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## Denoising Of DT-MR Images With An Iterative PCA

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### Abstract

Nowadays most of the clinical applications uses Magnetic Resonance Images(MRI) for diagnosing neurological abnormalities. During MR image acquisition the emitted energy is converted to image by using some mathematical models, and this may cause addition of noise. Therefore we need to denoise the image. Currently most of the clinical application uses Diffusion Tensor-MR Images for tracking neural fibres by extracting features from the images. Noise in DT-MR Images make fibre tracking and disease diagnosing tougher. So our work aims to denoise the Diffusion Tensor MR images with better visual quality. In this paper, we propose a denoising technique that uses Structural Similarity Index Matrix (SSIM) for grouping similar patches and performs Iterative Principal Component Analysis on each group. By performing the weighted average on Principal Component, we have obtained the denoised DT-MR Image. For getting better visual quality of the denoised images we employ Iterative Principal component Analysis technique.

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*Keywords:* DT-MR Images; SSIM; Iterative PCA.

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### 1. INTRODUCTION

Diffusion MRI (*DMRI*) is a magnetic resonance imaging (*MRI*) method which came into existence in the mid-1980s. It allows the mapping of the diffusion process of molecules, mainly water, in biological tissues, in vivo and non-invasively. The Diffusion MRI, is also referred to as Diffusion Tensor Imaging or DTI has been extraordinarily successful in the field of medical science. It is mainly used for the study and treatment of neurological disorders, mostly for the patients with acute stroke. Because it can find the abnormalities in white matter fiber structure and provide models of brain connectivity. With Diffusion Tensor Imaging (*DTI*), diffusion anisotropy effects can be fully

extracted, characterized, and exploited, providing even more exquisite details on tissue microstructure. DTI is used to demonstrate subtle abnormalities in a variety of diseases (including stroke, schizophrenia), for finding the issues related to fibre connectivity and is currently becoming part of many routine clinical protocols.

The image acquisition process of DT-MR Images is by the patient is positioned within an MRI scanner where it forms a strong magnetic field around the area to be imaged. The medical applications rely on detecting a radio frequency signal emitted by excited hydrogen atoms in the body using energy from an oscillating magnetic field applied at the appropriate resonant frequency. The orientation of the image is controlled by varying the main magnetic field using gradient coils. As these coils are rapidly switched on and off they create the characteristic repetitive noises of an MRI scan. The contrast between different tissues is determined by the rate at which excited atoms return to the equilibrium state.

During the tissue movement to equilibrium state and the anisotropic movement of water molecules, generated DT-MRI may contain unwanted intensity values. This unwanted intensity values are called as Noise. The presence of noise makes disease diagnosing tougher. Therefore we need to denoise the image. The existing denoising algorithms may provide a denoised image but it may not yield good quality . ie, sometimes the denoised image may got blurred and may loss the fine structure. So our proposed Iterative PCA based method aims to denoise the DT-MRI which will result in good quality images.

In this paper we propose a technique to perform denoising which is able to remove the noise with better quality thereby making the disease diagnosing easier. The denoising technique exploits the structural similarity of the patches and uses an iterative principal component analysis for the denoising process. The method takes advantage of the fact that there is a high degree of redundancy in the content of images.

The proposed Iterative PCA based denoising technique is composed of four steps. In the first step, the similar patches are extracted and grouped by using Structural Similarity Index Matrix (SSIM) <sup>4</sup> and there is one-to-one correspondence between patches and groups (sparse denoising). The actual denoising is done in the second step ie, by finding the principal component of each group. In the third step, patch restoration is performed by using the principal component. After the third step a denoised image will be obtained, but the first iteration helps us to remove only the most significant error portion of the image. The output of the first iteration needs to be refined for a better quality image. The amount of noise present in the output of first iteration is estimated and use this noise variance along with the denoised output of the first iteration is fed as the input to the second iteration. Iteration need to be stopped in the second stage, otherwise it may cause the loss of fine structures.

## 2. RELATED WORK

Image denoising technique has been well studied before. Most of the denoising algorithms emphasize on the fact that denoising can be achieved by averaging. This averaging can be done via locally by Gaussian smoothing filter, by anisotropic filtering, by neighbourhood filtering, by calculus of variations or in the frequency domain by wiener filters or by wavelet thresholding methods. However, denoising should not alter the original image I.

Wavelet Transform <sup>1</sup> technique has been used in the noise removal from the images. The technique decomposes the input signal into multiple scales, each scale represents different time-frequency components of the original signal. thresholding <sup>11 12</sup> and statistical modelling <sup>13 14</sup> are performed on each scale of signal for removal of noise. The inverse transformation is performed for getting the denoised signal is done by using the fixed wavelet basis such as Dilation and Translation. Dabov et.al <sup>2</sup> exploits the idea of sparse representation for image denoising strategy whose input is a 2D noisy image. The collaborative filtering consists of three steps, 3D transformation of the group, shrinkage of the transform spectrum, and the inverse 3D transformation. Here the noise is separated by the shrinkage operation. Even though this technique deals with the denoising, this may lead to the loss of edges and its

structures.

The SSIM based Sparse representation can be used for the image denoising. Rehman et.al<sup>5</sup> remove the noise from distorted image by training a dictionary, for which the original image can be represented sparsely in its domain. The method uses KSVD method to train the dictionary. In this method the dictionary, which is trained directly over the noisy image and denoising is done in parallel. PCA is a statistical technique used for transforming the data set with higher dimension to the lower dimension. The paper<sup>6</sup> deals with the denoising of the image using the principal component analysis approach with local pixel grouping (LPG). By transforming the original dataset into PCA domain and preserving only the several most significant principal components, the noise and non-trivial information can be removed.

The image denoising with the diffusion weighted image using an overcomplete PCA is explained in Jose et.al<sup>19</sup>. The methodology proposed in this paper exploit the feature of multi component nature of the multi directional DWI. The method discussed on<sup>7</sup> is to remove the speckle noise present in the images. The homomorphic filtering is done by using a fourth order complex diffusion technique. The methodology discussed in<sup>3</sup> deals with nonlocal means denoising which is adaptive based on the noise level. Chandrika Saxena et.al<sup>8</sup> deals with the existing image denoising algorithms like filtering approach, wavelet approach, and multifractal approach. The filtering approach for denoising of the image is best suited when the image is corrupted with salt and pepper noise. The wavelet based approach is best suited when the image is corrupted with Gaussian noise. If the image is corrupted with complex characteristics based noise then the multifractal approach will be best suiting.

The most common method for the denoising of MR images is the Non Local Means method. The method in<sup>9</sup> deals with the denoising of MR Images which uses a Robust Non Local Means Maximum Likelihood estimation, which helps to avoid the loss of fine structures and edge boundaries. For maintaining these features during the denoising they use Geman-McClure function for the weight calculation.

Lijun Bao et al.<sup>10</sup> deal with denoising of the diffusion tensor MR images by exploiting the concept of sparsity. In this they are using the diffusion tensor MR image as the input data and grouping the patches based on their similarity, and uses a structure adaptive window method to avoid the edge blurring effect and loss of fine structures. Grouping of the patches is based on a structural similarity index matrix. The noise component of the resulting structure-adaptive arrays is attenuated by Wiener shrinkage in a transform domain. From each group they are finding the principal component and then perform a 1D Haar Transformation. Finally performing a weighted average to get denoised Diffusion Tensor MR image. By using this technique, we will get a denoised image with less noise presence. But we need to remove that minute noise presence. So in our proposed Iterative PCA based denoising method, we are going for a second stage, where it helps to group the patches more correctly thereby helps to find the principal component of each group more precisely makes the more minute noise to be reduced.

### 3. THE PROPOSED MODEL FOR DENOISING OF DT-MR IMAGES

The goal of image denoising methods is to recover the original image from a noisy measurement,

$$X(i) = I(i) + n(i) \quad (1)$$

where  $X(i)$  is the observed value,  $I(i)$  is the true value and  $n(i)$  is the noise at a pixel  $i$ . Several methods have been proposed to remove the noise and recover the true image  $I$ .

The proposed model for denoising of diffusion tensor MR images is shown in Figure 1. As shown in figure, initial stage performs the grouping of patches based on the similarity. The similarity measure used here is the structural similarity index matrix (SSIM) which helps to ensure the local consistency. The next step is actual denoising operation. It is performed by using Principal Component analysis. For this the set of patches are needed to be projected to the PC domain and perform hard thresholding of the PC projection coefficient. Then an inverse transformation gives patch estimates as result.

Formally we can write the 3D transform and the inverse 3D transformation as 3D and 3D<sup>10</sup>. This operation is performed for all the patches and then create denoised image by performing a cumulative weighted average of denoised patch. Up to this point it includes first level of denoising. The above discussed existing work removes high frequency noise components to form denoised image. In our proposed denoising technique a second iteration<sup>18</sup> of above discussed steps is performed to remove low frequency noise component for getting a better quality denoised image. High frequency noise component is removed in the first phase.

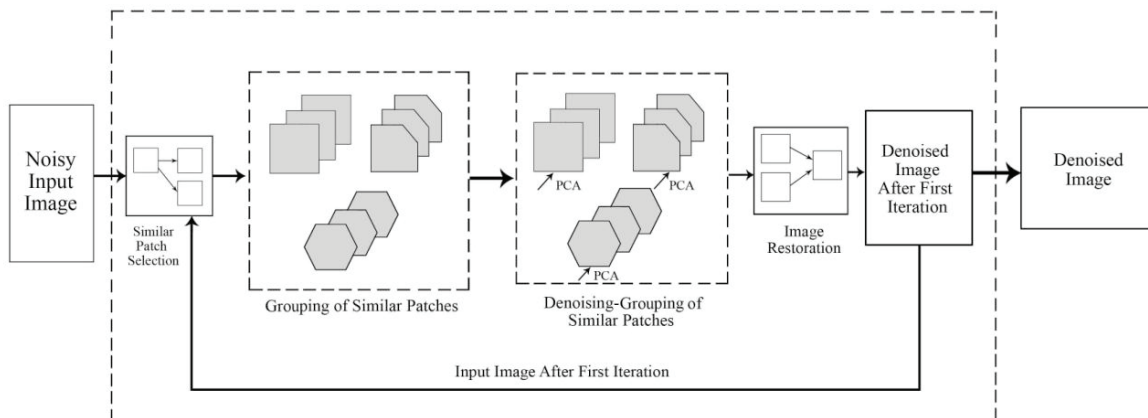


Fig. 1. Proposed Iterative PCA Based Denoising Model

Grouping the patches is done more precisely in the second phase, thereby we can represent each group with more accurate principal component making the images to be reformed by projecting to more accurate principal component. Iteration should be stopped at the second phase since increase in number of iteration may cause loss of signal.

The different modules for denoising of Diffusion Tensor MR Image can be stated as:

### 3.1. Grouping of Similar Patches

The grouping of similar patches are done by using a similarity measure called as the Structural Similarity Index Matrix (SSIM)<sup>10</sup>. The grouping of similar patches can be made efficient by using the SSIM and also by thresholding the index and placing bounds on the means of the candidate patch. The similarity of two patches can be found out by using the equation:

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{2}$$

$\mu_x$  and  $\mu_y$  are the mean of patches  $x$  and  $y$ ,  $\sigma_x^2$  and  $\sigma_y^2$  are the variance of patches  $x$  and  $y$ , and  $\sigma_{xy}$  is the covariance between patches  $x$  and  $y$ .  $c_1$  and  $c_2$  are constants which is used to avoid instability when mean and variance are close to zero. SSIM ensures local consistency. The dissimilar patches is needed to be preclude during weighted averaging. The grouping of patches is performed as  $N_k$  on a square region of size  $n$  with centre pixel as  $k$ . These grouped patches are stacked as a rectangular array  $N_i$ .

### 3.2. Denoising each group of similar patches

Initially process an image I having noise components,

$$X(i) = I(i) + n(i) \tag{3}$$

and  $X_1, X_2, X_3 \dots, X_m$  are the group of similar patches we get from the first step. In each group the patches are arranged as a rectangular array. Figure 2 shows the actual denoising by the use of PCA<sup>10</sup>. Each group of patches are transformed into the PCA domain which are easier to de-correlate the noise and the original signal. For the denoising of images, calculate the covariance matrix.

$$\Sigma = \frac{1}{n} \sum_{k=1}^n y_k y_k^T - \hat{y} \hat{y}^T \tag{4}$$

Here  $\hat{y}$  represents the mean,  $\Sigma$  is the covariance matrix. n is the count of patches in each group and we are decomposing the group of patches into a PC basis  $v(i) = \{v_1(i), v_2(i), v_3(i) \dots, v_k(i)\}$ .

PCA( $S_{X_{N_i}}^P$ ) =  $\{\alpha_1(j), \alpha_2(j), \alpha_3(j) \dots, \alpha_k(j)\}$ ;



Fig. 2. Denoising of Patches

Here  $\{\alpha_1(j), \alpha_2(j), \alpha_3(j) \dots, \alpha_k(j)\}$  the coordinate vector of  $Y(N_j^{SA})$  in  $v(i)$  and then perform the hard thresholding because, for a given patch x only a few axes are relevant, but the relevant axes may vary from one patch to another. Therefore, hard thresholding is a reasonable strategy to process the principal coefficient according to their magnitude. This process is repeated for each group of patches and results in the principal component for each group.

### 3.3. Creation of denoised image

After the inverse transformation of the patches from the PCA domain, it results in the principal component of each group. We need to assemble these patches to form an image. For that performs the weighted average of patches<sup>10</sup>. The equation for weighted averaging of patches is as follows:

$$W(i) = \frac{1}{(\sigma^2 N_c(i))} ; \text{ if } N_c(i) \geq 1 \tag{5}$$

Otherwise  $W(i)=1$ .

Here  $N_c(i)$  is the major principal component used to represent the patch.

### 3.4. Iterative PCA

We have modified the existing denoising technique by using an Iterative PCA. In the proposed technique for denoising of DT-MR images is done in two iteration. In the first iteration of denoising high frequency noise components are removed. Due to the presence of so many low frequency noise components in the denoised image after first iteration the quality of image is not so good.

In order to improve quality of image some refinement on the output of first iteration is to be performed. So in proposed Iterative PCA denoising technique, second iteration of the above mentioned steps are to be performed. We denote  $I$  as the denoised image of  $I_v$  after the first iteration and the  $v_x$  is the noise residual of the denoised image; ie,  $I = I + v_x$ . The pre-requirement for the second iteration is to estimate the level of this noise residual  $v_x$ , denoted as  $\bar{x} = E[v_x^2]$  and input it to the second iteration of denoising. The input for second iteration is an image only with low frequency noise components. It makes the grouping of similar patches efficiently. So the shrinkage operation on these groups yields us a better principal component by which we are able to generate the better quality denoised image after the second iteration. Iterative process need to be stopped in the second stage, when it has removed the low frequency noise components. Any further iteration may cause loss of true signal values thereby degrading the quality of images.

#### 4. PERFORMANCE ANALYSIS

From the literature survey, it is evident that the denoising technique plays a vital role in disease diagnosing. Many denoising algorithms are there for DT-MR Images, but most of them are not up to the mark for a practical use. It makes the disease diagnosing tougher. Most of the noise in the medical images are following the rician distribution. So we are focusing on methods that deal with rician distributed noise.

##### 4.1. Dataset

We have simulated dataset by using the simulator DSI studio. Experimentation have been done on synthetic data generated by the simulator as well as with the real datasets, with different noise variance levels such as 5, 10, 15, 20, 30 etc.. . The proposed denoising method was compared with conventional Principal Component Analysis and BLS-GSM method to evaluate the performance of the proposed method. BLS-GSM method<sup>15</sup> uses the prior knowledge about image, noise and observed image as the elements for the denoising under a Bayesian Frame. DT-MR images dataset was obtained from Brain Atlas. Images were of size 256X256. Algorithm was coded in Matlab. Different criteria was taken into consideration for comparison. The performance of the proposed method was compared with the existing in terms of PSNR, SSIM and Visual Quality Comparison.

##### 4.2. Generation of Rician Noise

DT-MR images are magnitudes of complex valued signals. If the real and imaginary components of the signal are assumed to have a Gaussian noise, the resulting magnitude image will have Rician distributed noise<sup>16</sup>. A signal is said to be corrupted with Rician noise if the pdf of the noisy signal has a Rice distribution.

P.D.F of Rice Distribution,

$$P(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{xA}{\sigma^2}\right) \quad (6)$$

Rician noise is not additive, but data dependent<sup>17</sup>. Consider a set of random numbers, which we take to be the intensity values of a noise-free image  $A$  defined on a discrete grid  $I$  so that  $A = a_i | i \in I$ . Let  $\sigma$  be the standard deviation of Gaussian noise. We generate two sets of Gaussian distributed random numbers  $X = x_i | i \in I$  and  $Y = y_i | i \in I$  with zero mean and identical standard deviation  $\sigma$ . Then the following  $M = m_i | i \in I$  are Rician distributed.

$$M_i = \sqrt{(a_i + x_i)^2 + y_i^2} \quad (7)$$

#### 4.3. Noise Variance

Noise variance was estimated from the DT-MR image. There are different methods for estimation of noise variance from DT-MR data. Compute sum of absolute values by performing the convolution and scale the variance with the coefficient. The estimated noise variance was used as a parameter in the proposed iterative principal component analysis method to obtain the noise free image.

#### 4.4. Result Analysis

The sparse denoising of diffusion tensor MR images are validated on two types of dataset. One is the real Diffusion Tensor MR image and on the simulated data. The dataset images are DICOM Format. The simulated data will give us a noisy human brain DT-MR Image. The validation of the proposed Iterative PCA denoising technique is done on the simulated DICOM format image. The metric used for validation of proposed Iterative PCA based denoising are PSNR and MSSIM.

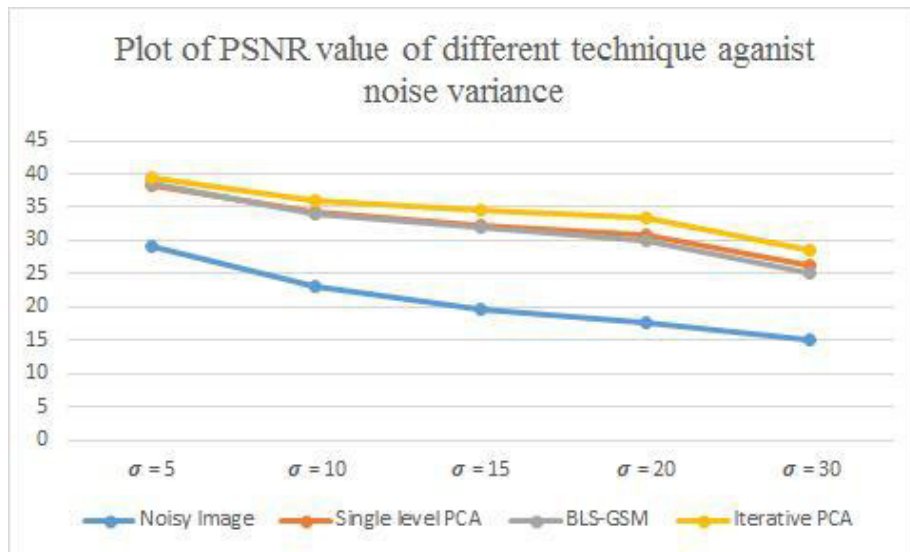


Fig. 3. Plot of PSNR against noise variance.

##### 4.4.1. PSNR (Peak Signal to Noise Ratio)

PSNR is the ratio between the signal and the noise. By using PSNR value we can found the presence of noise component of the image. If the PSNR value is a higher value means that the signal component is larger than noise component in the image.

$$\text{PSNR} = 20 \log_{10} \left( \frac{L}{\|X - Y\|_2} \right) \quad (8)$$

Table 1. PSNR values of different denoising techniques for different noise variance level for DT-MR brain image

INDEX	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=30$
Noisy Image	29.05	23.02	19.53	17.53	14.97
Single Level PCA	38.43	34.24	32.18	30.86	26.36
BLS-GSM	38.48	34.02	31.88	29.85	25.03
Proposed Iterative PCA	39.58	36.07	34.52	33.38	28.63

4.4.2. MSSIM (Mean Structural Similarity Index Matrix)

The MSSIM is the another performance evaluation metric which is used for finding the structural similarity between the noisy input image and the denoised output image. The MSSIM value which is equal to 1 implies that the denoised image has no loss of structural similarity between the noisy input and the denoised output image.

$$MSSIM = \frac{1}{|\Omega_i|} \sum_{i \in \Omega_i} SSIM(X(N_i), X(N_i)) \tag{9}$$

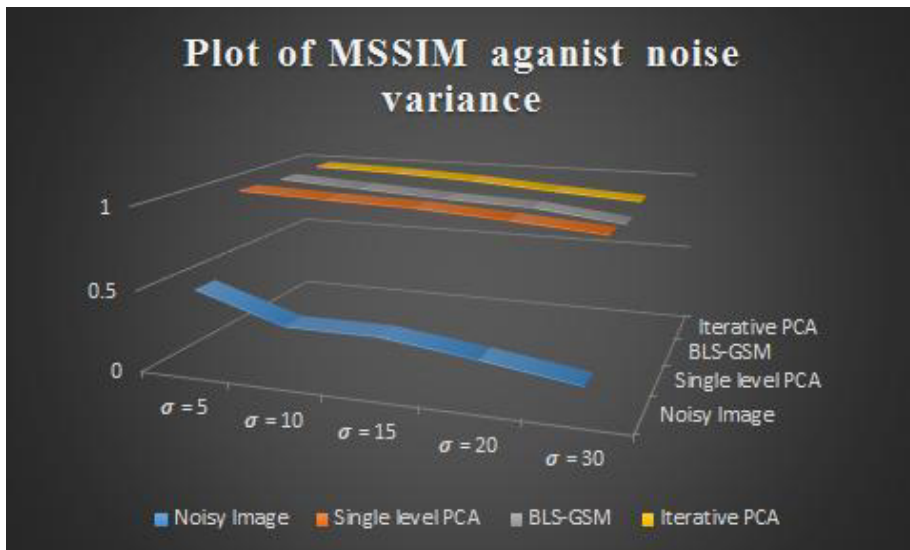


Fig. 4. Plot of MSSIM against noise variance.

SSIM considers image degradation as perceived change in structural information as defined in (2). We analysed the performance of the above methods on DT-MR images of brain with different noise variance level and the results of our observation are shown in the table below.



Table 2. MSSIM values of different denoising techniques for different noise variance level for DT-MR brain image

INDEX	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=30$
Noisy Image	0.49	0.32	0.32	0.251	0.171
Single Level PCA	0.987	0.966	0.950	0.915	0.87
BLS-GSM	0.974	0.942	0.910	0.888	0.823
Proposed Iterative PCA	0.980	0.966	0.941	0.90	0.865

As shown in the figure 4 and table 2 it is understood that when compared to other denoising techniques the proposed Iterative PCA based denoising will not cause any loss of structure of image after denoising.

#### 4.4.3. Visual Quality Comparison

The different denoising techniques are compared visually. Algorithms were applied on different MR images. Results show that images denoised with proposed Iterative PCA are more clearly visible than by Single Level PCA and BLS-GSM algorithm. The fine structural details and boundaries are more preserved in the Iterative PCA denoising method. In visual analysis the expectations are

1. Image edges and corners should be well preserved
2. Texture detail should not be lost
3. Few or ideally no artefacts.
4. Flat regions should be smooth as possible

Figure 5 and 6 shows the denoised effect of single level PCA and proposed Iterative PCA based denoising algorithms on DT-MRI brain images with different noise variance.

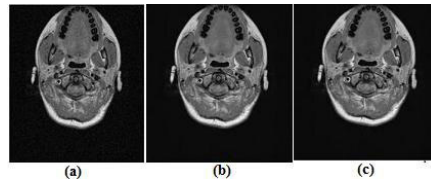


Fig. 5. Denoising of DT-MR image using different techniques a)Noisy Input image with variance=30 b)Single Level PCA c)Iterative PCA

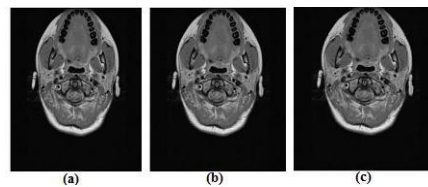


Fig. 6. Denoising of DTMR image using different techniques a)Noisy Input image with variance=20 b)Single Level

PCA c)Iterative PCA

## 5. CONCLUSION

The proposed method was to reduce Rician noise in DT-MR images with an Iterative Principal Component Analysis. By using Iterative Principal Component Analysis method we can refine the output of first phase makes the low frequency noise components got eliminated in the second phase. The proposed method was compared with Classical Principal Component Analysis Method and BLS-GSM Method. The performance was measured by calculating the PSNR and MSSIM. The method was tested on DT-MR medical images and natural DT-MR images. It outperformed the existing methods in terms of better preserving edge boundaries and retaining the fine structural details while removing noise. The computational complexity can be further reduced by patch based estimation.

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