals with a small value. On the other hand, residual with a large value usually followed a residual with another large value. However, this is not typical for a white noise process. Therefore, this assumption lead us to think about stochastic model for volatility. The suitability for using stochastic volatility model was also accepted by performed heteroskedasticity test. ARCH test confirmed the series was heteroskedastic since the null hypothesis of homoscedasticity had been rejected at 5% and so the residuals were characterized by the presence of ARCH effect which was quite a frequent phenomenon at financial time series. Therefore, we applied a stochastic volatility model into the basic model. According to correlogram of squared residuals of EUR/GBP differences we quantified ARCH(4) model for volatility. After quantification of ARCH(4) model, the residuals were characterized by the absence of conditional heteroskedasticity: the ARCH-LM statistics were strictly less than the critical value at 5%. In addition, the standardized residuals tested with Ljung-Box Q-test confirmed there were no significant coefficients in residuals of this model. Finally, we defined the final AR(0)+ARCH(4) volatility model as the appropriate model for forecasting EUR/GBP time series with conditional volatility. The volatility part of the model is defined as follows

$$\begin{aligned} \sigma^2 = & 0.00000438 + 0.104930\varepsilon_{t-1}^2 + 0.101053\varepsilon_{t-2}^2 \\ & + 0.150503\varepsilon_{t-3}^2 + 0.085457\varepsilon_{t-4}^2 \end{aligned} \quad (1)$$

2.4. Neural Network Approach

Non-linearity modelling is one of the drawbacks of Box-Jenkins models. According to studies such as that by Gooijer (Gooijer, de Hyndman, 2006), artificial neural networks (ANN) are the machine learning models having the biggest potential in forecasting time series with non-constant volatility. This is due to the fact that these models are extremely helpful in modelling non-linear processes which have a priori unknown functional relations or this system of relations is very complex to describe mathematically (Darbellay, Slama, 2000). ANN is based on human neural

system and is a universal functional black-box approximator of non-linear type (Hornik, 1993; Hornik, Stinchcomber, White, 1989 and Maciel, Ballina, 2008). The reason for attractiveness of ANNs for financial prediction can be found in the work of Hill et al. (Hill, Marquez, O'Connor, 1994). Here, the authors showed that the ANNs worked best in connection with high-frequency dynamic data. The competitive performance of ANN is also documented on a large number of time series (see Liao, Fildes, 2005 and Zhang, Qi, 2005). In this part we show a new approach of estimation of forecasting function for time series with conditional volatility modelled by feedforward neural network of RBF type combined with genetic algorithms.

A fully connected feed-forward neural network was selected to be used as the forecasting function, due to its conceptual simplicity, and computational efficiency (Marcek, Marcek, 2006). We proposed the architecture of the neural network with only one hidden layer due to the fact that according to Cybenko theorem (1989) the network with one hidden layer is able to approximate any continuous function. This hidden layer made a previous nonlinear transformation of the data so as to facilitate resolution of the problem in hand such as regression, classification, etc. The neural network used for this research was the network of RBF type (Orr, 1996). This network is one of the most frequently used networks for regression (Marcek, Marcek, 2006). RBF has been widely used to capture a variety of nonlinear patterns (Hornik, Stinchcombe, White, 1990) thank to their universal approximation properties (Leshno, Lin, Pinkus, Schocken, 1993). The most popular optimization method in neural networks is back-propagation (Bryson and Ho) (Bryson, Yu-Chi, 1969). However, there are some drawbacks to back-propagation. One of them is the convergence of this algorithm. Due to this reason, we also used the combination of back-propagation with the standard unsupervised technique called K-means (see MacQueen, 1967), which belongs to a group of unsupervised methods and is a nonhierarchical exclusive clustering method. The K-means was used in the phase of non-random initialization of weight vector w performed before the phase of network learning. We assumed that in many cases it was not necessary to interpolate the output value by radial functions, it was quite sufficient to use one function for a set of data (cluster), whose center was considered to be a center of activation function of a neuron.

Since BP also features some other problems such as "scaling problem" we decided to implement genetic algorithm as other optimizing method for our RBF neural network too. Adopted from biological systems, GA, which are algorithms for optimization and machine learning, are stochastic search techniques that guide a population of solutions towards an optimum using the principles of evolution and natural genetics (Dharmistha, 2012). They are based loosely on several features of biological evolution (Holland, 1975) and have become a popular optimization tool in various areas. GA are characterized by basic genetic operators, i.e. reproduction, crossover and mutation (Whitley, 1988). Given these genetic operators and components, a GA operates according to the steps stated in (Montana, Davis, 1989).

3. Results and Discussion

In our tests, we used one-step-ahead, frequently called as static, forecasts, i.e. the horizon of predictions was equal to one day. Firstly, we estimated and tested the ARCH(4) model for volatility defined in (1). However, we also tested some other statistical models modelling conditional variance such as GARCH(1,1) model (Engle, 1982) which is supposed to be so-called universal model in financial domain. We also tested EGARCH(1,1,1) defined in (8). Important to remember that the estimation of these models was only based on 1306 in-sample observations, in order to make ex-post predictions with remaining 1304 observations. We used the Marquardt optimization procedure for finding the optimal values of ARCH/GARCH parameters. The forecasting ability of particular networks was measured by the MSE criterion of ex post forecast periods (validation data set).

As for models based on neural networks, we implemented three models, each of them was an implementation of feedforward neural network of RBF type (Orr, 1996). We implemented three different optimization techniques for adaptation of weights (parameters) of this network – genetic algorithm, standard back-propagation algorithm (BP) as well as a combination of K-means clustering combined with the back-propagation (KM+BP). We implemented all of these algorithms and models by ourselves using the JAVA programming language. The results for out-of-sample (ex-post) predictions are stated in Table 1.

The standard back-propagation algorithm for weights adaptation showed to be a weakness of the network. The convergence was very slow (cca 5000 epochs) and in addition to that, it generally converged to any local minimum

on the error surface. In addition, this algorithm was very dependent on the initialized random weights. Due to this, generally a lot of more epochs was needed to achieve reasonable accuracy compared to K-means + BP.

Bearing in mind these disadvantages of BP, we also tested K-means, that was used in the phase of non-random initialization of weight vector w performed before the phase of network learning. Besides lower MSE, another advantage of using K-means upgrade was the consistency of predictions. Moreover, the biggest strength of K-means was in the speed of convergence of the network. Without K-means, it took considerably longer time to achieve the minimum. However, when the K-means was used, the time (number of epochs) for reaching the minimum was much shorter (cca 500 to 1000 epochs). Therefore, the advantage of using K-means together with back-propagation is in the speed of adaptivity rather than in better predictions.

Table 1. Prediction accuracy of tested models measured by MSE (out-of-sample predictions).

Optimizing method			
NN configuration	RBF (BP)	RBF (KM)	RBF (GA)
(4 – 3 – 1)	4,1471*10 ⁻⁹	3,9282*10 ⁻⁹	4,2028*10 ⁻⁹
(4 – 5 – 1)	4,1590*10 ⁻⁹	4,2222*10 ⁻⁹	4,1587*10 ⁻⁹
(4 – 7 – 1)	4,2156*10 ⁻⁹	1,1480*10 ⁻⁸	4,1170*10 ⁻⁹
(4 – 10 – 1)	4,5086*10 ⁻⁹	7,8121*10 ⁻⁹	4,9923*10 ⁻⁹
Error Distribution			
Statistical model	Gaussian	Student	GED
ARCH(4)	3,8203*10 ⁻⁹	3,8054*10 ⁻⁹	3,8143*10 ⁻⁹
GARCH(1,1)	3,5058*10 ⁻⁹	3,5175*10 ⁻⁹	3,5099*10 ⁻⁹
EGARCH(1,1,1)	3,4809*10 ⁻⁹	3,4836*10 ⁻⁹	3,4851*10 ⁻⁹

Having tested also GA in weights adaptation, we found out the convergence was also considerably faster than at back-propagation. In addition to that, GA did not have the same problem with scaling as back-propagation. One reason for this is that GA generally improves the current best candidate monotonically. It does this by keeping the current best individual as part of their population while they search for better candidates.

However, the accuracy results were not very different from the other two optimization techniques. As according to the theory, GA are not bothered by local minimum problem such as BP and as GA are also especially capable of handling problems in which the objective function is discontinuous or non-differentiable, nonconvex, multimodal or noise; we expected better results than we got. This could, however, be caused due to non-optimized parameters of GA. Except for our experiments with the best configuration of GA parameters, we also tested the optimization procedure stated in (DeJong, 1990) which was in our case not very helpful. Maybe, testing some other optimization procedure for the best parameters of GA would lead to better results of genetic algorithm. The second reason could be that the standard unbiased crossover function was used. The biased crossover function stated in (Montana, Davis, 1989) could enhance our solution. The final comparison of all models is stated in Table 2.

Table 2. Final Comparison of out-of-sample (ex-post) predictions.

Model	Numerical characteristics		
	MSE E	RMSE E	Rank
RBF (4 – 3 – 1) (BP)	4,1471*10 ⁻⁹	0,00006439	6
RBF (4 – 3 – 1) (KM)	3,9282*10 ⁻⁹	0,00006267	4
RBF (4 – 7 – 1) (GA)	4,1170*10 ⁻⁹	0,00006416	5
ARCH(4) (Student)	3,8054*10 ⁻⁹	0,00006168	3
GARCH(1,1) (Gauss)	3,5058*10 ⁻⁹	0,00005920	2
EGARCH(1,1,1) (Gauss)	3,4809*10 ⁻⁹	0,00005899	1

On the validation set the best results were achieved with EGARCH(1,1,1) model (see Table 2) while the worst results were achieved with neural network combined with the standard algorithm. However, the differences between the results were small and the difference in results between the best and worst model is only about nine per cent.

Following from that, our suggested neural network model showed to be an efficient and accurate way of forecasting time series with conditional volatility in financial domain. But, generally speaking, the statistical models

achieved a little bit higher accuracy than the neural networks models. However, the difference was very small and the results were almost of the same accuracy is still reasonable and acceptable for use in forecasting volatility which plays an important role in managerial decision processes in the finance area. Moreover, a little bit worse results of neural network models can be the result of the following factors:

- non-optimized parameters of GA, which could cause a little bit worse final solution than expected
- back-propagation as the non-ideal optimization technique for parameters optimization
- The non-ideal inputs of the neural network coming from the statistical ARCH(4) model
- The data we chose for our experiments were not “representative“. One cannot eliminate the assumption that if we used other data for our experiments the neural network models would outperform the ARCH/GARCH.

Coming from that, there are more options of how to upgrade this model in the future:

- Apply other known optimization procedure than into our ANN models.
- Use and implement more advanced version of this algorithm (to avoid the local imprisonment)
- Implement the recurrent neural network based on GARCH model, not ARCH (recurrent GARCH-RBF).
- Implement the error-correction part, i.e. smoothing the error (residual) of the RBF neural network by using *m*-period weighted or exponential or simple moving average such as $\varepsilon_t^{RBF} = e_t + u_t$, $u_t \approx iid(0,1)$ where

$$e_t = \sum_{i=1}^q \theta_i \varepsilon_{t-i}^{RBF}, \quad \sum_{i=1}^q \theta_i = 1. \quad (2)$$

4. Conclusion

We investigated the volatility dynamics of EUR/GBP exchange rate differences. We examined two approaches for forecasting high-frequency series with conditional volatility –models based on statistics and ANN models. We evaluated the effectiveness of various models with respect to forecasting market risk in the exchange rate market.

We evaluated three most common statistical models for volatility forecasting – the universal GARCH model, the basic ARCH and EGARCH model which is able to model leverage effects. In addition to that, all three models were evaluated with Gaussian, Student and GED error distributions. We also suggested several models for forecasting time series with conditional volatility with artificial neural networks. Moreover, we constructed neural network with three different types of optimization techniques; except for the standard back-propagation, we combined a K-means clustering into the ANN to achieve higher accuracy of the network. The reason for incorporating other algorithms into the network was that the back-propagation was considered a weakness of the RBF. In addition, we also eliminated the back-propagation algorithm by using the genetic algorithm instead. In the final comparison of the selected optimization methods both of these upgrades showed to be helpful in the prediction process.

In our experiment, the statistical approach was more accurate than the ANN models. However, the differences in accuracy were very small. None of the considered models performed significantly better than the rest with respect to the considered criteria. So the achieved ex post accuracy of neural network models is still reasonable and acceptable for use in forecasting systems that routinely predict volatility in managerial decision processes. A little bit higher error could be caused by non-optimizing parameters of GA, non-ideal inputs of ARCH model or just due to type of data we used. Moreover, neural networks are capable of providing information in the form of forecasts with an acceptable degree of uncertainty. They are relatively fast and have the ability to generalize. The implemented ANN has also such attributes as computational efficiency, simplicity, and ease adjusting to changes in the process being forecast. ARCH statistical models require more costs of development, installation and operation in a management system, management comprehension and cooperation, and often a lot of computational time.

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